

# Panel Unit Root Tests under Cross Section Dependence with Recursive Mean Adjustment \*

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## Abstract

Utilizing recursive mean adjustment we provide two unit root tests: the covariate-recursive mean adjusted unit root test and the panel feasible generalized recursive mean adjusted unit root test. The first test uses the cross sectional average of the panel data to test for non-stationarity in the common factors of the panel. The second test is designed for testing non-stationarity in the idiosyncratic errors. The proposed panel unit root tests are precise and powerful, especially when  $T$  is larger than  $N$ .

*Keywords:* Recursive detrending, Dynamic factors, Panel unit root test. Covariate unit root test, Cross section dependence

*JEL Classification Numbers:* C33 Panel Data

## 1 Motivation and Models

This paper utilizes So and Shin (1999)'s and Shin and So(2001)'s recursive mean adjustment method for testing panel unit roots under cross section dependence. The cross section dependence is modelled through a common factor structure, given by

$$y_{it} = \lambda_i' F_t + y_{it}^o \text{ for } i = 1, \dots, N \text{ and } t = 1, \dots, T \quad (1)$$

where  $F_t$  is a  $k \times 1$  vector of common factors,  $\lambda_i$  is a  $k \times 1$  vector of factor loading coefficients,  $k$  is the number of common factors, and  $y_{it}^o$  is an idiosyncratic error. According to this model, the panel data

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$y_{it}$  is  $I(0)$  only when both  $F_t$  and  $y_{it}^o$  are  $I(0)$  :

$$\begin{aligned} y_{it} &\sim I(0) \iff F_t \sim I(0) \text{ and } y_{it}^o \sim I(0) \\ y_{it} &\sim I(1) \iff F_t \sim I(1) \text{ and/or } y_{it}^o \sim I(1) \end{aligned}$$

Bai and Ng (2004; hereafter BN) were the first to propose panel unit root tests based on this factor model of cross section dependence. BN propose two panel unit root tests based on principal component estimation of the factor structure. However, BN's method works only when both  $T$  and  $N$  are large. When either  $T$  or  $N$  is less than 20, both the number of common factors and the factors themselves are difficult to estimate accurately.

We suggest a simple panel unit root test to fill in this gap. The proposed panel unit root test consists of two sequential tests. The first test is a 'covariate unit root test' with recursive mean adjustment (CRMA) to test if the common factors are  $I(1)$ , we use the cross sectional average of  $y_{it}$ . The second test is a panel feasible generalized unit root test combined with recursive mean adjustment (PF-RMA), which tests if the idiosyncratic components,  $y_{it}^o$ , are non-stationary. Here we explain the second test first under the assumption that the common factor is stationary. Of course, if the first null hypothesis is not rejected, then there is no need to test the second null hypothesis.

## 2 PFGLS-RMA Test

The latent model for  $y_{it}$ , is given by

$$\begin{aligned} \text{Constant (M1):} \quad & y_{it} = a_i + x_{it}, \quad x_{it} = \rho x_{it-1} + \varepsilon_{it} & (2) \\ \text{Linear Trend (M2):} \quad & y_{it} = a_i + b_i t + x_{it}, \quad x_{it} = \rho x_{it-1} + \varepsilon_{it} & (3) \end{aligned}$$

where  $\varepsilon_{it}$  is stationary process, and the initialization of  $x_{it}$  is taken to be  $x_{i0} = O_p(1)$  and uncorrelated with  $\{\varepsilon_{it}\}_{t \geq 1}$ . Under the null of unit root,  $\rho = 1$  for all  $i$ . Define  $c_{it-1} = \frac{1}{t-1} \sum_{s=1}^t y_{is}$ . We start from the following simple panel AR(p) models under the assumption of homogeneity:

$$\begin{aligned} y_{it} - c_{it-1} &= \rho (y_{it-1} - c_{it-1}) + \sum_{j=1}^p \phi_{ij} \Delta y_{it-j} + e_{it} & \text{for M1} \\ y_{it} - 2c_{it-1} &= \beta_i + \rho (y_{it-1} - 2c_{it-1}) + \sum_{j=1}^p \phi_{ij} \Delta y_{it-j} + e_{it} & \text{for M2} \end{aligned} \quad (4)$$

