Test 4: (10 points, 2 extra bonus points)

T=100,N=3. $a = \beta = 0$, $a_i \sim iidN(0,1)$

Construct Matlab codes for the following questions.

$$y_{it} = a + \beta x_{it} + u_{it}, \quad \text{for } u_{it} = (a_i - a) + \varepsilon_{it}, \quad \varepsilon_{it} \sim iidN(0,1)$$

1. Obtain MLE for $a$ and $\beta$. (6 points)
Likelihood:

$$(2\pi)^{-n/2} |\mathbf{\Sigma}|^{-1/2} \exp\left(\frac{1}{2}(\mathbf{y} - a - \boldsymbol{\beta}\mathbf{x})' \mathbf{\Sigma}^{-1} (\mathbf{y} - a - \boldsymbol{\beta}\mathbf{x})\right)$$

where

$$\Sigma_{T\times T} = \begin{bmatrix} \sigma_a^2 + \sigma^2 & \sigma_a^2 & \cdots & \sigma_a^2 \\ \sigma_a^2 & \sigma_a^2 + \sigma^2 & \cdots & \sigma_a^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_a^2 & \sigma_a^2 & \cdots & \sigma_a^2 + \sigma^2 \end{bmatrix}.$$

$z(1) = \sigma_a$. $z(2) = \sigma^2$, $z(3) = a$, $z(4) = \beta$

```
function llh = mymle1(z)
global y x t n
sigma = ones(t,t).*z(1) + eye(t).*z(2);
dsigma = det(sigma);
llh = -((t*n)/2)*log(2*pi*dsigma^2);
llh1 = 0;
for i = 1:n;
llh1 = llh1 - sum( (y(:,i) - z(3) - z(4)*x(:,i))'*inv(sigma)*(y(:,i) - z(3) -
z(4)*x(:,i)).*(1/2) );
end;
llh = llh +llh1;
llh = - llh;
return;
```

2. Set $a_i = a$. Obtain GMM estimator for $a$ and $\beta$. Use the following moment conditions (6 points)

$$E(u_{it}x_{it}) = E([y_{it} - a - \beta x_{it}]\, x_{it}) = 0$$

```
clear;
global t y x n %define global variables
t = 100; n = 10;
x = randn(t,n);
y = randn(t,n);
y = x + y;
init_w =[0.1 0.9];
```

```
format short;
%[param,fval,exitflag,output] = fminsearch('mymle', init_w, optimset('Display','iter','TolX',1e-2,'TolFun',1e-3));
% fminsearch(fun,x0,options,P1,P2,...): function is fun(x,P1,P2,...) with min over x and P1,P2,... held constant
% in options can add: 'MaxIter',500,'Display','iter','TolX',1e-2,'TolFun',1e-3,'MaxFunEvals',5000,'MaxIter',1000
% Level of display: 'off' displays no output; 'iter' displays output at each iteration; 'final' displays just
% the final output; 'notify' dislays output only if the function does not converge.
[param,fval,exitflag,output,grad1,hess1] = fminunc('mygmm3', init_w, optimset('Display','iter','TolX',1e-3,'TolFun',1e-8));
format short g;
std = diag(inv(hess1));
std
std = sqrt(std);
trat = param'./std;
[param' trat]

function llh = mygmm3(z)
global y x t n
my = zeros(t,n);
for i = 1:n;
my(:,i) = ( y(:,i) - z(1) - z(2)*x(:,i) ).*x(:,i) ;
end;
sigy = my'*my/t;
psi = mean(my)';
llh = psi'*inv(sigy)*psi*t;
return
```