Optical precursors with finite rise and fall time

J. F. Chen, M. M. T. Loy, G. K. L. Wong, and Shengwang Du*
Department of Physics, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China
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We report results of both theoretical and experimental studies of optical precursors generated from a square-modulated probe laser pulse having a finite rise and fall time and propagating through a cold atomic ensemble, under the conditions of either a two-level Lorentz absorber system or a three-level system with electromagnetically induced transparency (EIT). Because of the finiteness of the rise (and fall) time, the precursor signal is observed to decrease with increasing optical depth ($\alpha_0 L$). We find that the precursor can experience little absorption even at high optical depth if the rise (and fall) time is sufficiently short. At an optical dpeth of $\alpha_0 L$=42, the normalized precursor peak intensity is observed to increase from 9% to 27% when the rise (and fall) time is shortened from 7 to 3 ns. Meanwhile, we reaffirm that there is no violation of Einstein's causality principle in light propagation through both slow and fast light media. In the EIT system with high optical depth, the main field propagates with a subluminal group velocity and it is separated from the precursor. In the two-level system, the effect of negative group velocity in the anomalous dispersion regime is observed, but we detect no advancement in the rising edge of the precursors. In both cases, the leading edges of the precursors show no detectable delay to that through vacuum.

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I. INTRODUCTION

In 1914, Sommerfeld and Brillouin studied theoretically the propagation of a step-modulated optical pulse through a linear dispersive medium and found the remarkable fact that the wave front at the rising edge always travels at the speed of light in vacuum c [1, 2]. This wave front, in the form of a transient wave, is now known as the optical precursor. Since then, theoretical [3–9] and experimental [10–14] confirmation of the existence of optical precursors has been a subject of fundamental interest because of its connection to Einstein's causality principle [15]. But until recently, some researchers still questioned whether precursors had indeed been observed in previous experiments [16–18]. Very recently, we reported the observation of precursors that are clearly separated from the main pulse when a long square laser pulse passes through a cold atomic medium with electromagnetically induced transparency (EIT) [19, 20] and at high optical depth [21]. Our results also confirm that in a two-level narrow-line Lorentz absorber, when the resonance main field is completely absorbed, the left-over standing-alone transient spikes at the rising and falling edges are indeed the precursors [21].

Theoretically, the wave front of the precursor at the rising edge of an ideal step-modulated optical pulse is expected to propagate with zero absorption at all optical depths [7–9]. However, our experiment showed that, for a real step pulse with finite rise time, the precursor signal decreases as we increase the optical depth [21]. Oughstun has theoretically investigated this finite rise-time effects on the precursor field formation [22]. This propagation loss of optical precursors has also been studied indirectly in broadband femtosecond pulses [23, 24]. In this paper, we report the direct measurement of the dependence of propagation loss of optical precursors on rise (fall) time. Our results show that the transmission loss of precursors can be significantly reduced by shortening the rise and fall time. At a high optical depth of about 42, the precursor peak transmission is observed to increase from 9% to 27%when the rise (fall) time is shorten from 7 to 3 ns. The results suggest that precursors may have possible applications in under-water optical communication and biomedical imaging [25].

The paper is organized as follows. In Sec. II, we present a simple theoretical model that we use to describe optical precursors generated from a step pulse with finite rise time. We then show in Sec. III the experimental results obtained at different optical depths and with rise (fall) time of 7 and 3 ns. In Sec. IV, we show that slow and fast light effects can be observed in the main pulse and in both cases the main pulse is well behind the precursor, reaffirming that there is no violation to Einstein's causality principle. Finally, we draw a conclusion in Sec. V.

