Using Mixed-Integer Programming to Reduce Label Changes in the Coors Aluminum Can Plant

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Valley Metal Container (VMC), a joint venture between the Coors Brewing Company and American National Can, operates the world's largest single-site facility for aluminum can production. Located in Golden, Colorado, the plant manufactures over 4 billion cans per year on six production lines. Coors Brewery produces seven products, each requiring a distinct label. We developed an optimization-based decision support system with a spreadsheet user interface that helps VMC planners to determine the weekly can production schedule, aiming to meet brewery demand while minimizing the number of label changes and related costs. VMC has been using the new system since October 1998 and estimates the reduction in direct costs (scrap, labor, and inventory) at over $150,000 per year.

Valley Metal Container (VMC), a joint venture between the Coors Brewing Company and American National Can, operates the world's largest single-site facility for aluminum can production. Located in Golden, Colorado, the plant manufactures over 4 billion cans per year on six production lines. Currently VMC supplies cans for seven Coors beer labels to three filling facilities. All three are directly linked to three Coors breweries in Golden, Colorado; Shenandoah, Virginia; and Memphis, Tennessee. The facilities' weekly production schedules are variable and of-

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ten unpredictable, and the lack of the necessary cans can shut down a fill line, which costs approximately $65 per minute of downtime.

Demand for beer varies with the seasons, peaking in the summer and tapering off in the winter (Figure 1).

A common way to cope with seasonal demand is through *anticipation inventory*—inventory carried for meeting future demand. The brewery, however, cannot carry much anticipation inventory of beer because its freshness is crucial to customers. Brewery production load must fluctuate throughout the year to meet varying demand.

The can plant relies on anticipation inventory to smooth its load more than the brewery does. It operates at full capacity (three shifts per day seven days per week) during peak summer months, and it usually runs three shifts five days a week the rest of the year. Overall, VMC builds up anticipation inventory of all seven labels during winter and spring and depletes these inventories during the summer. The cost of fill-line downtime makes coordination between container production and filling facilities crucial.

Demand from the brewery changes weekly, and the can plant scheduling team, also weekly, has to decide
—Which label to produce on each can line during each shift,
—Where to store the finished cans, and
—How to meet brewery demand.
Each line can produce multiple labels (although not all lines can produce all labels). When a line is switched from one label to another, a *label change* occurs. Since label changes do not vary in magnitude depending on production sequence, they are *setups*.

Finished cans go into short-term inventory, to be shipped to a filling facility and used in production in the very near future (within a day) or into one of two types of longer-term inventory. The VMC manage-

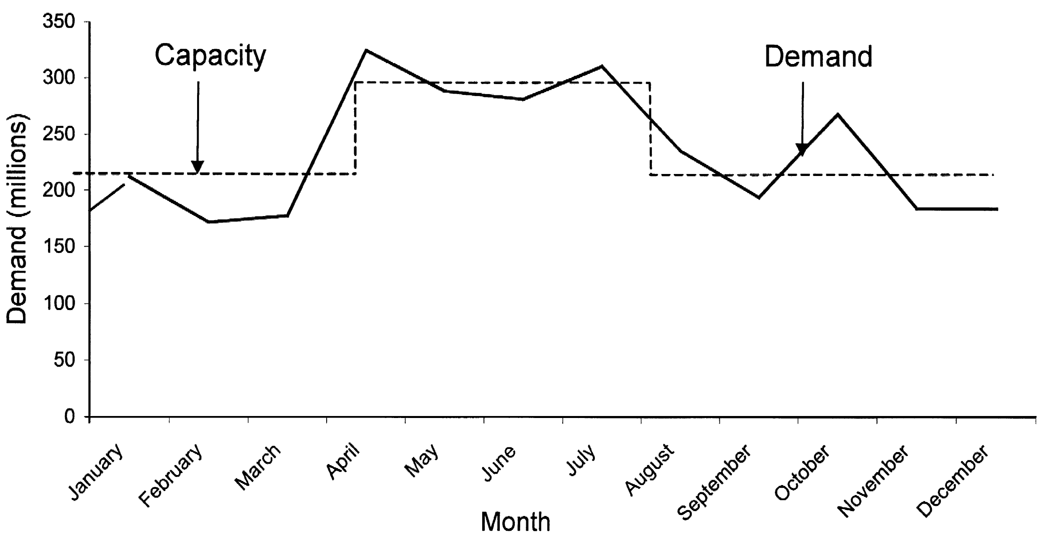

Figure 1: The typical annual demand for beer declines in winter and peaks in summer. The Valley Metal Container capacity profile responds to the demand peak.

