Managing Inventory in the Information Age: a system with product life-cycle uncertainty and stochastic time-varying demand

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Abstract

We analyze a problem commonly encountered in technology and information industries where upgrading causes short and uncertain product life spans and random and non-stationary demands during different stages of the product life cycle. One example is the publishing industry, especially the publishing of technical reference materials. We formulate the problem as a dynamic programming (DP) model and propose several simple, yet effective, heuristic inventory control policies. We also derive the upper and lower bounds of the optimal cost function. We compare these heuristic methods using two sets of systematically generated test problems, and a third problem set based on the real data from Jeppesen Sanderson, Inc., a major aviation information provider. Numerical results show that all the heuristics are effective and locate near optimal solutions, with the best of those generating the solutions within 0.15% of optima, on average. Our heuristics are also robust against different parameter settings and require only a small fraction of computational efforts for the optimal solutions. We incorporated many of the ideas in this paper into a decision support system for inventory management that we implemented at Jeppesen and generated actual annual cost reductions of over $800,000.

1 Introduction

The ever increasing pace of technological innovation has added a new level of complexity to the task of managing inventory. The typical inventory management problem involves balancing the cost of holding inventory with such considerations as customer service level and ordering cost. Standard stochastic inventory models usually assume the planning horizon is either fixed and finite or infinite, and the uncertainty is caused by that of customer demand or leadtime.

But when the product itself is frequently upgraded, as is the case, for example, with computer software, the uncertainty about the product's life-cycle length has an impact on inventory decisions. Consider, for example, the problem faced by publishers of reference books about the Visual Basic programming language. Between 1996 and and 1998 Microsoft released three generations of the language in rapid succession. While Visual Basic 4 (VB4) barely lasted one year, the current version of the language, Visual Basic 6 (VB6), has not changed in over three years. When VB6 came out, reference books written about VB5 quickly lost a large portion of their value–an example of obsolescence due to upgrading.

Our work was inspired by a real problem faced by a company called Jeppesen Sanderson, Inc. (Jeppesen). A Boeing subsidiary since April 2000, Jeppesen is the largest provider of aviation information products in the world. Jeppesen manufactures flight simulators, aircraft maintenance manuals, training videos, and flight manuals that are used by over 80% of pilots worldwide. These manuals are composed of aviation charts that cover the entire world (see Figure 1), and Jeppesen manufactures, distributes, and maintains inventory of over 160,000 individual charts used in over 2,000 different flight manuals. Virtually all of the world's airlines (United, American, Delta, British Airways, Lufthansa, and many others) depend on the accuracy and the timely delivery of Jeppesen charts for the safety of millions of passengers who fly every day.

Jeppesen flight manuals contain different types of aviation charts of various sizes and colors. Aviation charts are maps that include information specific to pilots, such as detailed pictures of runways, approach and landing routes, radio frequencies, and global positioning system (GPS) coordinates. The nature of much of this aviation information is such that it changes, or in Jeppesen's terminology, it *revises*, quite often, and these revisions are generally unpredictable. For example a chart for the Dallas International Airport revised for seventeen consecutive periods one year, while charts for similar large airports sometimes (admittedly, not very often) remain unchanged for as long as a year. To keep its manuals current, Jeppesen re-issues charts immediately following their revision, and ships new versions of these charts to all its customers who subscribe to manuals that contain these charts. Jeppesen sends out between 5 million and 35 million charts to over 200,000 customers every week.

Demand for charts depends on the demand for manuals that contain them. Every day Jeppesen ships out over 900 new flight manuals, most assembled to order from individual charts. When an order comes in and one or more of the charts needed to assemble the manual is not available, these charts are immediately printed in-house or ordered from an outside vendor. There is typically a high fixed cost associated with such orders, the so-called *plating cost*, or the cost to assemble a plate for offset printing and setup the printing press for a new run. Plating cost includes the cost of labor for mounting negatives onto a plate and setting up a plate on a printing press, the cost of a plate itself, the cost of film for negatives, and the cost of scrap for fine-tuning a press run. So in summary, optimal stock levels for Jeppesen charts depend on:

Figure 1: The majority of Jeppesen charts are $5\frac{1}{2} \times 8\frac{1}{2}$ black and white sheets, such as the one on the right in the figure. These charts are printed in-house and are relatively inexpensive. Some charts, however, are large and multi colored, and are often outsourced. They are relatively more expensive to produce.

1. The demand for new manuals.
2. The specific chart's revision history.
3. The fixed plating cost and the variable printing cost that will have to be incurred if the chart is re-ordered.

Additional issues that are not pertinent to Jeppesen but that we expect to be pertinent in general include inventory holding cost and customer service level. The models we develop in this paper are applicable to inventory management situations likely to occur in the information and technology industries, where managers often face random planning horizons, stochastic and non-stationary demands, fixed ordering costs, and linear production and inventory holding costs. In the remainder of this section we review the relevant literature, and summarize our contributions and major results. In Section 2 we define the problem, and present the dynamic programming formulation. In Section 3 we describe three computationally efficient heuristic solution methods. We discuss bounds in Section 4, present computational results in Section 5, and conclude the paper in Section 6.

1.1 Background Literature

When Jeppesen issues a chart, it is not known in advance what information will change and when, so a chart's lifetime is usually not known in advance. This situation resembles that of *obsolete* and *perishable* inventory. The first to study the obsolete inventory problem were Barankin and Denny (1965). Pierskalla (1969) shows that obsolescence has a significant impact on the optimal policy. Nahmias (1977) derives an analytical expression for expected outdating, and two approximations, for problems where both demands and lifetimes are stochastic and the ordering cost is linear (this is in contrast to the Jeppesen problem, where the ordering cost is fixed). Rosenfeld (1989) further examines the problem where inventory has salvage value. Finkin (1989) suggest ways to manage obsolete inventory in practice.

The notion of *perishability* is slightly different from that of obsolescence and applies to products that deteriorate over time. This time may or may not be known in advance. Perishable inventory problems tent to occur in the food and the health-care industries. An example that is sited often in the literature is a blood bank's inventory problem (see Elston (1970), Jennings (1973), and Prastacos (1980)), where perishability has a major impact on the optimal inventory policies. Nahmias and Periscalla (1973) studied the perishable inventory model where product life-span is known to be two periods, and Fries (1975) and Nahmias (1975) extended the model to the case of three or more periods. These models, just as Nahmias (1977), assume linear ordering cost. Nahmias (1978) considers fixed ordering cost in conjunction with deterministic life-span. We are unaware of any analytical models or applications combining uncertain life spans, non-statinary stochastic demands and fixed ordering costs–the situation Jeppesen faces.

1.2 Contributions and Major Results

A setting where a product becomes obsolete, not as a result of natural degradation (as is the case in the food industry) but as a result of some external event, such as an upgrade, is one that is becoming increasingly common in industries where technology plays an important role. Inventory management in situations where frequent upgrades cause uncertain product life spans and non-stationary demand has not been extensively analyzed in the operations management literature.

In this paper we consider a variation of an inventory problem with obsolescence, where products are subject to upgrades, and hence have uncertain life spans and non-stationary demand. Our work was inspired by a problem faced by Jeppesen. Since aviation information is constantly evolving, Jeppesen is faced with the problem of deciding on stock levels for aviation charts that may revise in the future.

We formulate the obsolete inventory problem with non-stationary demand as a dynamic programming model, and propose three simple, yet effective, heuristic methods. The first heuristic, the FE policy, is based on the relaxation of the service constraint and reduces the problem to a single-period, newsvendor-type problem. The second heuristic, the OUTP policy, uses a modified Wagner-Whitin algorithm to solve the deterministic version of the problem and then adds safety stock to the average demand to determine the order-up-to quantity. The third heuristic, the hybrid policy, is a combination of the first two heuristics. We also derive the upper and lower bounds of the optimal cost function.

We evaluate the performance of the heuristics by comparing their solutions with the optimal solutions for a large number of test problems under various system configurations. Numerical results show that all the heuristics locate near optimal solutions, with the best heuristic, the hybrid heuristic, generating the solutions within 0.15% of optima, on average. Our heuristics are also robust against different system configurations and require only a small fraction of computational efforts for the optimal solutions. In addition, we report the test results of the heuristic methods using slightly modified real data sets from Jeppesen. The best heuristic finds solutions that are only 0.17% worse than the optima, on average.

We incorporated many of the ideas in this paper into a decision support system for inventory management that we implemented at Jeppesen and that generated actual annual cost reductions of over \$800,000 (see Katok, Lathrop, Tarantino, and Xu (2001)). For more details on how Jeppesen successfully uses Operations Research see Katok, Tarantino, and Tiedeman (2001).

2 Problem Definition and Formulation

2.1 Problem Definition

We model the problem as a periodic-review inventory system with a random duration R, where random variable R is understood as the lifetime of the product. Random lifetime R can assume values kt, $t = 1, 2, \ldots, b$, where k is a given integer representing the length of the upgrading cycle, t denotes the cycle index with an upper bound b. In other words, the end of the product life can only occur at the end of the kt-th period, $t = 1, \ldots, b$. In the case of Jeppesen, 99% of the time the charts we consider are revised on an 8 week cycle basis, each 8-week cycle consisting of four two-week periods, and last no more than 6 cycles, so $k = 8$, $b = 6$, and a chart can be revised $8,\ 16,\ \ldots, 48$ weeks after issuance. If the product lifetime does not have the cyclic structure, then we set $k = 1$ and the product can end its life at the end of any period.

We denote demand in period n by D_n, where D_n, $n = 1, 2, \ldots, kb$, are independent random variables with density and cumulative distribution functions $f_n(x)$ and $F_n(x)$, respectively. Note that by assuming the demand in a period follows its individual distribution, we reflect the non-stationary demand pattern during the various stages of the product life cycle.

Let A be the fixed cost incurred whenever an order is placed. Let c be the unit variable production cost. Let $c(y)$ be the procurement cost if quantity y is ordered. Then,

$$c(y) = \begin{cases} A + cy & y > 0, \\ 0 & y = 0. \end{cases} \tag{2.1}$$

We assume that orders can be filled instantaneously. We also assume that if shortage occurs at the end of product life R, one must place an order to satisfy the shortage. The unit inventory holding cost is h per period.

We do not assume a variable shortage cost. However, we assume that the system must maintain a required service level, α, defined as the proportion of periods in which all demand is met (also known as the Type 1 Service). Finding the optimal policy subject to service level constraints for periodic review system with positive setup cost is considered computationally complex, even for the problem with stationary demands and an infinite product lifetime. Different approaches for finding approximate (s, S) and (R, Q) policies satisfying the service constraint are available in the literature (Schneider (1978), Tijms and Groenevelt (1984), Yano (1985)). As pointed out by Schneider (1978), the computational efforts for determining the reorder point satisfying the service constraint, for the system with stationary demand and infinite product lifetime, is so prohibitively expensive that one often prefers to use (2.2) (given below) instead of more accurate approximations. For simplicity in this paper we determine the reorder point of period n by finding the smallest integer s_n^α satisfying

$$F_n(s_n^\alpha) = P(D_n \le s_n^\alpha) \ge \alpha. \tag{2.2}$$

We assume that one must reorder if the stock on hand at the beginning of a period is less than s_n^α. We note that this is a conservative estimate of the reorder point that ensures the desired service level α.

2.2 Dynamic Programming Formulation

We formulate the problem as a finite-period dynamic programming (DP) problem. The stage of the DP model, denoted by n, is defined as the "age" of the product lifetime, that is, the number of periods since the product lifetime began. We say that the process is in period n, $n = kt + i$, $t = 0, 1, \ldots, b-1$, and $i = 1, \ldots, k$, if the age of the product lifetime is n. The state of the system in a period, denoted by x, is the starting inventory in that period.

