Sub-Convergence with Multiple Factors

Donggyu Sul

December 28, 2019

Erasmo Papagni asked a question regarding the relationship between multivariate factors and the relative convergence. Before answering the question, we study underlying econometric models for the relative convergence. Assume that the variable of interest, y_{it}, has a single common factor.

$$y_{it} = a_i + \lambda_{1i} F_{1t} + y_{it}^o, \tag{1}$$

where y_{it}^o is a pure idiosyncratic component. Further assume that F_{1t} has a stochastic trend. That is, $F_{1t} = c_0 + c_1 t + u_{1t}$ with $u_{1t} = u_{1t-1} + \varepsilon_{1t}$ where ε_{1t} is a stationary variable. Next, observe this.

$$y_{it} = \left(\lambda_{1i} + \frac{a_i + y_{it}^o}{F_{1t}} \right) F_{1t} = \lambda_{1i,t} F_{1t}. \tag{2}$$

Phillips and Sul (2007) modeled y_{it} as a product between a time varying factor loading $\lambda_{1i,t}$ and a common factor.

Consider the case with two factors:

$$\begin{aligned} y_{it} &= a_i + \lambda_{1i} F_{1t} + \lambda_{2i} F_{2t} + y_{it}^o \\ &= \left(\frac{a_i + \lambda_{1i} F_{1t} + \lambda_{2i} F_{2t} + y_{it}^o}{F_t} \right) F_t = b_{it} F_t, \end{aligned} \tag{3}$$

where F_t can be either F_{1t} or F_{2t}. It does not matter at all how to rewrite a model.

Next, consider (1) again to evaluate the meaning of the relative convergence. If $\lambda_{1i} = \lambda_1$ for all i, then the relative convergence holds since

$$\lambda_{1i,t} = \lambda_1 + \frac{a_i + y_{it}^o}{F_{1t}} \to \lambda_1 \text{ as } t \to \infty.$$

Note that $a_i/F_{1t} = a_i/(c_0 + c_1 t + u_{1t}) \to 0$ as $t \to \infty$. Similarly, $y^o_{it}/(c_0 + c_1 t + u_{1t}) \to 0$ as well. Suppose that

$$\lambda_{1i} = \begin{cases} \lambda_{11} & \text{if } i \in G_1 \\ \lambda_{12} & \text{if } i \in G_2 \end{cases},$$

then as $t \to \infty$,

$$y_{it} \to \begin{cases} \lambda_{11} F_t & \text{if } i \in G_1 \\ \lambda_{12} F_t & \text{if } i \in G_2 \end{cases}.$$

In other words, y_{it} has two sub-convergent clubs. The cross-sectional averages of y_{it} in G_1 and G_2 with a very large n must look like this.

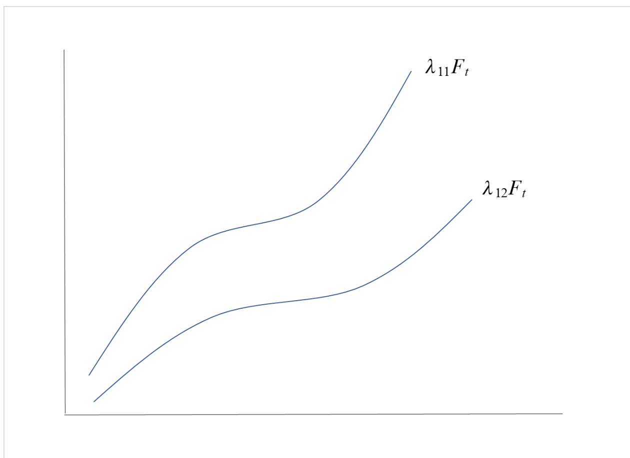

Figure 1: Estimated Common Factors in Sub-Convergent Panels

It is because the cross-sectional average becomes a good proxy for the common factor for each sub-convergent panel. As $n \to \infty$, the average approaches to its true value. Nonetheless, the gap between two factors must be increasing over time if F_t includes a trend component. Furthermore, the general pattern must be very similar.

With multiple factors, one may have various different cases. For example, assume that $\lambda_{1i} = \lambda_1$ for all i & $\lambda_{2i} = \lambda_2$ in (3). Further we assume that

$$y_{it} = b_{it} F_t = \begin{cases} \lambda_{1i} F_{1t} + y^o_{it} & \text{if } i \in G_1 \\ \lambda_{2i} F_{2t} + y^o_{it} & \text{if } i \in G_2 \end{cases}.$$

Then as $t \to \infty$, we have

$$y_{it} \to \begin{cases} \lambda_1 F_{1t} & \text{if } i \in G_1 \\ \lambda_2 F_{2t} & \text{if } i \in G_2 \end{cases}.$$

In this case, the time varying pattern of the cross-sectional average in each sub-convergent club must be different.

Nonetheless, the number of sub-convergent clubs is not necessarily equal to the number of common factors. With a single factor case in (1), we already showed that there are possibly many sub-convergent clubs. With two factors, there must be more than two sub-convergent clubs (as n and $t \to \infty$).