Push, Pull, or Both? A Behavioral Study of How the Allocation of Inventory Risk Affects Channel Efficiency

Andrew M. Davis
Samuel Curtis Johnson Graduate School of Management, Cornell University, Ithaca, New York 14853, adavis@cornell.edu
Elena Katok
Jindal School of Management, University of Texas at Dallas, Richardson, Texas 75080, ekatok@utdallas.edu
Natalia Santamaría
Samuel Curtis Johnson Graduate School of Management, Cornell University, Ithaca, New York 14853, nsantam@cornell.edu

In this paper we experimentally investigate how the allocation of inventory risk in a two-stage supply chain affects channel efficiency and profit distribution. We first evaluate two common wholesale price contracts that differ in which party incurs the risk associated with unsold inventory: a push contract in which the retailer incurs the risk and a pull contract in which the supplier incurs the risk. Our experimental results show that a pull contract achieves higher channel efficiency than that of a push contract, and that behavior systematically deviates from the standard theory in three ways: (1) stocking quantities are set too low, (2) wholesale prices are more favorable to the party stocking the inventory, and (3) some contracts are erroneously accepted or rejected. To account for these systematic regularities, we extend the existing theory and structurally estimate a number of behavioral models. The estimates suggest that a combination of loss aversion with errors organizes our data remarkably well. We apply our behavioral model to the advance purchase discount (APD) contract, which combines features of push and pull by allowing both parties to share the inventory risk, in a separate experiment as an out-of-sample test, and we find that it accurately predicts channel efficiency and qualitatively matches decisions. Two practical implications of our work are that (1) the push contract performs close to standard theoretical benchmarks, which implies that it is robust to behavioral biases, and (2) the APD contract weakly Pareto dominates the push contract; retailers are better off and suppliers are no worse off under the APD contract.

Data, as supplemental material, are available at http://dx.doi.org/10.1287/mnsc.2014.1940.

Keywords: behavioral operations management; inventory risk allocation; supply chain contracts

History: Received November 15, 2012; accepted February 21, 2014, by Serguei Netessine, operations management. Published online in Articles in Advance August 7, 2014.

1. Introduction

Location and ownership of inventory is one of the key drivers of supply chain performance. Even in a simple supply channel—single retailer, single supplier, and full information—researchers and companies have found that common wholesale price contracts with different inventory allocations affect channel efficiency (e.g., Lariviére and Porteus 2001, Cachon 2003, Kaya and Özer 2012). Determining the best channel design and inventory allocation in supply chains involves difficult trade-offs that can have a direct effect on a firm’s survival. For instance, Randall et al. (2002) provide examples of companies in which the difference between success and bankruptcy may be attributed to different inventory allocation strategies, whereas Randall et al. (2006) identify empirical relationships between a firm’s decision to own the inventory and several key performance indicators.

Traditional channels use a push structure in which the retailer makes stocking decisions, owns the inventory, and thus incurs the holding cost, as well as the cost of any unsold product. However, Internet-enabled technologies now permit other supply chain arrangements for allocating inventory ownership and risk (the cost of unsold inventory), which may affect channel profitability (see Cachon 2004, Netessine and Rudi 2006).

One such arrangement is the pull inventory system. Under this system the supplier makes the stocking (production) decision and therefore incurs the holding costs and inventory risk. The retailer provides a storefront (real or virtual), but products flow from the supplier to the end customer with minimal exposure of the retailer to inventory risk. One practical implementation of the pull inventory system is a drop-shipping arrangement—the retailer is never exposed to the inventory at all; the
supplier ships to customers directly. Such arrangements are prevalent in e-commerce; Randall et al. (2006) report that between 23% and 33% of Internet retailers use drop-shipping exclusively, and the U.S. Census estimates that e-commerce sales by retailers totaled $194 billion in 2011, up 16.4% compared with 2010 (U.S. Census Bureau 2013). Additionally, supply chains selling specialty products utilize pull structures (Klein 2009). The popularity of these contracts has even created opportunities for companies to specialize in providing drop-shipping services for businesses (Davis 2014 provides the example of CommerceHub). Less extreme pull arrangements exist as well, including just-in-time delivery, where the supplier delivers in small batches, thus becoming effectively responsible for holding cost and inventory risk; and vendor-managed inventory, in which the supplier makes stocking decisions but the retailer may hold the physical inventory.

Another inventory structure, often referred to as an advance purchase discount (APD) contract, combines aspects of the push and the pull systems so that both parties share the inventory risk (Cachon 2004). Cachon (2004) provides the example of O’Neill Inc., a manufacturer of water-sports apparel, which successfully uses the APD contract. In another study, Tang and Girotra (2010) evaluate how an APD structure impacts Costume Gallery, a privately owned wholesaler of dance costumes, and estimate that the company could increase its net profits by 17% if it adopted an APD contract.

The question of how to structure the channel to best allocate inventory risk, and the effect of inventory risk on channel performance, has been extensively studied analytically (Cachon 2004, Netessine and Rudi 2006, Özer and Wei 2006, Özer et al. 2007). Because in practice these channel design decisions are strategic, involve difficult trade-offs, and cannot be automated, senior managers must rely on their judgment and experience when they make these decisions. Consequently, it is important to understand how decisions—made by humans under different inventory risk structures—affect profits.

To gain insight into the role human judgment plays in channel design decisions, we conduct a set of laboratory experiments to explore human behavior in push and pull settings. We find that the pull contract delivers higher efficiency than does the push contract. Although this finding is in line with the standard theory, we also identify three ways in which behavior deviates from the normative prediction: (1) stocking quantities are lower than they should be, (2) wholesale prices systematically favor the party stocking the inventory, and (3) some profitable contracts are rejected, and vice versa. Therefore, we extend the standard model to account for these three behavioral regularities. We characterize and derive the equilibrium predictions for three behavioral

models that have been identified in the more recent literature (Cui et al. 2007, Ho and Zhang 2008, Su 2008, Ho et al. 2010), structurally estimate their parameters, and find that a simple model of loss aversion with random errors fits the data remarkably well.

We proceed to further investigate the loss aversion with a random errors model in the context of the APD contract, which includes push and pull features, and find that it accurately predicts channel efficiency and qualitatively matches decisions. We consider this an important contribution to the literature, because identifying systematic deviations from standard theory, and incorporating these behavioral regularities into analytical models, helps us to understand their causes and provides insights that result in designing contracting mechanisms that are behaviorally robust. Therefore, using our model, managers can make better decisions when designing channel structures.

We test the robustness of the loss aversion model by fitting it to two data sets from the literature, Katok and Wu (2009) and Becker-Peth et al. (2013), and we find that the estimated loss aversion parameters are of a similar magnitude to those in our data. We also conduct an additional APD treatment, with more symmetric bargaining power of the two players, and observe that the loss aversion parameter estimates are somewhat sensitive to the relative bargaining power in the channel.

Our experimental results highlight a number of managerial insights. First, we find that the performance of the push contract is closest to standard theoretical predictions. In that sense, one might say that of the three contracts we study, the push contract is most robust to behavioral biases. Second, we find that the APD contract, contrary to theory, fails to deliver higher supply chain efficiency than the pull contract. Our third practical finding is that the APD contract results in the most equitable profit distribution of the three contracts—under the APD contract, the supplier is as well off as under push, but the retailer is significantly better off. Thus, the APD contract Pareto weakly dominates the traditional push contract. This indicates that in supply chain settings with powerful suppliers, suppliers may wish to consider using an APD arrangement in favor of the simple push contract because it does not damage their own profitability but generally improves the retailer’s profit. Furthermore, fairer profit distribution may well result in additional benefits stemming from more cooperative long-term relationships.

2. Experimental Design and Standard Theory

2.1. Experimental Design

We evaluate three supply chain contracts, each in a separate between-subjects experimental treatment.
In the push and pull contracts, one party offers the wholesale price and the other party sets the stocking quantity (or rejects the contract). To be consistent with this structure, in our push treatment, the supplier offers the wholesale price and the retailer decides on the order quantity. Conversely, in our pull treatment, the retailer offers the wholesale price and the supplier decides on the quantity.

The APD contract differs from the push and pull contracts in that it includes two wholesale prices (the regular wholesale and discount wholesale prices), and both parties may share the inventory risk. The retailer incurs the inventory risk for a quantity ordered in advance of realized demand (called the prebook), and the supplier incurs the inventory risk on the difference between its production amount and the retailer’s prebook quantity. Specifically, in the APD treatment, the supplier moves first and proposes the two wholesale prices. After observing these prices, the retailer commits to paying for the prebook quantity (or rejects the contract). Next, the supplier sets the production quantity. Finally, demand and profits are realized for both players. We discuss more details of the APD contract in §4. Figure 1 depicts the decision sequence for all three treatments in our study.