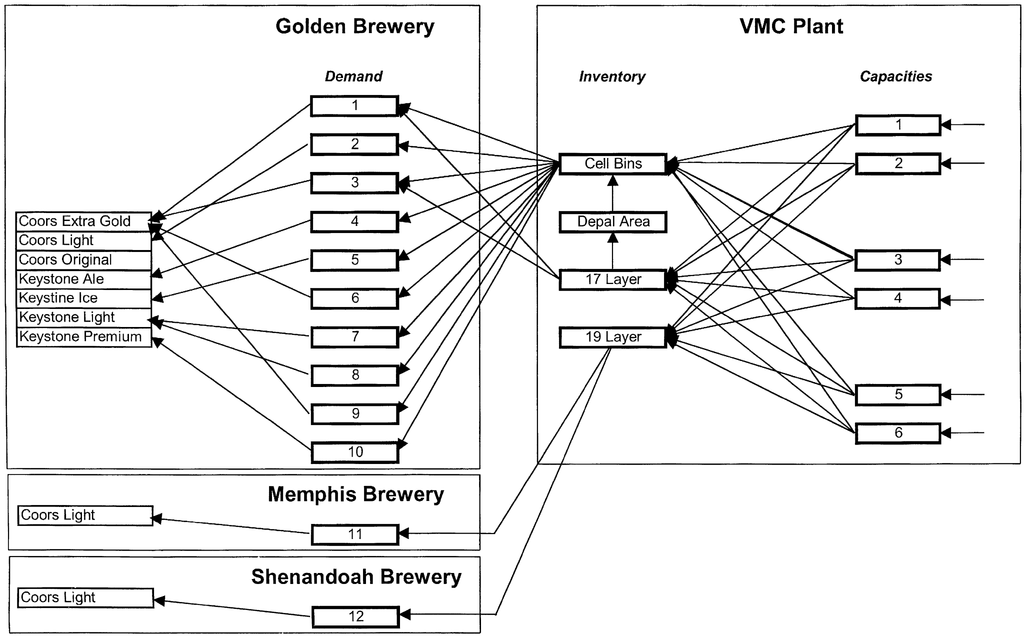

Figure 2: Six can lines at the VMC plant manufacture cans for seven Coors labels. Not all fill lines can handle all labels. VMC sends cans to cell-bin inventory, 17-layer pallet inventory, or 19-layer pallet inventory. Cell-bin inventory goes directly to the Golden brewery fill lines and the 19-layer inventory is used to fill demand from the Shenandoah and Memphis breweries for the Coors Light label. Coors Light is the only product brewed at Shenandoah and Memphis. The 17-layer inventory can be converted to cell-bins in the "depal" area, or used in two of the Golden brewery fill lines.

ment asked us to develop a system that would construct production schedules to reduce label changes and that would help manage the three types of inventory.

Aluminum Cans Production and Distribution

Coors operates three filling facilities, the largest in Golden, Colorado and the two smaller ones in Memphis, Tennessee and Shenandoah, Virginia. VMC in Golden supplies all three with aluminum cans (Figure 2).

The Golden brewery uses 10 fill lines to manufacture seven beer labels: Coors Extra Gold, Coors Light, Coors Original, Keystone Light, Keystone Ale, Keystone Ice, and Keystone Premium. The facilities in Memphis and Shenandoah brew Coors Light only. The three breweries transmit their weekly requirements to the can plant, where, prior to the implementation of our system, schedulers developed production schedules manually. The main concern always was to avoid undersupplying the breweries, while label changes and inventory considerations were strictly secondary.

Finished cans are placed either into cell-bin trailers or stored on pallets. Cell-bins are used for short-term storage and for immediate delivery to the Golden filling facility two miles away. A cell-bin trailer is a tractor or semi-truck trailer that has been modified for the automatic conveying of the cans to the brewery. The top of the trailer has been removed, and the trailer is divided width-wise with each division being a little larger than the length of the can. A trailer is divided into 90 cells or bins that each hold 1,300 cans. As the last step in the can production process, a conveyer moves the cans to a top filler that automatically locates itself above an empty cell bin, and drops (gravity feeds) the cans into each bin until it is full. The

> The schedulers found it difficult to explain what the real constraints were until they saw a solution that violated these constraints.

top filler then indexes itself to the next empty cell bin. The cans in the trailer are neatly stacked on their sides in a cell (or bin), so they roll down into position and are shipped on their sides. A full trailer may sit for awhile, but usually it immediately proceeds to the filling facility for unloading. At the filling facility trailers unload themselves again by gravity. The trailer is tipped sideways into a conveying system that eventually feeds the fillers. A tarp covers the cans (top of the cell-bin trailer) during shipment.