Note that for M1, the use of the *common* recursive mean is first suggested by Shin and So (2001). For M2, subtracting the double recursive mean method is a new idea. To eliminate the linear trend term completely, we consider the following simple modification. Note that under M2,  $\bar{y}_{it-1} = (t-1)^{-1} \sum_{s=1}^{t-1} y_{is} = a_i + \frac{1}{2} b_i (t-1) + \bar{x}_{it-1}$ . Subtracting two times of  $\bar{y}_{it-1}$  from  $y_{it-1}$  and  $y_{it}$  yields

$$y_{it-1} - 2\bar{y}_{it-1} = -a_i + (x_{it-1} - 2\bar{x}_{it-1}), \quad y_{it} - 2\bar{y}_{it-1} = -a_i + b_i + (x_{it} - 2\bar{x}_{it-1}).$$

Hence the linear trend term is completely eliminated. Even under the null of  $\rho_i = 1$ , the trend is eliminated but the constant is still present. Taking an overall mean adjustment yields

$$y_{it} - \bar{y}_i - 2(\bar{y}_{it-1} - \mu_i) = \rho_i [y_{it-1} - \bar{y}_{i,-1} - 2(\bar{y}_{it-1} - \mu_i)] + (e_{it} - \bar{e}_i),$$

where  $\mu_i = (T-1)^{-1} \sum_{t=2}^T \bar{y}_{it-1}$ ,  $\bar{y}_{i,-1} = (T-1)^{-1} \sum_{t=2}^T y_{it-1}$  and  $e_{it} = -2(1-\rho_i)\bar{x}_{it-1} + \varepsilon_{it}$ . By means of Monte Carol simulation, we found that the proposed new estimator works very well.

When  $y_{it}$  is cross sectionally dependent, the PFGLS-RMA estimator requires the estimation of the covariance matrix. To obtain a more accurate covariance estimator, we consider the following additional step. The first stage estimator of  $\hat{\rho}_{\text{PRMA}}$  in (4) is treated as the true value of  $\rho$ , and we run the following regression to obtain  $\hat{\varepsilon}_{it}^\dagger$ . If  $\hat{\rho}_{\text{PRMA}} > 1$ , set  $\hat{\rho}_{\text{PRMA}} = 1$ .

$$\begin{aligned} y_{it} - \hat{\rho}_{\text{PRMA}} y_{it-1} &= a_i + \sum_{j=1}^p \phi_{ij} \Delta y_{it-j} + \varepsilon_{it}^\dagger & \text{for M1} \\ y_{it} - \hat{\rho}_{\text{PRMA}} y_{it-1} &= a_i + \beta_i t + \sum_{j=1}^p \phi_{ij} \Delta y_{it-j} + \varepsilon_{it}^\dagger & \text{for M2} \end{aligned} \quad (5)$$

Let  $\mathbf{y}_t = (y'_{1t}, \dots, y'_{Nt})'$ ,  $\mathbf{c}_{t-1} = (c'_{1t}, \dots, c'_{Nt})'$ , and  $\boldsymbol{\varepsilon}_t = (\varepsilon'_{1t}, \dots, \varepsilon'_{Nt})'$ . Denote  $\hat{\boldsymbol{\Sigma}}_{\varepsilon, \text{prma}}$  as the estimated covariance matrix using  $\hat{\varepsilon}_{it}^\dagger$ , and consider the decomposition  $\hat{\boldsymbol{\Sigma}}_{\varepsilon, \text{prma}}^{-1} = \boldsymbol{\Lambda} \boldsymbol{\Lambda}'$ . Further define the transformed vector  $\mathbf{y}_t^+ = \boldsymbol{\Lambda} \mathbf{y}_t$ ,  $\mathbf{c}_{t-1}^+ = \boldsymbol{\Lambda} \mathbf{c}_{t-1}$ . Let  $y_{it}^+$ ,  $c_{it-1}^+$  and  $e_{it}^+$  denote the  $i$ th elements of  $\mathbf{y}_t^+$ ,  $\mathbf{c}_{t-1}^+$  and  $\mathbf{e}_t^+$ , respectively. Then we have

