CS 4384 **Automata Theory** SAMPLE
FALL 2012 MIDTERM

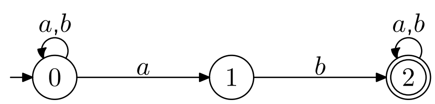
This exam is closed book and closed notes, except that you may bring one sheet of paper with anything written on it front and back. In your answers, any results proved in class or on homeworks may be applied without re-proving or re-deriving them. Do not write answers on this exam paper; nothing written on this exam paper will be graded.

(1) **(14 pts)** Let $B \subseteq \mathbb{N}_1$ (given) be a finite set of positive integers and define $\Sigma = \{a, b\}$. **Mathematically** define a DFA A that accepts a string $s \in \Sigma^*$ if and only if for all $i \in B$, the ith symbol of s is b. For example, if $B = \{1, 3\}$ then A accepts all strings whose 1st and 3rd symbols are b's (and s must be long enough to have a 1st and 3rd symbol). If $B = \emptyset$ then A accepts every string in Σ^*. Your definition of DFA A should be mathematically expressed in terms of set B.

(2) **(8 pts) Draw** an NFA that accepts the language denoted by the following regular expression: $(ab + cd)^*(c\varepsilon + \emptyset b)$.

(3) **(9 pts)** Write a regular expression that denotes the language of all strings over alphabet $\{a, b\}$ whose length is *not* a multiple of 3.

(4) **(8 pts)** Convert the following NFA to a DFA using the subset construction.



(5) **(12 pts)** Consider the language L that consists of all strings over alphabet $\{a, b\}$ that have exactly twice as many a's as b's. Use the pumping lemma to prove that L is not regular.

(6) **(9 pts)** Define $E(L)$ to be the set of even-length strings in language $L \subseteq \Sigma^*$. That is, $E(L) = \{s \mid s \in L,\ |s| \bmod 2 = 0\}$. Prove that E is a closure property of the regular languages. That is, prove that if L is regular then $E(L)$ is also regular. In your proof, you may use any of the closure properties we proved in class without re-proving them.

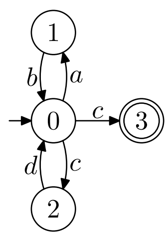
(7) **(12 pts)** Define $D(L) = \{s_1 s_2 \mid s_1 \sigma s_2 \in L, \sigma \in \Sigma\}$. That is, $D(L)$ is the language of strings that can be obtained by deleting exactly one symbol from some string in L. Prove that if L is regular then $D(L)$ is also regular.

Solutions

(1) Define $n = \max(B \cup \{0\})$. Then define $A = (Q, \Sigma, \delta, 0, \{n\})$ where $Q = [0, n] \cap \mathbb{N}_0$ and

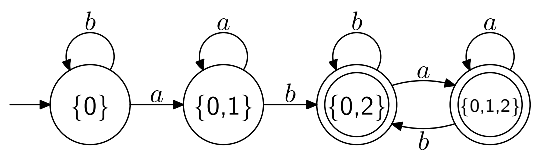
$$\begin{aligned}\delta = &\{((q, b), q{+}1) \mid q, q{+}1 \in Q\} \cup \\ &\{((q, a), q{+}1) \mid q \in Q,\ q{+}1 \in Q - B\} \cup \\ &\{((n, \sigma), n) \mid \sigma \in \Sigma\}\end{aligned}$$

(2) One correct solution is as follows:



(3) $((a + b)(a + b)(a + b))^*(a + b)(a + b + \varepsilon)$

(4) Omitting unreachable states, the subset construction yields:



(5) *Proof.* Expecting a contradiction, assume L is regular and let $p \geq 1$ be its pumping length. Define $s = a^{2p}b^p$ and let $xyz = s$ be a partitioning satisfying $|xy| \leq p$ and $|y| \geq 1$. Since $|xy| \leq p$, partition y consists entirely of a's. String xy^2z therefore has $2p + |y|$ a's and p b's. Since $|y| \geq 1$, string $xy^2z \notin L$, contradicting the pumping lemma. We conclude that L is not regular. $\square$

(6) *Proof.* Define L_1 to be the language of all even-length strings. L_1 is regular because there is a DFA that accepts it: $A = (\{0, 1\}, \Sigma, \delta, 0, \{0\})$ where $\delta = \{((0, \sigma), 1), ((1, \sigma), 0) \mid \sigma \in \Sigma\}$. For any language L, observe that $E(L) = L \cap L_1$. Thus, if L is regular then $E(L)$ is regular by regular closure under intersection. $\square$

(7) *Proof.* Since L is regular, there exists a DFA $A = (Q, \Sigma, \delta, q_0, F)$ that accepts L. Construct a new NFA $A' = (Q \times \{0, 1\}, \Sigma, \Delta, (q_0, 0), F \times \{1\})$ where

$$\Delta = \{(((q, i), \sigma), (q', i)),\ (((q, 0), \varepsilon), (q', 1)) \mid ((q, \sigma), q') \in \delta, i \in \{0, 1\}\}$$

NFA A' essentially consists of two copies of A with ε-edges from the first copy to the second. The start state is in the first copy and the final states are all in the second, so every accepting path of A' includes exactly one ε-edge. Each ε-edge serves to delete exactly one symbol from a string in L; therefore A' accepts exactly language $D(L)$. We conclude that $D(L)$ is regular. $\square$