Pallets are used for longer-term storage and for shipments to Shenandoah and Memphis. The Shenandoah and Memphis facilities accept cans on 19-layer pallets; while VMC stores its regular inventory on 17-layer pallets. Eight of the Golden fill lines accept cans in cell-bin trailers only, one accepts 17-layer pallets only, and one is flexible and accepts cans either on pallets or in cell-bin trailers. To use cans from 17-layer inventory on any of the eight lines that accept cans in trailers only, the plant must remove the cans from the pallets and load them into cell-bin trailers (*depaling*). This procedure is performed manually in a separate depal area and adds cost.

Can-line capacity, which ranges from 1/2 million to 1 million cans per eight-hour shift on a line, limits can-line production, and the number of cans that can be depaled is limited to 840,000 cans per shift. The total cell-bin capacity is limited to 50 trailers at 100,000 cans each for a total of 5 million cans.

When the plant switches from producing one label to producing another, an operator resets the line's can decorator, and tunes the colors. Such label changes take on average one hour and generate a large amount of scrap because the operators tune the colors using otherwise saleable cans. Also, during a label change, the line is out of production, and the entire crew with the exception of decorator operators is essentially idle. Avoided label changes reduce cost through decreasing scrap and increasing effective can-line production rates. Since Coors is not unionized, the workforce is flexible, and during peak summer months there is plenty of often unwanted overtime. Consequently, increasing effective production rates directly decreases labor costs by reducing the number of shifts needed to fill demand. There are also inventory-related benefits, because label changes eliminated during peak summer months allow a decrease in anticipation inventory levels.

The system has several technical constraints that limit the output of the six can lines. Line 1 (Figure 2) has the only five-color decorator in the facility (all others are four color) and is therefore the only line that can produce the Coors Extra Gold label (the only five-color label). Lines 5 and 6 produce Coors Light and Keystone Light only. Lines 3 and 4 share a washer, and consequentially during any one shift, they can produce either *full-bodied* labels or *light* labels but not a combination. The light labels are Coors Light and Keystone Light, and all other labels are full-bodied, so whenever either line 3 or 4 produces a light label both lines must produce a light label. Cans go into washers in preparation for decoration and internal coating. Full-bodied labels have decoration for the full body of the can; Coors Light and Keystone Light have much of the aluminum undecorated. As such, a less aggressive acid etch is used in the washers for the light labels because the etch used for full-bodied cans mars the appearance of the aluminum. This is not an issue for the fully decorated cans because the ugly etching is covered by decorating inks.

Decision Support System

Since label changes are expensive, our major concern was to decrease their number. As we started implementation, however, we discovered that schedulers were concerned with managing inventory even more than with decreasing the number of label changes. The final system minimizes total production cost while meeting demand and not exceeding available capacity of any of the resources (including the six can lines, the depal area, and the cell-bin inventory) (Figure 3).

Demand and inventory-level data changes every week, so schedulers must be able to edit data without changing the model. Schedulers are familiar with Microsoft Excel, so we developed an Excel-based user interface for data entry. They can easily enter and adjust all data without having to make any changes in the optimization code. The user interface (Figure 4) consists of an Excel workbook with two worksheets. One worksheet has demand, capacity, and inventory data that users can adjust. The second worksheet displays the production schedule. We modeled the problem as a mixed-integer program (Appendix) and implemented the model in GAMS [Brooke, Kendrick, and Meeraus 1988] with spreadsheet linking accomplished via SSLINK [Rutherford 1998]. The user can change values inside the tables. All other cells are protected. The "Run GAMS" button is linked to a macro that copies table contents into a new Excel file, saves the file in Excel 4 format (this is currently the type compatible with SSLINK), and runs GAMS. GAMS reads the data from the spreadsheet using the SSIMPORT module of SSLINK and writes the solution data onto another spreadsheet using the SSEXPORT module. The "View Schedule" button activates a macro that opens the spreadsheet with GAMS output and copies its content into a worksheet containing special report formatting. The user can view, print, and edit the schedule.

We first attempted to model the VMC problem as a pure network, as in Glover, Klingman, and Phillips [1992], because pure-network models are extremely efficient to solve. Unfortunately, pure-network formulations cannot handle logical constraints of the form "if line X

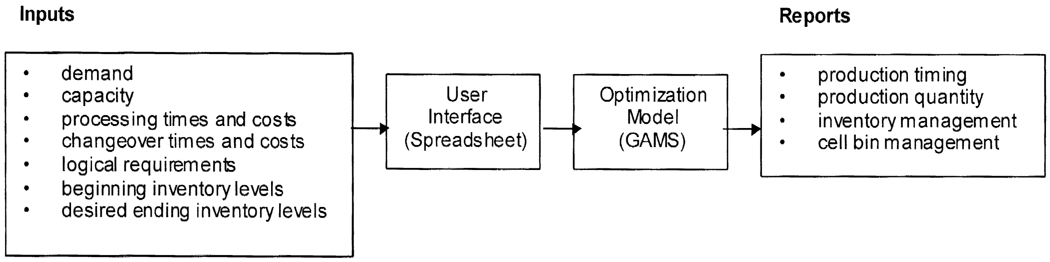

Figure 3: In the decision support system, users enter demand, capacity, inventory, and production data into a spreadsheet. Some of the data (demand and inventory) changes weekly, while the rest is stable. The GAMS model minimizes system-operating cost while meeting demand and preserving capacity and logical constraints. System outputs include reports on the quantity and timing of production and on inventory management.

