Test 3

Consider the following data generating process.

$$y_{it} = a_i + \beta x_{it} + u_{it}$$

where

$$\begin{bmatrix} x_{it} \\ u_{it} \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & \phi \end{bmatrix} \begin{bmatrix} x_{it-1} \\ u_{it-1} \end{bmatrix} + \begin{bmatrix} e_{it} \\ v_{it} \end{bmatrix},$$

$$\begin{bmatrix} e_{it} \\ v_{it} \end{bmatrix} \sim iidN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right).$$

Let $a_i = \beta = 0$ but $\rho = \phi = 0.95$. Generate $T \times N$ pseudo random variables of y_{it}, x_{it} and u_{it}. Treat them as if they were actual data. Let $T = 30$, $N = 10$.

Q1. Construct the ordinary t-ratio, and its boostrap critical value. (8 points)

```
clc
clear all;
format short g;
% Example of Bootstrap: t-value starts ============================
clear all;
% =========================== DGP and get sample t-value ====================
t = 30; k = 100; n = 10;
rho = 0.95;
% hstr = randstream('mt19937ar', 'seed', 12455); % set the seed number for random number generation
u = randn(t+k,n);
e = randn(t+k,n);
y = u;
x = e;
for i = 2:t+k;
y(i,:) = y(i-1,:)*rho + u(i,:);
x(i,:) = x(i-1,:)*rho + e(i,:);
end;
y = y(k+1:t+k,:);
x = x(k+1:t+k,:);
y1 = y - repmat(mean(y),t,1);
x1 = x - repmat(mean(x),t,1);
beta = inv(x1(:)'*x1(:))*x1(:)'*y1(:);
resi = y1 - x1*beta;
fixe = y - x*beta;
fixe = mean(fixe);
vres = var(resi(:));
trat1 = beta./sqrt(vres*inv(x1(:)'*x1(:)));
```

```
trat1
% get rho and phi from AR1 regression
X1 = x(2:t,:); X2 = x(1:t-1,:);
X11 = X1 - repmat(mean(X1),t,1);
X21 = X2 - repmat(mean(X2),t,1);
phi = inv(X12(:)'*X12(:))*X12(:)'*X11(:);
resix = X11 − X21*phi;
same as resi here.
%==============================================================
% In order to do bootstrap, you need to know if or not resi and x are serially
% correlated. Here we don't account for this. Hence the bootstrap becomes
% invalid.
msim = 1000;
csim = zeros(msim,2);
for m = 1:msim;
e = rand(t+k,1)*(t-1);
e = 1 + floor(e);
us = resiu(e,:);
xs = resix(e,:);
for i = 2:t+k;
xs(i,:) = xs(i-1,:)*phi + ...
us(i,:) = us(i-1,:)*rho + ...
end;
us = us - repmat(mean(us),t+k,1); %recentering
ys = repmat(fixe,t+k,1) + beta*xs + us;
ys = ys(k+1:t+k,:);
xs = xs(k+1:t+k,:);
ys = ys - repmat(mean(ys),t,1);
xs = xs - repmat(mean(xs),t,1);
betas = inv(xs(:)'*xs(:))*xs(:)'*ys(:);
resis = ys - xs*betas;
vress = var(resis(:));
trat2s = (betas - beta)./sqrt(vress*inv(xs(:)'*xs(:)));
% for q2, vress = panelcov(xs,resis); trat2s = (betas - beta)./sqrt(vress);
csim(m,:) = [trat1s trat2s];
end;
csim = sort(abs(csim));
bootcv = csim(msim*0.95,:);
[ bootcv trat1 ]
```

Q2. Construct the t-ratio based on the panel robust covariance estimator and its bootstrap critical value. (2 points)

```
function omega = panelcov(x,u)
[t,n] = size(u);
[t,nk] = size(x);
w = [];
xi = [];
```

```
k = nk/n;
for i = 1:k;
xx = x(:,(i-1)*n+1:i*n);
w = [w xx(:)];
xi = [xi; sum(xx.*u)];
end;
omega = inv(w'*w)*(xi*xi')*inv(w'*w);
end;
```

Q3. Suppose that

$$\begin{bmatrix} e_{it} \\ v_{it} \end{bmatrix} = \begin{bmatrix} \theta_t \\ \eta_t \end{bmatrix} + \begin{bmatrix} e_{it}^o \\ v_{it}^o \end{bmatrix}, \quad \begin{bmatrix} \theta_t \\ \eta_t \end{bmatrix} \sim iidN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

$$\begin{bmatrix} e_{it}^o \\ v_{it}^o \end{bmatrix} \sim iidN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

Then e_{it} and v_{it} are cross sectionally correlated. Is your bootstrap in Q1 and Q2 still valid? Discuss. (5 points)

Yes. As long as pseudo e_{it}^* and v_{it}^* are drawn randomly from column vector of $\hat{e}_{it}$ and $\hat{v}_{it}$, the sample cross sectional dependence are presented in e_{it}^* and v_{it}^*.