Our objective is to find the optimal ordering policy that minimizes the expected total cost incurred during the lifetime of the product. Let $V_n(x)$ be the minimum expected total costs of the system from period (age) n to the end of its life, given that the initial inventory at the beginning of period n is x. Thus $V_1(0)$ represents the minimum expected total cost incurred during the lifetime of the product.

We call a period *interior* if the product cannot become obsolete at the end of that period. An interior period has a time index $n = kt + i, i = 1, 2, \ldots, k-1, t = 0, 1, 2, \ldots, b-1$. We call a period *boundary* if the product may end its life at the end of that period. In other words, a boundary period is the last period in a cycle and hence has a time index $k(t+1), t = 0, 1, \ldots, b-1$. In the case of Jeppesen, boundary periods are 8, 16, 24, and so on, and interior periods are 1,2,...,7, 9,...,15, 17,...,23, and so on.

Denote the remaining life of the product, with age kt, by

$$R_{kt} = R - kt | R > kt, \qquad t = 0, 1, \ldots, b-1.$$

Then the conditional probability that the product lifetime, with age kt, has a remaining life of t' cycles, is given by

$$P(R_{kt} = kt') = P(R = k(t+t') | R > kt), \qquad t = 0, 1, \ldots, b-1, t' = 1, \ldots, b-t. \tag{2.3}$$

By (2.3), the product with age $n = kt + i$ has the remaining life:

$$\text{The remaining life of the product with age } kt + i = R_{kt} - i. \tag{2.4}$$

It further implies,

$$\text{the ending period of the product life, given age } kt + i = kt + R_{kt}. \tag{2.5}$$

For example, suppose cycle length $k = 8$. If age $n = kt + i = 8 \cdot 1 + 2 = 10$ and $R_8 = 16$, then by (2.4) the remaining life of the product with age 10 is $R_8 - 2 = 16 - 2 = 14$ periods. By (2.5), the product ends its life at the end of period $8 + 16 = 24$.

Next, we derive the optimality equations for interior and boundary periods. The DP recursion corresponding to an interior period is

$$V_{kt+i}(x) = \min_{y \ge x, y \ge s_{kt+i}^\alpha} \{c(y-x) + hE[y - D_{kt+i}]^+ + EV_{kt+i+1}(y - D_{kt+i})\}, \tag{2.6}$$
$$i = 1, 2, \ldots, k-1, \; t = 0, 1, \ldots, b-1.$$

where $[a]^+ = \max\{a, 0\}$. Here, $c(y-x) + hE[y - D_{kt+i}]^+$ are the procurement and holding costs incurred in the interior period $kt + i$ if we order up to quantity y, and $EV_{kt+i+1}(y - D_{kt+i})$ is the expected costs from period $kt + i + 1$ onward, assuming we act optimally in the future. The problem in period $kt + i$, then, is to choose y that minimizes the sum of the present and future expected costs. The constraints on the order quantity y in (2.6) mean that we cannot scrap stock on-hand, and when it falls below the reorder point, we must place an order.

The DP recursion corresponding to a boundary period (the last period of a cycle) is

$$V_{k(t+1)}(x) = \min_{y \geq x, y \geq s^{\alpha}_{k(t+1)}} \{c(y-x) + hE[y - D_{k(t+1)}]^+ + P(R_{kt} > k)EV_{k(t+1)+1}(y - D_{k(t+1)}) \\ +P(R_{kt} = k)(AP(D_{k(t+1)} > y) + cE[D_{k(t+1)} - y]^+\}, \quad t = 0, 1, \ldots, b-1, \quad (2.7)$$

where the distribution function of R_{kt} is defined by (2.3). The above optimality equation is understood as follows: As in (2.6), $c(y-x) + hE[y - D_{k(t+1)}]^+$ is the procurement plus holding costs incurred in the boundary period $k(t+1)$, given that stock on hand before ordering is x and after ordering is y. Future costs depend on whether the product becomes obsolete at the end of the period. With probability $P(R_{kt} > k)$, the product with age kt survives at least another cycle, and the process evolves to the next interior period $k(t+1)+1$ with the minimum expected cost $EV_{k(t+1)+1}(y - D_{k(t+1)})$. With probability $P(R_{kt} = k)$, the product ends its life at the end of the $(t+1)$st cycle, in which case the system incurs reorder cost A with probability $P(D_{k(t+1)} > y)$ and production cost $cE[D_{k(t+1)} - y]^+$. Note that when $t = b-1$ in (2.7), $V_{kb}(x)$ gives the boundary condition of the DP recursion, with $P(R_{k(b-1)} = k) = 1$.

Theoretically, one can solve (2.6) and (2.7) using a standard DP computational technique such as value iteration. However, such an approach becomes impractical for large-scale problems. For example, a typical Jeppesen chart lasts up to 5 or 6 cycles, with each cycle consisting of 8-weeks, and each period consisting of two weeks (because charts are always handled on either odd or even weeks) meaning that the planning horizon is 20 or 24 periods long. The bi-weekly demand D_i can usually be approximated by a normal random variable $D_i \sim N(\mu_i, \sigma_i^2)$, where mean demand μ_i can take a value between 1 and 1000, resulting in a DP problem with an extremely large state space. In addition, because of the positive setup cost, $V_n(x)$ is not convex and may have multiple local minima. To overcome computational complexity and facilitate implementation, we develop several simple heuristic methods in the following section.

3 Heuristic Ordering Policies

In this section we propose three heuristic ordering policies. We focus on the class of (s_n, S_n) policies, and propose several methods to approximate the optimal order-up-to-levels S_n^*, with reorder points s_n^* approximated by (2.2).

The three heuristic ordering policies developed in this section can be briefly described as follows.

1. The Front-End (FE) Policy: This heuristic policy determines the order-up-to quantity based on the relaxation that reorders are only required in the *front* (the current period) and at the *end* of the product life. This heuristic method is the simplest among the three, as it only requires a solution to a single-period, newsvendor-type problem.

2. The Order-Up-To-Period (OUTP) Policy: This policy first uses a modified Wagner-Whitin algorithm to determine the optimal order-up-to-period, based on the simplifying assumption that demand in each period is deterministic and equals its mean, then adds safety stock to the mean demand from the current period to the order-up-to-period.

3. The Hybrid Policy: This policy can be considered as a combination of the above two heuristics. It first obtains, via the modified Wagner-Whitin algorithm, the optimal order-up-to-period using mean demands, then determines the order-up-to quantity using the Front (the current period)-End (the order-up-to-period) heuristic.

3.1 The Front-End Ordering Policy

We first describe an ordering policy whose order-up-to quantity in a given period, say n, $n = 1, \dots, kb$, is based on the relaxation that reorders are only required in the *front* (the beginning of the current period n), if the stock on hand is less than the reorder point, and at the *end* of the product life, if there is a stock out. We expect the heuristic method performs well when the product's life is short and inventory holding cost h is small.

We need the following notation. Let $D^{(m,n)}$ denote the demand from period m to period n. Then the remaining demand from period $kt+i$ until the end of the product life, $kt + R_{kt}$, can be expressed by

$$D^{(kt+i,kt+R_{kt})} := \sum_{n=kt+i}^{kt+R_{kt}} D_n, \quad t = 0, 1, \dots, b-1, \ i = 1 \dots, k.$$

Let $f^{(kt+i,kt+R_{kt})}$ and $F^{(kt+i,kt+R_{kt})}$ be the density and distribution functions of $D^{(kt+i,kt+R_{kt})}$, respectively. Then, for $t = 0, 1, \dots, b-1, \ i = 1 \dots, k$,

$$F^{(kt+i,kt+R_{kt})}(y) = \sum_{t'=1}^{b-t} F^{(kt+i,k(t+t'))}(y) P(R_{kt} = kt'),$$

and

$$f^{(kt+i,kt+R_{kt})}(y) = \sum_{t'=1}^{b-t} f^{(kt+i,k(t+t'))}(y) P(R_{kt} = kt').$$

Suppose in period $kt+i$, we follow the $(s^{\alpha}_{kt+i}, \bar{S}^1_{kt+i})$ policy where s^{α}_{kt+i} is the reorder point and $\bar{S}^1_{kt+i}$ is the order-up-to quantity. Quantity $\bar{S}^1_{kt+i}$ is determined by assuming that in the subsequent periods of $kt+i$ one defers orders, even if the stock on hand is less than the reorder point, until the end of the product life, at which time one must place an order if there is a stock out. As such, $\bar{S}^1_{kt+i}$ is obtained by solving the following single-period model:

$$\bar{S}^1_{kt+i} = \text{argmin}_{y \geq s^{\alpha}_{kt+i}} \{L^1_{kt+i}(y)\}, \qquad t = 0, 1, ...b-1, \ i = 1, 2, \dots, k,$$

where

$$\begin{aligned} L^1_{kt+i}(y) &= A + cy + hE \sum_{n=kt+i}^{kt+R_{kt}} [y - D^{(kt+i,n)}]^+ \\ &\quad + AP(D^{(kt+i,kt+R_{kt})} > y) + cE[D^{(kt+i,kt+R_{kt})} - y]^+. \end{aligned} \tag{3.8}$$

where $D^{(i,i)} = D_i$. Unfortunately, $L^1_{kt+i}(y)$ is generally not a convex function, even for some commonly used demand distributions such as normal and binomial. Indeed, $L^1_{kt+i}(y)$ often admits several local maximums and local minimums. To find the global minimum of $L^1_{kt+i}(y)$, we take the first derivative of $L^1_{kt+i}(y)$ and obtain

$$\begin{aligned} \frac{d}{dy}L^1_{kt+i}(y) &= c - Af^{(kt+i,kt+R_{kt})}(y) + hE\sum_{n=kt+i}^{kt+R_{kt}} F^{(kt+i,n)}(y) - c(1 - F^{(kt+i,kt+R_{kt})}(y)) \\ &= cF^{(kt+i,kt+R_{kt})}(y) - Af^{(kt+i,kt+R_{kt})}(y) + hE\sum_{n=kt+i}^{kt+R_{kt}} F^{(kt+i,n)}(y). \end{aligned} \tag{3.9}$$

Setting the above derivative to zero yields

$$\frac{f^{(kt+i,kt+R_{kt})}(y)}{F^{(kt+i,kt+R_{kt})}(y)} = \frac{cF^{(kt+i,kt+R_{kt})}(y) + hE\sum_{n=kt+i}^{kt+R_{kt}} F^{(kt+i,n)}(y)}{AF^{(kt+i,kt+R_{kt})}(y)}. \tag{3.10}$$

Then one must evaluate $L^1_{kt+i}(y)$ for all y that are the solutions of (3.10) and choose $\bar{S}^1_{kt+i} \geq s^{\alpha}_{kt+i}$ that minimizes $L^1_{kt+i}(y)$.

Remark 3.1 The *reversed hazard rate* of a random variable Y with density f and distribution F is defined as (see Shaked and Shanthikumar (1994))

$$\tilde{r}(y) = \frac{d}{dy}logF(y) = \frac{f(y)}{F(y)}.$$

Hence, the left-hand side of (3.10),

$$\tilde{r}_{kt+i}(y) = \frac{f^{(kt+i,kt+R_{kt})}(y)}{F^{(kt+i,kt+R_{kt})}(y)},$$

is the reversed hazard rate of $D^{(kt+i,kt+R_{kt})}$, the total demand from period $kt+i$ to the end of the product life. The intuitive interpretation of $\tilde{r}_{kt+i}(y)$ is as follows. Given that the total remaining demand is less than y, then for small $\epsilon > 0$, $\epsilon \cdot \tilde{r}_{kt+i}(y)$ is the approximate probability that the remaining demand is at least $y - \epsilon$. On the other hand, the numerator of the right-hand side (RHS) of (3.10) is understood as the expected unit overage cost, including the expected unit production cost $cF^{(kt+i,kt+R_{kt})}(y)$, and the expected unit holding cost from now to the end of the product life, $hE\sum_{n=kt+i}^{kt+R_{kt}} F^{(kt+i,n)}(y)$. The denominator of the RHS of (3.10) is the expected underage cost because an additional production run must be made to make up the shortage. Thus, (3.10) states that the reverse hazard rate at $\bar{S}^1_{kt+i}$, should balance the ratio between the overage cost and underage cost. This result resembles the solution of the newsvendor model.