$$\begin{aligned} y_{it}^+ - c_{it-1}^+ &= \rho (y_{it-1}^+ - c_{it-1}^+) + \sum_{j=1}^p \phi_{ij} \Delta y_{it-j}^+ + e_{it}^+ & \text{for M1} \\ y_{it}^+ - 2c_{it-1}^+ &= \beta_i + \rho (y_{it-1}^+ - 2c_{it-1}^+) + \sum_{j=1}^p \phi_{ij} \Delta y_{it-j}^+ + e_{it}^+ & \text{for M2} \end{aligned} \quad (6)$$

and denote

$$t^{rc} = \frac{\hat{\rho}^{rc}}{\sqrt{V(\hat{\rho}^{rc})}} \quad \text{and} \quad t^{r\tau} = \frac{\hat{\rho}^{r\tau}}{\sqrt{V(\hat{\rho}^{r\tau})}} \quad (7)$$

where  $\hat{\rho}^{rc}$  and  $\hat{\rho}^{r\tau}$  are point estimates in (6). The null and alternative hypotheses are given by

$$H_0^2 : \rho_i = 1 \text{ for all } i, \quad \text{Against } H_A^2 : \rho_i < 1 \text{ for all } i.$$

Note that we may alternatively consider a heterogeneous panel unit root test. See Shin, Kang and Oh (2004) for more on this issue. We do not consider the heterogeneous panel unit root test here since (preliminary simulations showed) the power and size of the two tests are very similar.

We now consider the local asymptotic power of the pooled test. Let  $\rho = 1 + \gamma/T$ , and derive the limit distributions of  $t^{rc}$  and  $t^{r\tau}$ .

**Proposition 1 (PFGLS-RMA Unit Root Tests)** *The limiting distribution of the test statistics are given by*

$$t^{r\kappa} \rightarrow^d A^\kappa + \gamma B^\kappa, \quad \text{for } \kappa = c \text{ and } \tau \quad (8)$$

where

$$\begin{aligned} A^\kappa &= \left[ \sum_{i=1}^N \int_0^1 J_i^\kappa dW_i \right] \left[ \sum_{i=1}^N \int_0^1 (J_i^\kappa)^2 dr \right]^{-\frac{1}{2}}, \\ B^\kappa &= \sum_{i=1}^N \left\{ \Phi_i \left[ \int_0^1 (J_i^\kappa)^2 dr + \int_0^1 \tilde{J}_i^\kappa dr \right] \right\} \left[ \sum_{i=1}^N \int_0^1 (J_i^\kappa)^2 dr \right]^{-\frac{1}{2}} \end{aligned}$$

and

$$\begin{aligned} \Phi_i &= \left( 1 - \sum_{j=1}^{p-1} \phi_{ij} \right)^{-1}, \quad J_i = J_i(r) = \int_0^r e^{c(r-s)} dW_i(s) \\ \bar{J}_i &= \bar{J}_i(r) = r^{-1} \int_0^r J_i(s) ds, \quad \dot{J}_i = 2\bar{J}_i - \int_0^1 J_i(s) ds + 2 \int_0^1 \bar{J}_i dr \\ J_i^c &= J_i - \bar{J}_i, \quad J_i^\tau = J_i - \dot{J}_i, \quad \tilde{J}_i^c = J_i^c \bar{J}_i, \quad \tilde{J}_i^\tau = J_i^\tau \left( \bar{J}_i - \int_0^1 \bar{J}_i dr \right) \end{aligned}$$

where  $J_i(r)$  is the Ornstein-Uhlenbeck process.

The proof of Proposition 1 is straightforward and hence it is omitted. The critical values for RMA unit root test can be obtained by letting  $\gamma = 0$  and replacing  $J$  by the standard Brownian motion  $W$ . The asymptotic critical values for the Brownian motion case are as follows: For M1, the 5% critical values are -1.88, -1.86, -1.83, -1.77, -1.73 meanwhile for M2, they are -1.86, -1.82, -1.80, -1.75, -1.73 with  $N=1,2,3,10,20$ , respectively.