II. THEORY

The schematics of a three-level EIT system is shown in Fig. 1. The atoms are prepared in the ground state $|1\rangle$. In presence of a strong coupling laser (ω_c), on resonance with the transition $|2\rangle \rightarrow |3\rangle$, the medium becomes transparent at the probe laser (ω_p) transition $|1\rangle \rightarrow |3\rangle$ [19, 20]. The steep linear dispersion in the EIT narrow transparency window results in a slow group velocity [26]. Obviously, when we turn off the coupling laser, it becomes a two-level system where the probe field sees on-resonance absorption and anomalous dispersion with

*Electronic address: dusw@ust.hk

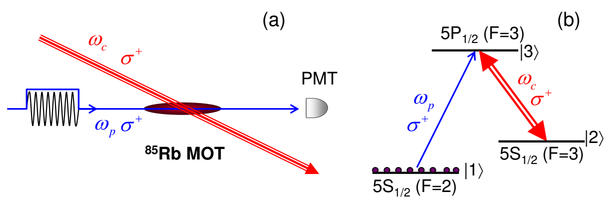

FIG. 1: (color online). Schematics of optical precursor generation. (a) Experimental configuration. (b) ^{85}Rb energy level diagram in a three-level Λ system with electromagnetically induced transparency (EIT).

negative group velocity. Therefore, the system allows us to study optical precursors in both slow and fast light media by controlling the coupling laser in a single experimental apparatus.

We use the formal notation $\frac{1}{2}E(z,t)e^{i[k_0 z-\omega_0 t]}+c.c.$ to describe the probe laser field, where $E(z,t)$ is the complex envelope. $\omega_0=\omega_{31}$ and $k_0=\omega_0/c$ are the probe carrier angular frequency and wave number in vacuum. For linear propagation of a weak probe pulse, the output optical field envelope can be obtained from the integral

$$E(t)=\frac{1}{2\pi}\int E_0(\omega)e^{i[\Delta k(\omega)L-\omega t]}d\omega, \tag{1}$$

where $E_0(\omega)=\int E_0(t)e^{i\omega t}d\omega$ is the spectrum of the initial input pulse envelop at $z=0$, L is the medium length, and $\Delta k=k_0\sqrt{1+\chi}-k_0\simeq k_0\chi/2$. With ω denoted as frequency detuning from the carrier ω_0, the EIT linear susceptibility (complex) is given as

$$\chi=\frac{\alpha_0}{k_0}\frac{4(\omega+i\gamma_{12})\gamma_{13}}{|\Omega_c|^2-4(\omega+i\gamma_{12})(\omega+i\gamma_{13})}, \tag{2}$$

where α_0 is the on-resonance absorption coefficient when the coupling laser is off, and Ω_c is the coupling laser Rabi frequency. γ_{ij} is the atomic dephasing rate between $|i\rangle$ and $|j\rangle$. In an ideal EIT system, the ground state dephasing rate is typically very small, i.e., $\gamma_{12}\simeq 0$. When the coupling laser is switched off ($\Omega_c=0$), Eq. (2) reduces to that of a two-level system.

An ideal step-modulated input probe laser pulse can be written as $E_{0\pm}(t)=E_0\Theta(\pm t)$, where $\Theta(t)$ is the Heaviside function and the sign + (-) represents the step-on (-off) pulse with a rising (falling) edge. There have been many theoretical studies on propagation of a step-modulated pulse through a resonant two-level Lorentz absorber [5, 9] or an EIT system [7, 8]. At high optical depth, both yield nearly identical Sommerfeld-Brillouin precursors envelops [8, 21]:

$$E_{SB\pm}(t)\simeq\pm E_0 J_0\big(\sqrt{2\alpha_0 L\gamma_{13}\tau}\big)\Theta(\tau)e^{-\gamma_{13}\tau}, \tag{3}$$

where $\tau=t-L/c$, J_0 is the 0-order first kind Bessel function. A more detailed derivation using uniform asymptotic expansion can be found in Refs. [4, 9].

In this paper, we study the optical pulse whose carrier frequency is on resonance at the transition $|1\rangle\rightarrow|3\rangle$. Therefore in the two-level system when the coupling laser is off, the carrier frequency (ω_p) is on absorption band and its behavior has been studied theoretically using the uniform asymptotic analysis [6]. At high optical depth, this main field is absorbed and only the precursor field is left over. For the EIT system, the main field within the narrow transparency window, traveling with a slow light group delay of $\tau_g\simeq 2\alpha_0 L\gamma_{13}/|\Omega_c|^2$, can be approximated as [21]:

$$E_{M\pm}(t)\simeq\frac{E_0}{2}\big(1\pm\text{erf}\big[\frac{\sqrt{\alpha_0 L}(\tau-\tau_g)}{2\sqrt{2}\tau_g}\big]\big)e^{i\Delta k(0)L}, \tag{4}$$

where erf is the error function. The total field envelop is $E_\pm(t)=E_{SB\pm}(t)+E_{M\pm}(t)$.