In summary, we have the following heuristic ordering policy.

The Front-End (FE) Ordering Policy. Let x be the starting inventory in period $kt+i$, $t = 0, \ldots, b-1$, $i = 1, 2, \ldots, k$. Then if $x \geq s^{\alpha}_{kt+i}$, do not order; otherwise order up to $\bar{S}^1_{kt+i}$, where

$$\bar{S}^1_{kt+i} = \text{argmin}_{y \geq s^{\alpha}_{kt+i}}\{L^1_{kt+i}(y),\ y \text{ is a solution of (3.10)}\}, \qquad t = 0, 1, ...b-1,\ i = 1, 2, \ldots, k, \tag{3.11}$$

and $L^1_{kt+i}(y)$ is defined in (3.8).

Example 3.2 Let $k = 8$, $c = 1$, $h = 0$ and $A = 50$. Note that with $h = 0$, (3.10) reduces to

$$\frac{f^{(kt+i,kt+R_{kt})}(y)}{F^{(kt+i,kt+R_{kt})}(y)} = \frac{c}{A}. \tag{3.12}$$

Let R follow a uniform distribution with $P(R = 8t) = 1/3,\ t = 1, 2, 3$. That is, the maximal life of the product is three cycles, with each cycle lasting 8 periods. Also let D_n be iid normal random variables with mean 5 and standard deviation 1. Suppose the required service level is $\alpha = 0.9$. Let us compute $s_n^{0.9}$, $\bar{S}_1^1$, $\bar{S}_9^1$ and $\bar{S}_{17}^1$ under the FE policy. Since $D_n \sim N(5, 1)$ for all n, we need to select the smallest integer $s_n^{0.9}$ such that be $s_n^{0.9} - 5 \geq 1.28$, or, $s_n^{0.9} = 7$, $n = 1, 2, \ldots, kb$. To compute $\bar{S}_1^1$, we condition on the product life, which is equally likely to be 8, 16 and 24 periods, and get from (3.12)

$$\frac{f^{(1,R)}(y)}{F^{(1,R)}(y)} = \frac{\frac{1}{3}(f^{(1,8)}(y) + f^{(1,16)}(y) + f^{(1,24)}(y))}{\frac{1}{3}(F^{(1,8)}(y) + F^{(1,16)}(y) + F^{(1,24)}(y))} = 0.02.$$

The above equation admits the solutions 46, 73, 86, 115 and 124, of those 73 and 115 correspond to the local maximums of $L_1^1(y)$ and 46, 86 and 124 correspond to the local minimums of $L_1^1(y)$. Thus

$$\bar{S}_1^1 = \text{argmin}_{y\geq 7}\{L_1^1(46), L_1^1(86), L_1^1(124)\} = 86.$$

Now consider $\bar{S}_9^1$. Conditioning on R_8, which is equally likely to be 8 and 16, we get from (3.12),

$$\frac{f^{(9,8+R_8)}(y)}{F^{(9,8+R_8)}(y)} = \frac{\frac{1}{2}(f^{(9,16)}(y) + f^{(9,24)}(y))}{\frac{1}{2}(F^{(9,16)}(y) + F^{(9,24)}(y))} = 0.02.$$

The above equation admits the solutions 35, 45, 73, and 86, where 35 and 73 correspond to the local maximums of $L_9^1(y)$ and 45 and 86 correspond to the local minimums of $L_9^1(y)$. Thus

$$\bar{S}_9^1 = \text{argmin}_{y\geq 7}\{L_9^1(45), L_9^1(86)\} = 86.$$

Note that the mean remaining demand is $\frac{1}{2}40 + \frac{1}{2}80 = 60$. The probability that $\bar{S}_9^1 = 86$ is sufficient to meet remaining demand is

$$\frac{1}{2}P(D^{(9,16)} \leq 86) + \frac{1}{2}P(D^{(9,24)} \leq 86) \approx \frac{1}{2}\cdot 1 + \frac{1}{2}(0.933) = 0.967.$$

Finally,

$$\frac{f^{(17,16+R_{16})}(y)}{F^{(17,16+R_{16})}(y)} = \frac{f^{(17,24)}(y)}{F^{(17,24)}(y)} = 0.02$$

has a unique minimum at $y = 46$, so $\bar{S}_{17}^1 = 46$. Since the demand in the last cycle is $D^{(17,24)} \sim N(40, 8)$, the probability that $\bar{S}_{17}^1 = 46$ is sufficient to meet $D^{(17,24)}$ is

$$P(D^{(17,24)} \leq 46) \approx 0.983.$$

Next assume a positive holding cost $h = 0.2$, with all other parameters the same as before. For simplicity we only compute $\bar{S}_9^1$. Equation (3.10) becomes

$$\frac{\frac{1}{2}(f^{(9,16)}(y) + f^{(9,24)}(y))}{\frac{1}{2}(F^{(9,16)}(y) + F^{(9,24)}(y))} = \frac{[\frac{1}{2}F^{(9,16)}(y) + \frac{1}{2}F^{(9,24)}(y)] + 0.2[\frac{1}{2}\sum_{n=9}^{16} F^{(9,n)}(y) + \frac{1}{2}\sum_{n=9}^{24} F^{(9,n)}(y)]}{50[\frac{1}{2}F^{(9,16)}(y) + \frac{1}{2}F^{(9,24)}(y)]},$$

The above expression is solved at $\bar{S}_9^1 = 42$, which can satisfy demand for the current cycle with probability 0.76. In contrast, for $h = 0$, the FE heuristic orders $\bar{S}_9^1 = 86$ units, which can satisfy the maximal remaining demand with probability 0.967.

3.2 The Order-Up-To-Period (OUTP) Policy

This heuristic method consists of two steps. In the first step we determine the *order-up-to period* under the simplifying assumption that demand in period n is *deterministic* and equals its mean $\mu_n = E[D_n]$, $n = 1, 2, \ldots, kb$. In the second step we determine the order-up-to quantity which equals the mean demand until the next order plus safety stock. To determine safety stock, we take into account the relative magnitudes of A and $c+h$. The intuition here is that if A is low relative to $c+h$, we should accept a higher probability of stock out before the end of the order-up-to period than if A is high relative to $c+h$. The OUTP heuristic is suitable when demand follows a normal distribution.

If the average demand in period $n = kt + i$ is $\mu_n = E(D_n)$, $n = 1, 2, \ldots, kb$, (2.6) and (2.7) become

$$V_{kt+i}(x) = \min_{y \geq \mu_{kt+i}, y \geq x} \{c(y-x) + h(y - \mu_{kt+i}) + V_{kt+i+1}(y - \mu_{kt+i})\}, \quad i = 1, 2, \ldots, k-1, \ t = 0, 1, \ldots, b-1. \tag{3.13}$$

$$V_{k(t+1)}(x) = \min_{y \geq \mu_{k(t+1)}, y \geq x} \{c(y-x) + h(y - \mu_{k(t+1)}) + P(R_{kt} > k) V_{k(t+1)+1}(y - \mu_{k(t+1)})\}, \quad t = 0, 1, \ldots, b-1. \tag{3.14}$$

Observe that under this simplifying assumption our model becomes a variant of Wagner-Whitin dynamic lot sizing problem with a random lifetime that may end at the end of a boundary period $k(t+1)$, $t = 0, 1, \ldots, b-1$. It is well-known that the standard Wagner-Whitin algorithm has the properties that it places an order only when starting inventory is zero and, when ordering, its order size must cover demand for an integer number of periods. It turns out that our simplified problem has the properties that resemble that of the Wanger-Whitin dynamic lot sizing problem, as stated in the following proposition.

Proposition 3.3 Let demand in period n be deterministic $\mu_n = E(D_n)$, $n = 1, 2, \ldots, kb$.

1. One does not place an order in a period, given that the product has not become obsolete, if stock on hand is sufficient to meet demand in that period.

2. When ordering one brings stocks to such a level that they exactly satisfy demand for an integer number of periods.

Proof.

1. Suppose $\pi^D = \{S_n^D, n = 1, 2, \ldots, kb\}$ is an optimal ordering policy, where S_n^D is stock on hand after ordering in period n, provided that the product has not become obsolete by n. Suppose that initial stock x in period $n = kt + i$ is sufficient to meet the demand for the period, that is $x \geq \mu_n$, but policy π^D prescribes $S_n^D > x \geq \mu_n$. Consider another policy $\bar{\pi}^D = \{\bar{S}_n^D, n = 1, 2, \ldots, kb\}$ that is the same as π^D except that it would not place an order in period n, but would order the stock up to level $(S_n^D - x) + S_{n+1}^D$ in the next period $n+1 = kt + i + 1$, if the product were still alive by then. If the product became obsolete in period $n+1$ ($n = k(t+1)$ for some t), then policy $\bar{\pi}^D$ would save a

combined ordering, production and holding cost $A + (c+h)(S_n^D - x)$. If the product were not obsolete in period $n+1$ and policy π^D does not place an order in period $n+1$, then policy $\bar{\pi}^D$ would save a holding cost $h(S_n^D - x)$. If the product were not obsolete in period $n+1$ and policy π^D places an order in period $n+1$, then policy $\bar{\pi}^D$ would realize a saving of $A + h(S_n^D - x)$.

2. Let π^D be defined as in the above. Suppose that in period n policy π^D orders enough to satisfy demand before period $m > n$, but not enough to satisfy demand in period m; i.e.,

$$S_n^D = \sum_{j=n}^{m-1} \mu_j + \delta,$$

where $0 \leq \delta < \mu_m$. Note that by Proposition 3.3 (1), policy π^D would not place an order until period m. Consider another policy $\bar{\pi}^D$ that is the same as π^D, except that it would order up to quantity $S_n^D - \delta$ in periods n and up to quantity $S_m^D + \delta$ in period m, if the product were still alive by them. If the product lifetime ended between periods n and m, say in period ℓ, $n < \ell < m$, policy $\bar{\pi}^D$ would save production and holding costs $c\delta + h(\ell - n)\delta$. Otherwise in period m one must order. In this case policy $\bar{\pi}^D$ would save holding cost $h(m-n)\delta$. Hence $\delta = 0$ in an optimal policy. ■

The above results imply that if the demand in each period is deterministic, then an order is placed only when inventory on hand is zero and the order quantity must cover the demand for an integer number of periods. Thus, at the beginning of period $kt+i$, we seek to find the optimal *order-up-to period* l_{kt+i}, $l_{kt+i} \geq kt+i$, such that

$$S_{kt+i}^D = \sum_{n=kt+i}^{l_{kt+i}} \mu_n.$$

To determine l_{kt+i}, define function $L_{kt+i}^2(l)$ as the total ordering, production and holding costs from period $kt+i$ to period l or $kt+R_{kt}$ (recall this is the end of the product life), whichever occurs first. Then,

$$L_{kt+i}^2(l) = A + c\sum_{n=kt+i}^{l} \mu_n + hE[\sum_{n=kt+i}^{\min\{kt+R_{kt},l\}} (n-kt-i)\mu_n], \quad kt+i \leq l \leq kb. \tag{3.15}$$

We can obtain l_{kt+i} by solving the optimality equation:

$$V_{kt+i}(0) = \min_{kt+i \leq l \leq kb} \{L_{kt+i}^2(l) + P(kt+R_{kt} > l)V_{kt+i+l}(0)\}, \tag{3.16}$$

with the boundary condition:

$$V_{kb}(0) = L_{kb}^2(1) = A + c\mu_{kb}. \tag{3.17}$$

Expression (3.15) can be easily evaluated by conditioning on R_{kt} whose distribution function is given by (2.3). The cost function (3.16) can be calculated recursively starting from the boundary equation (3.17).