Run GAMS

View Schedule

Demand: 17-Layer Pallets

Product	Mon-G	Mon-D	Mon-S	Tue-G	Tue-D	Tue-S	Wed-G	Wed-D	Wed-S	Thr-G	Thr-D
Coors Light	479,133	479,133	-	547,581	479,133	-	-	-	-	205,343	479,133
Coors Original	-	68,448	547,581	-	-	68,448	547,581	547,581	547,581	342,238	-
Keystone Premium	-	-	-	-	-	-	-	-	-	-	-
Keystine Ice	-	-	-	-	-	-	-	-	-	-	-
Keystone Light	-	-	-	-	-	-	-	-	-	-	-
Coors Extra Gold	-	-	-	-	-	-	-	-	-	-	-
Keystone Ale	-	-	-	-	-	-	-	-	-	-	-

Demand Cell Bins

Product	Mon-G	Mon-D	Mon-S	Tue-G	Tue-D	Tue-S	Wed-G	Wed-D	Wed-S	Thr-G	Thr-D
Coors Light	3,082,457	2,348,051	1,624,225	2,273,651	1,848,747	1,806,422	1,896,693	2,629,116	2,782,214	994,640	1,149,722
Coors Original	173,930	608,754	296,276	613,714	186,495	687,121	409,694	-	-	68,448	777,724
Keystone Premium	-	145,162	1,161,295	613,714	613,714	586,269	1,026,714	1,106,735	662,983	2,182,718	1,007,866
Keystine Ice	-	-	-	-	-	-	-	-	-	-	-
Keystone Light	-	-	-	-	-	-	-	-	-	-	-
Coors Extra Gold	-	-	-	-	-	-	-	-	-	-	-
Keystone Ale	-	-	-	-	-	-	-	-	-	-	-

Capacities

Line	Mon-G	Mon-D	Mon-S	Tue-G	Tue-D	Tue-S	Wed-G	Wed-D	Wed-S	Thr-G	Thr-D
1	8	8	8	8	8	8	8	8	8	8	8
2	8	8	8	8	8	8	8	8	8	8	8
3	8	8	8	8	8	8	8	8	8	8	8
4	8	8	8	8	8	8	8	8	8	8	8
5	8	8	8	8	8	8	8	8	8	8	8
6	8	8	8	8	8	8	8	8	8	8	8

Inventory

Product	BegTot	EndTot	BegCell	EndCell	Beg19	End19
Coors Light	16,714,740	21,800,000	1,500,000	1,500,000	15,000,000	-
Coors Original	8,625,120	12,000,000	500,000	500,000		
Keystone Premium	14,758,380	21,500,000	-	-		
Keystine Ice	3,727,080	3,727,080	-	-		
Keystone Light	3,919,860	3,919,860	-	-		
Coors Extra Gold	4,326,840	4,326,840	-	-		
Keystone Ale	1,213,800	1,213,800	-	-		

Figure 4: With this Excel user interface the user can change values in the data tables, and start GAMS by pressing a button. The "Run GAMS" button is linked to a macro that copies all data into a different Excel file, saves it, closes it, and runs GAMS through a batch file. The "View Schedule" button opens an Excel output file created by the SSEXPORT module of SSLINK and copies the data into a worksheet that contains report formatting.

produces A then line Y must produce B." Such constraints require the use of binary variables, and so we developed a mixed-integer formulation (Appendix). Our final formulation is an extension of the multi-level capacitated lot-sizing problem (MLCLSP) originally introduced by Billington, McClain, and Thomas [1983]. The main difference is that in the VMC problem a lot can be produced over multiple time periods. Maes, McClain, and Van Wassenhove [1991] show that the problem of finding even a feasible solution in the multi-item lot-sizing setting with multiple constrained resources is NP-complete when setups consume capacity. Therefore, we had major concerns with speed and solution quality. Heuristic solution procedures for some lot-sizing problems do exist [Harrison and Lewis 1996; Katok, Lewis, and Harrison 1998; and Tempelmeier and Destroff 1996], but so far none have been implemented using an algebraic modeling language, such as GAMS.