To study the finite rise-time effects, Oughstun suggested to use a hyperbolic tangent function to describe a real step pulse [22]. However, the theoretical analysis of such a hyperbolic tangent function based on the asymptotic approach is complicated [22]. Here we take a simple approach and model a realistic step pulse by turning on (off) the field amplitude linearly with a finite rise (fall) time of Δt. Mathematically, this can be done by convoluting the ideal step pulse and a quare function having a time duration of Δt:

$$\begin{aligned}\widetilde{E}_0(t) &= \frac{1}{\Delta t}E_0(t)*\Pi(t,\Delta t)\\ &= \frac{1}{\Delta t}\int E_0(t-t')\Pi(t',\Delta t)dt'\\ &= \frac{1}{\Delta t}\int_0^{\Delta t}E_0(t-t')dt'.\end{aligned} \tag{5}$$

The unit square function is defined as $\Pi(t,\Delta t)=1$ for $t\in[0,\Delta t]$ and otherwise zero. Then the output precursor field becomes

$$\begin{aligned}\widetilde{E}_{SB\pm}(t) &= \frac{1}{\Delta t}E_{SB\pm}(t)*\Pi(t,\Delta t)\\ &= \frac{1}{\Delta t}\int_0^{\Delta t}E_{SB\pm}(t-t')dt'.\end{aligned} \tag{6}$$

Equation (6) gives a clear picture in time domain: the averaging effect within the rise (fall) time window reduces the peak values of the precursors. There are two important characteristic times that determine the shape of precursors. At first, the duration of first peak of the precursor is determined by the Bessel function in Eq. (3): $\tau_s=\pi^2/(2\alpha_0 L\gamma_{13})$ [21, 27]. The second is the precursor (intensity) decay time constant $\tau_\gamma=1/(2\gamma_{13})$ determined by the atomic natural linewidth. Therefore, in order to detect the optical precursor with high visibility, it requires that the finite rise (fall) time is shorter than the both characteristic times, *i.e.*, $\Delta t<\{\tau_s,\tau_\gamma\}$.

In frequency domain, the effect of finite rise (fall) time works as a low-pass filter with a bandwidth determined by $1/\Delta t$:

$$\begin{aligned}\Phi(\omega) &= \frac{1}{\Delta t}\int\Pi(t,\Delta t)e^{-i\omega t}dt\\ &= \text{sinc}(\omega\Delta t/2)e^{-i\omega\Delta t/2},\end{aligned} \tag{7}$$

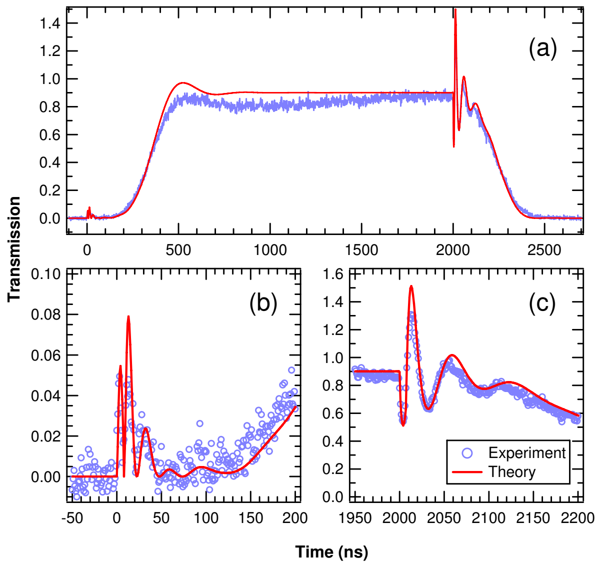

FIG. 2: (color online). Observation of optical precursors from a weak long-square pulse propagating through an EIT system with a finite rise and fall time of 7 ns. $\Omega_c = 4\gamma_{13}$, $\alpha_0 L = 42$. The red solid lines are numerical simulation using FFT. (b) and (c) are zoomed views around the rising and falling edges.