In all three treatments a rejection results in both parties earning 60. In theory, this outside option is not binding for the push and pull contracts given our parameters, but it is binding under the APD contract, because the supplier has the ability to extract the entire channel profit (which we will show when we outline the APD theory). Our setting makes the APD prediction somewhat more realistic because the proposer should now extract most but not all of the channel profit.

Figure 1 Decision Sequence for Each Contract in the Experiment

We used the same demand distribution, per-unit revenue, and cost parameters in all three treatments, which were common knowledge to all participants. Specifically, customer demand is an integer uniformly distributed from 0 to 100, $U[0, 100]$; the retailer receives revenue of $r = 15$ for each unit sold; and the supplier incurs a cost of $c = 3$ for each unit produced.

In each treatment, we provided participants with a decision support tool. The player in the proposer role could test wholesale prices between 3 and 15 (the unit cost and revenue per unit) using a scroll bar. For each test wholesale price, the computer would show the proposer the stocking quantity that would maximize the other player’s expected profit. We made it clear that this quantity was best in terms of average profit for the other player for this test wholesale price but that they were playing with a human who may not necessarily stock this amount. Similarly, for the player setting a stocking quantity, once a wholesale price was offered, he or she could test different stocking quantities using a scroll bar (from 0 to 100). Each time the player moved the scroll bar, a line graph would display his or her profit, given the proposed wholesale price; this was calculated for every realization of demand (from 0 to 100). We provided this decision support tool to ensure that participants could comprehend the task and also allow the standard theory a viable chance of being confirmed. Screenshots of the participants’ decisions are included in the sample instructions and are available from the authors upon request.

In total, 120 human subjects participated in the study, 40 in each treatment. We randomly assigned subjects to a role (retailer or supplier) at the beginning of each treatment. To reduce the complexity of the game, roles remained fixed for the duration of each session. Subjects made decisions in 30 rounds. Retailers and suppliers were placed into a cohort of six to eight participants, and a single retailer was randomly rematched with a single supplier within the cohort in each round, replicating a one-shot game. To mitigate reputation effects, subjects were unaware that their cohort size was six to eight participants; they were simply told that they would be randomly rematched with another person each round. Each experimental treatment had six cohorts. Because subjects were placed into a fixed cohort for an entire session, we use the cohort as the main statistical unit of analysis.

We conducted the experiment at the Laboratory for Economics, Management and Auctions (LEMA) at Penn State University in 2010. Participants in all treatments were students, mostly undergraduates, from a variety

1 This value is slightly below the minimum of any party’s profits, in any contract, in equilibrium. The experimental profit predictions will be illustrated in the next section.

2 In the APD treatment this support was similar. Suppliers had to set two wholesale prices, then retailers could test the prebook quantity with a graph, and finally suppliers could also test production amounts with a graph.
of majors. Before each session, the subjects read the instructions themselves for a few minutes. Following this, we read the instructions orally and answered any questions to ensure common knowledge about the rules of the game. Each individual participated in a single session. We recruited subjects through an online system, offering cash participation. Subjects earned a $5 show-up fee plus an additional amount that was proportional to their total profits from the experiment. Average compensation for the participants, including the show-up fee, was $25. Each session lasted approximately 1 to 1.5 hours, and we programmed the software using the z-Tree system (Fischbacher 2007).

2.2. Theoretical Benchmarks
In all treatments, a retailer $R$ receives revenue $r$ for each unit sold, incurs no fixed ordering costs, and loses sales if demand exceeds inventory. A supplier $S$ produces inventory at a fixed per-unit cost of $c$. Customer demand $D$ is a continuous uniform random variable between 0 and $D$. Note that in our experiment, values are actually integers drawn uniformly (between 0 and 100). We assume that the application of the continuous theory to a discrete implementation is sufficiently precise. There is full information of all cost parameters, and we assume that retailers and suppliers are risk-neutral expected-profit maximizers. Finally, we measure efficiency by the percent ratio between the decentralized supply chain expected profit and the centralized supply chain expected profit. To distinguish between push and pull contracts, we mark the push contract by a caret ($\hat{\cdot}$).

For the push contract, a supplier offers a per-unit wholesale price $w$ to a retailer. The retailer sets a stocking quantity $q$ for a given $w$ that maximizes its expected profit, $\pi_s(q) = rS(q) - wq$. Let $S(q) = \mathbb{E}[\min(q, D)] = q - q^2/(2D)$ represent the expected sales for a stocking quantity $q$, and let $q^*$ be the quantity that maximizes the retailer’s expected profit under the push contract. In this case, the best-response stocking quantity for the retailer must satisfy the critical fractile

$$q^* = D\left(\frac{r - w}{r}\right).$$

The supplier’s decision under a push contract is the wholesale price $w$, where $\hat{w}$ maximizes the supplier’s push contract profit $\hat{\pi}_s(w) = (w - c)q$ and simplifies to

$$\hat{w} = \frac{r + c}{2}.$$

Under the pull contract, the decisions of the retailer and supplier are reversed; the retailer offers a per-unit wholesale price $w$, and the supplier then sets a stocking quantity $q$ that maximizes its expected profit, $\pi_s(q) = wS(q) - cq$. Let $q^*$ be the stocking quantity that maximizes the supplier’s expected profit. Then $q^*$ must satisfy the critical fractile

$$q^* = \frac{D}{w}.\frac{w - c}{w}.$$

The retailer’s decision under the pull contract is the wholesale price. Let $w^*$ be defined as the wholesale price that maximizes the retailer’s expected profit in the pull contract, where $\pi_s(w) = (r - w)S(q)$; $\pi_s(w)$ is unimodal in $w$ if demand has the increasing generalized failure rate property (Cachon 2004), so the optimal solution can be characterized using the first-order condition. Therefore, $w^*$ must satisfy $(w^*)^3 = c^2(2r - w^*)$.

Note that the push and pull contracts not only differ in who incurs the risk of unsold inventory but also in how the demand risk is shared. Under a pull contract, a retailer makes an order from a supplier in advance of demand, where a supplier can determine exactly what the retailer will order, produce that amount, and avoid any demand uncertainty. However, under a pull setting, the retailer pulls product from the supplier as demand is realized. Therefore, in a pull context, both parties share the risk associated with random demand.

2.3. Experimental Predictions
Table 1 summarizes the theoretical predictions for expected retailer and supplier profits and supply chain efficiency given our experimental parameters ($r = 15$, $c = 13$, demand $U[0, 100]$, and the outside option of 60). For wholesale prices, in the push contract, it is clear that $(15 + 3)/2 = 9.00$. In the pull contract, the first-order condition $w^3 = 270 - 9w$ is satisfied at a wholesale price of 6.00. Substituting these wholesale prices into the critical fractile solutions from the previous section leads to integer valued stocking quantities of 40 in the push contract and 50 in the pull contract. These predictions work well because, in our experiment, participants were allowed to enter their decisions up to two decimal places for the wholesale prices and integers for stocking quantities.

3. Results of the Push and Pull Contracts
We begin by presenting the results from the push and pull treatments separately. Following this, we present a number of behavioral models and structurally estimate their parameters.
with uncertain demand (they have only a push contract) in which both sides are human. They, too, find that the profit distribution is more equitable than the standard theory predicts. In a more recent paper, Kalkancı et al. (2014) also conduct an all-human study but focus on contract complexity involving asymmetric information.

3.2. Decisions
Table 3 summarizes the average wholesale prices and stocking quantities for the push and pull contracts for agreements. For both contracts, proposers set wholesale prices that are significantly different from the theoretical predictions ($p = 0.028$ for both push and pull). Specifically, for both contracts, the party setting the wholesale price made offers that were more generous than theory predicts. In the pull contract, the average wholesale price is below the prediction; in the pull contract, it is above the prediction.

To interpret the observed stocking quantities correctly, we calculate the optimal stocking quantities conditioned on the proposers’ wholesale prices, and we then average them for the predicted values. The second row of values in Table 3 shows these results. There are significant differences between observed and best-reply values in the pull contract ($p = 0.046$). In the push contract we find that the observed quantity is not statistically different from predicted ($p = 0.463$).