We were pleased to find that our formulation solves in under a minute on a 400 MHz Pentium II under Windows NT, essentially to optimality (we set tolerance at 0.003). Input data files are small, so by far the majority of the time is spent in the branch-and-bound portion of the algorithm (of the 10 problem instances for which we recorded solution times, less than 0.5 seconds was spent on importing data with SSIMPORT, 0.25 seconds were spent on problem generation, and 5.0 seconds on solving the initial linear program). We selected the 0.003 tolerance by trial and error because smaller tolerances seem to increase solution time disproportionately, while larger tolerances rarely generate solutions that could be improved by inspection.

Implementation

We started developing the production-scheduling system in March 1998. Our first step was to understand how cans are manufactured, how the can plant and the brewery are related, and what impact this relationship has on scheduling. We spent some time working on the can line to gain an understanding of scheduling issues. We also worked closely with the schedulers to develop a better understanding of the issues they face. The most difficult and time-consuming part of implementation was eliciting complete information on production-system restrictions. After we developed the first version of the model and showed the solution to the inventory manager, it took us several months to develop a model that represented the actual system to such an extent that schedulers became willing to use it. We think that the implementation took as long as it did because the schedulers found it difficult to explain to us what the real constraints of the system were until they saw a solution that violated these constraints.

Another important implementation step was report generation. Originally we assumed that the schedulers' main concern was label changes, so we generated reports that showed for each shift which label was produced on which line. We later learned that although schedulers care about label changes, they care even more about understanding what happens with the three types of inventory. They were particularly interested in cell-bin inventory because cell-bin capacity turns out to be most binding. We developed reports that show how inventory levels change for each label and each shift. These reports were key: once the schedulers saw them, they become truly committed to using our system. The ownership of the system passed from us to the scheduling team in October 1998. As part of the business case (a briefing of the senior management during which we described system benefits and requested funding), we justified the purchase of a dedicated computer system

(400 MHz Pentium II with 384 MB of RAM) housed in the scheduling area and a commercial GAMS license to run our model. We also implemented GAMS interface with Excel through SSLINK at that time.

Results

To show the impact our model has on scheduling the can plant in general and on label changes in particular, we will describe the week of March 9, 1998. During that week, line 1 was scheduled to work for five days only, line 2 was down for maintenance, and the other four lines were scheduled for seven days (Figure 5).

The schedule included 12 label changes. Although the schedules for lines 3 and 4 are similar, and those for 5 and 6 are similar, there was no reason for this similarity other than the convenience of the schedulers. We developed a schedule for the same week using our DSS (Figure 6).

The proposed schedule produced with the DSS has only eight label changes and uses the depal area for three shifts only instead of the 13 shifts in the actual schedule. The workforce in the depal area is flexible, consisting of part-time employees and employees cross-trained at other tasks, and has always been scheduled as needed. Therefore, cutting down on its use translates to proportionally lower labor costs. Our model took advantage of the fact that the Coors Extra Gold inventory level was higher than necessary and produced Coors Extra Gold label cans for only two shifts instead of eight. Since the old scheduling system had no way of tracking inventory by label, Coors Extra Gold cans were depaled unnecessarily for

Shift/Line	1	3	4	5	6	Depal Area
Mon-morning	COORS LIGHT	COORS ORIGINAL	COORS ORIGINAL	COORS LIGHT	COORS LIGHT	COORS LIGHT
Mon-swing	KEYSTONE PREM	COORS ORIGINAL	COORS ORIGINAL	COORS LIGHT	COORS LIGHT	COORS LIGHT
Mon-night	KEYSTONE PREM	COORS ORIGINAL	COORS ORIGINAL	COORS LIGHT	COORS LIGHT	COORS LIGHT
Tue-morning	KEYSTONE PREM	COORS ORIGINAL	COORS ORIGINAL	COORS LIGHT	COORS LIGHT	COORS LIGHT
Tue-swing	KEYSTONE PREM	COORS ORIGINAL	COORS ORIGINAL	COORS LIGHT	COORS LIGHT	COORS LIGHT
Tue-night	KEYSTONE PREM	KEYSTONE ALE	KEYSTONE ALE	COORS LIGHT	COORS LIGHT	
Wed-morning	KEYSTONE PREM	KEYSTONE ALE	KEYSTONE ALE	COORS LIGHT	COORS LIGHT	COORS LIGHT
Wed-swing	EXTRA GOLD	KEYSTONE ICE	KEYSTONE ICE	COORS LIGHT	COORS LIGHT	COORS LIGHT
Wed-night	EXTRA GOLD	KEYSTONE ICE	KEYSTONE ICE	KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT
Thu-morning	EXTRA GOLD	KEYSTONE ICE	KEYSTONE ICE	KEYSTONE LIGHT	KEYSTONE LIGHT	COORS LIGHT
Thu-swing	EXTRA GOLD	KEYSTONE ICE	KEYSTONE ICE	KEYSTONE LIGHT	KEYSTONE LIGHT	
Thu-night	EXTRA GOLD	COORS LIGHT	COORS LIGHT	KEYSTONE LIGHT	KEYSTONE LIGHT	EXTRA GOLD
Fri-morning	EXTRA GOLD	COORS LIGHT	COORS LIGHT	KEYSTONE LIGHT	KEYSTONE LIGHT	COORS LIGHT
Fri-swing	EXTRA GOLD	COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT
Fri-night	EXTRA GOLD	COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT
Sat-morning		COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sat-swing		COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sat-night		COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sun-morning		COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sun-swing		COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sun-night		COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	