Remark 3.4 If the holding cost $h = 0$, then following the proof similar to that of Proposition 3.3, one can show that an order is placed only at the beginning of a *cycle*, i.e., in periods $n = kt+1,\ t = 0, 1, ...b-1$, and when inventory on hand is zero. In addition, the order quantity must cover demand for an integral number of *cycles*.

Next we compute the safety stock to be added to S^D_{kt+i}. Let demand in period n be normally distributed with mean μ_n and standard deviation σ_n. Thus, the total demand between periods $kt+i$ and l_{kt+i} is normally distributed with mean demand $\sum_{n=kt+i}^{l_{kt+i}} \mu_n$ and standard deviation $\sqrt{\sum_{n=kt+i}^{l_{kt+i}} \sigma_n^2}$. The order-up-to quantity is determined by adding safety stock to the mean demand:

$$\bar{S}^2_{kt+i} = S^D_{kt+i} + \text{safety stock} = \sum_{n=kt+i}^{l_{kt+i}} \mu_n + z_{1-c/A}\sqrt{\sum_{n=kt+i}^{l_{kt+i}} \sigma_n^2}, \tag{3.18}$$

where $z_{1-\frac{c+h}{A}}$ is $(1-\frac{c+h}{A}) \times 100$ percentile of the standard normal distribution, that is, for $Z \sim N(0,1)$, $z_{1-\frac{c+h}{A}}$ is chosen so that

$$P(Z \leq z_{1-\frac{c+h}{A}}) = 1 - \frac{c+h}{A}. \tag{3.19}$$

The result is that the value of the safety stock is determined by the ratio of overage cost $c+h$ and underage cost A.

The OUTP policy is summarized as follows.

The Order-Up-To-Period (OUTP) Policy:

Step 1 For each $kt+i$, $t = 0, 1, \ldots, b-1$, $i = 1, 2, \ldots, k$, determine l_{kt+i} from (3.15)-(3.17);

Step 2 If inventory on hand in period $kt+i$ is greater than or equal to s^{α}_{kt+i}, do not order; otherwise order up to $\bar{S}^2_{kt+i}$, where $\bar{S}^2_{kt+i}$ is given by (3.18) and $z_{1-\frac{c+h}{A}}$ satisfies (3.19).

Example 3.5 As in Example 3.2, let R be uniformly distributed with $P(R = 8t) = 1/3$, $t = 1, 2, 3$. Suppose demand is stationary, with period demand distributed as $D_i \sim N(5, 1)$. Let $A = 50$, $c = 1$ and $h = 0$.

Since $h = 0$, by Remark 3.4, we need to determine the order-up-to cycles l_{8t+1}, $t = 0, 1, 2$. Equation (3.15), now understood as the total expected cost from cycle t to cycle l , becomes

$$L^2_{kt+1}(l) = A + c\sum_{n=kt+1}^{kl} \mu_n, \qquad t = 0, 1, \ldots, l = 1, \ldots, b-t. \tag{3.20}$$

Also, (3.16) becomes

$$V_{kt+1}(0) = \min_{l \leq b-t+1} \{L^2_{kt+1}(l) + P(kt + R_{kt} > kl)V_{kl+1}(0)\}. \tag{3.21}$$

For simplicity denote $V_{kt+1} =: V_{kt+1}(0)$. First we compute the distribution of the product remaining life:

$$P(R > 8) = 2/3,\ P(R_8 > 8) = P(R > 16 | R > 8) = 1/2,\ P(R_{16} = 8) = 1.$$

Then, by (3.20) amd (3.21), we have

$$\begin{aligned}
V_{2\cdot 8+1} &= V_{17} = 50 + 40 = 90, \\
V_{8+1} &= V_9 = \min\{50 + 40 + \frac{1}{2}V_{17}, 50 + 80\} \\
&= \min\{90 + 45, 130\} = 130, \\
V_1 &= \min\{50 + 40 + \frac{2}{3}V_9, 50 + 80 + \frac{1}{3}V_{17}, 50 + 120\} \\
&= \min\{176.66, 160, 170\} = 160
\end{aligned}$$

Hence, the optimal order-up-to periods are

$$l_1 = 16,\ l_9 = 24,\ l_{17} = 24,$$

with mean demands during the order cycles being 80, 80, and 40, respectively.

Next we determine the order-up-to quantity $\bar{S}^2_{8t+1}$, using (3.18). With $A = 50$ and $c + h = 1$, $z_{1-c/A} = z_{0.98} = 2.06$. Using the OUTP heuristic method, the order-up-to quantity in period 1 is

$$\bar{S}^2_1 = \mu_i \cdot l_1 + z_{1-\frac{c+h}{A}} \cdot \sigma_i \sqrt{l_1} = 80 + 2.06(\sqrt{16}) \approx 88.$$

The probability that $\bar{S}^2_1 = 88$ is sufficient to cover demand for the next two cycles is

$$\frac{1}{2}P(D^{(1,8)} \leq 88) + \frac{1}{2}P(D^{(1,16)} \leq 88) = \frac{1}{2} + \frac{1}{2}0.98 = 0.99.$$

Similarly, we can compute $\bar{S}^2_9 = 88$ and $\bar{S}^2_{17} = 47$. Recall that under the FE policy, with the same parameters, $\bar{S}^1_1 = 86$, $\bar{S}^1_9 = 86$ and $\bar{S}^1_{17} = 46$, all are lower than their counterparts under the OUTP policy. Indeed, the OUTP policy tends to hold a large amount of safety stock, especially for high demand variability and high values of $1 - \frac{c+h}{A}$.

3.3 The Hybrid Policy

This policy is a combination of the OUTP policy and the FE policy. It attempts to take the advantages, while readdress the shortcomings, of those two policies.

The hybrid policy consists of two steps. The first step is the same as the first step in the OUTP policy, which determines the order-up-to period l_{kt+i}, $i = 1, \ldots, k$, $t = 0, \ldots, b-1$. The second step is similar to the FE policy, which determines the order-up-to quantity $\bar{S}^3_{kt+i}$, by minimizing the cost from period $kt + i$ to $\min\{kt + R_{kt}, l_{kt+i}\}$, under the relaxation that in the subsequent periods of $kt+i$ we are allowed to defer orders, even if stock on hand is below the reorder point s^{α}_{kt+i}, until period $\min\{kt + R_{kt}, l_{kt+i}\}$, at which time one must place an order to fill shortages, if any.

More specifically, quantity $\bar{S}^3_{kt+i}$ under the hybrid policy is determined by

$$\bar{S}^3_{kt+i} = \text{argmin}_{y \geq s^{\alpha}_{kt+i}} \{L^3_{kt+i}(y)\}, kt + i = 1, 2, \ldots, kb, \tag{3.22}$$

where

$$\begin{aligned} L^3_{kt+i}(y) \quad &= \quad A + cy + hE \sum_{n=kt+i}^{\min\{kt+R_{kt}, l_{kt+i}\}} [y - D^{(kt+i,n)}]^+ \\ &\quad + AP(\sum_{n=kt+i}^{\min\{kt+R_{kt}, l_{kt+i}\}} D_n > y) + cE[\sum_{n=kt+i}^{\min\{kt+R_{kt}, l_{kt+i}\}} D_n - y]^+ \end{aligned} \tag{3.23}$$

where l_{kt+i} is computed using (3.15)-(3.17). Thus we need to find the solutions of $\frac{d}{dy}L^3_{kt+i}(y) = 0$, or equivalently,

$$\frac{f^{(kt+i, \min\{kt+R_{kt}, l_{kt+i}\})}(y)}{F^{(kt+i, \min\{kt+R_{kt}, l_{kt+i}\})}(y)} = \frac{cF^{(kt+i, \min\{kt+R_{kt}, l_{kt+i}\})}(y) + hE \sum_{n=kt+i}^{\min\{kt+R_{kt}, l_{kt+i}\}} F^{(kt+i,n)}(y)}{AF^{(kt+i, \min\{kt+R_{kt}, l_{kt+i}\})}(y)} \tag{3.24}$$

The reader may wish to compare the similarities between (3.24) and (3.10)(also see Remark 3.6). As before, the above equation may have several solutions. We will choose the one that minimizes $L^3_{kt+i}(y)$, for $y \geq s^{\alpha}_{kt+i}$.

This gives us the third heuristic method.

The Hybrid Policy

Step 1 For each $kt+i$, $t = 0, 1, \ldots, b-1$, $i = 1, 2, \ldots, k$, determine l_{kt+i} from (3.15)-(3.17);

Step 2 If the stock on hand in period $kt+i$ is greater than or equal to s^{α}_{kt+i}, do not order; otherwise order up to $\bar{S}^3_{kt+i}$, where $\bar{S}^3_{kt+i}$ is a solution of (3.24) and satisfies (3.22) and $L^3_{kt+i}(y)$ is given by (3.23).

Remark 3.6 Compared with the FE policy, the hybrid policy takes a "myopic" viewpoint: It attempts to minimize the cost from the current period, $kt+i$, to the end of either the order-up-to-period or the product life, $\min\{kt+R_{kt}, l_{kt+i}\}$, and ignore the cost incurred thereafter, under the relaxation that shortages can be filled at the end of period $\min\{kt+R_{kt}, l_{kt+i}\}$. In contrast, the FE policy aims at minimizing the cost from the current period $kt+i$ to the end of product life $kt+R_{kt}$, under the relaxation that shortages can be filled at the end of the product life. If the product life is long and demand variability is low, we expect that the hybrid policy outperforms the FE policy. Computationally, the FE policy is more attractive since it does not require to solve the dynamic recursions (3.15)–(3.16).

Example 3.7 Let us again use the data given in Example 3.2. Let $h = 0$. We wish to compute $\bar{S}^3_1$, $\bar{S}^3_9$ and $\bar{S}^3_{17}$, using the hybrid policy. Consider $\bar{S}^3_1$ first. From Example 3.5, $l_1 = 16$. To evaluate

$$\frac{f^{(1,\min\{R,l_1\})}(y)}{F^{(1,\min\{R,l_1\})}(y)} = \frac{f^{(1,\min\{R,16\})}(y)}{F^{(1,\min\{R,16\})}(y)} = 0.02,$$

conditioning on R, which equally likely takes on values 8, 16 and 24, we get

$$\frac{\frac{1}{3}[f^{(1,8)}(y) + 2f^{(1,16)}(y)]}{\frac{1}{3}[F^{(1,8)}(y) + 2F^{(1,16)}(y)]} = 0.02,$$

We obtain the solutions 46,72 and 86. Of those, 46 and 86 correspond to the local minima of $L^3_1(y)$. Thus $\bar{S}^3_1 = \text{argmin}_{y \geq 7}\{L^3_1(46), L^3_1(86)\} = 86$. To compute $\bar{S}^3_9$, we note from Example 3.5 that $l_9 = 24$. Then, (3.24) becomes

$$\frac{f^{(9,\min\{8+R_8,16\})}(y)}{F^{(9,\min\{8+R_8,16\})}(y)} = \frac{\frac{1}{2}[f^{(9,16)}(y) + f^{(9,24)}(y)]}{\frac{1}{2}[F^{(9,16)}(y) + F^{(9,24)}(y)]} = 0.02,$$

but this is the same expression that determines $\bar{S}^1_9$ under the FE policy. Therefore $\bar{S}^3_9 = \bar{S}^1_9$=86. Similarly, one can show that $\bar{S}^3_{17} = \bar{S}^1_{17} = 46$. Thus, for this particular example, the order-up-to quantities under the FE and hybrid policies are identical, both are lower than that with the OUTP policy. ■

4 Bounds

In this section we derive several upper and lower bounds for the optimal value function $V_{kt+i}(x)$, $t = 0, 1, \ldots, b-1$, $i = 1, 2, \ldots, k$, given by (2.6)–(2.7).