Recall that the party setting the stocking quantity, when receiving a wholesale price, had the option to reject the contract so that both parties earn an outside option of 60. In Table 3 we provide the predicted rejection rates, conditioned on the observed wholesale prices, along with the observed rejection rates. In the push contract, retailers rejected significantly more than predicted ($p = 0.028$), whereas in the pull contract, there are no significant differences in the predicted and observed rejection rates. Overall, the party stocking the inventory made the correct accept/reject decision 92.8% of the time in the push contract and 94.7% of the time in the pull contract. Moreover, rejection rates do not appear to change over time in either contract (based on a logit regression with random effects with

### Table 2: Average Profits for the Channel, Retailers, and Suppliers

<table>
<thead>
<tr>
<th>Profit</th>
<th>Push Predicted</th>
<th>Push Observed</th>
<th>Pull Predicted</th>
<th>Pull Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel profit</td>
<td>360.00</td>
<td>336.85</td>
<td>412.50</td>
<td>402.37</td>
</tr>
<tr>
<td>(Efficiency in %)</td>
<td>(75.00)</td>
<td>(17.54)</td>
<td>(85.94)</td>
<td>(6.79)</td>
</tr>
<tr>
<td>Retailer profit</td>
<td>120.00</td>
<td>130.14</td>
<td>337.50</td>
<td>257.32**</td>
</tr>
<tr>
<td></td>
<td>(75.00)</td>
<td>(85.00)</td>
<td>(12.35)</td>
<td></td>
</tr>
<tr>
<td>Supplier profit</td>
<td>240.00</td>
<td>206.71*</td>
<td>75.00</td>
<td>145.04**</td>
</tr>
<tr>
<td></td>
<td>(15.96)</td>
<td></td>
<td>(10.30)</td>
<td></td>
</tr>
</tbody>
</table>

*Note. Standard errors are reported in square brackets. **p < 0.05; *p < 0.10 (indicates significance of Wilcoxon signed-rank test compared with the predictions).

### Table 3: Average Wholesale Prices and Quantities for Agreements, and Average Rejection Rates

<table>
<thead>
<tr>
<th></th>
<th>Push Predicted</th>
<th>Push Observed</th>
<th>Pull Predicted</th>
<th>Pull Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale price</td>
<td>9.00**</td>
<td>8.26</td>
<td>6.00**</td>
<td>8.02</td>
</tr>
<tr>
<td></td>
<td>[0.16]</td>
<td></td>
<td>[0.28]</td>
<td></td>
</tr>
<tr>
<td>Quantity</td>
<td>w</td>
<td>44.93</td>
<td>42.40</td>
<td>61.26**</td>
</tr>
<tr>
<td>(w)</td>
<td>[1.04]</td>
<td>[2.86]</td>
<td>[1.30]</td>
<td>[1.72]</td>
</tr>
<tr>
<td>Rejection rate</td>
<td>w</td>
<td>0.020**</td>
<td>0.082</td>
<td>0.048</td>
</tr>
<tr>
<td>(w)</td>
<td>[0.011]</td>
<td>[0.020]</td>
<td>[0.019]</td>
<td>[0.009]</td>
</tr>
</tbody>
</table>

*Notes. Standard errors are reported in square brackets. Predicted quantities and rejections are conditioned on observed wholesale prices. **p < 0.05 (indicates significance of Wilcoxon signed-rank test).
the decision period as the independent variable). These results suggest that, with the provided decision support tools, subjects were generally able to comprehend the task.

Focusing only on the players who set the stocking quantity, Table 4 shows how much supply chain efficiency was lost based on (1) incorrect rejections, (2) incorrect acceptances, and (3) incorrect stocking quantities, with respect to the standard theoretical predictions. In the push contract, 3.15% of the predicted efficiency was lost from rejecting favorable offers, whereas 5.49% was lost from suboptimal stocking quantities. In the pull contract, only 0.94% efficiency was lost as a result of rejecting favorable offers, but 7.59% was lost as a result of low stocking quantities.

We emphasize that the numbers in Table 4 are calculated against the normative benchmark. It is possible that an incorrect rejection might actually be correct if decision makers know that they will not stock in line with the standard theory. With this in mind, it still appears that accept/reject decisions and quantities play a role in efficiency losses; therefore, we consider both of these effects in our behavioral models that we develop in the next section.

### 3.3. Behavioral Models

Our goal in this section is to formulate a parsimonious behavioral model that can explain the deviations we observe in our data. These deviations are (1) order quantities that are below predicted levels in both contracts, (2) wholesale prices that are below predicted in the push contract and above predicted in the pull contract, and (3) incorrect responder accept/reject decisions. We consider behavioral models that have been proposed in recent literature: loss aversion from leftover inventory (Ho et al. 2010, Becker-Peth et al. 2013), inequality aversion (Cui et al. 2007), anchoring toward the mean (Schweitzer and Cachon 2000, Benzion et al. 2008), and random errors in accept/reject decisions (Su 2008).

Because of the nature of these behavioral regularities, it is natural to assume that the proposer may have none of these biases. In particular, losses and anchoring cannot happen since proposers do not hold inventory, and inequality aversion is unlikely to play a major role because proposers work under advantageous inequality (it has been shown that advantageous inequality aversion is virtually nonexistent in the laboratory; see De Bruyn and Bolton 2008, Katok and Pavlov 2013). Also, Katok and Wu (2009) find that when suppliers in a push wholesale contract are matched with automated retailers programmed to place optimal orders, suppliers quickly learn to set wholesale prices optimally. Considering that the behavioral regularities we investigate are unlikely to be present for proposers, we begin by making an assumption that proposers are fully rational. We introduce the following notation for our behavioral parameters:

- $\beta > 1$: The degree of loss aversion that the party stocking the inventory experiences from having leftover inventory. Note that $\beta > 1$ implies loss aversion, and $\beta = 1$ corresponds to rational behavior (see Ho et al. 2010 and Becker-Peth et al. 2013 for related formulations).
- $\alpha > 0$: The degree of disadvantageous inequality aversion (see Cui et al. 2007). We assume that decision makers do not have disutility from advantageous inequality.
- $0 \leq \delta \leq 1$: The degree of anchoring toward the mean (see Benzion et al. 2008 and Becker-Peth et al. 2013 for a similar approach).

We consider each of the above behavioral issues separately but will add random errors in accept/reject decisions when we discuss our parameter estimation.

### 3.3.1. Push Behavioral Models

In Table 5 we outline each of the behavioral models for the push setting and demand following a continuous uniform distribution between 0 and $\bar{D}$. We relegate the derivations to the appendix.

In Table 5, $\mu = 50$, $\hat{u}_R(q)$ denotes the retailer’s expected utility, $\pi_S(w) = (w - c)q$, $\tilde{u}_R(q) = rS(q) - wq$, $\hat{w} = (r(1 + \alpha) + c(2 + \alpha)) / (3 + 2\alpha)$, and $\tilde{w} = (r + 2c) / 3$. The three cases in the inequality aversion model stem

| Table 4 Efficiency Impacts in the Push and Pull Contracts |
|-------------|-------------|
|             | Push   | Pull   |
| Efficiency given correct accept/reject and quantity | 78.65% | 90.96% |
| Efficiency lost from incorrect rejection          | 3.15%  | 0.94%  |
| Efficiency lost from incorrect acceptance         | 0.16%  | -1.40% |
| Efficiency lost from incorrect quantity           | 5.49%  | 7.59%  |
| Observed efficiency                               | 70.18% | 83.83% |

<table>
<thead>
<tr>
<th>Table 5 Push Contract: Retailer Expected Utility Functions and Optimal Stocking Quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss aversion $\tilde{u}_R(q) = (r - w)S(q) - \mu(S(q) - S(q))$; $\hat{q} = \frac{\bar{D} (r - w)}{\bar{D} \bar{D} - \bar{D} - 1}$.</td>
</tr>
<tr>
<td>Inequality aversion $\hat{u}_R(q) = rS(q) - wq - a(\pi_S(w) - \pi_S(q))$; $\hat{q} = \frac{\bar{D} (r - w + a(1 + \alpha) - 2w)}{r(1 + \alpha)}$; $\hat{w} = \frac{\bar{D} (2r + c - 2w)}{r - 2w}$; $\hat{q} = \frac{\bar{D} (r - w)}{\bar{D}}$; $w &lt; \hat{w}$.</td>
</tr>
<tr>
<td>Anchoring $\hat{u}_R(q) = rS(q) - wq$; $\hat{q} = (1 - \delta) \left( \frac{\bar{D} (r - w)}{\bar{D}} \right) + \delta \mu$.</td>
</tr>
</tbody>
</table>
from the final term in the retailer’s expected utility function, \((\hat{\pi}_S(w) - \hat{\pi}_R(q))^+\). When \(w \geq \bar{w}\), the retailer stocks in a way that earns him less than (or the same as) the retailer or the supplier in terms of expected profit; hence \((\hat{\pi}_S(w) - \hat{\pi}_R(q)) \geq 0\). When \(\bar{w} < w < \bar{w}\), the retailer stocks so that the two parties make the same expected profit, and \((\hat{\pi}_S(w) - \hat{\pi}_R(q)) = 0\). Finally, when \(w \leq \bar{w}\), the retailer stocks so that supplier earns less than (or the same as) the retailer; therefore \((\hat{\pi}_S(w) - \hat{\pi}_R(q)) \leq 0\). Note that in this last case, the retailer has no inequality concerns, and the optimal quantity corresponds to the critical fractile from §2.2.