Figure 5: This is the actual schedule for the week of March 9, 1998. A week has 21 shifts (three shifts per day), shown in the first column. Columns labeled 1,3,4,5, and 6 display production scheduled for the five can lines that were operating during the week, and the last column displays production scheduled for the depal area. This schedule has 12 label changes and 13 shifts in the depal area.

Shift/Line	1	3	4	5	6	Depal Area
Mon-morning	COORS LIGHT	COORS ORIGINAL	COORS ORIGINAL	COORS LIGHT	COORS LIGHT	
Mon-swing	COORS LIGHT	COORS ORIGINAL	KEYSTONE ICE	COORS LIGHT	COORS LIGHT	
Mon-night	COORS LIGHT	COORS ORIGINAL	KEYSTONE ICE	COORS LIGHT	COORS LIGHT	COORS ORIGINAL
Tue-morning	COORS LIGHT	COORS ORIGINAL	KEYSTONE ICE	COORS LIGHT	COORS LIGHT	
Tue-swing	KEY ALE	COORS ORIGINAL	KEYSTONE ICE	COORS LIGHT	COORS LIGHT	
Tue-night	KEYSTONE ICE	COORS ORIGINAL	KEYSTONE ICE	COORS LIGHT	COORS LIGHT	KEYSTONE PREM
Wed-morning	KEYSTONE ICE	COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Wed-swing	EXTRA GOLD	COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Wed-night	EXTRA GOLD	COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Thu-morning	KEYSTONE PREM	COORS LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Thu-swing	KEYSTONE PREM	KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	KEYSTONE LIGHT
Thu-night	KEYSTONE PREM	KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Fri-morning	KEYSTONE PREM	KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Fri-swing	KEYSTONE PREM	KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Fri-night	KEYSTONE PREM	KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sat-morning		KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sat-swing		KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sat-night		KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sun-morning		KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sun-swing		KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	
Sun-night		KEYSTONE LIGHT	COORS LIGHT	COORS LIGHT	COORS LIGHT	

Figure 6: Our proposed schedule for the week of March 9, 1998 has eight label changes and three shifts in the depal area, saving approximately $3,000 compared to the actual schedule.

one shift in the actual schedule. Our system also produced Keystone Ale labeled cans for a single shift only, instead of the four shifts in the actual schedule. Producing less of the smaller labels allows us to dedicate lines 5 and 6 to Coors Light and to produce mostly Coors Light on line 4. This heavy Coors Light production allowed us to avoid using the depal area for Coors Light entirely.

We estimated cost savings from our system with the help of Coors accountants, and they approved the final numbers. Coors rewards its employees for finding and implementing ways to save money by returning to them a small percentage of these savings. Not surprisingly, Coors accountants scrutinize projected savings figures and sign off only on very conservative savings estimates. Their estimates of savings from our model are based on the cost of $240 per avoided shift in the depal areas (three people for eight hours at $10 per hour) and $400 per avoided label change ($250 in scrap reduction, $100 in labor, and $50 in inventory benefits). Overall, the difference between the actual and proposed schedules for the week of March 9, 1998 represents savings of over $3,000 ($2,400 in depal area labor reductions and $1,600 in avoided label changes).

We used the week of March 9, 1998 to validate the model. Afterwards, we ran it in parallel with the existing system for nine additional weeks, from April 6, 1998 through June 1, 1998. For each week, we compared our system's schedule with the actual schedule. Based on the 10 test weeks, we determined that on average our system saved four label changes per week and practically eliminated depal, for total cost savings averaging about $3,000 per week and annual savings of $169,230. Coors validated this annual savings figure when Dennis Ott submitted our project to the employee incentive program.

Conclusions

Our work has implications beyond aluminum can manufacturing. Consumer products ranging from dog food to breakfast cereals and from milk to engine oil are brought to fill sites where high speed automated equipment conveys containers into fill positions. Sometimes the only difference between containers for different products is the label on the container, so our mathematical model and the general approach have the potential of benefiting companies in many different industries.

