Consider the following models.

Regression equation	$y_{it} = a_i + \beta x_{it} + u_{it}$	(1)
Regressor	$x_{it} = \alpha_i + \rho x_{it-1} + e_{it},$	(2)
Regression error	$u_{it} = \phi u_{it-1} + \varepsilon_{it},$	(3)
Error distribution	$e_{it} \sim N(0, \Omega_e)$	(4)
	$\varepsilon_{it} \sim N(0, \Omega_\varepsilon)$	(5)

Part I: Matlab Code: Define y as a $T \times N$ matrix of y_{it}, x as a $T \times N$ matrix of x_{it}, and u as a $T \times N$ matrix of $\hat{u}_{it}$.

Q1 You want to impose $a_i = a$, and run POLS. (5 points)

```
x1 = x(:);
y1 = y(:);
dd = ones(t*n,1);
xx = [dd x1];
bpols = inv(xx'*xx)*xx'*y1;
```

Q2 Now you let $a_i \neq a$, and want to run LSDV. (10 points)

```
dd = ones(t,1);
dd = kron(eye(n),dd);
xx = [dd x1];
blsdv = inv(xx'*xx)*xx'*y1;
```

Q3 You have residuals of $\hat{u}_{it}$. You want to estimate the panel robust covariance matrix. Provide the function. (10 points)

see http://www.utdallas.edu/~dxs093000/Econo2/matlab_ex/panelcov.m

Q4 You want to bootstrap $t_{\hat{\beta}}$. Define rx = $\hat{\rho}$ and ru = ϕ. Find out the programming errors in the below. First, write down line numbers of which contain errors. Second, explain briefly the problems and solutions (don't write up codes) (25 points)

See http://www.utdallas.edu/~dxs093000/Econo2/matlab_ex/boot/ex3.m

```
% Starting seive bootstrap
1 msim = 1000;
```

2 csim = zeros(msim,2);

3 for m = 1:msim;

4 e = rand(t,1)*(t-1); e = 1 + floor(e); Should generate t+k

5 us = u(:,e); xs = x(:,e); Should choose column

6 for ii = 2:t;

7 us(ii,:) = us(ii-1,:).*ru + us(ii,:); need to define another variable rather than 'us' before the do-loop such that uss = us.

8 xs(ii,:) = xs(ii-1,:).*rx + xs(ii,:);

9 end;

10 for i = 2:t; i should start from 1. + requires recentering of uss.

11 ys(i,:) = b(1:n,:)' + us(i,:);

12 end;

13 xx = xs(:);

14 bs = inv(xx'*xx)*xx'*ys(:); Requires either POLS or LSDV.

15 uh1 = ys - xs*bs;

16 ome2 = panelcov(xs,uh1,0); %ordinary t-ratio

17 tr2 = bs./sqrt(ome2);

18 csim(m,:) = [tr1 tr2]; tr1 is not defined

19 end;

20 crtic = csim(0.95*msim,:); should be sorted first.

Part II: Theory

Part II-A: Derivation

Q1 Derive the limiting distribution of $\hat{\beta}_{LSDV}$. Assume $\Omega_e = \Omega_\varepsilon = I$, and $\rho = \phi = 0$. (20 points)

$$\begin{aligned}\sqrt{NT}\left(\hat{\beta}-\beta\right) &= \frac{\frac{1}{\sqrt{NT}}\sum^N\sum^T e_{it}}{\frac{1}{NT}\sum^N\sum^T\left(e_{it}-\frac{1}{T}\sum^T e_{it}\right)^2} - \frac{\frac{1}{\sqrt{NT}}\sum^N\frac{1}{T}\left(\sum^T e_{it}\right)\left(\sum^T u_{it}\right)}{\frac{1}{NT}\sum^N\sum^T\left(e_{it}-\frac{1}{T}\sum^T e_{it}\right)^2} \\ &= I + II\end{aligned}$$

but we have

$$II = -\frac{\frac{1}{\sqrt{NT}}\sum^{N}\frac{1}{T}\left(\sum^{T}e_{it}\right)\frac{1}{T}\left(\sum^{T}u_{it}\right)}{\frac{1}{NT}\sum^{N}\sum^{T}\left(e_{it}-\frac{1}{T}\sum^{T}e_{it}\right)^{2}} = -\frac{\frac{1}{\sqrt{NT}}\sum^{N}\left(\frac{1}{\sqrt{T}}\sum^{T}e_{it}\right)\left(\frac{1}{\sqrt{T}}\sum^{T}u_{it}\right)}{\frac{1}{NT}\sum^{N}\sum^{T}\left(e_{it}-\frac{1}{T}\sum^{T}e_{it}\right)^{2}}$$

$$= -\frac{\frac{1}{\sqrt{NT}}\sum^{N}\bar{e}_i\bar{u}_i}{\frac{1}{NT}\sum^{N}\sum^{T}\left(e_{it}-\frac{1}{T}\sum^{T}e_{it}\right)^{2}} = -\frac{\frac{1}{\sqrt{NT}}\sum^{N}\xi_i}{\frac{1}{NT}\sum^{N}\sum^{T}\left(e_{it}-\frac{1}{T}\sum^{T}e_{it}\right)^{2}}, \text{ let say}$$

where

$$\bar{e}_i = \frac{1}{\sqrt{T}}\sum^{T}e_{it} = O_p(1), \text{ and } \bar{u}_i = \frac{1}{\sqrt{T}}\sum^{T}u_{it} = O_p(1).$$

Hence

$$II = -\frac{\frac{1}{\sqrt{T}}\left(\frac{1}{\sqrt{N}}\sum^{N}\xi_i\right)}{\frac{1}{NT}\sum^{N}\sum^{T}\left(e_{it}-\frac{1}{T}\sum^{T}e_{it}\right)^{2}} = \frac{O\left(\frac{1}{\sqrt{T}}\right)O_p(1)}{O_p(1)} = O_p\left(\frac{1}{\sqrt{T}}\right).$$

I didn't provide the rest of the derivation.

Part II-B: True/False questions. Answer if the statement is true or false, and explain your answer.

Q2 Assume $\Omega_e = \Omega_\varepsilon = I$, $\rho = 0$ but $\phi \neq 0$. Then $\hat{\beta}_{LSDV}$ is consistent (**true upto here**) but $t_{\hat{\beta}}$ based on the ordinary t-ratio becomes inconsistent. (**False. t-ratio is consistent since** $\xi_{it} = x_{it}u_{it}$ **is not serially correlated**. (10 points)

Q3 Assume $\Omega_e \neq I$, $\Omega_\varepsilon \neq I$, $\rho \neq 0$ and $\phi \neq 0$. Then $\hat{\beta}_{LSDV}$ is consistent but $t_{\hat{\beta}}$ based on the panel robust covariance matrix becomes consistent as long as $T > N$. (10 points)

False $\hat{\beta}_{LSDV}$ **is consistent as long as** x_{it} **is strongly exogeneous. However panel robust covariance matrix becomes inconsistent if** $T > N$. + **Cross section dependence can't be controlled by panel robust covariance matrix (by clustering over i).**

Q4 Assume that $E(a_i\alpha_i) \neq 0$. Then $\hat{\beta}_{LSDV}$ is not consistent. (10 points)

False. $\hat{\beta}_{LSDV}$ **is the within group estimator, so it is free from the correlation among fixed effects**