In terms of the supplier’s optimal wholesale price, the supplier takes into account the retailer’s stocking quantity bias (and errors in accept/reject decisions, outlined in §3.4) and sets the wholesale price in a way that maximizes his expected profit. We compute this optimal wholesale price numerically.

3.3.2. Pull Behavioral Models. As with the push contract, we outline each of the behavioral models for the pull setting and demand following a continuous uniform distribution between 0 and \(D\), as shown in Table 6. Please see the appendix for details.

In Table 6, \(\mu = 50\), \(\pi_R(w) = (r - w)S(q)\), \(\pi_S(q) = wS(q) - cq\), \(\bar{w} = (r + \sqrt{r^2 + 8cr})/4\), and

\[
\hat{w} = \frac{1}{4(1 + 2\alpha)} \left[ 4ar + r + 2ac \right. \\
\left. + \sqrt{(4ar + r + 2ac)^2 + 8r(1 + 2\alpha)(c(1 - \alpha) - ar)} \right].
\]

The three cases in the inequality aversion model for the pull contract, as with the push contract, come from the final term in the supplier’s expected utility function, \((\pi_S(w) - \pi_S(q))^+\). When \(w \leq \bar{w}\), the supplier stocks in a way that earns him less than (or the same as) the retailer in terms of expected profit; hence \((\pi_S(w) - \pi_S(q)) \geq 0\). When \(\bar{w} < w < \bar{w}\), the supplier stocks such that the two parties make the same expected profit, and \((\pi_S(w) - \pi_S(q)) = 0\). Finally, when \(w \geq \bar{w}\), the supplier stocks so that the retailer earns less than (or the same as) the supplier, \((\pi_R(w) - \pi_S(q)) \leq 0\); in this case, the supplier has no inequality concerns, and the optimal quantity is the same as the standard critical fractile in §2.2.

In terms of the retailer’s optimal wholesale price, the retailer takes into account the supplier’s stocking quantity bias (and errors in accept/reject decisions, outlined later) and sets the wholesale price in a way that maximizes his expected profit, where we compute this optimal wholesale price numerically.

3.4. Structural Estimation of the Behavioral Models

We fit the stocking quantities, accept/reject decisions, and wholesale prices to find the levels of loss aversion, inequality aversion, and anchoring that match our data best using maximum likelihood estimation (MLE). This allows us to compare the overall model fits and determine which behavioral factors are likely driving the regularities we observe.

For stocking quantities, we assume errors follow a normal distribution with left-side truncation 0 and right-side truncation 100. Let \(\psi(\cdot)\) denote the density of the truncated normal distribution for quantities, with the optimal quantity as the mean and variance \(\sigma_q^2\). Also, as previously noted, subjects exhibited errors with respect to their accept/reject decisions. Therefore, we assume that the utility of the party stocking the inventory has an extreme value error term so that the probability of accepting a wholesale price follows a logistic form with precision parameter \(\tau\):

\[
\frac{\exp[u_V(q)/\tau]}{\exp[u_V(q)/\tau] + \exp[u'_V/\tau]}.
\]

Let \(V\) denote the party setting the stocking quantity, and let \(w'_V\) reflect the outside option (60 in our experiment). As \(\tau \to 0\), the party stocking the inventory accepts any wholesale price that results in his expected utility exceeding the outside option. Similarly, as \(\tau \to \infty\), the party stocking the inventory accepts with probability 1/2. As mentioned previously, for wholesale prices, we assume that a proposer takes the responder’s stocking bias and noisy accept/reject decision into account and sets the wholesale price in a way that maximizes his expected profit. For the estimation of wholesale prices, we assume errors follow a normal distribution with left-side truncation \(c = 3\) and right-side truncation \(r = 15\). Let \(\psi(\cdot)\) denote the density of the truncated normal distribution for wholesale prices, with the optimal wholesale price as the mean and variance \(\sigma_w^2\).
Table 7  Results of the Structural Estimations for Each of the Outlined Behavioral Models on the Aggregated Push and Pull Data

<table>
<thead>
<tr>
<th>Fit</th>
<th>Baseline</th>
<th>Errors</th>
<th>Errors + Loss aversion</th>
<th>Errors + Inequality aversion</th>
<th>Errors + Anchoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>−7,783.4</td>
<td>−6,954.6</td>
<td>−6,925.4</td>
<td>−6,935.5</td>
<td>−6,946.3</td>
</tr>
<tr>
<td>Push predictions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{w}$</td>
<td>9.00</td>
<td>8.21</td>
<td>8.13</td>
<td>8.16</td>
<td>8.32</td>
</tr>
<tr>
<td>$\hat{q}$</td>
<td>44.93</td>
<td>44.93</td>
<td>41.06</td>
<td>43.32</td>
<td>45.61</td>
</tr>
<tr>
<td>Pull predictions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{w}$</td>
<td>6.00</td>
<td>7.51</td>
<td>7.55</td>
<td>7.64</td>
<td>7.55</td>
</tr>
<tr>
<td>$\hat{q}$</td>
<td>61.26</td>
<td>61.26</td>
<td>57.30</td>
<td>59.05</td>
<td>59.35</td>
</tr>
<tr>
<td>Estimates</td>
<td>$\beta$</td>
<td></td>
<td>1.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\delta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tau$</td>
<td></td>
<td>29.3</td>
<td>24.3</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\theta$</td>
<td>17.2</td>
<td>17.2</td>
<td>16.6</td>
<td>16.9</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\omega$</td>
<td>2.17</td>
<td>1.38</td>
<td>1.38</td>
<td>1.36</td>
</tr>
</tbody>
</table>

Notes: Standard errors are reported in square brackets. The prediction rows calculate the optimal wholesale prices $\hat{w}$ and quantities $\hat{q}$, given the maximum-likelihood estimates. The predicted quantities also consider the observed wholesale prices.

The joint-likelihood function, where $t$ is a single decision period and $T$ denotes the total number of periods, is given by

$$L(\beta, \alpha, \delta, \tau, \sigma_\theta, \sigma_\omega) = \prod_{t \in T} \varphi(q_t)^{A_t} \psi(w_t)^{A_t} \Pr(\text{Accept})^{A_t} \cdot (1 - \Pr(\text{Accept}))^{1 - A_t},$$

where $A_t = 1$ if the proposed wholesale price was accepted in period $t$ and 0 otherwise.

In Table 7 we present the estimates for a baseline model, an errors model, and three errors models with loss aversion, inequality aversion, or anchoring, respectively. The baseline model assumes that quantities are set according to the normative critical fractiles outlined in §2.2, and the wholesale prices that best reply to this behavior, and estimates $\sigma_\theta$ and $\sigma_\omega$. According to the log-likelihood values (LL), allowing errors in the accept/reject decision improves the fit a great deal over the baseline model, as does errors plus loss aversion (a likelihood ratio test yields $\chi^2 = 1674.21$ between the baseline model and errors model, and $\chi^2 = 58.36$ between the errors model and errors plus loss aversion model, both $p < 0.001$). This is in line with our experimental data in that subjects did not always make correct accept/reject decisions, and parties set stocking quantities too low, as if the cost of unsold inventory was greater than its true value.

To compare the three behavioral models with errors more rigorously, we conducted a Vuong (1989) test. The Vuong test results show that the loss aversion model is significantly better than both the anchoring model ($p = 0.004$) and inequality aversion model ($p = 0.02$), though the inequality aversion model is a marginally better fit than the anchoring model ($p = 0.056$). We focus on the errors plus loss aversion model in the out-of-sample test section (presented in §4) because it provides the best overall fit.

Table 7 also shows the predicted order quantities (and wholesale prices) given the maximum-likelihood estimates for each model. One can see that the predictions for most all of the models evaluated, when compared with the observed values in Table 3, are a substantial improvement over the baseline, which assumes the standard theoretical predictions. In regard to the best-fitting model, errors plus loss aversion, we find that it matches our data well. We denote predictions with a tilde ($\tilde{\cdot}$): $\tilde{\hat{w}} = 41.06$ versus $q = 42.60$ for the push contract and $\tilde{\hat{q}} = 57.30$ versus $q = 55.34$ for the pull contract. We also evaluated whether the errors and loss aversion model fit the observed rejection rates. Figure 2 plots the predicted rejection rates along with the observed rejection data for the push and pull contracts. Although there are some deviations in

\[\text{Notes: Standard errors are reported in square brackets. The prediction rows calculate the optimal wholesale prices } \hat{w} \text{ and quantities } \hat{q}, \text{ given the maximum-likelihood estimates. The predicted quantities also consider the observed wholesale prices.} \]
both directions, it appears that the model provides a reasonable fit.