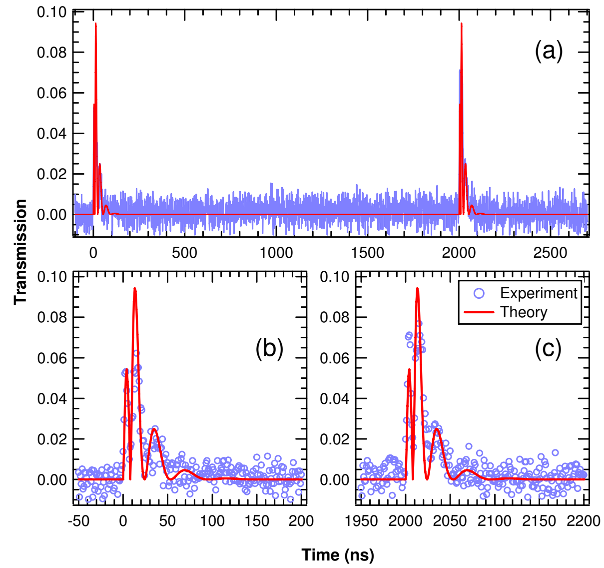

FIG. 3: (color online). Observation of optical precursors from a weak long-square pulse propagating through a two-level system with a finite rise and fall time of 7 ns. $\Omega_c = 0$, $\alpha_0 L = 42$. The red solid lines are numerical simulation using FFT. (b) and (c) are zoomed views around the rising and falling edges.

where $\text{sinc}(x) = \sin(x)/x$. For high optical depth, the Sommerfeld and Brillouin precursor frequency saddle points move far away from the atomic resonance [7, 9] and are attenuated by the filter effect caused by the finite rise and fall time. We notice that, the detection bandwidth of the equipment has also similar effects on reducing the measured precursor transients [12, 28]. As described in next section, to avoid the effect from the finite detection bandwidth, we use a detector with much higher response speed than the rising (falling) edge of the square pulse.

III. EXPERIMENT

The experimental system used in this work is similar to the one reported in Ref. [21]. We work with cold atoms in a two-dimensional (2D) ^{85}Rb magneto-optical trap (MOT) with a longitudinal length L=1.5 cm and a temperature of about 100 μK. The energy level configuration is taken as the following in ^{85}Rb D1 line (795 nm): $|1\rangle = |5S_{1/2}, F = 2\rangle$, $|2\rangle = |5S_{1/2}, F = 3\rangle$, and $|3\rangle = |5P_{1/2}, F = 3\rangle$. The dephasing rates are $\gamma_{13} = 2\pi \times 3$ MHz and $\gamma_{12} = 0.01\gamma_{13}$. To ensure that linear propagation effect is studied, we keep the intensity of the probe laser, locked to the resonance of the $|1\rangle \rightarrow |3\rangle$ transition, sufficiently low that the atomic population remains primarily in the state $|1\rangle$. The coupling and probe beams cross each other at the MOT with a 2 degree angular separation so that they can be separated when they reach the detector (PMT, Hamamatsu, H6780-20, 0.78 ns rise time). The data is recorded using a 1 GHz realtime digital oscilloscope (Tektronix, TDS684B) and averaged over 30 traces. To study optical precursor transient response at both the rising and falling edges in a single measurement, we used a probe laser beam that is modulated by a long square pulse with a length of 2 μs.

