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PROBING PARING SYMMETRY OF SUPERCONDUCTORS BY USING r.f. SQUID

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An experiment is suggested to study the symmetry of order parameter of high temperature superconductors. The experiment is based on the resonant characteristics of the radio frequency superconducting interference device operating in the inductive mode. By analyzing the effects of device parameters, the presence of multiple junctions in series and the residual magnetic field, the feasibility of the experiment is discussed. © 1998 Elsevier Science Ltd. All rights reserved

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The symmetry of superconducting order parameter is an important factor for understanding the mechanism of high temperature superconductivity (HTS). The early experiment on the amplitude suggested that the HTS have a non *s*-wave paring symmetry [1, 2]. Many phase sensitive experiments were carried out to verify the existence of this non conventional symmetry, after the suggestion of Sigrit and Rice [3]. The experiments revealed some remarkable evidences of *d*-wave paring state for $YBa_2Cu_3O_{7-\delta}$ (YBCO) and $Tl_2Ba_2CuO_{6+\delta}$ cuprate superconductors [4–7]. The observation of the spontaneous half-flux quantum in superconducting rings on tri-crystals, which contained one π-junction, strongly support the *d*-wave model [8–10]. However, some ambiguities are remained to be clarified. The tunneling between Pb and *c*-axis YBCO with diluted twin boundaries imposed that YBCO have $s + id$ symmetry in *c*-axis [11]. HgBaCuO in the point-contact tunneling [12] and electronic HTS $Nd_{1.85}Ce_{0.15}CuO_4$ in the penetration depth measurement [13] seem to behave more like conventional *s*-wave superconductors. The phase sensitive experiments on HTS were all carried out in the liquid helium temperatures so far. It is also important to know whether the symmetry of the cuprate superconductors at the low temperatures is the same as that near transition temperature.

The present paper suggests an experiment which can be performed at liquid nitrogen temperature to study the parameter symmetry based on the resonant characteristics of radio frequency superconducting quantum interference device (r.f. SQUID). We demonstrate that the acquired phase across the junction associated with the symmetries of the superconductors can be detected through the applied magnetic flux, at which the SQUID coupled to a tank circuit has the minimum (or maximum) resonant frequency. The geometry and parameter of the devices and tank circuit for this phase sensitive experiment are also given. Moreover, we have showed that the contribution of the junctions in series with the studied junction may be negligible provided they are much larger than the studied one. This is important as it is not possible to produce a superconducting ring with no other but a single π junction.

The r.f. SQUID is a superconducting loop containing one Josephson junction, as shown in Fig. 1. The inductance of the SQUID loop is L_S and the critical current of the Josephson junction is I_C. It couples to a tank circuit of a capacitance C and an inductance L, through which the radio frequency bias I_{rf} is supplied and the output voltage V_{rd} is detected. As we shall discuss, the resonant characteristics of the circuit is sensitive to the magnetic flux enclosed by the superconducting loop. At a certain biasing level, the output voltage is the periodic function of the flux.

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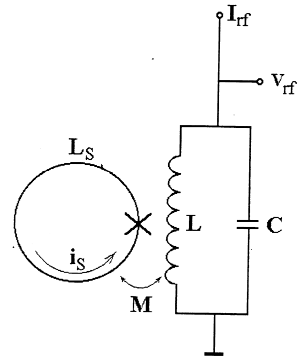

Fig. 1. Schematic of an r.f. SQUID coupled to a tank circuit.

Two different operation modes of the r.f. SQUID are recognized. For $\beta = 2\pi L_S I_c/\Phi_0 > 1$, the circulating supercurrent I_S is a multiple valued function of the external Φ_e, so is the total flux Φ (the sum of the external flux Φ_e and the induced flux $L_S I_S$), where $\Phi_0 = 2.07 \times 10^{-15}$ Wb is the superconducting flux quantum. We shall not discuss this mode here. For $B < 1$, the Φ vs Φ_e curve is single valued, known as inductive model [14]. In this mode, the superconducting loop with one junction inserted can be described as an effective inductance depending on the external applied flux.

$$L_{eff} = L_S\left[1 + \frac{1}{\beta \cos \varphi_e}\right], \tag{1}$$

where $\varphi_e = 2\pi\Phi_e/\Phi_0$ is phase shift of the order parameter caused by the external flux. Equation (1) is obtained when the critical current of the junction satisfies the conventional Josephson equation (0-junction), viz.

$$I_c = I_S \sin \varphi, \tag{2}$$

where φ is the phase difference across the junction. If the Josephson junction inserted is not a 0-junction, an extra phase shift δ is acquired. The effective should be written as

$$L_{eff} = L_S\left[1 + \frac{1}{\beta \cos(\varphi_e + \delta)}\right]. \tag{3}$$