4.1 Lower Bound

We provide an easily assessable, crude lower bound for $V_{kt+i}(x)$, $t = 0, 1, \ldots, b-1$, $i = 1, 2, \ldots, k$, based on the FE heuristic. Recall that the FE policy is based on the relaxation that, for the subsequent periods of $kt+i$ we are allowed to defer orders until the end of the product life, at which time one must place an order to fill shortages, if any. It is clear that the minimal cost under this relaxation, denoted by $V^{L_1}_{kt+i}(x)$, is a lower bound of $V_{kt+i}(x)$. Therefore,

$$V_{kt+i}(x) \quad \geq \quad V^{L_1}_{kt+i}(x) = \min_{y \geq x}\{L^1_{kt+i}(y)\} - cx, \tag{4.25}$$

where $L^1_{kt+i}(y)$ is given by (3.8).

Example 4.1 We work with the data given by Example 3.2, assuming $h = 0$. Let us compute the lower bound of $V_1(0)$. From Example 3.2, $\bar{S}^1_1 = 86$. Hence

$$\begin{aligned}
V_1(0) \quad &\geq \quad 50 + 86 + 50P(D^{(1,R)} > 86) + E[D^{(1,R)} - 86]^+ \\
&= \quad 136 + \frac{50}{3}\sum_{t=1}^{3} P(D^{(1,8t)} > 86) + \frac{1}{3}\sum_{t=1}^{3} E[D^{(1,8t)} - 86]^+ \\
&= \quad 136 + \frac{50}{3}(P(D^{(1,8)} > 86) + P(D^{(1,16)} > 86) + P(D^{(1,24)} > 86)) + \\
&\quad\quad + \frac{1}{3}(E[D^{(1,8)} - 86]^+ + E[D^{(1,16)} - 86]^+ + E[D^{(1,24)} - 86]^+) \\
&= \quad 136 + \frac{50}{3}(0 + 0.067 + 1.000) + \frac{1}{3}(0 + 0.117 + 34.000) \\
&= \quad 165.15.
\end{aligned}$$

4.2 Upper Bounds

First note that the cost function under any of the heuristic ordering policies discussed in Section 3 provides an upper bound for $V_{kt+i}(x)$, because no policy can do better than the optimal policy. The computation of those bounds, however, is not simple. Here we aim at finding some upper bounds of $V_{kt+i}(x)$ that can be evaluated easily. The bounds developed in this section is based on the assumption that demand in each period is normally distributed.

The first upper bound is the cost associated with a greedy-like ordering policy (the G-policy, in short). The G-policy, for each given period $kt+i$, sets the order-up-to quantity, say $\bar{S}^G_{kt+i}$, that is almost certain to meet the entire remaining demand, and orders up to $\bar{S}^G_{kt+i}$ as long as inventory on hand is less than that. The order-up-to quantity in period $kt+i$ under the G-policy is given by

$$\bar{S}^G_{kt+i} = \sum_{n=kt+i}^{kb} \mu_n + z_G \sqrt{\sum_{n=kt+i}^{kb} \sigma^2_n}, \tag{4.26}$$

where z_G is chosen such that

$$P(Z \leq z_G) \approx 1.$$

Thus, for each given period $kt+i$, the G-policy only places a single order during the entire remaining life of the product. The cost under the policy, denoted by $V^{U_1}_{kt+i}(x)$, $x < \bar{S}^G_{kt+i}$, can be easily computed:

$$V_{kt+i}(x) \le V^{U_1}_{kt+i}(x) = A + c(\bar{S}^G_{kt+i} - x) + hE \sum_{n=kt+i}^{kt+R_{kt}} [\bar{S}^G_{kt+i} - D^{(kt+i,n)}]^+. \tag{4.27}$$

Although the G-policy is a legitimate ordering policy, we do not expect it to be competitive against the three heuristic policies derived in Section 3. Indeed, this policy only facilitates the computation of the closed form upper bounds of $V_{kt+i}(x)$.

Example 4.2 We compute the upper bound of $V_1(0)$ and $\bar{S}^G_1$, using the data given by Example 3.2. Let $h=0$. If we select $z_G = 3$, (4.26) gives us

$$\bar{S}^G_1 = 5(24) + 3\sqrt{24} \approx 135$$

The probability that the demand during the entire product life is met by $\bar{S}^G_1$ is

$$\begin{aligned} P(D^{(1,R)} \le 135) &= \frac{1}{3}[P(D^{(1,8)} \le 135) + P(D^{(1,16)} \le 135) + P(D^{(1,24)} \le 135)] \\ &\approx \frac{1}{3}[1 + 1 + 0.9987] = 0.99957 \end{aligned}$$

Hence,

$$V_1(0) \le V^{U_1}_1(0) = 50 + 135 = 185.$$

The upper bound obtained by the G-policy is simple but very crude. Our next upper bound is based on a policy that is a combination of the OUTP policy and the G-policy (call it the OUTP-G policy). The bound is computed as follows.

An Upper Bound Based on the OUTP-G Policy

Step 1 For a given period, say period $l^0_{kt+i} + 1 := kt+i$, use (3.15)-(3.16) to determine its order-up-to period $l^1_{kt+i} := l_{kt+i}$. Given $l^1_{kt+i}+1$, again use (3.15)-(3.16) to compute its order-up-to period $l^2_{kt+i} := l_{l^1_{kt+i}+1}$. Repeat the process for m steps such that $l^m_{kt+i} = kb$. Those order-up-to periods l^j_{kt+i}, $j = 0, 1, \ldots, m$ partition the maximal remaining product life $kb - kt - i$ into m non-overlapping subintervals:

$$[l^0_{kt+i}+1, l^1_{kt+i}], [l^1_{kt+i}+1, l^2_{kt+i}], \ldots, [l^{m-1}_{kt+i}+1, kb].$$

Step 2 Apply the G-policy in each subinterval and compute its cost. Let $\bar{S}^{G_j}_{kt+i}$ be the order-up-to quantity for subinterval j, $j = 1, 2, \ldots, m$. Then

$$\bar{S}^{G_j}_{kt+i} = \sum_{n=l^{j-1}_{kt+i}+1}^{l^j_{kt+i}} \mu_n + z_G \sqrt{\sum_{n=l^{j-1}_{kt+i}+1}^{l^j_{kt+i}} \sigma^2_n}, \quad j = 1, 2, \ldots, m.$$

The expected cost in the jth subinterval under the OUTP-G policy, given the product is still alive at the beginning of the subinterval and initial inventory is x, is

$$g^j_{kt+i}(x) := \begin{cases} A + c[\bar{S}^{G_j}_{kt+i} - x] + hE\sum_{n=l^{j-1}_{kt+i}+1}^{l^j_{kt+i}}[\bar{S}^{G_j}_{kt+i} - D^{(l^{j-1}_{kt+i}+1,n)}]^+, & x < \bar{S}^{G_j}_{kt+i} \\ \\ hE\sum_{n=l^{j-1}_{kt+i}+1}^{l^j_{kt+i}}[x - D^{(l^{j-1}_{kt+i}+1,n)}]^+ & x \geq \bar{S}^{G_j}_{kt+i} \end{cases} \quad j = 1, 2, \ldots, m.$$

Step 3 Compute the weighted sum of the costs in those m subintervals, where the weight of a subinterval is the probability that the product is still alive at the beginning of that subinterval:

$$V^{U_2}_{kt+i}(x) \quad = \quad g^1_{kt+i}(x) + \sum_{j=2}^{m} Eg^j_{kt+i}(\bar{S}^{G_j}_{kt+i} - D^{(l^{j-1}_{kt+i}+1,l^j_{kt+i})})P(kt + R_{kt} > l^{j-1}_{kt+i} + 1). \quad (4.28)$$

In general we expect that $V^{U_2}_{kt+i}(x)$ is a tighter upper bound than $V^{U_1}_{kt+i}(x)$, as illustrated by the following example.

Example 4.3 We again use the data given by Example 3.2, assuming $h = 0$. We compute the upper bound of $V_1(0)$ using the OUTP-G policy.

From Example 3.5, $l^1_1 = 16$ and $l^2_1 = 24$. Thus we can partition the maximal product life to two non-overlapping intervals $[1, 16]$ and $[17, 24]$. Selecting $z_G = 3$, we obtain the order-up-to levels in those subintervals as

$$\bar{S}^{G_1}_1 = 5 \cdot 16 + 3\sqrt{16} = 92$$

and

$$\bar{S}^{G_2}_1 = 5 \cdot 8 + 3\sqrt{8} \approx 49.$$

Using (4.28), we obtain

$$\begin{aligned} V_1(0) \leq V^{U_2}_1(0) \quad &= \quad g^1_1(0) + P(R > 16)(g^2_1(0) - cE[\bar{S}^{G_1}_1 - D^{(1,16)}]) \\ &= \quad 50 + 92 + \frac{1}{3}[50 + 49 - (92 - 80)] = 171 \end{aligned}$$

This upper bound substantially improves the first upper bound (= 185) developed in Example 4.2. The bound seems tight, judging from the lower bound (= 165.15) given in Example 4.1.

5 Computational Results

We test our heuristic methods using three separate problem sets. The first is a set of small test problems where we vary setup costs, demand distributions, holding costs, and product lifetime distributions systematically. The second is a set of large problems where the demand distribution has a larger mean and higher variance. The third is a set of problems based on the actual inventory problem for ordering Jeppesen Enroute and Area charts. We have to scale down the mean demand of the actual problems to be able to compute the optimal solutions, but except for demand distributions, we preserve all other problem characteristics.

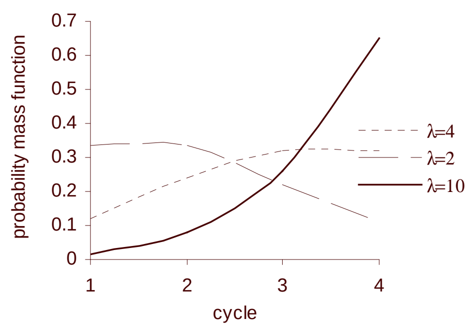
5.1 Test Problems with Low Mean Demand

To create the first problem set we systematically varied parameters that can have an impact on performance:

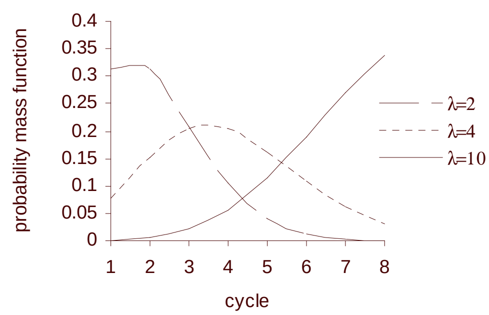
1. The ordering cost A, the production cost c, and holding cost h,
2. the distribution of unit demand,
3. the length of each cycle and the number of cycles,
4. the probability distribution of the product lifetime, *which we call the lifetime pattern.*

We set $c = 1$ and vary $A = 50, 250, 2000$. We let the holding costs $h = 0.00$ and $h = 0.03$. We assume that demand is stationary, $D_n = D$ for all n and choose three distributions for D: Normal with the mean of 5 and the standard deviation of 1, Uniform between 1 and 9, and Triangular with parameters 1 (minimum), 3 (mode) and 11 (maximum). We restrict demand to be integers, so all three distributions have the mean of 5. The normal distribution has the standard deviation of 1, the uniform distribution has the standard deviation of 2.58, and the triangular distribution is skewed and has the standard deviation of 2.52.