Finally, recall that we assumed wholesale prices were offered by fully rational parties who have beliefs about the party stocking the inventory and best reply to this behavior. We can test how close this assumption is to reality by comparing the predicted best reply wholesale prices, given the estimates, with the average observed wholesale prices in Table 3. First, it is worth noting that the errors model greatly improves wholesale prices by adding only a single parameter, \( \tau \). Second, in terms of the errors plus loss aversion model, we observe accurate predictions as well: \( \hat{w} = 8.13 \) versus \( w = 8.26 \) for push and \( \hat{w} = 7.55 \) versus \( w = 8.02 \) for pull. As with the stocking quantities and accept/reject decisions, the predicted wholesale prices are remarkably close to our data, indicating that an errors plus loss aversion model is useful in organizing all the decisions.

4. Advance Purchase Discount Contract

The push and pull wholesale price contracts cannot coordinate the channel because of double marginalization. However, Cachon (2004) shows that the APD contract can coordinate the channel by distributing inventory risk between the supplier and the retailer. Next we will review the theory for the APD contract under our behavioral model, and we develop a number of experimental hypotheses that we will proceed to test in a separate, out-of-sample experiment.

4.1. APD Behavioral Model

Under the APD contract, a supplier begins by proposing two wholesale prices, a regular wholesale price \( w \) and a discount wholesale price \( w_d \). It is reasonable, although not necessary, to assume \( w \geq w_d \) (see Özer and Wei 2006 for a slightly different setting where this is relaxed). A retailer then sets a prebook quantity \( y \), where the retailer commits to purchasing the entire prebook quantity regardless of demand and pays \( w_d \) for each unit of the prebook quantity. Following this, a supplier sets a production amount \( q \), where \( q \geq y \). We will outline the APD contract for our behavioral model, noting that the standard theory is the special case of \( \beta_R = \beta_S = 1 \), where \( \beta_R \) represents the retailer’s loss aversion and \( \beta_S \) represents the supplier’s loss aversion.

Table 8 shows the expected utility functions and the corresponding optimal order quantities when demand is uniformly distributed between 0 and \( \bar{D} \) (please see the appendix for corresponding derivations).

In Table 8, \( K(y, q) \) corresponds to the expected number of units the supplier sells when the retailer prebooks \( y \) units and the supplier produces \( q \) units,

\[
K(y, q) = E[\min(\max(y, D), q)] = q - \left( \frac{q^2 - y^2}{2D} \right).
\]

The first term in the expected utility function for the supplier represents immediate revenue from the retailer’s prebook quantity, the second term represents the additional revenue from selling any units above the prebook quantity, the third term is the supplier’s production cost for units sold, and the last term represents the cost and disutility from any potential leftover units. Similarly, the first term in the expected utility for the retailer represents the profits from prebook sales, the second term the additional profits from units sold

\[
\begin{array}{l}
\text{Table 8 } \\
\text{APD Contract: Expected Utility Functions for Suppliers and Retailers, and Their Optimal Stocking Quantities}
\end{array}
\]

<table>
<thead>
<tr>
<th>Supplier</th>
<th>( u_S(y, q) = w_Sy + w(K(y, q) - y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-cK(y, q) - \beta_Sc(q - K(y, q)));</td>
</tr>
</tbody>
</table>
|          | \( q^* = \left\{ \begin{array}{ll}
(\bar{D} - w - c)/(w + c (\beta_S - 1)) & w \geq \bar{D} + y (\beta_S - 1) \\
y & w < \bar{D} + y (\beta_S - 1)
\end{array} \right. \)
| Retailer | \( u_R(y, q) = (r - w_S)S(y) + (r - w) \) |
|          | \(-S(q) - S(y) - \beta_Rw_S(y - S(y))\); |
|          | \( y^* = \bar{D}(w - w_S)/(w + w_S (\beta_R - 1)) \). |
beyond the prebook order, and the third term is the cost and disutility of any leftover prebooked units.

Finally, we allow errors to affect the APD contract the same way that they affect the push and pull contracts. The retailer, faced with a proposed set of wholesale prices \( w \) and \( w_d \), accepts with probability

\[
\frac{\exp[u_y(y, q)/\tau]}{\exp[u_y(y, q)/\tau] + \exp[u_y^d(y)/\tau]},
\]

where \( u_y^d \) is the retailer’s outside option profit.

A few comments are in order regarding the wholesale prices in the APD contract. Consider the special case of the standard theory, such that \( \beta_R = \beta_S = 1 \) and \( \tau \to 0 \). Under this setting, the supplier can achieve 100% channel efficiency by setting \( w = r \) and producing the first-best order. If the retailer plays the best response, \( w_d \) determines the division of channel profits. For example, if both parties set \( y = y^* \) and \( q = q^* \), then the supplier can extract 100% of the channel profits by setting \( w_d = w \). On the other hand, if the supplier sets \( w_d = c^* \), the retailer would earn 100% of the channel profits because the retailer would be induced to set \( y \) to the first-best order quantity, which the supplier would produce.

For our experimental setting, the standard theory predicts \( y^* = 28.30, q^* = 80.00 \), and optimal wholesale prices of \( w^* = 15.00 \) and \( w_d^* = 10.75 \). It also predicts 100% channel efficiency, where the split of profits is 419.79 for the supplier and 60.21 for the retailer. Note that the standard theory results in \( r > w_d^* \) for our experiment, as retailers will accept only if their expected profit is greater than 60, the value of the outside option.

4.2. Out-of-Sample Test

We now formulate several hypotheses for the APD contract that follow from the behavioral model. Our goal here is not to identify the best-fitting model for the APD contract but instead to evaluate how a model that fits the push and pull contracts performs under alternative structures, such as an APD contract. This leads to our first hypothesis.

**Hypothesis 1 (Model).** The loss aversion with errors model will fit the data better than the baseline model.

In some ways the APD contract can be considered a combination of the push and pull contracts; the retailer’s prebook, and its associated cost of unsold inventory, is essentially a push contract. Additionally, the difference between the supplier’s production amount and the prebook, and its cost of unsold inventory, is similar to a pull contract. To further develop a hypothesis about the APD contract’s performance, we use the push and pull data to estimate the loss aversion parameter under the two contracts separately. It turns out that under push, \( \beta = 1.08 \) (SE = 0.03), but under pull, \( \beta = 1.26 \) (SE = 0.04). And the rationality parameters under the two contracts are similar to one another: \( \tau = 23.5 \) under push and \( \tau = 24.3 \) under pull. Combining the difference in loss aversion estimates with the fact that under the APD contract the optimal prebook quantity is decreasing in \( \beta_R \) and the optimal production amount is decreasing in \( \beta_S \) (see Table 8) leads to our second formal hypothesis.

**Hypothesis 2 (Quantities).** Retailers will set prebook quantities only slightly below the standard theoretical benchmark, and suppliers will set production levels below the standard theoretical benchmark, where the standard theoretical benchmarks are conditioned on the observed wholesale prices.

Given predictions about prebook quantities and production amounts, we can determine the optimal wholesale prices for the supplier. Cachon (2004) notes that in a fully rational model, the supplier maximizes his expected utility by coordinating the channel. Therefore, in this case he sets \( w = r \) and then sets \( w_d \) in a way that splits the channel profits between the two parties (higher \( w_d \) leads to more profit for the supplier, and vice versa). However, when the party setting the stocking quantity is loss averse and makes errors, \( w = r \) may not maximize the supplier’s expected utility. Therefore, the supplier’s optimal wholesale prices \( w \) and \( w_d \) can be computed by replacing the optimal order quantities in his utility function and using the first-order conditions to solve for the two wholesale prices simultaneously. The resulting expression is a third-degree polynomial, and therefore closed-form solutions cannot be readily interpretable. However, for any specific set of parameters and demand distributions, one can compute the optimal wholesale prices.

Figure 3, panel (a) plots the optimal wholesale prices given our experimental parameters, when loss aversion is equal for both parties and cases of \( \tau = 1 \) and \( \tau = 24 \), we chose these levels of \( \tau \) to illustrate how prices behaved when there is minimal noise and noise that is similar to the estimates from the push and pull data. Figure 3, panel (b) depicts a similar plot for \( \tau = 24 \) but fixes the retailer’s loss aversion parameter at \( \beta_R = 1.08 \) (the estimate from the push data) and allows the supplier’s loss aversion parameter to vary.

In Figure 3, panel (a), when the loss aversion is restricted to be the same across both parties, the optimal wholesale prices converge for small levels of loss.

---

5 We also fit the errors plus anchoring and errors plus inequality aversion plus models to the push and pull data separately (four estimations). We find that the errors plus loss aversion model generates a higher log-likelihood than both models for either data set, although the difference is not significant in two of the four comparisons.
In regard to predicting the wholesale prices of the APD contract, we can take our structural estimates from push and pull separately, and those observations mentioned above on wholesale prices, and articulate them into our third hypothesis.