For a superconducting loop with a π-junction, we have $\delta = \pi$. The equivalent inductance of the tank circuit coupled to the superconductor loop is

$$L_T = L[1 - k^2 L_S/L_{eff}] = L\left[1 - \frac{k^2\beta \cos(\varphi_e + \delta)}{1 + \beta \cos(\varphi_e + \delta)}\right], \tag{4}$$

where k is the coupling coefficient between the SQUID and the tank circuit, defined as $k^2 = M^2/L_S L$, M is the mutual inductance. The resonance frequency of the tank circuit coupled to the SQUID is

$$\omega = \frac{1}{\sqrt{L_T C}} = \omega_0\left(1 - \frac{k^2\beta \cos(\varphi_e + \delta)}{1 + \beta \cos(\varphi_e + \delta)}\right)^{-1/2}, \tag{5}$$

where $\omega_c = 1/\sqrt{LC}$. It indicates that the resonant frequency is the periodic function of the applied external magnetic flux. This behaviour has been observed for the r.f. SQUID fabricated from conventional superconductor [14]. When the external flux satisfies $2\pi\Phi_e/\Phi_0 + \delta = 2n\pi$, the equivalent inductance takes its minimum value and the resonant frequency is maximum,

$$\omega_M = \omega_0\left(1 - \frac{k^2\beta}{1 + \beta}\right)^{-1/2}. \tag{6}$$

We can derive the acquired phase δ from the lowest applied flux, Φ_M, corresponding to the maximum resonant frequency, viz.

$$\delta = -2\pi\Phi_M/\Phi_0. \tag{7}$$

For a conventional superconducting ring, $\delta = 0$, the maximum resonant frequency is obtained at the zero external magnetic field. Similarly, we can determine the phase from the flux, Φ_m, at the minimum resonant frequency,

$$\delta = \pi - 2\pi\Phi_m/\Phi_0. \tag{8}$$

If the junction has $\delta = \pi$, the resonant frequency has its minimum at zero magnetic field.

The SQUID can be fabricated from a single slab of cuprate superconducting thin film with artificial grain boundary. The geometry of the SQUID with a superconducting resonator is designed as shown in Fig. 2. The

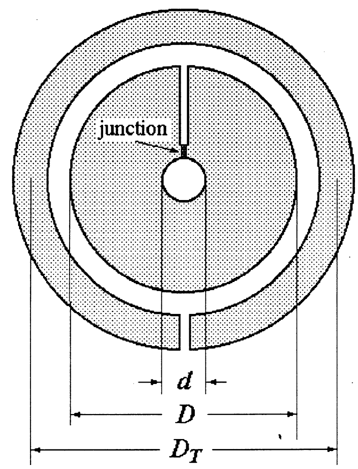

Fig. 2. Structure of the r.f. SQUID (center part) used for the phase sensitive experiment with a modified hairpin superconducting resonator (outer broken ring). It can be patterned from a single slab of thin film of high temperature superconductor.

edge length of the inner hole $d = 10\ \mu$m. The inductance of the SQUID is $L_S \approx \mu_0 d = 12$ pH, if the outer edge length D is more than three times larger than the inner [15]. For $\beta < 1$, we should have $I_c < \Phi_0/(2\pi L_S) \approx 27\ \mu$A. This can be fulfilled in a junction of a couple of micrometers wide at a temperature about 10 degrees below the transition temperature of the material.

The experiment suggested is to measure the resonant characteristics of the SQUID coupled to a tank circuit with respect to the external magnetic field. It is required that the difference of the maximum and the minimum frequencies is larger than the bandwidth of the tank circuit, $\Delta\omega_T = \omega_0/Q$, where Q is the quality factor of the system. The maximum difference change of the frequency is

$$\frac{\Delta\omega}{\omega_0} \approx \frac{k^2\beta}{(1-k^2)(1-\beta^2)}. \tag{9}$$

The condition of $\Delta\omega > \Delta\omega_T$ leads to

$$\frac{k^2\beta Q}{(1-k^2)(1-\beta^2)} > 1. \tag{10}$$

This is the requirement for the coupling coefficient k and the quality factor Q.