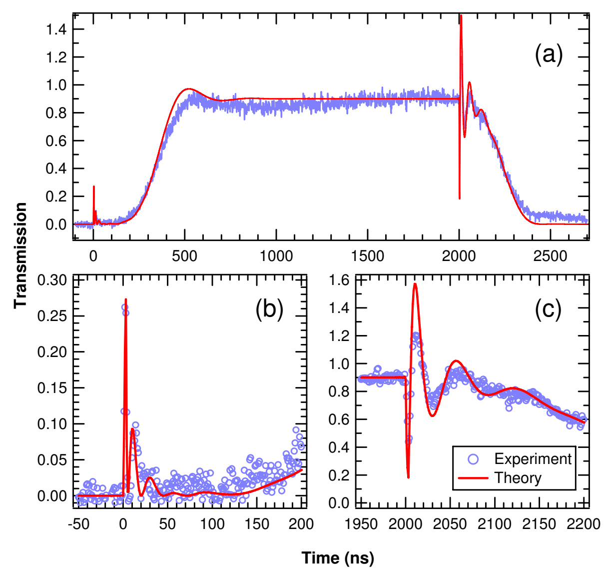

FIG. 4: (color online). Observation of optical precursors from a weak long-square pulse propagating through an EIT system with a finite rise and fall time of 3 ns. $\Omega_c = 4\gamma_{13}$, $\alpha_0 L = 42$. The red solid lines are numerical simulation using FFT. (b) and (c) are zoomed views around the rising and falling edges.

We first measure the optical precursors from a weak square pulse with a rise (and fall) time of $\Delta t = 7$ ns, gen-

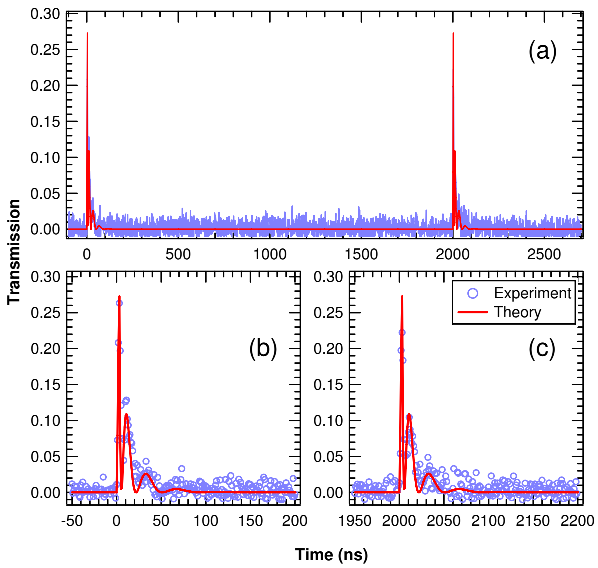

FIG. 5: (color online). Observation of optical precursors from a weak long-square pulse propagating through a two-level system with a finite rise and fall time of 3 ns. $\Omega_c = 0$, $\alpha_0 L = 42$. The red solid lines are numerical simulation using FFT. (b) and (c) are zoomed views around the rising and falling edges.

erated using a 3 GHz acoustic-optical modulator (AOM, Brimrose) modulated by a digital delay/pulse generator (SRS, DG645, 3 ns rise time). Figure 2 shows the output signal obtained from the EIT system with an optical depth of $\alpha_0 L = 42$ and a coupling laser Rabi frequency $\Omega_c = 4\gamma_{13}$. At the rising edge, the precursor field is separated from the delayed main field, consistent with the theoretical prediction of Sec. II. Due to the finiteness of the rise time, the peak intensity of the precursor transient peak is only about 9% of that of the input probe beam. At the falling edge, the precursor signal is, however, amplified due to the beating between the precursor field and the delayed main field. Measurement result in the two-level system (prepared by turning off the coupling laser) is shown in Fig. 3. As expected, the main field is absorbed and the precursor signals at the rising and falling edges are identical. These optical transient spikes are also nearly the same as that observed at the rising edge in the EIT system. The red solid lines are obtained numerically from the integral given in Eq. (1) using fast Fourier transform (FFT, 1ns resolution) and taking into account the finite rise and fall time. The theoretical curves agree well with the experimental data.

We then repeat the measurements using a square pulse with a shorter rise (and fall) time of $\Delta t = 3$ ns, generated using an electro-optical modulator (EOM, EOSpace) driven by the same digital delay/pulse generator. The results obtained from the EIT and two-level systems are shown in Figs. 4 and 5, respectively. The data are qualitative the same as those shown in Figs. 2 and 3. However, by shortening the rise and fall time from 7 to 3 ns, the normalized peak intensity of the precursor is observed to increase from 9% to about 27%.

