

Assignment 3 & 4.

Model (two regressors fixed effects model)

$$y_{it} = a_i + bx_{it} + cz_{it} + u_{it}$$

where

$$u_{it} = \rho u_{it-1} + \epsilon_{it}$$
$$W_{it} = \begin{bmatrix} x_{it} \\ z_{it} \end{bmatrix} = \begin{bmatrix} \rho_1 & 0 \\ 0 & \rho_1 \end{bmatrix} \begin{bmatrix} x_{it-1} \\ z_{it-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{it} \\ \epsilon_{it} \end{bmatrix}$$

Q1. (Theory) Derive the limiting distribution of LSDV estimator of $\hat{\beta} = (\hat{b} \ \hat{c})'$.

Q2. (Theory) Show that the ordinary variance estimator for $\hat{\beta}$, $\hat{\sigma}_u^2 (W'W)^{-1}$, becomes inconsistent estimator for the true variance of β .

Q3. (Theory) Construct panel infeasible GLS estimator. We assume that you know ρ value. Derive its limiting distribution.

Q3. (Monte Carlo) Consider $N = 25, 50, 100, 200$ and $T = 10, 20, 40, 80, 200$. Assume $e_{it} \sim iidN(0, 1)$, $\varepsilon_{it} \sim iidN(0, 1)$, and $\epsilon_{it} \sim iidN(0, 1)$. Next, consider the cases where $\rho = 0.1, 0.5, 0.9, 0.95$ and $\rho_1 = 0, 0.9$.

1. Show $\hat{\beta}$ is consistent.
2. Show the ordinary $t_{\hat{\beta}}$ has serious size distortion (set $b = c = 0$).
3. Show the modified t-ratio (by using panel robust covariance estimation) reduces the size distortion when $T < N$.
4. Construct panel feasible GLS estimator. Compare PFGLS estimator with LSDV estimator in terms of bias and mse.