APPENDIX

We present the mathematical programming formulation of the VMC problem. First we introduce the following notation:

Index Sets

$\mathcal{P}$—Beer labels = {*Coors Extra Gold*, *Coors Light*, *Coors Original*, *Keystone Light*, *Keystone Ale*, *Keystone Ice*, *Keystone Premium*}

$\mathcal{R}$—Can lines = {1,2,3,4,5,6}

$\mathcal{T}$—Weekly shifts = {*start*, *Mon-morning*, *Mon-swing*, *Mon-night*, *Tue-morning*, *Tue-swing*, *Tue-night*, *Wed-morning*, *Wed-swing*, *Wed-night*, *Thu-morning*, *Thu-swing*, *Thu-night*, *Fri-morning*, *Fri-swing*, *Fri-night*, *Sat-morning*, *Sat-swing*, *Sat-night*, *Sun-morning*, *Sun-swing*, *Sun-night*, *end*}

Data

$beg19_p$—beginning 19-layer inventory of label p (in 1,000s cans)

$begcell_p$—beginning cell-bin inventory of label p (in 1,000s cans)

$beg17_p$—beginning 17-layer inventory of label p (in 1,000s cans)

cap_{rt}—number of hours of line r available during shift t

$ccost_{pr}$—cost (in dollars) incurred when line r is switched to label p

cellcap—number of cans (in 1000s) that can be held in cell-bin inventory at any one time

$ctime_{pr}$—fixed consumption of line r capacity when switched over to product p

$dcost_p$—depalletizer cost in dollars per 1,000 cans for label p

$demand_{pt}$—demand in 1,000s of cans from cell bins for the Golden brewery for label p in shift t

$demand17_{pt}$—demand in 1,000s of cans from 17-layer pallets of label p in shift t

depalcap—the maximum number of cans that can be depalletized during a shift

$end19_p$—target ending 19-layer inventory of label p (in 1,000s of cans)

$endcell_p$—target ending cell-bin inventory of label p (in 1,000s of cans)

$finalinv_p$—target ending 17-layer inventory of label p (in 1,000s of cans)

$invflex_p$—percentage product p is allowed to vary from 17-layer inventory target

$prod_{prt}$—upper bound on label p production on line r for shift t

$pcost_{pr}$—cost to produce 1,000 cans of label p on line r

$ptime_{pr}$—consumption of line r capacity in hours to produce 1,000 cans of label p

Decision Variables

$cellbin_{pt}$—number of cans (in 1,000s) of label p put into cell-bin inventory during shift t

$cellinv_{pt}$—number of cans (in 1,000s) of label p in cell bins for shift t

$decell_{pt}$—number of cans (in 1,000s) of product p consumed from cell bins during period t

$depal_{pt}$—number of cans (in 1,000s) of label p depaled from 17-layer inventory during shift t

$delayer17_{pt}$—number of cans (in 1,000s) of label p used from 17-layer pallets during shift t

$inv19_{pt}$—number of cans (in 1,000s) of label p on 19-layer pallets for at the beginning of shift t

$layer19_{pt}$—number of cans (in 1,000s) of label p put into 19-layer inventory during shift t

$inv17_{pt}$—number of cans (in 1,000s) of label p on 17-layer pallets for at the beginning of shift t

$layer17_{pt}$—number of cans (in 1,000s) of label p put into 17-layer inventory during shift t

$prod_{prt}$—production of label p on line r during shift t

$smdepal_{pt}$—number of cans (in 1,000s) of label p depalletized from 19-layer inventory during shift t

$y_{prt} = 1$ if $prod_{prt} > 0$; 0 otherwise

$z_{prt} = 1$ if line r was not producing label p during shift $t - 1$ and started producing label p during shift t, and 0 otherwise

Mathematically, the VMC production scheduling problem can be represented as follows:

$$\text{Min} \sum_{p\in\mathcal{P}}\sum_{r\in\mathcal{R}}\sum_{t\in\mathcal{T}}(pcost_{pr}prod_{prt} + ccost_{pr}Z_{prt}) + \sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}} dcost_p(depal_{pt} + smdepal_{pt}) \quad (1)$$

subject to:

$$inv19_{CoorsLight,t-1} + layer19_{CoorsLight,t} - smdepal_{CoorsLight,t} - inv19_{CoorsLight,t} = 0 \quad \forall\ (t = Mon\text{-}morning\ldots Sun\text{-}night), \quad (2)$$

$$cellinv_{p,t-1} + cellbin_{p,t} + depal_{p,t} - decell_{p,t} - cellinv_{p,t} = 0\ \forall\ (p \in \mathcal{P}, t = Mon\text{-}morning\ldots Sun\text{-}night), \quad (3)$$