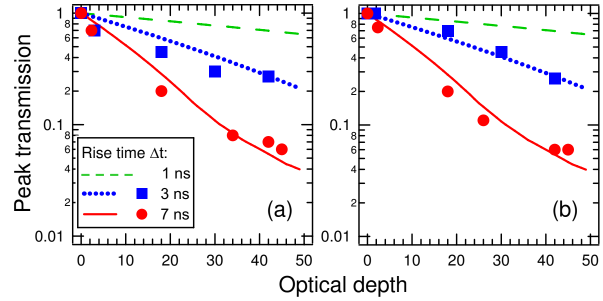

FIG. 6: (color online). The transmission transient peak values at the rising edge for the (a) EIT and (b) two-level systems as functions of optical depth.

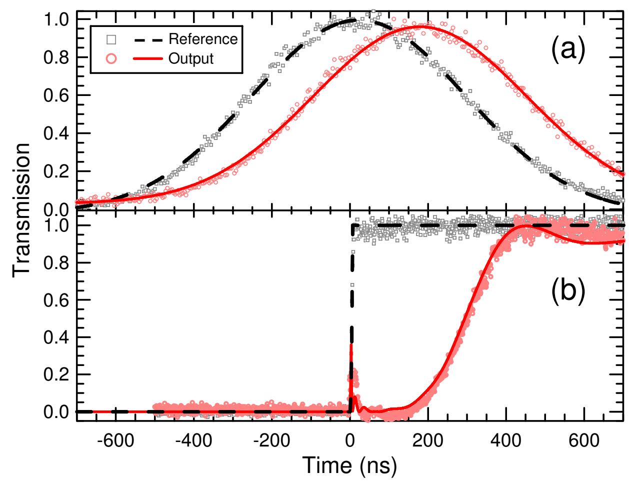

FIG. 7: (color online). Propagation of Gaussian and step pulses in the EIT system with an optical depth of $\alpha_0 L = 30$. $\Omega_c = 4\gamma_{13}$.

By varying the optical depth ($\alpha_0 L$) from 0 up to 45, we measure the attenuation of optical precursors for Δt =3 and 7 ns. Figure 6 shows normalized transient peak intensity at the rising edge as a function of optical depth for both the EIT and two-level systems. As expected, faster rise time results in higher transient peak intensity. We also show in Fig. 6 the theoretical curves calculated for 1 ns rise, which suggest that still higher precursor intensity can be achieved by using a faster light modulator. These experimental results suggest that precursors may have potential applications in optical communication through lossy medium.

IV. SLOW AND FAST LIGHT

Equation (4) in Sec. II shows that in the EIT slow light medium, the main field is delayed by the group delay time τ_g. This makes physical sense for slow light. However, if one extends this idea to the case of negative

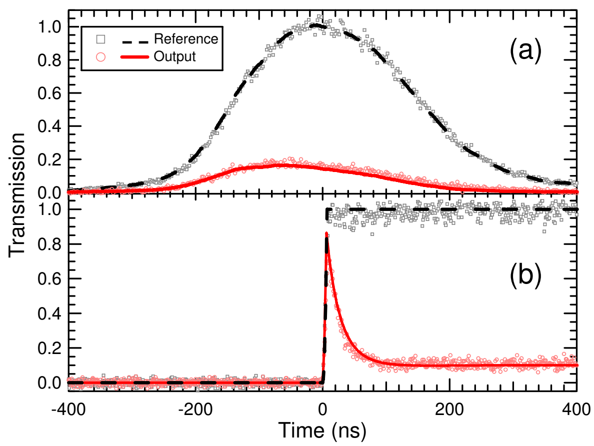

FIG. 8: (color online). Propagation of Gaussian and step pulses in the two-level system with an optical depth of $\alpha_0 L = 2.3$. $\Omega_c = 0$.

group velocity in a fast light medium, it would seem that the main field may advance ahead of the rising edge of the precursor, leading to violation of Einstein's causality principle. In this section, we show experimentally that this does not happen and Einstein's causality is upheld in both slow and fast light media.

First, we consider the EIT slow light case. Figure 7 shows the propagation of a Gaussian as well as a step pulses through an EIT medium having an optical depth of $\alpha_0 L = 30$. In the case of Gaussian pulse, we measured a group delay of about 200 ns with no attenuation and distortion [Fig. 7(a)]. This pulse delay is consistent with the observation that the delayed main field that turns on smoothly after 200 ns [Fig. 7(b)]. Meanwhile, the leading edge of the precursor shows no detectable delay to the step pulse.