### 3 Unit Root Tests on Cross Sectional Average

Note that either when  $N$  or  $T$  is less than 20, it is hard to estimate both the factor number and the common factors. To test if the common factors stationary or not, we suggest the use of the cross sectional average of  $y_{it}$  to approximate the common factors. We assume that

$$\bar{\lambda}_s = N^{-1} \sum_{i=1}^N \lambda_{is} \neq 0. \quad (9)$$

Under this assumption, the cross section average,  $\bar{y}_t$ , is given by

$$\bar{y}_t = \frac{1}{N} \sum_{i=1}^N y_{it} = \sum_{s=1}^K \bar{\lambda}_s F_{st} + \frac{1}{N} \sum_{i=1}^N y_{it}^o := \bar{F}_t + M_t, \text{ let say} \quad (10)$$

Even though  $M_t = O_p(N^{-1/2})$ , we will show later that the cross sectional average of  $y_{it}$  approximates  $\bar{F}_t$  well even with a small  $N$ . Note that one can use Shin and So (2001)'s univariate unit root test by using  $\bar{y}_t$  to examine if  $\bar{F}_t$  is  $I(1)$  or not. Here we provide a better way to improve the power of the univariate unit root test by combining Hansen's covariate ADF (CADF) test with the recursive mean adjustment method.

When a nonstationary covariate,  $g_t$ , is available, the principle of RMA can be directly applied to obtain more power. Following Hansen (1995), consider the following covariate augmented DF CADF( $p, q_1, q_2$ ) regressions for the unknown constant:

$$\bar{y}_t = \alpha + \rho \bar{y}_{t-1} + \sum_{j=1}^p \phi_j \Delta \bar{y}_{t-j} + \sum_{j=-q_1}^{q_2} \phi_j \Delta g_{t+j} + u_t$$

Define  $\varepsilon_t = \sum_{j=-q_1}^{q_2} \phi_j \Delta g_{t+j} + u_t$ ,

$$\Omega = \sum_{l=-\infty}^{\infty} E \left[ \begin{pmatrix} \varepsilon_t \\ u_t \end{pmatrix} \begin{pmatrix} \varepsilon_{t-k} & u_{t-k} \end{pmatrix} \right] = \begin{pmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon u} \\ \sigma_{\varepsilon u} & \sigma_u^2 \end{pmatrix}$$

and  $\varphi^2 = \sigma_{\varepsilon u}^2 [\sigma_u^2 \sigma_\varepsilon^2]^{-1}$  and  $R^2 = \sigma_u^2 / \sigma_\varepsilon^2$ .

The RMA-modified covariate augmented DF regression is given by

$$\begin{aligned} \bar{y}_t - c_{t-1} &= \rho (\bar{y}_{t-1} - c_{t-1}) + \sum_{j=1}^p \phi_j \Delta \bar{y}_{t-j} + \sum_{j=-q_2}^{q_1} \psi_j \Delta g_{t-j} + u_t & \text{for M1} \\ \bar{y}_t - 2c_{t-1} &= \beta + \rho (\bar{y}_{t-1} - 2c_{t-1}) + \sum_{j=1}^p \phi_j \Delta \bar{y}_{t-j} + \sum_{j=-q_2}^{q_1} \psi_j \Delta g_{t-j} + u_t & \text{for M2} \end{aligned} \quad (11)$$

The covariate RMA (CRMA) test statistics are defined as

$$t_{\text{crma}}^{rc} = (\hat{\rho}_{\text{crma}}^{rc} - 1) / \sqrt{V(\hat{\rho}_{\text{crma}}^{rc})}, \quad t_{\text{crma}}^{r\tau} = (\hat{\rho}_{\text{crma}}^{r\tau} - 1) / \sqrt{V(\hat{\rho}_{\text{crma}}^{r\tau})}.$$

where  $\hat{\rho}_{\text{crma}}^{rc}$  and  $\hat{\rho}_{\text{crma}}^{r\tau}$  are point estimates in (11) for M1 and M2, respectively. Let  $\rho = 1 + \gamma/T$ .