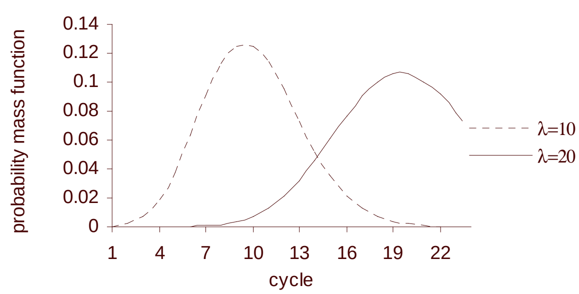
We let the maximum lifetime of the product be $kb = 24$ periods, and vary the number and the length of cycles. We use problems with 4 cycles and 6 periods per cycle, 8 cycles and 3 periods per cycle, and 24 cycles and 1 period per cycle. We let the product lifetime R be a Poisson random variable. For the problems with 4 and 8 cycles we use the Poisson distribution with $\lambda = 2$, $\lambda = 4$, and $\lambda = 10$ (see Figure 2a and Figure 2b). For the problems with 24 cycles we use the Poisson distribution with $\lambda = 10$ and $\lambda = 20$ (see Figure 2c).



(a) Lifetime pattern for problems with 4 cycles.



(b) Lifetime pattern for problems with 8 cycles.

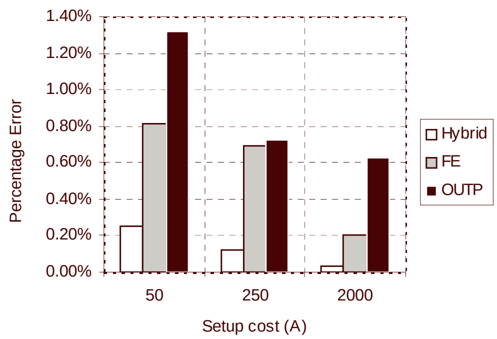


(c) Lifetime pattern for problems with 24 cycles.

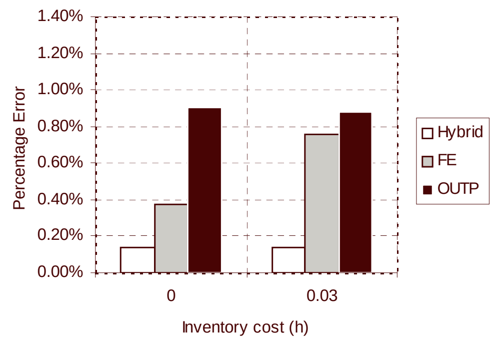
Figure 2: Distributions for the Product Lifetime R

In total the first problem set contains 144 test problems (3 ordering cost levels, 2 holding cost levels, 3 demand distributions, and 3 cycle lengths–with 4 and 8 cycle problems having three lifetime patterns and 24-cycle problems having 2 lifetime patterns).

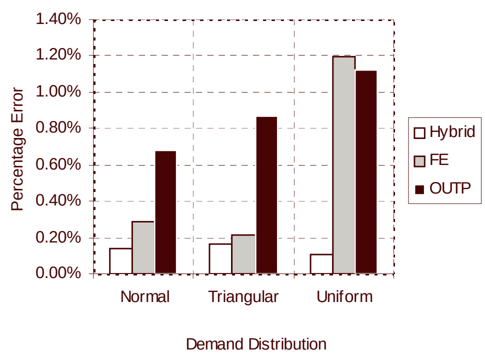
To compare the three heuristics we first solved the 144 problems to optimality using a dynamic programming solver called General Purpose Dynamic Programming GPDP (see (Kennedy 1986)). The data for GPDP is stored in text files, with one line (record) for each state/stage/action/next stage combination. Each data file for a single test problem contains over 3 million records, and takes up more than 3 gigabytes of disk space. We generated all GPDP data files using Microsoft Excel 2000 and Visual Basic for Applications (VBA). Each test problem took about 20 minutes to generate and 10 minutes to solve to optimality with GPDP on a 700 MHz IBM ThinkPad 20a with 512 MB of memory.



(a) Percentage error of the four heuristics, grouped by setup cost A. For $A = 50$ the percentage error for the OUTP heuristic is 1.32%, for FE 0.81% and for Hybrid 0.25%. For $A = 250$ the percentage error for the OUTP heuristic is 0.72%, for FE 0.69% and for Hybrid 0.12%. For $A = 2000$ the percentage error for the OUTP heuristic is 0.62%, for FE 0.20% and for Hybrid 0.03%.



(b) Percentage error of the four heuristics, grouped by holding cost h. For $h = 0$ the percentage error for the OUTP heuristic is 0.90%, for FE 0.38% and for Hybrid 0.14%. For $h = 0.03$ the percentage error for the OUTP heuristic is 0.88%, for FE 0.76% and for Hybrid 0.14%.



(c) Percentage error of the four heuristics, grouped by demand distribution. For Normal the percentage error for the OUTP heuristic is 0.68%, for FE 0.28% and for Hybrid 0.14%. For Triangular the percentage error for the OUTP heuristic is 0.86%, for FE 0.21% and for Hybrid 0.16%. For Uniform the percentage error for the OUTP heuristic is 1.11%, for FE 1.11% and for Hybrid 0.10%.

Figure 3: Percentage Error of the Heuristics

To evaluate heuristics' performance we also solved each problem with the three heuristic methods and used GPDP to compute the cost of each policy. We implemented the FE policy in Microsoft Excel 2000 and VBA. For the other two heuristics, we first used GPDP to evaluate equations (3.15)-(3.17), based on the mean demand of 5. We then used these results to compute the policies using Excel. For each problem we compute the relative difference between the cost of the optimal policy and the cost of the heuristic policy, as follows:

$$\text{Percentage Error} = \frac{\text{Cost of Heuristic Policy - Cost of Optimal Policy}}{\text{Cost of Optimal Policy}}.$$

Figure 3 shows the average percentage error grouped in various ways. Figure 3a groups the test problems by different setup cost levels. In Figure 3b we compare the performance of the heuristics with two levels of holding costs, $h = 0.03$ and $h = 0.00$. The holding cost level of 0 is meant to represent short-term problems, where each period corresponds to one or two weeks (for example), and the higher holding cost level of 0.03 is meant to represent medium-range problems, where each period corresponds to one month, and the annual holding cost rate is about 36%. Finally, in Figure 3c, we compare the performance with different demand distributions.

In addition, we compare the performance by both the number of cycles and product lifetime patterns. This comparison is shown in Figure 4.

We also computed the bounds of $V_1(0)$ for this problem set. On average the lower bound computed using (4.25) is 4.07% below the optimum, and the upper bound based on the OUTP-G policy is 1.48 % above optimum.

In summary, we gained the following insights from the first test problem set:

- Figure 3a shows that the magnitude of setup costs has an impact on the performance under all three heuristics, and all do better when setup costs are higher.

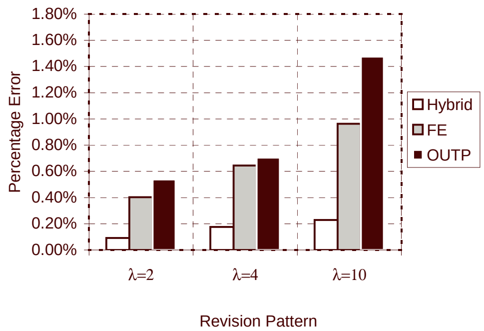
- Figure 3b indicates that the performance of the OUTP and hybrid policies are fairly robust against holding costs, while the presence of holding costs has an adverse impact on the performance of the FE policy.

- Figure 3c suggests that the FE and OUTP heuristics perform slightly worse against the uniform demand, which has the largest demand variability among the three demand distributions. The hybrid heuristic performs equally well against all three demand distributions.

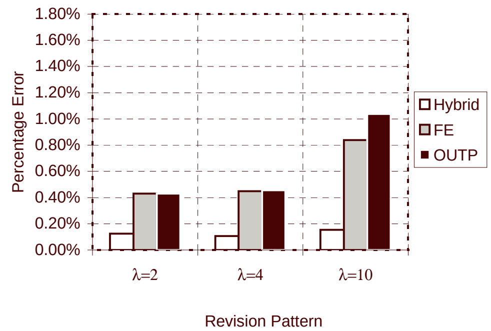
- Figure 4 shows that all three heuristics perform slightly worse when R is more likely to assume large values (i.e, the performance of each heuristic deteriorates when the Poisson parameter λ increases).

- The lower bound is 4.07% below the optimum, and the upper bound is 1.48% above the optimum.

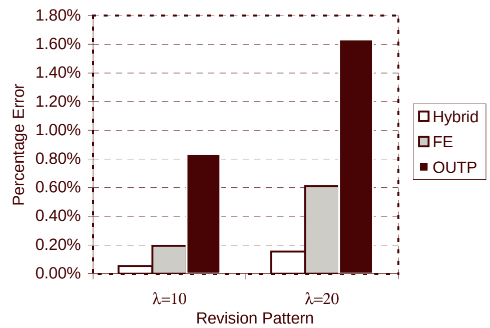
- Overall, the hybrid heuristic performs by far better than the other two heuristics, both in terms of the percentage error measures and robustness against different parameter settings. The hybrid heuristic is followed by the FE heuristic, and then the OUTP heuristic.



(a) 4-cycle problems: Percentage error for λ = 2 for OUTP 0.54%, for FE 0.40% and for Hybrid 0.09%. For λ = 4 OUTP 0.70%, for FE 0.64% and for Hybrid 0.17%. For λ = 10 OUTP 1.48%, for FE 0.96% and for Hybrid 0.11%.



(b) 8-cycle problems: Percentage error for λ = 2 OUTP 0.43%, for FE 0.43% and for Hybrid 0.13%. For λ = 4 for OUTP 0.46%, for FE 0.45% and for Hybrid 0.11%. For λ = 10 OUTP 1.04%, for FE 0.84% and for Hybrid 0.15%.



(c) 24-cycle problems: Percentage error for λ = 10 for OUTP 0.84%, for FE 0.19% and for Hybrid 0.05%. For λ = 20 for OUTP 1.63%, for FE 0.61% and for Hybrid 0.15%.

Figure 4: Percentage error grouped by the number of cycles and lifetime patterns

5.2 Test Problems with High demand

All problems in the first test set have a mean period demand of 5, and the number of distinct demand levels is limited to at most 11 (in the triangular distribution case). We also wish to assess the performance of the heuristics using the problems with a higher number of demand values. Toward this end, we created additional 24 problems where demand per period is normally distributed with mean of 10 and standard deviation of 3.

As in the previous problem set, we vary setup costs at $A = 50, 250, 2000$, the numbers of cycles at $t = 4, 8, 24$, three lifetime patterns for the 4 and 8 cycle problems and two lifetime patterns for the 24-cycle problems. All the 24 problems have $h = 0.03$ per period.

The larger number of demand values (20 instead of 11) makes these problems significantly more difficult to solve to optimality than the smaller problems. In particular, each GPDP data file has more than 30 million records (vs. 3 million for a smaller problems), and the size of the file ranges between 9 and 20 gigabytes. Each data file takes about 2.5 hours to generate (vs. 20 minutes for a smaller problems) and about 90 minutes to solve (vs. 10 minutes for a smaller problems) on the same IBM ThinkPad we used to solve the smaller problems.

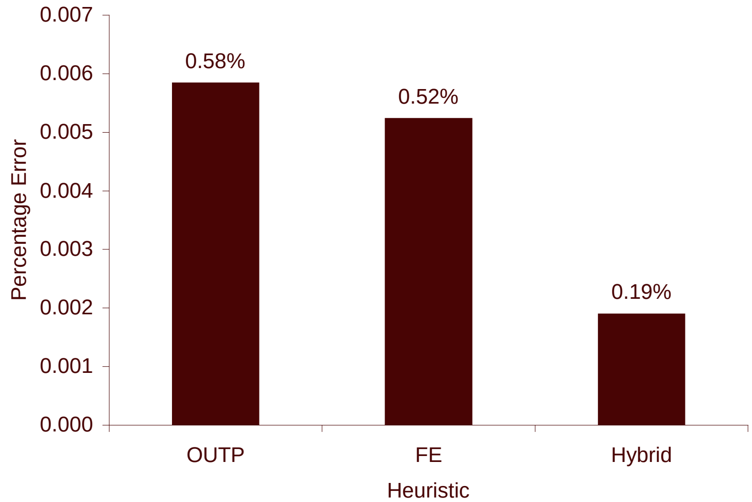


Figure 5: Results for Problems with High Demand.