For a conventional tank circuit of lump elements of Fig. 1, the quality factor is normally less than 100. If it couples to an r.f. SQUID, k^2 is of the order of magnitude of 10^{-3}. It is difficult to fulfill the requirement in (10). Since a superconductor microstrip resonator has a high quality factor, it can be employed as a tank circuit for r.f. SQUID. Both the S-shaped $\lambda/2$ resonator [16] and the modified hairpin resonator [17] were demonstrated to have quality factors up to 6000, operating at frequencies of giga-hertz. For a modified hairpin resonator sketched in Fig. 2, one can estimate its coupling coefficient by $k^2 = A_{eff}/A_T$, where $A_{eff} = 2Dd/\pi$ is the effective area of the SQUID and A_T is the area enclosed by the resonator ring. If $d = 10\ \mu$m, $D = 5$ mm and $D_T = 8$ mm, we have $k^2 = 6 \times 10^{-4}$. The critical current of the junction can be trimmed to have a β less than and close to one. Thus the requirement in (10) can be fulfilled.

In order to eliminate the influence of the earth magnetic field, the measurement should be performed in the ‘‘zero’’ field space. A mu-metal shielding could provide a space with residual field down a couple of nano-tesla. The phase caused by an external field is $\varphi_e = 2\pi\Phi_e/\Phi_0$. For a SQUID of an effective area 3×10^{-8} m^2, the residual field gives a phase error in the order of $10^{-2}\pi$. A SQUID coupled to an S-shaped resonator may give a smaller phase error, due to its smaller effective area.

The experiment suggested is for the superconducting ring with a single junction. However, it is not possible to produce a ring with no other junction but a single π-junction for d-wave superconductors, since a π-junction must be fabricated from one d-wave and one s-wave superconductors [5] or two d-wave superconductors with different orientation [6]. Fortunately, as we will show, a single junction ring can be approximated by making the desired junction much smaller than the other junctions. For a ring inserted $N+1$ junctions, the critical current of the junctions are I_{cn} $(n = 0, 1, \ldots, N)$. The desired junction has critical current $I_{c0} \ll I_{ck}$ (k 0). We have

$$\varphi_0 + \sum_{k=1}^{N} \varphi_k = 2n\pi - 2\pi\frac{\Phi}{\Phi_0} \tag{11}$$

where $\Phi = \Phi_e + L_S I_S$ and φ_n is the phase difference across the nth junction, which is related to the critical current of the junction by $I_S = I_{cn}\sin\varphi_n$. Since $I_S \le I_{co} \ll I_{ck}$, $I_S = I_{ck}\sin\varphi_k \approx I_{ck}\varphi_k$. Equation (11) is approximated by

$$\begin{aligned}\varphi_0 &= 2n\pi - \sum_{k=1}^{N}\frac{I_S}{I_{ck}} - \frac{2\pi}{\Phi_0}(\Phi_e + L_S I_S)\\ &= 2n\pi - \frac{2\pi}{\Phi_0}(\Phi_e + L_1 I_S),\end{aligned} \tag{12}$$

where

$$L_1 = L_S\left(1 + \sum_{k=1}^{N}\frac{\Phi_0}{2\pi L_S L_{ck}}\right). \tag{13}$$

When I_{ck} approaching infinity, the system is reduced to the single junction ring. One sees that the effect of the large junctions in series with a small junction in a superconducting ring is only to increase the inductance of the ring, but not alter the other behaviour of the system. A junction with a critical current of 10 mA contributes to an extra inductance of 30 fH, which is negligible compared to the ring inductance, if the critical current of the desired junction is $10\ \mu$A.

The experiment suggested above can also be employed to study the superconductors with complex symmetries, such as $s + id$, or $d_{x^2+y^2} + id_{xy}$. In these cases, the δ is neither 0 nor π, but depends on the oritation of the junction relative to the crystal axis. The acquired phase can be derived according to equation (10) or (11). Since the applied field can be measured with a high accuracy and the contribution of the residual field is small as discussed above, we may determine the different symmetries from the phase shift. We shall point out that this acquired phase could be measured through the experiments of d.c. SQUID [4] or tri-crystal ring [6].

However, some other phase shifts may exist. In d.c. SQUID, the inductances of the two arms, or the critical currents of the two junctions may be different. It will lead to an extra phase shift. In tri-crystal ring, the phase shift caused by the other junctions may not be small and the β of the ring is not infinity. These will also affect the spontaneous flux measurement. As these phase shifts are unknown, thus not able to be disentangled from the phase caused by the complex symmetry. The suggested experiment of r.f. SQUID is of the advantage for determining the phase in studying the superconductors of complex symmetries, in comparison with other experiments.

In conclusion, we have proposed a phase sensitive experiment based on the resonant characteristics of r.f. SQUID to study the symmetry of the pairing in high temperature superconductors. The experiment can be performed at liquid nitrogen temperature, or close to the transition temperature of the materials. Moreover, it has an advantage of low error in the phase shift compared to the experiments reported.

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