**Proposition 2 (Covariate-RMA test)** *The limiting distribution of the test statistics are given by*

$$t_{crma}^{r\kappa} \xrightarrow{d} \varphi A^\kappa + \frac{\gamma}{R} B^\kappa + (1 - \varphi^2)^{1/2} \mathcal{N}(0, 1), \text{ for } \kappa = c, \tau$$

where  $A^\kappa$  and  $B^\kappa$  are defined in Proposition 1.

The proof of Proposition 2 is straightforward and hence it is omitted.

For CADF test case, the critical value is very sensitive to  $\varphi$ . Meanwhile the 5% critical values of the RMA unit root tests for unknown constant and linear trend cases are given by -1.88 and -1.86, respectively, which are equivalent to the 5% critical values for  $\varphi = 1$  but to the 3% critical values for  $\varphi = 0$ . This implies that practitioners do not need to estimate  $\varphi$  to pin down the critical value. They can use the asymptotic critical value of the RMA unit root test, which makes the tests slightly conservative. For choice of a covariate, practitioners may choose the covariate of which the long run correlation is highest.

## 4 Monte Carlo Simulation and Summary

We consider the following data generating process

$$y_{it} = \lambda_i F_t + y_{it}^o, \quad F_t = \vartheta F_{t-1} + v_t, \quad y_{it}^o = \rho y_{it-1}^o + \varepsilon_{it} \quad (12)$$

$$v_t = \sqrt{1 - \varphi^2} \Delta g_t + \sqrt{\varphi^2} w_t, \quad \Delta g_{it} = \lambda_i \Delta g_t \quad (13)$$

where  $\Delta g_t \sim iid\mathcal{N}(0, 1)$ ,  $w_t \sim iid\mathcal{N}(0, 1)$ ,  $\varepsilon_{it} \sim iid\mathcal{N}(0, I_N)$ , but  $\lambda_i$  is generated either from  $U(0, 1)$  and  $iid\mathcal{N}(0, 1)$ . The latter case is violating the assumption in (9). Two null hypotheses are tested are our proposed unit root test.

$$\mathcal{H}_0^1 : F_t \sim I(1) \text{ or } \vartheta_s = 1 \text{ for all } s, \quad \mathcal{H}_0^2 : y_{it}^o \sim I(1) \text{ or } \rho_i = 1 \text{ for all } i$$

We consider two cases: ‘No-cointegrated’ and ‘cointegrated’ panel. The no-cointegrated panel under the null is given by  $\rho = \vartheta = 1$ . The cointegrated panel under the null is given by  $\rho < 1$  but  $\vartheta = 1$ . We set  $\varphi^2 = 0.2$ . To investigate the size properties of tests, we consider three cases: Case I: (no-cointegrated panel)  $\rho = \vartheta = 1$ , Case II (cointegrated panel):  $\rho = 0.95$ ,  $\vartheta = 1$ , and Case III (alternative):  $\rho = \vartheta = 0.95$ . For all cases, we set  $T \in [50, 100]$ . For the CRMA regression, we use the cross sectional average of  $g_{it}$  as the covariate for the cross sectional average of  $y_{it}$ . The autoregressive order is assumed to be known.

We compare the finite sample performance of our tests with BN’s tests. For BN’s univariate ADF tests with the principal-component estimated  $F_t$ , we assume the number of common factors are known (in this case  $K = 1$ ). We grant the BN method this advantage since preliminary simulations showed that the selected number of common factors are always equal to the maximum number tested for. However, when we construct BN’s pooled tests with the estimated  $\varepsilon_{it}$ , we estimate the number of common factors based on  $BIC_3$  in Bai and Ng (2002). The maximum number of common factors is set to be 4 for  $N = 5$  and 8 for  $N = 15$ .

