Introduction to Model-checking
CS 6301-001: Language-based Security

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Software Verification Approaches

- **Unit Testing / Fuzzing**
  - Throw many test inputs (often randomly generated) at software and see whether it fails.
  - Good for fault detection. Inadequate for security.
    - input space usually infinite
    - attackers seek out and exploit untested inputs

- **Program-Proof Co-Development (Coq)**
  - Implement software in a “nice” (e.g., functional) language.
  - Write formal correctness properties and proofs.
  - Proofs are *machine-checked* (not trusted).
  - Pros: highest assurance, covers infinite state space
  - Cons: painful to write proofs

- **Today: Model-checking**
  - a middle-ground between random fuzzing and formal proofs
  - Express software as an abstract, finite-state model $\mathcal{M}$.
  - Express security property as a logical predicate $\phi$.
  - Decide $\mathcal{M} \models \phi$ by exhaustive state-space search.
Some History

- First developed in 1980s by Clarke, Emerson, and Sifakis (Turing Award 2007)
  - primarily targeted hardware verification
  - disillusionment with proofs in 80s and 90s
  - found previously undetected errors in 1992 IEEE Future+ cache coherence protocol
  - 1994 Intel Pentium floating-point bug
    - passed unit testing
    - cost Intel $400–500 million
    - could have been detected by model-checking
  - model-checking now routinely used by Intel, AMD, IBM, Lucent, etc.
- Rise of Software Model-checking in late 90s
  - VeriSoft (Lucent), SPIN (Holtzmann, Bell Labs)
  - Big challenge: state-space explosion
Example (from JavaPathFinder documentation)

```java
Random random = new Random();
int a = random.nextInt(2);
System.out.println("a=" + a);

// lots of code here

int b = random.nextInt(3);
System.out.println("b=" + b);
int c = a/(b+a-2);
System.out.println("c=" + c);
```

▶ Sample run:

▶ a = 1
▶ b = 0
▶ c = −1
State Space

start

\[ a = 0 \]
\[ b = 0 \]
\[ c = 0 \]

\[ a = 0 \]
\[ b = 1 \]
\[ error \]

\[ a = 0 \]
\[ b = 2 \]

\[ a = 1 \]
\[ b = 0 \]
\[ error \]

\[ a = 1 \]
\[ b = 1 \]
\[ c = -1 \]

\[ a = 1 \]
\[ b = 2 \]
\[ c = 1 \]
State Spaces

- Not always (or even usually) trees
  - conditionals = multiple in-edges
  - program loops = cycles
- Does not always match control-flow graph structure
  - One program line could correspond to many different states, depending on the values of its variables.
  - Abstracting coalesces states (more on this later...)
- Can be huge
  - How many states if we change the “2” argument in line 2?
Properties

- Typically expressed in a temporal logic
- Flagship example: Linear Temporal Logic (LTL)
- Assertions: \( \pi \models \phi \) — path \( \pi \) models property \( \phi \)
  - atomic propositions (e.g., is\_error, \( a = 2 \), etc.)
  - \( \neg \phi \) — negation
  - \( \phi_1 \lor \phi_2 \) — disjunction
  - \( X(\phi) \) — next \( \phi \)
  - \( U(\phi_1, \phi_2) \) — \( \phi_1 \) until \( \phi_2 \)
  - \( F(\phi) \) — finally \( \phi \)
  - \( G(\phi) \) — globally \( \phi \)

Exercise: Do all paths from “start” model the following?
- \( X(a = 0) \)
- \( U(\neg\text{is\_error}, b > 0) \)
- \( F(U(\text{false}, b \leq 2)) \)
Branching Temporal Logics

- LTL cannot express most existential properties
  - Example: “for every state there exists a non-error step”
- Solution: Branching Temporal Logics
- Flagship example: Modal $\mu$-Calculus
- Assertions: $s \models \psi$ — state $s$ is a member of the set of all states denoted by $\psi$
  - $\psi_1 \land \psi_2$ — conjunction (intersection)
  - $\psi_1 \lor \psi_2$ — disjunction (union)
  - $[a] \psi$ — all outgoing $a$-transitions model $\psi$
  - $\langle a \rangle \psi$ — some outgoing $a$-transitions model $\psi$
  - $\mu X . \psi$ — least fixed point
  - $\nu X . \psi$ — greatest fixed point
- What are least and greatest “fixed points”? 
Fixed Point Semantics

**Definition:** A *fixed point* of a function \( f : A \rightarrow A \) is a value \( x \in A \) such that \( f(x) = x \).

- **Examples:**
  - What is a fixed point of \( f(x) = x + 1 \)?
  - What is a fixed point of \( g(x) = x^2 \)?
  - What is a fixed point of \( h(S) = \{x^2 | x \in S\} \)?

- When \( f \) is a function from sets to sets, we say \( S \) is...
  - ...a *least fixed point* if \( S \) is a fixed point and all other fixed points are supersets of \( S \).
  - ...a *greatest fixed point* if \( S \) is a fixed point and all other fixed points are subsets of \( S \).

- Can a function have multiple least fixed points or multiple greatest fixed points?
Fixed Point Operators

- Back to modal \(\mu\)-calculus:
  - \(\mu X . \psi\) is the least set \(S\) such that \(S = \psi[X := S]\)
  - \(\nu X . \psi\) is the greatest set \(S\) such that \(S = \psi[X := S]\)

- Finding least/greatest fixed points:
  - Find \(\mu X . \psi\) inductively:
    - start with \(X = \emptyset\)
    - keep adding things to \(X\) until no progress
  - Find \(\nu X . \psi\) co-inductively:
    - start with \(X =\text{universe of all states}\)
    - keep removing things from \(X\) until no progress

- Examples:
  - What is \(\mu X . (X \lor \langle\text{is_error}\rangle)\) ?
  - What is \(\nu X . (\text{is_error} \lor \langle X\rangle)\) ?
State Space Explosion Problem

- Main challenge: What if the state space is huge?
- Example: How many states does the following program have?

```plaintext
int i = 0;
while true do
    i := i + 1;
```

- Solution: Abstract Interpretation
  - Instead of having one state for every mapping of variables to values, label states with abstract properties.
  - Example: What if we only care about whether `i` is zero (e.g., to avoid division-by-zero)?
  - Could instead just have one state for each possible `sign` of `i`
    - `zero + positive` =?
    - `positive + positive` =?
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    - \( zero + positive = positive \)
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  - Could instead just have one state for each possible sign of \( i \)
    - \( zero + positive = positive \)
    - \( positive + positive = positive \)
  - We’re finished with only 2 states to explore!
Counterexample Guided Abstraction Refinement (CEGAR)

- Over-abstraction Problem
  - If model-check succeeds on abstract model, then we’re done. But...
  - Abstracting often forgets information needed to prove correctness.
  - Results in false rejection (model-checker signals fault where there is none)

- Solution: Iteratively Abstract and Refine
  1. Abstract until search space is feasible.
  2. Exhaustively search the space. If model-check rejects...
  3. Test the counterexample on the original (non-abstract) search space. If it’s a real counterexample, we found a real bug. Otherwise...
  4. We must have abstracted too much. Refine (opposite of abstract) and repeat.

- Next time: Binary code analysis