Hypothesis 3 (Wholesale Prices). The regular wholesale price will equal 15.00 and the discount wholesale price will be less than 10.75.

Our last hypothesis relates to channel efficiency. Overall supply chain efficiency is driven primarily by the supplier in setting production quantities and the retailer’s accept/reject decision. Based on the pull loss aversion estimate of 1.26, and considering that from Table 8 optimal production amounts are decreasing in $\beta_S$, one would suspect that production quantities will be set lower than the standard theory prediction, driving channel efficiency to less than 100%. This reduction in efficiency will be further exacerbated by a retailer making erroneous accept/reject decisions.

Hypothesis 4 (Efficiency). The presence of loss aversion on the supplier’s side, and errors in the retailer’s accept/rejection decision, will drive channel efficiency below 100%.

4.3. Results of the APD Contract

Before evaluating our formal hypotheses, we find that in the APD experiment, the channel profit is 403.75 (SE = 5.93), which leads to a channel efficiency of 84.11%, significantly below the normative efficiency prediction of 100%. Unlike the push and pull results, the APD contract performs far below the standard theoretical prediction in terms of efficiency. In fact, the observed supply chain efficiency is virtually identical between the APD contract and pull contract (see Table 3). This suggests, counter to standard theory, that moving from a pull contract to an APD contract does not improve overall efficiency.

In terms of average profits, we observe first that profits are split in a more equitable way than the standard theory predicts. The average retailer profit under the APD contract is 188.33 (SE = 10.24)—significantly above the prediction of 60—and average supplier profit is 215.41 (SE = 10.96)—significantly below the prediction of 420. Second, comparing the APD contract to the push contract, we observe that the APD contract weakly Pareto dominates the push contract. Specifically, retailers are better off under the APD contract compared with the push contract, and suppliers are no worse off.

Unlike the push and pull contracts, we do observe that prices change with experience in the APD treatment. In Figure 4, we plot average wholesale prices over time. It is apparent from the figure that both wholesale prices increase rather quickly—suppliers
learn to design more profitable contracts. This learning may be because the APD contract is more complex than the push and pull contracts. A set of linear regressions with random effects, with the period as the independent variable and the two wholesale prices as dependent variables (in separate regressions), confirms this, with the coefficient on period being positive and significant.

We now turn to evaluating our formal hypotheses. Because there are changes in subjects’ decisions over time, we partition the data into thirds (10 rounds each) and conduct estimations on the first third and final third. We repeat this for the baseline model and the errors plus loss aversion model, and we report results in Table 9. Consistent with Hypothesis 1, we observe the considerable improvement in fit over the baseline model, given by the larger log-likelihood values (a likelihood ratio test yields \( \chi^2 = 372.74 \) for the first third and \( \chi^2 = 379.26 \) for the final third, both \( p < 0.001 \)).

We can also gain a preliminary sense of the performance of the other hypotheses from the estimates in Table 9. When looking at the first third of the data, estimated loss aversion parameters are close to those we estimated from the push and pull data, \( \beta_R = 1.11 \) and \( \beta_S = 1.22 \) (compared with 1.08 under push and 1.26 under pull). However, the degree of loss aversion increases throughout the session, and in the last 10 periods we see \( \beta_R = 1.23 \) and \( \beta_S = 1.53 \). Comparing the two players’ estimates, the retailer’s loss aversion bias is smaller than the supplier’s. One potential reason for this could be because the retailer is less susceptible to having leftover inventory than the supplier, as the prebook quantity is always equal to or smaller than the supplier’s production amount. In other words, the retailer observes leftover inventory less frequently than

\[ \tau = 49.3 \] \[ 4.79 \] \[ 3.54 \]

\[ a_f = 23.2 \] \[ 0.19 \]

\[ a_o = 21.0 \] \[ 1.35 \]

\[ a_w = 1.22 \] \[ 0.91 \]

\[ \sigma_d = 3.99 \] \[ 0.14 \]

\[ \mu(\sigma_d) = 0.87 \] \[ 0.20 \]

\[ \mu(\sigma_d^2) = 0.75 \] \[ 0.04 \]

\[ \mu(\sigma_d^3) = 0.78 \] \[ 0.03 \]

\[ \mu(\sigma_d^4) = 0.57 \] \[ 0.00 \]

Note. Standard errors are reported in square brackets.

The overall fit of our behavioral model is better when looking at the final third of the data, which accounts for learning and, more importantly, suggests that subjects updated their behavior over time in a way that is more in line with our behavioral model (LL of \(-2,534.5 \) versus \(-2,490.5 \)). Therefore, we will use the last third of the data (the final column in Table 9) to generate predicted quantities, wholesale prices, and profits for the behavioral model.

Table 10 presents these predictions, along with the observed data, conditional theory, and standard theory. The column labeled “Standard theory” highlights the original experimental predictions based on the special case of \( \beta_R = \beta_S = 1 \) outlined in §4.1. The column labeled “Conditional theory” represents the standard theory’s best reply when conditioned on decisions. Specifically, prebook quantities and production amounts are conditioned on observed wholesale prices, which are then used to generate profits. We provide these two

\[ 7 \] Despite prices changing in the APD contract, efficiency is constant over time (85.7% for the first third and 84.5% for the final third), and the APD weakly Pareto dominates the push contract even when looking at the final third of the data; see the next paragraph for more details on partitioning. The distribution of profits slightly varies though, with retailers making around 170 in the final third.

\[ 8 \] Because the supplier sets both prices simultaneously, we assume a bivariate normal distribution for prices with correlation \( \rho_{(w, w')}. \)
we find that a model with errors but no loss aversion what occurs in the data; thus this is consistent with we calculate predicted production levels based on the predicted presence of loss aversion for the supplier, 14.06, and the predicted discount wholesale price is parameters, the predicted regular wholesale price is an improvement over the standard theory but that are significantly lower than the standard retail prices are slightly below predicted. We compared with the data (66.05 versus 59.26). Thus, confirming this hypothesis. When the MLEs, the behavioral model is relatively accurate so) as a result of the loss aversion parameter for suppliers being close to 1.00. Furthermore, production amounts are significantly lower than the standard theory predicts, thus confirming this hypothesis. When we calculate predicted production levels based on the MLEs, the behavioral model is relatively accurate compared with the data (66.05 versus 59.26).

Turning to our third hypothesis, which deals with wholesale prices, we find that the behavioral model is an improvement over the standard theory but that there are still some differences. Given the loss aversion parameters, the predicted regular wholesale price is 14.06, and the predicted discount wholesale price is 8.92—both significantly above observed prices. Therefore, we reject our third hypothesis.

Finally, our fourth hypothesis deals with efficiency. The predicted presence of loss aversion for the supplier, combined with a retailer’s errors, should drive the expected supply chain efficiency below 100%, which is what occurs in the data; thus this is consistent with the hypothesis. More precisely, the observed supply chain efficiency for the final third of decisions is 84.5%, whereas the loss aversion plus errors model predicts efficiency of 84.1%. To determine whether loss aversion or errors is the primary driver for lower efficiency, we find that a model with errors but no loss aversion ($\beta_S = \beta_S = 1$) leads to 94.9% efficiency, whereas a model with loss aversion but no errors generates efficiency of 88.8%, suggesting that loss aversion is the primary culprit of efficiency reductions.

Overall, we find qualitative support for three of our four hypotheses but not Hypothesis 3, which deals with wholesale prices. In this case, the observed wholesale prices are slightly below predicted. We offer two informal explanations for the lower prices: random errors and learning. First, because the optimal wholesale price is 14.06, which is close to the selling revenue per unit of 15, if suppliers make random errors, one would expect the average observed wholesale price to be below 14.06, because there is more room for errors below 14.06 than between 14.06 and 15.00.

The second informal explanation relates to Figure 4, in that both wholesale prices are trending up over time to the behavioral predictions. There was a substantial improvement (about 70% in terms of supplier expected utility) between the first third and last third of the session. It may be that the wholesale prices would be closer to the behavioral predictions if given more decisions.

### 4.4. Robustness Checks

To check the robustness of the behavioral model, we conducted two additional analyses: (1) we fit the model to data sets from two existing research studies, and (2) we conducted an additional APD contract treatment in which we balanced the bargaining power of the two players.