Table 1 reports the rejection rates of the two tests for the first null hypothesis. For the CRMA tests, we use the asymptotic critical values of -1.88 and -1.86 for the constant and linear trend cases. Here is a summary of the findings: First, when  $\lambda_i \sim U(0, 1)$ , the sizes of all tests are accurate. When

$\lambda_i \sim \mathcal{N}(0, 1)$ , the assumption in (9) does not hold so that the cross sectional average of  $y_{it}$  in (10) is not well defined asymptotically. However in the finite sample, the size distortion of CRMA test is mild. Note that the power of CRMA test decreases also when  $\lambda_i \sim \mathcal{N}(0, 1)$ . Second, for all cases CRMA test provides more power than BN's test.

Table 2 reports the joint testing results of both  $\mathcal{H}_0^1$  and  $\mathcal{H}_0^2$ : 'RMA' stands for the joint tests of CRMA and PF-RMA and 'BN' is the joint tests of BN. Only joint testing results for Case I and III are reported. Here is a summary of the findings: First, for the no-cointegrated panel, both BN's and our tests are seriously under sized regardless of  $N$  and  $T$ . Second, the joint tests of CRMA and PF-RMA produce good power for both the constant and linear trend cases.

This paper suggests new panel unit root tests to somewhat restore the panel power gain by using recursive mean adjustment under the situation where  $T$  is larger than  $N$ . If  $N$  is larger than  $T$ , the suggested panel unit root tests may be used indirectly by forming panel subgroups.

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**Table 1: Rejection Rates for  $H_0^1 : F_t - I(1)$** 

Size of Tests (5%); Case I: $\rho = \vartheta = 1$									
		$\lambda_i \sim U(0, 1)$				$\lambda_i \sim \mathcal{N}(0, 1)$			
		Constant		Trend		Constant		Trend	
T	N	CRMA	BN	CRMA	BN	CRMA	BN	CRMA	BN
50	5	5.3	5.5	5.1	6.7	5.6	5.6	5.5	6.4
50	15	4.8	5.5	4.9	6.4	5.3	5.8	5.1	6.9
100	5	5.2	5.8	5.0	5.7	5.4	5.8	5.3	6.1
100	15	4.3	6.2	4.2	5.9	5.2	5.4	5.2	6.0
Size of Tests (5%): Case II: $\rho = 0.95, \vartheta = 1$									
50	5	4.8	6.4	5.5	6.8	7.0	6.0	5.8	6.9
50	15	4.9	6.1	4.6	6.8	7.8	6.0	6.3	6.9
100	5	4.3	7.8	6.2	7.3	11.4	6.3	7.9	6.7
100	15	5.1	6.7	4.7	6.2	11.0	6.1	7.1	6.1
Size Adjusted Power: Case III: $\rho = \vartheta = 0.95$									
50	5	30.7	8.3	11.9	7.7	16.6	8.9	9.9	7.8
50	15	31.4	7.5	17.9	7.3	16.3	8.7	10.8	8.1
100	5	61.4	13.5	23.8	9.8	33.6	13.8	18.9	10.5
100	15	64.0	13.5	41.0	10.2	34.2	12.5	18.6	9.7

**Table 2: Joint Rejection Rates for RMA against BN's Tests**

Empirical Size: $\rho = \vartheta = 1$									
		$\lambda_i \sim U(0, 1)$				$\lambda_i \sim \mathcal{N}(0, 1)$			
		Constant		Trend		Constant		Trend	
T	N	RMA	BN	RMA	BN	RMA	BN	RMA	BN
50	5	0.6	0.5	0.7	0.9	0.7	0.6	0.7	1.3
50	15	0.2	0.4	0.2	1.0	0.2	0.3	0.2	0.4
100	5	0.6	0.4	0.7	0.8	0.5	0.8	0.6	1.0
100	15	0.1	0.6	0.2	0.9	0.5	0.2	0.3	0.3
Size Unadjusted Power: $\rho = \vartheta = 0.95$									
50	5	8.4	2.9	2.3	1.7	7.4	3.1	2.1	1.8
50	15	19.7	4.3	2.4	2.0	10.9	3.6	1.6	0.9
100	5	36.4	9.1	9.9	4.2	29.7	9.6	7.9	4.8
100	15	63.3	12.4	25.5	6.8	34.1	11.7	12.3	4.0