$$inv17_{p,t-1} + layer17_{p,t} - depal_{p,t} - delayer17_{p,t} - inv17_{p,t} = 0\ \forall\ (p \in \mathcal{P}, t = Mon\text{-}morning\ldots Sun\text{-}night), \quad (4)$$

$$\frac{finalinv_p}{invflex_p} \leq inv17_{p,end} \leq finalinv_p \times invflex_p\ \forall\ (p \in \mathcal{P}), \quad (5)$$

$$\sum_{p\in\mathcal{P}} inv17_{p,end} \geqslant \sum_{p\in\mathcal{P}} finali_p, \quad (6)$$

$$\sum_{r\in\mathcal{R}} prod_{prt} - cellbin_{pt} - layer17_{pt} - layer19_{pt} = 0\ \forall\ (p \in \mathcal{P}, t \in \mathcal{T}), \quad (7)$$

$$prod_{prt} \leq \overline{prod}_{prt}y_{prt}\ \forall\ (p \in \mathcal{P}, r \in \mathcal{R}, t \in \mathcal{T}), \quad (8)$$

$$z_{prt} \geqslant y_{prt} - y_{pr(t-1)}\ \forall\ (p \in \mathcal{P}, r \in \mathcal{R}, t = Mon\text{-}morning,\ldots Sun\text{-}night), \quad (9)$$

$$\sum_{p\in\mathcal{P}} y_{prt} \leq 1\ \forall\ (r \in \mathcal{R}, t \in \mathcal{T}), \quad (10)$$

$$\sum_{p\in\mathcal{P}} ptime_{pr}prod_{prt} + \sum_{p\in\mathcal{P}} ctime_{rt}z_{prt} \leq cap_{rt}\ \forall\ (r \in \mathcal{R}, t \in \mathcal{T}), \quad (11)$$

$$\sum_{p\in\mathcal{P}} cellinv_{pt} + decell_{pt} \leq cellcap\ \forall\ (t \in \mathcal{T}), \quad (12)$$

$$\sum_{p\in\mathcal{P}} (depal_{pt} + smdepal_{pt}) \leq depalcap\ \forall\ (t \in \mathcal{T}), \quad (13)$$

$$decell_{pt} \geqslant demand_{pt}\ \forall\ (p \in \mathcal{P}, t \in \mathcal{T}), \quad (14)$$

$$delayer17_{pt} \geqslant demand17_{pt}\ \forall\ (p \in \mathcal{P}, t \in \mathcal{T}), \quad (15)$$

$$inv19_{p,end} - end19_p = smdemand\ \forall\ (p \in \mathcal{P}), \quad (16)$$

$$y_{Coors\ Light,4,t} + y_{Keystone\ Light,4,t} - y_{Coors\ Light,3,t} - y_{Keystone\ Light,3,t} = 0\ \forall\ (t \in \mathcal{T}), \quad (17)$$

$$y_{prt} \in \{0,1\}\ \forall\ (p \in \mathcal{P}, r \in \mathcal{R}, t \in \mathcal{T}), \quad (18)$$

$$inv19_{p,start} = beg19_p, \quad (19)$$

$$inv17_{p,start} = beg17_p, \quad (20)$$

$$cellinv_{p,start} = begcell_p. \quad (21)$$

All other variables are positive.

The objective function (1) minimizes the total weekly cost of changing labels and depalletizing cans. Equations (2), (3) and (4) are inventory flow constraints for 19-layer, 17-layer, and cell-bin inventory. Equation (5) ensures that the actual ending inventory level for each label is within a specified tolerance of the target level. Equation (6) makes certain that the ending 17-layer inventory for all labels combined is at least as high as the cumulative target inventory level. Equation (7) states that all can production must go into cell-bins, 17-layer inventory, or 19-layer inventory. Equation (8) links variables *prod* and *y*, and Equation (9) ensures that the setup variable z is 1 whenever a label change occurs. Equation (10) limits the production during each shift on each line to one label. Equations (11), (12) and (13) are capacity constraints for the can lines, cell-bins, and the depal area. Equations (14) and (15) make sure that all brewery demand is met, and Equation (16) makes sure that the requirements from Shenandoah and Memphis facilities are met with product on 19-layer pallets. Equation (17) ensures that lines 3 and 4 both produce either a light-label or a full-bodied label, but not a combination. Equation (19) restricts the variable y to be binary. Although the variable z is binary also, it is defined in equation (9) in such a way that there is no need to constrain it to be binary. Equations (19), (20), and (21) are initial inventory conditions.

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Susan Schulze, Manager of Logistics, Coors Container Operations, Coors Brewing Company, Golden, Colorado 80401-1295, writes: "This letter is to verify the business case that justified implementation of the Mixed Integer Program developed and formulated by Dennis Ott and Elena Katok. The annual savings computed and verified by our Finance Department was $169,230."