For our first robustness check, we obtained data from Katok and Wu (2009) and Becker-Peth et al. (2013). Both of these studies focus exclusively on push contracts. Specifically, Katok and Wu investigate wholesale price, buyback, and revenue sharing push contracts, and Becker-Peth et al. study buyback push contracts. Because there are no accept/reject or pricing decisions in these experiments, and contract parameters are exogenously set, we took the stocking quantity data and fit our loss aversion model (without errors) to these decisions. Despite a number of other differentiating factors between ours and these studies, such as a lack of decision support, varying feedback, 1 round or 200 rounds of decisions, different cost and price parameters, and no accept/reject or price decisions, we find that the loss aversion model fits both of these data sets well. The estimates are $\hat{\beta} = 1.45$ for the Katok and Wu data set and $\hat{\beta} = 1.89$ for the Becker-Peth et al.

\[ R = \frac{\beta_d}{\beta_d} = 1 \]
data set, both significantly greater than 1. Additionally, in both cases, the fit from the loss aversion model is statistically better than the normative benchmark (likelihood ratio test yields $\chi^2 = 624.4$ and $\chi^2 = 170.4$, both $p < 0.001$). The loss aversion levels are not identical to those estimated from the data in this paper, but when considering the differences between our studies, these results indicate that the loss aversion model is generally robust.\footnote{There are two other behavioral operations management studies that use loss aversion to explain behavior in a setting in which the retailer, who is averse to a fixed fee under the two-part-tariff contract, is a monopolist rather than a news-vendor. Ho and Zhang (2008) report the loss aversion parameter of 1.37 under the two-part-tariff contract and 1.27 under the mathematically identical quantity discount contract. Haruvy et al. (2013) report the loss aversion parameter of 1.44 under the ultimatum bargaining protocol and 1.23 under a structured bargaining protocol.}

Our second robustness check involved an additional experiment treatment. In all our previous three treatments, one party has considerable bargaining power by proposing the wholesale price(s) (i.e., suppliers set the price in push, retailers set the price in pull, and suppliers set both prices in APD). To determine whether our results vary when there is a more equitable bargaining split in how parties set prices, we collected data on a new APD contract, called the APD alternative, with the same ability to split inventory risk but different bargaining power structure. Under this APD alternative, the supplier proposes the discount wholesale price, then the retailer offers the regular wholesale price and prebook, and finally the supplier decides on a production quantity. This way, both parties set a wholesale price and quantity. The APD alternative cannot fully coordinate the channel, but given our experimental parameters, it can achieve roughly 94% efficiency, with the retailer earning about 78% of the profits.

Interestingly, under the APD alternative structure, suppliers set the discount price too high, at a level nearly identical to the wholesale price observed in the push contract. The retailers then respond with a wholesale price that is also too high but is nearly identical to the wholesale price we observe in the pull contract. Suppliers set the prebook higher than expected, thus assuming more inventory risk than they should. Suppliers still produce less than predicted, agreeing with our earlier APD data. In short, the overall performance of this alternative APD contract is similar to the pull and original APD contracts in terms of efficiency (about 80%) and to pull in terms of the profit split (the retailer earns 55% of the profit). Also, similar to the regular APD treatment, we find learning effects over time.

We fit the loss aversion with errors model to the final third of the APD alternative data set and find that the model fits the data significantly better than the standard theory. However, the retailer’s level of loss aversion is higher than under the original APD contract, $\beta_s \approx 2.5$ (but the supplier’s parameter is similar; $\beta_s \approx 1.3$). The retailer’s estimate is driven largely by including the wholesale prices in the estimation process, because the wholesale price is now the retailer’s decision. We conclude that the errors plus loss aversion model is an improvement over the standard theory but that the loss aversion parameter seems quite sensitive to different bargaining structures, as well as to framing (Ho and Zhang 2008). We believe that more research is needed to explore the effect of framing, bargaining protocols, and relative bargaining power on contract performance.

5. Conclusion

In this study we evaluate three wholesale price contracts, each differing in how inventory risk is allocated across the supply chain. Managers, who rely on human judgment in making these strategic decisions, design supply chain contracts. Therefore, understanding how people make decisions that involve inventory risk is a key step in helping managers design behaviorally robust contracts.

We begin by testing the performance of the push and pull contracts in the laboratory and find that, consistent with the standard theory, the pull contract results in higher channel efficiency. However, standard theory fails to capture some important quantitative predictions—specifically, that orders are lower than they should be, the wholesale prices are far from the normative benchmark, and the rejection rates are incorrect. We proceed to estimate and compare several behavioral models that have been used in the literature: random errors alone and random errors combined with loss aversion, anchoring, and inequality aversion. Ultimately, a simple model with random errors and loss aversion fits the data well.

We further test our model through an out-of-sample test with the APD contract. In this additional experiment, we find that our behavioral model provides accurate predictions of the most critical decision for channel efficiency, production amounts. It also makes the correct qualitative prediction that average discount wholesale prices should be significantly lower than average regular wholesale prices. However, it fails to correctly predict the levels of wholesale prices. There are two suggestive explanations for this. First, errors for the regular wholesale price are generally one-sided, leading to lower wholesale prices; second, wholesale prices increase throughout the session so that suppliers are able to increase their expected utility by roughly 70% from the start to the end of the session. Ultimately, by the end of the session, suppliers’ expected utility is
close to the utility achieved by the optimal wholesale prices predicted by our behavioral model.

In an effort to test our model even further, we obtained data sets from two independent studies. These studies investigated revenue sharing and buyback contracts, and they differ from our experiment in a number of systematic ways (such as decision support and experimental parameters). Nevertheless, we find that our model fits the data from both studies well.

A limitation of our work is that our data do not allow us to separately estimate the effect of the different behavioral irregularities. This is because many of these motivations have a similar effect on order quantities and best-response wholesale prices. Separating the effect of these behavioral factors is an important direction for future research. One possibility might be to create a competitive market where there are an unequal number of suppliers and retailers and one side is therefore at a disadvantage, similar to Leider and Lovejoy (2013).

Another opportunity for future research might be to better understand why the loss aversion estimates differ between the retailer and supplier in the APD contract. Finally, one additional limitation stems from our second robustness check, where we manipulate the bargaining structure to be more equitable between the two parties. The results of that experiment and analysis suggest that our model is sensitive to different bargaining structures. We feel that identifying a behavioral model for settings with more equitable bargaining arrangements is an opportunity for future research.

A key practical implication of our work pertains to which inventory structure performs best for the supply chain and the parties involved. Generally speaking, both bargaining power and the bargaining protocol play an important role in determining the efficiency and profit allocation of the various contractual arrangements (see Leider and Lovejoy 2013 for initial results regarding the role of bargaining power, and see Haruvy et al. 2013 and Davis and Leider 2013 for studies that allow more dynamic bargaining structures). As mentioned previously, our experiments feature extreme levels of bargaining power, with one powerful party proposing a take-it-or-leave-it offer. In these types of contexts, our results indicate that powerful retailers are best off under pull contracts, and powerful suppliers are roughly indifferent between push or APD contracts. The APD contract delivers higher efficiency than the push contract and weakly Pareto dominates the push contract. Thus, powerful suppliers who may value long-term collaborative relationships with their retailers may favor an APD contract over a push contract because of its equitable profit distribution.

Many retailers often feel that they must keep physical product on shelf in their stores. It is important to note that does not necessarily preclude a brick-and-mortar retailer from implementing a pull or APD contract. For example, the retailer may use a pull contract under one form of vendor-managed inventory, where the retailer has product on shelf but the supplier retains ownership of it until the point of sale. Similarly, the APD contract elegantly addresses this issue by allowing the retailer to carry the prebook quantity in store while ordering more only if necessary.

In conclusion, our study suggests that retailers and suppliers should carefully evaluate their inventory arrangements because the location of the inventory in the supply chain can have serious consequences on profits for both parties and the overall supply chain.

Supplemental Material
Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2014.1940.

Acknowledgments
The authors gratefully acknowledge financial support of the Deutsche Forschungsgemeinschaft through the DFG-Research Group “Design and Behavior” and its members for useful comments. E. Katok also acknowledges financial support from the National Science Foundation [Award 1243160]. The authors thank seminar participants at the University of Texas at Dallas, three anonymous referees, an anonymous associate editor, and Serguei Netessine, whose suggestions greatly improved the paper. Last, the authors thank Özalp Özer, Srinagesh Gavirneni, Ben Greiner, Michael Becker-Peth, and Ulrich Thonemann for providing useful feedback on the paper. All remaining errors are the responsibility of the authors.

Appendix. Behavioral Model Derivations
Here, we present the derivation of the optimal quantities for the push, pull, and APD contracts. Since closed-form solutions for a general distribution cannot be explicitly stated, we derive all the optimal quantities assuming demand follows a general distribution cannot be explicitly stated, we derive all the optimal quantities assuming demand follows a general distribution. Here, we present the derivation of the optimal quantities for the push contract under loss aversion.