We summarize the results in Figure 5. The results look very similar to those for small problems. Overall, the hybrid heuristic produces the best results, followed by the FE heuristic, and then the OUTP heuristic. The percentage errors are fairly close to the ones for small problems.

5.3 Jeppesen Test Problem

To demonstrate the performance of our heuristics for a wider range of problems, we created a third data set based on the actual characteristics of Jeppesen's Enroute and Area charts. These charts are large and multicolored, and Jeppesen usually out-sources their production. Each time a chart is ordered, regardless of whether the chart revised, Jeppesen pays a large fixed charge. This fixed charge varies depending on the chart characteristics (size, the number of colors, type of paper, etc.). The variable printing cost similarly varies. The distributions of bi-weekly demand and revision history are forecasted based on historical data.

Jeppesen charts revise on an 8-week cycle, and at most they last for 48 weeks (six 8-week cycles). Each specific chart is always produced on either odd or even weeks. So for Jeppesen, an 8-week cycle has 4 periods, where each period contains two weeks. Bi-weekly demand varies from as few as one or two charts in two weeks, to over 1,000. The coefficient of variation of bi-weekly demand ranges between 0.03 to 0.28. The fixed order cost is either 383, 500, 900, 1006 or 1600 dollars, and the variable printing cost is either 2, 15, 20 or 45 cents per chart.

To create problems that we can solve to optimality we had to scale the bi-weekly demands and approximate them by a discrete random variables with no more than 11 demand levels. To do this, we introduce the notion of a "unit" of demand. Each unit can contain more than one chart, and bi-weekly demand and printing cost are converted to be in terms of units. We select the unit size for each chart in such a way that approximates the demand with a mean of 5 "units." We select the standard deviation of bi-weekly demand that preserves its original coefficient of variation. For example, if a chart has the average demand of 50 and standard deviation of 10, the printing cost of 20 cents and setup cost of $900, the scaled problem of that chart uses the unit size $50/5 = 10$, so the new average demand is $50/10 = 5$ "units" (each unit represents 10 charts), the new standard deviation is $10/10 = 1$, the new printing cost is $0.20 \times 10 = 2$, and the new setup cost remains at 900. The original data for the Jeppesen problems, along with the conversion factors (the number of charts in each "unit") and the resulting parameters for the scaled problems is presented in the Appendix. The data in the Appendix also contains revision patterns for charts. Inventory holding costs for the Jeppesen problems are 0.

Results for Jeppesen problems are displayed in Figure 6. For these realistic problems, both the FE and the hybrid heuristics locate near-optimal solutions (0.29% and 0.17% above the optimum). The OUTP heuristic locates the solutions 4.65% above the optimum.

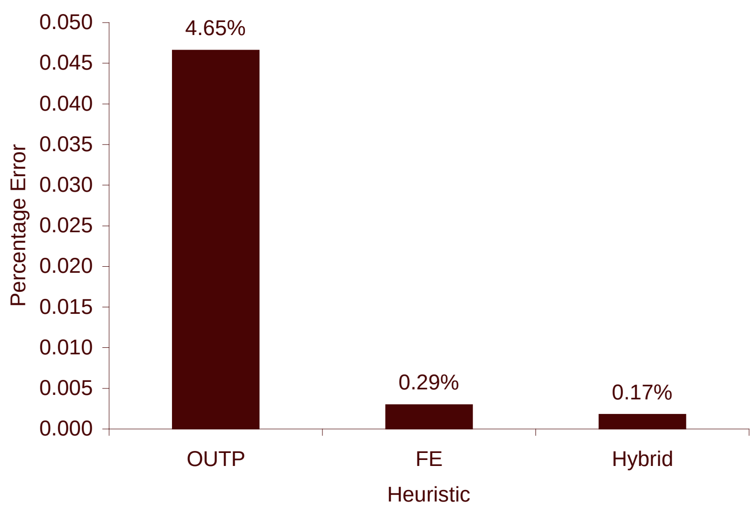


Figure 6: Results for Jeppesen problems. The relative gap 4.65% for the OUTP, 0.29% for FE and 0.17% for Hybird.

6 Conclusion

In this paper we consider a variation of an inventory problem with obsolescence, where products are subject to upgrades, and hence have uncertain life spans and non-stationary demand. Our work was inspired by a real-world problem faced by Jeppesen Sanderson, Inc., a Boeing subsidiary and the major provider of aviation information worldwide. Since aviation information is constantly evolving, Jeppesen is faced with the problem of deciding on stock levels for aviation charts that may revise in the future.

We formulate the problem as a dynamic programming model, and present formal analysis of the problem with deterministic demand. Since the size of real problems may be too large to solve to optimality using standard dynamic programming solution methods, we develop three simple, yet effective, heuristic methods. We compare these heuristic methods using two sets of systematically generated test problems, and a third set based on the actual Jeppesen problems.

The performance of all heuristics is generally robust against different levels of holding costs, lifetime patterns and demand distributions. All three heuristics perform the best on problems with high setup costs. Most heuristics do slightly worse for the problems where the probability of obsolescence is high towards the end of the product's life span.

Overall, the best of the three heuristics–the hybrid heuristic–finds solutions that are about 0.15% worse than the optimum on average. While large-scale problems cannot be solved to optimality with standard dynamic programming algorithms, the heuristics we present in this paper can be easily implemented on a spreadsheet, even for large-scale problems. We built and implemented a Decision Support System that incorporates many of the ideas in this paper (see Katok, Lathrop, Tarantino, and Xu (2001)).