In the case of a two-level Lorentz absorber system, the on-resonance group velocity becomes negative and the main field experiences absorption. One may argue that Einstein's causality can not be violated in the propagation of step pulse because the main field is essentially completely absorbed at high optical depth as shown in Figs. 3 and 5. But this may not be true for low optical depth. In Fig. 8, we show the propagation of both Gaussian and step pulses at a low optical depth $\alpha_0 L = 2.3$ where substantial main field (more than 10% of the input pulse) is present. In this case, we observe a significant peak advancement of 50 ns in the Gaussian pulse propagation [Fig. 8(a)] with obvious attenuation and distortion. However, for the step pulse, we observe no advancement of the rising edge [Fig. 8(b)]. Since information is encoded at a non-analytic wave front [29–31], our experiment confirms that there is no violation of Einstein's Causality principle in light propagation through fast light medium and the information velocity is different from the group velocity.

In our experiment, the long Gaussian pulses have narrow linewidths compared to the medium dispersion, thus their peaks propagate with the group velocity. As the pulse temporal length becomes short and the pulse spectrum is wide, it reduces to the energy velocity description [32].

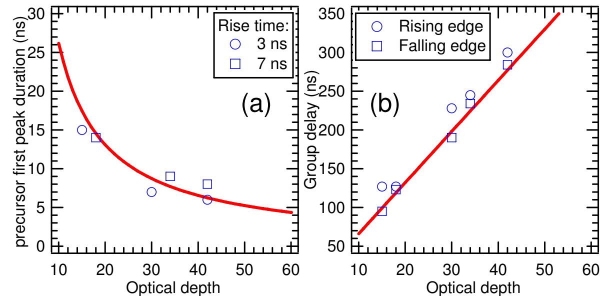

FIG. 9: (color online). (a) Precursor peak duration and (b) group delay of the main field as functions of optical depth in the EIT system. $\Omega_c = 4\gamma_{13}$.

In Fig. 9, we show the precursor peak duration and group delay as a function of the optical depth for a square pulse propagating in the EIT medium. The peak duration is measured from the first peak of the precursors, which can be fitted well by the curve calculated from $\tau_s = \pi^2/(2\alpha_0 L\gamma_{13})$. The group delay is measured from the rising edge [Fig. 9(b): circle points] and falling edge [Fig. 9(b): square points] to the point where the main field transmission is 25% [see Eq. (4)]. Both set of data agree very well with the theoretical curves obtained directly from the EIT linear dispersion. The above results show clearly that precursors do not propagate with the group velocity of the main field.

V. CONCLUSION

In summary, we have studied, in both theory and experiment, optical precursors generated from a low-intensity square-modulated on-resonance laser pulse having a long pulse duration and a finite rise (and fall) time. Our experimental system allows us to switch between the EIT system and two-level Lorentz absorber system by turning on and off the coupling laser. This enables us to modify the medium dispersion parameters over a wide range. Additionally the optical depth can be varied from 0 up to 45. Our experiments show that the optical precursors in both the EIT and two-level systems can experience little absorption even at high optical depth if the rise (and fall) time is sufficiently short. For example, at an optical depth $\alpha_0 L = 42$, the normalized precursor peak intensity is observed to increase from 9% to 27% when the rise (and fall) time is shortened from 7 to 3 ns. In time domain, shortening the rise (fall) time effectively reduces the averaging time [Eq. (6)]. In frequency domain, a step pulse with a shorter rise (fall) time contains a wider spectrum and more spectral components fall outside of the absorption band [Eq. (7)]. As a result, the precursors get absorbed less. Our simple theoretical analysis of the finite rise (fall) time effect agree well with the experimental observation in a wide parameter range. Our results suggest that precursors may find potential applications in optical communication through lossy medium.

We also reaffirm that Einstein's causality principle is upheld in light propagation through both slow and fast light media. In the EIT slow light case, the main field of a step pulse is delayed from the precursor, consistent with the pulse delay measurement we performed using a Gaussian pulse. In the two-level fast light case, we measured a negative pulse delay of about -50 ns for a Gaussian pulse, but no advancement in the rising edge of the precursor of the step pulse is detected.

Acknowledgments

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