For the push contract under loss aversion,

\[
\max_{q \geq 0} \bar{u}_q(q) = \left( r + w(\beta - 1) \right) \left( q - \frac{q^2}{2} \right) - \beta w q, \quad \hat{q}^* = \frac{r - w}{r + w(\beta - 1)}.
\]

In turn, the problem the retailer must solve in a push contract under loss aversion is

\[
\max_{q \geq 0} \bar{u}_q(q) = \left( q - \frac{q^2}{2} \right) - w q - \alpha \left( \bar{\pi}_s(w) - \bar{\pi}_g(q) \right)^+.
\]

Under inequality aversion, we need to consider two cases to find the optimal quantities:

i. When \( q \geq (2(r + c - 2w))/r, \) \( \bar{\pi}_s(w) \geq \bar{\pi}_g(q), \) implying that \( (\bar{\pi}_s(w) - \bar{\pi}_g(q))^+ = (2w - c)q - r(q - \frac{q^2}{2}) \geq 0. \)

ii. When \( q \leq (2(r + c - 2w))/r, \) \( \bar{\pi}_s(w) < \bar{\pi}_g(q), \) and therefore \( (\bar{\pi}_s(w) - \bar{\pi}_g(q))^+ = 0. \)

Note that in both cases, when \( q = (2(r + c - 2w))/r, \) then \( \bar{\pi}_s(w) = \bar{\pi}_g(q). \)

Using the restriction defined in cases (i) and (ii), we can solve a separate problem for each condition to find the
optimal quantities and determine the ranges of wholesale prices for which the optimal solution corresponds to an interior solution, or to a boundary solution. This way, given a wholesale price, we can identify the corresponding optimal quantity.

The problem defined by condition (i) is

$$\max_{q \in (2/3 - 2w)/(r - 2w), q \geq 0} \hat{u}_S(q) = r \left( q - \frac{q^2}{2} \right) - wq - \alpha \left( 2wq - cq - r \left( q - \frac{q^2}{2} \right) \right).$$

When \( \hat{w} \geq w \geq \hat{w} \), where \( \hat{w} \) and \( \hat{w} \) are given by Equations (1) and (2), respectively, \( w > \hat{w} \). Thus the optimal quantity corresponds to the interior solution defined by Equation (3):

$$\hat{w} = \frac{r + \alpha(r + c)}{1 + 2\alpha}, \quad \hat{w} = \frac{r(1 + \alpha) + c(2 + \alpha)}{3 + 2\alpha}, \quad \hat{q}^* = \frac{r - w + \alpha(r + c - 2w)}{r(1 + \alpha)}.$$

If the wholesale price is above the threshold \( \hat{w} \), the optimal quantity is equal to 0, and if \( w < \hat{w} \), the optimal quantity corresponds to the boundary solution given by \( \hat{q}^* = 2(r + c - 2w)/r \).

The problem defined by condition (ii) is as follows:

$$\max_{q \in (2/3 - 2w)/(r - 2w), q \geq 0} \hat{u}_S(q) = r \left( q - \frac{q^2}{2} \right) - wq.$$

When \( w \leq \hat{w} \), where \( \hat{w} \) is given by Equation (4), the optimal quantity corresponds to the interior solution defined by Equation (5):

$$\hat{w} = \frac{r + 2c}{3}, \quad \hat{q}^* = \frac{r - w}{r}.$$

If \( \hat{w} < w \), the optimal quantity corresponds to the boundary solution given by \( \hat{q}^* = 2(r + c - 2w)/r \).

In summary, combining the results from above, the optimal stocking quantity for the push contract, under inequality aversion, is as follows:

$$\hat{q}^* = \begin{cases} \frac{r - w + \alpha(r + c - 2w)}{r(1 + \alpha)} & w \geq \hat{w}, \\ \frac{2(r + c - 2w)}{r} & \hat{w} < w < \hat{w}, \\ \frac{r - w}{r} & w \leq \hat{w}. \end{cases}$$

A.2. Pull Contract

For the pull contract under loss aversion, the problem the supplier must solve to determine the optimal quantity and its solution is

$$\max_{q \geq 0} u_S(q) = \left( w + c(\beta - 1) \right) \left( q - \frac{q^2}{2} \right) - \beta cq, \quad \hat{q}^* = \frac{w - c}{w + c(\beta - 1)}. $$

In the case of a pull contract under inequality aversion, the problem the supplier needs to solve is defined as follows:

$$\max_{q \geq 0} u_S(q) = w \left( q - \frac{q^2}{2} \right) - cq - \alpha (\pi_R(w) - \pi_S(q))^+.$$ 

Similar to the push contract, in the pull contract under inequality aversion we need to consider two cases to find the optimal quantities:

i. When \( q \leq (2(r + c - 2w))/(r - 2w), \pi_R(w) \geq \pi_S(q) \), implying that \( (\pi_R(w) - \pi_S(q))^+ = (r - 2w)(q - q^2/2) + cq \geq 0 \).

ii. When \( q \geq (2(r + c - 2w))/(r - 2w), \pi_R(w) \leq \pi_S(q) \), and therefore \( (\pi_R(w) - \pi_S(q))^+ = 0 \).

Note that in both cases when \( q = (2(r + c - 2w))/(r - 2w) \), then \( \pi_R(w) = \pi_S(q) \).

As in the push contract, using conditions (i) and (ii) we can find the optimal quantities and determine the range of wholesale prices for which they will correspond to interior or boundary solutions.

The problem corresponding to case (i) is as follows:

$$\max_{q \in (2/r + c - 2w)/(r - 2w), q \geq 0} u_S(q) = w \left( q - \frac{q^2}{2} \right) - cq - \alpha \left( r - 2w \right) \left( q - \frac{q^2}{2} \right) + cq.$$

When \( l \geq w \), where \( l \) and \( w \) are given by Equations (6) and (7), respectively, \( l > w \). Thus the optimal quantity corresponds to the interior solution defined by Equation (8):

$$l = \frac{c + \alpha(r + c)}{1 + 2\alpha}, \quad \bar{w} = \frac{1}{4(1 + 2\alpha)} \left( 4ar + r + 2ac + \sqrt{\left( 4ar + r + 2ac \right)^2 + 8r(1 + 2\alpha)(c(1 - \alpha) - \alpha r) } \right), \quad q^* = \frac{w - c - \alpha(r + c - 2w)}{w - \alpha(r - 2w)}.$$

If the wholesale price is below the threshold \( l \), the optimal quantity is equal to 0; if \( w < l \), the optimal quantity corresponds to the boundary solution \( q^* = (2(r + c - 2w))/(r - 2w) \).

In turn, the problem for case (ii) is as follows:

$$\max_{q \in (2/r + c - 2w)/(r - 2w), q \geq 0} u_S(q) = w \left( q - \frac{q^2}{2} \right) - cq.$$ 

When \( w \geq \bar{w} \), where \( \bar{w} \) is given by Equation (9), the optimal quantity corresponds to the interior solution given by Equation (10):

$$\bar{w} = \frac{r + \sqrt{r^2 + 8cr}}{4}, \quad \bar{q}^* = \frac{w - c}{w}.$$

If \( w < \bar{w} \), the optimal quantity corresponds to the boundary solution \( q^* = (2(r + c - 2w))/(r - 2w) \).

In summary, combining the results from above, the optimal stocking quantity for the pull contract under inequality aversion is as follows:

$$q^* = \begin{cases} \frac{w - c}{w} & w \geq \bar{w}, \\ \frac{2(r + c - 2w)}{r - 2w} & w < \bar{w}, \\ \frac{w - c - \alpha(r + c - 2w)}{w - \alpha(r - 2w)} & w \leq \bar{w}. \end{cases}$$

Figure A.1 plots the optimal quantities in the push and pull contracts under the standard theory, loss aversion with \( \beta = 1.15 \), and inequality aversion with \( \alpha = 1 \), \( r = 15 \), and \( c = 3 \).
A.3. APD Contract

The problem the supplier needs to solve to determine the optimal quantity $q^*$ given the retailer's proposed quantity is as follows:

$$\max_{u(y, q)} u(y, q)_{y \geq 0} = w_d y + \bar{w} (K(y, q) - y) - cK(y, q)\left(1 - \beta \right),$$

where $K(y, q) = \mathbb{E} [\min \{\max (y, D), q\}] = y F(y) + \int_{y}^{q} x f(x) dx + q (1 - F(q)) = q - (q^2 - y^2)/2$.

The corresponding optimal production quantity is given by

$$q^* = \begin{cases} \frac{w - c}{w + c (\beta - 1)} & \frac{c (1 + y (\beta - 1))}{1 - y} \\ y & \frac{c (1 + y (\beta - 1))}{1 - y} \end{cases} \quad \text{for } w \geq c (1 + y (\beta - 1)).$$

We next consider the problem the retailer must solve to set the prebook quantity, given the proposed wholesale prices by the supplier, which can be stated as follows:

$$\max_{y \geq 0} u_S(y, q) = (r - \bar{w} ) S(y) + (r - w) (S(q) - S(y)) - \beta w y (y - S(y)).$$

The corresponding optimal prebook quantity is given by

$$y^* = \frac{w - \bar{w}}{w + \bar{w} (\beta_R - 1)}.$$