7 Appendix

		Original (bi-weekly)			Charts/	Scaled				Probability of Revision (week number)					
ID	Title	μ	σ	c	Unit	μ	σ	c	A	8	16	24	32	40	48
1	US (HI) 3/4 (DAL)	123.20	6.83	0.20	24.64	5.00	0.28	4.93	900.00	0.20	0.60	0.20	0.00	0.00	0.00
2	US (HI) 5/6	563.73	37.57	0.15	112.75	5.00	0.33	16.91	500.00	0.14	0.71	0.14	0.00	0.00	0.00
3	CA (HI) 3/4	367.73	35.52	0.15	73.55	5.00	0.48	11.03	500.00	0.14	0.29	0.57	0.00	0.00	0.00
4	CA (HI) 3/4 (NWA)	39.67	4.78	0.20	7.93	5.00	0.60	1.59	900.00	0.29	0.14	0.57	0.00	0.00	0.00
5	CA (HI) 5/6	98.93	27.33	0.15	19.79	5.00	1.38	2.97	500.00	0.22	0.56	0.22	0.00	0.00	0.00
6	CA (HI) 5/6 (UAL)	162.40	27.33	0.20	32.48	5.00	0.84	6.50	900.00	0.29	0.43	0.29	0.00	0.00	0.00
7	CA (LO) 3/4	452.67	34.16	0.15	90.53	5.00	0.38	13.58	500.00	0.00	0.33	0.00	0.00	0.67	0.00
8	LA (H/L) 1/2 (DAL)	79.33	6.15	0.20	15.87	5.00	0.39	3.17	900.00	0.50	0.00	0.25	0.13	0.13	0.00
9	LA (H/L) 1/2 (AAL)	104.07	10.93	0.20	20.81	5.00	0.53	4.16	900.00	0.44	0.11	0.33	0.11	0.00	0.00
10	LA (H/L) 3/4 (UAL)	67.67	4.10	0.20	13.53	5.00	0.30	2.71	900.00	0.25	0.38	0.25	0.13	0.00	0.00
11	LA (H/L) 3/4 (NWA)	18.20	2.05	0.20	3.64	5.00	0.56	0.73	900.00	0.33	0.44	0.11	0.11	0.00	0.00
12	SA (HI) 3/4	273.47	30.74	0.15	54.69	5.00	0.56	8.20	500.00	0.50	0.40	0.00	0.00	0.10	0.00
13	SA (LO) 7/8	222.60	15.71	0.15	44.52	5.00	0.35	6.68	500.00	0.20	0.40	0.00	0.20	0.00	0.20
14	SA (H/L) 13	49.93	2.73	0.20	9.99	5.00	0.27	2.00	900.00	0.67	0.00	0.00	0.00	0.00	0.33
15	AS (H/L) 1/2 (MNA)	15.00	1.18	0.20	3.00	5.00	0.39	0.60	900.00	0.00	0.00	0.00	0.00	0.50	0.50
16	AS (H/L) 5/6	136.73	12.98	0.20	27.35	5.00	0.47	5.47	900.00	0.33	0.00	0.17	0.33	0.17	0.00
17	ANCHORAGE, AS	385.93	23.23	0.02	77.19	5.00	0.30	1.23	383.00	0.25	0.00	0.50	0.00	0.00	0.25
18	CALI, COL	193.20	28.01	0.02	38.64	5.00	0.72	0.62	383.00	0.50	0.00	0.00	0.50	0.00	0.00
19	FUKUOKA, JAPAN	318.73	23.23	0.02	63.75	5.00	0.36	1.02	383.00	0.60	0.00	0.20	0.00	0.00	0.20
20	GUADALAJARA, MEX	512.40	80.61	0.02	102.48	5.00	0.79	1.64	383.00	0.33	0.00	0.00	0.00	0.00	0.67
21	HAVANA, CUBA	465.27	40.30	0.02	93.05	5.00	0.43	1.49	383.00	0.33	0.33	0.00	0.33	0.00	0.00
22	MELBOURNE, VIC, AUST	171.27	15.71	0.45	34.25	5.00	0.46	15.41	1006.00	0.00	0.25	0.25	0.25	0.00	0.25
23	PUERTO MONTT, CHILE	115.73	7.51	0.02	23.15	5.00	0.32	0.37	383.00	0.00	0.00	0.00	0.50	0.00	0.50
24	SEATTLE, WASH	1081.73	51.23	0.02	216.35	5.00	0.24	3.46	383.00	0.00	0.50	0.00	0.50	0.00	0.00
25	TAIPEI, TAIPEI/TAIWAN	380.80	38.94	0.02	76.16	5.00	0.51	1.22	383.00	0.33	0.00	0.00	0.00	0.00	0.67
26	AT (H/L) 1/2	231.93	19.13	0.20	46.39	5.00	0.41	9.28	900.00	0.00	0.00	0.60	0.40	0.00	0.00
27	P (HI) 1/P (H/L) 2	239.40	15.71	0.20	47.88	5.00	0.33	9.58	900.00	0.33	0.50	0.17	0.00	0.00	0.00
28	AT (HI) 5/BLK	58.80	5.47	0.20	11.76	5.00	0.46	2.35	900.00	0.00	0.25	0.25	0.00	0.00	0.50
29	CA (LO) 3/4 (DAL)	60.67	5.47	0.20	12.13	5.00	0.45	2.43	900.00	0.00	0.33	0.00	0.00	0.67	0.00
30	US (HI) 3/4 (AAL)	104.07	10.93	0.20	20.81	5.00	0.53	4.16	900.00	0.20	0.60	0.20	0.00	0.00	0.00
31	US (HI) 3/4 (AWE)	14.47	2.05	0.20	2.89	5.00	0.71	0.58	900.00	0.20	0.60	0.20	0.00	0.00	0.00
32	CA (LO)9/(H/L)10	337.40	32.79	0.15	67.48	5.00	0.49	10.12	500.00	0.00	0.60	0.20	0.00	0.00	0.20
33	AK (LO) 3/4 (DAL)	37.80	4.10	0.20	7.56	5.00	0.54	1.51	900.00	0.00	0.50	0.25	0.25	0.00	0.00
34	ENROUTE 1/2 (CRA)	5.13	1.37	0.02	1.03	5.00	1.33	0.02	383.00	0.87	0.13	0.00	0.00	0.00	0.00
35	ENROUTE 3/4 (CRA)	5.13	1.37	0.02	1.03	5.00	1.33	0.02	383.00	0.87	0.13	0.00	0.00	0.00	0.00
36	ENROUTE 5/6 (CRA)	5.13	1.37	0.02	1.03	5.00	1.33	0.02	383.00	0.87	0.13	0.00	0.00	0.00	0.00
37	ENROUTE 4A/BLK (CRA)	5.13	1.37	0.02	1.03	5.00	1.33	0.02	383.00	0.87	0.13	0.00	0.00	0.00	0.00
38	LA (H/L) 7/8 (UAL)	67.67	4.10	0.20	13.53	5.00	0.30	2.71	900.00	0.63	0.38	0.00	0.00	0.00	0.00
39	US (HI) 5/6 (AWE)	14.47	2.05	0.20	2.89	5.00	0.71	0.58	900.00	0.14	0.57	0.29	0.00	0.00	0.00
40	DAL CA (HI) 5/6	27.07	2.05	0.20	5.41	5.00	0.38	1.08	900.00	0.29	0.14	0.43	0.14	0.00	0.00
41	AU (LO) 1/2	67.67	4.78	0.20	13.53	5.00	0.35	2.71	1600.00	0.00	0.40	0.20	0.40	0.00	0.00
42	AU (LO) 7/8	42.47	4.10	0.20	8.49	5.00	0.48	1.70	1600.00	0.00	0.25	0.25	0.25	0.00	0.25
43	SA (LO) 1/2 (UAL)	57.40	4.10	0.20	11.48	5.00	0.36	2.30	900.00	0.00	0.50	0.33	0.17	0.00	0.00
44	CA (LO)9/(H/L)10(UAL)	64.40	4.10	0.20	12.88	5.00	0.32	2.58	900.00	0.00	0.60	0.20	0.00	0.00	0.20
45	FE (H/L) 1/2 (MNA)	15.00	1.18	0.20	3.00	5.00	0.39	0.60	900.00	0.38	0.25	0.13	0.25	0.00	0.00
46	FE (H/L) 3/4 (MNA)	15.00	1.18	0.20	3.00	5.00	0.39	0.60	900.00	0.38	0.50	0.00	0.00	0.00	0.13
47	US (HI) 5/6 (SWA)	84.47	2.73	0.20	16.89	5.00	0.16	3.38	900.00	0.00	0.67	0.33	0.00	0.00	0.00
48	HARRISBURG, PA	564.67	36.89	0.02	112.93	5.00	0.33	2.26	500.00	0.00	0.00	0.00	0.00	0.50	0.50
49	SA (HI) 1/2 (AAL)	94.73	5.47	0.20	18.95	5.00	0.29	3.79	900.00	0.20	0.20	0.20	0.20	0.20	0.00
50	EA (H/L) 3/4	32.20	1.37	0.20	6.44	5.00	0.21	1.29	900.00	0.20	0.00	0.40	0.20	0.20	0.00
51	EA (H/L) 9/10	82.60	4.78	0.20	16.52	5.00	0.29	3.30	900.00	0.00	0.33	0.33	0.17	0.17	0.00
52	US (HI) 1/2	818.07	42.35	0.15	163.61	5.00	0.26	24.54	500.00	0.44	0.33	0.11	0.00	0.11	0.00
53	US (HI) 1/2 (AWE)	14.47	2.05	0.20	2.89	5.00	0.71	0.58	900.00	0.33	0.33	0.33	0.00	0.00	0.00
54	US (HI) 1/2 (SWA)	84.47	2.73	0.20	16.89	5.00	0.16	3.38	900.00	0.33	0.33	0.33	0.00	0.00	0.00
55	DENVER, COLO	1178.33	53.97	0.02	235.67	5.00	0.23	3.77	383.00	0.00	0.00	0.00	0.50	0.00	0.50
56	SA (LO) 1/2 (COA)	36.87	5.47	0.20	7.37	5.00	0.74	1.47	900.00	0.00	0.20	0.40	0.40	0.00	0.00
57	SA (HI) 3/4 (COA)	40.13	4.78	0.20	8.03	5.00	0.60	1.61	900.00	0.56	0.22	0.00	0.11	0.11	0.00
58	ENROUTE 11/12 (CRA)	5.13	1.37	0.02	1.03	5.00	1.33	0.02	383.00	0.87	0.13	0.00	0.00	0.00	0.00
59	CA (HI) 1/2 (AWE)	13.07	2.05	0.20	2.61	5.00	0.78	0.52	900.00	0.25	0.38	0.38	0.00	0.00	0.00
60	US (LO) 33/34	1403.27	75.14	0.20	280.65	5.00	0.27	56.13	1600.00	0.00	0.33	0.00	0.67	0.00	0.00
61	US (LO) 45/46 (KAC)	187.50	14.73	0.20	37.50	5.00	0.39	7.50	900.00	0.00	0.50	0.50	0.00	0.00	0.00
62	US (LO) 3/4 (DAL)	92.40	6.15	0.20	18.48	5.00	0.33	3.70	900.00	0.00	0.00	0.67	0.00	0.00	0.33
63	US (LO) 11/11A/12	875.47	45.77	0.20	175.09	5.00	0.26	35.02	1600.00	0.17	0.33	0.17	0.17	0.17	0.00
64	US (LO) 11/11A/12(DAL)	87.73	6.15	0.20	17.55	5.00	0.35	3.51	900.00	0.20	0.40	0.00	0.20	0.20	0.00
65	US (LO) 27/28	1036.47	67.63	0.20	207.29	5.00	0.33	41.46	1600.00	0.33	0.00	0.33	0.33	0.00	0.00
66	US (LO) 37/38	1220.33	40.99	0.20	244.07	5.00	0.17	48.81	1600.00	0.00	0.25	0.25	0.50	0.00	0.00
67	US (LO) 43/44	1155.00	45.09	0.20	231.00	5.00	0.20	46.20	1600.00	0.00	0.00	0.00	0.67	0.00	0.33
68	US (LO) 49/50	1539.07	77.19	0.20	307.81	5.00	0.25	61.56	1600.00	0.00	0.00	0.67	0.33	0.00	0.00
69	MIAMI, FLA NASSAU	1223.13	58.75	0.02	244.63	5.00	0.24	3.91	383.00	0.33	0.00	0.00	0.00	0.33	0.33
70	AT (H/L) 1/2 (AAL)	85.87	5.47	0.20	17.17	5.00	0.32	3.43	900.00	0.00	0.00	0.25	0.50	0.00	0.25

		Original (bi-weekly)			Charts/	Scaled				Probability of Revision (week number)					
ID	Title	μ	σ	c	Unit	μ	σ	c	A	8	16	24	32	40	48
71	ENROUTE 14/BLK	5.13	1.37	0.02	1.03	5.00	1.33	0.02	383.00	0.88	0.13	0.00	0.00	0.00	0.00
72	CSN FE (HI) 8/BLK	15.00	1.18	0.20	3.00	5.00	0.39	0.60	900.00	0.43	0.43	0.00	0.00	0.14	0.00
73	MILAN, ITA	138.60	15.03	0.02	27.72	5.00	0.54	0.44	383.00	0.00	0.25	0.25	0.25	0.00	0.25
74	MUNICH, GER	117.60	15.03	0.02	23.52	5.00	0.64	0.38	383.00	0.00	0.25	0.25	0.25	0.25	0.00
75	NICE-COTE D'AZUR	64.40	13.66	0.02	12.88	5.00	1.06	0.21	383.00	0.50	0.00	0.00	0.00	0.00	0.50
76	GENEVA, SWITZ	62.07	13.66	0.02	12.41	5.00	1.10	0.20	383.00	0.50	0.00	0.00	0.00	0.00	0.50
77	MARSEILLE-PROV.	23.33	2.73	0.02	4.67	5.00	0.59	0.07	383.00	0.50	0.00	0.00	0.00	0.00	0.50
78	E (H/L) 3/4	124.60	14.35	0.20	24.92	5.00	0.58	4.98	900.00	0.14	0.43	0.29	0.14	0.00	0.00
79	E (LO) 7/8	198.33	19.13	0.20	39.67	5.00	0.48	7.93	900.00	0.25	0.25	0.25	0.25	0.00	0.00
80	ME (H/L) 1/2	100.80	15.03	0.20	20.16	5.00	0.75	4.03	900.00	0.00	0.20	0.20	0.60	0.00	0.00
81	ME (H/L) 5/6	91.47	14.35	0.20	18.29	5.00	0.78	3.66	900.00	0.00	0.17	0.17	0.50	0.17	0.00
82	EE (H/L) 1/2	59.73	14.35	0.20	11.95	5.00	1.20	2.39	900.00	0.13	0.63	0.25	0.00	0.00	0.00
83	E (HI) 1/2	210.00	19.81	0.20	42.00	5.00	0.47	8.40	900.00	0.00	0.78	0.11	0.11	0.00	0.00
84	E (HI) 7/8	129.27	18.44	0.20	25.85	5.00	0.71	5.17	900.00	0.00	0.88	0.13	0.00	0.00	0.00
85	US (LO) SE-1/SE-2	116.20	7.51	0.20	23.24	5.00	0.32	4.65	900.00	0.33	0.00	0.67	0.00	0.00	0.00
86	US (HI) 7/8 (SWA)	84.47	2.73	0.20	16.89	5.00	0.16	3.38	900.00	0.17	0.33	0.50	0.00	0.00	0.00
87	E (HI) 3/4 (UAL)	49.47	4.10	0.20	9.89	5.00	0.41	1.98	900.00	0.00	0.80	0.00	0.20	0.00	0.00
88	E (HI) 9/10 (UAL)	28.47	2.73	0.20	5.69	5.00	0.48	1.14	900.00	0.00	0.67	0.33	0.00	0.00	0.00
89	E (HI) 13/14	106.87	18.44	0.20	21.37	5.00	0.86	4.27	900.00	0.17	0.67	0.17	0.00	0.00	0.00
90	E (HI) 21/22	62.53	13.66	0.20	12.51	5.00	1.09	2.50	900.00	0.20	0.40	0.20	0.00	0.20	0.00
91	LA (H/L) 5/6 (NWA)	27.07	2.05	0.20	5.41	5.00	0.38	1.08	900.00	0.33	0.00	0.00	0.00	0.33	0.33
92	LA PAZ, MEX.	106.40	11.61	0.02	21.28	5.00	0.55	0.34	383.00	0.00	0.00	0.00	0.50	0.00	0.50
93	A (HI) 1/2	52.50	4.12	0.20	10.50	5.00	0.39	2.10	900.00	0.00	0.00	0.00	0.50	0.50	0.00
94	A (HI) 5/6	52.50	4.12	0.20	10.50	5.00	0.39	2.10	900.00	0.00	0.00	0.00	0.50	0.50	0.00
95	ME (H/L) 9/10	65.33	14.35	0.20	13.07	5.00	1.10	2.61	900.00	0.00	0.00	0.00	0.50	0.50	0.00
96	MERIDA, MEX.	168.00	8.20	0.02	33.60	5.00	0.24	0.54	383.00	0.50	0.00	0.00	0.00	0.00	0.50
97	SA (HI) 3/4 (JAL)	5.13	0.68	0.20	1.03	5.00	0.67	0.21	900.00	0.75	0.00	0.00	0.00	0.25	0.00

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