Lecture 3: Mutual Recursion & Tail Recursion

CS 6371: Advanced Programming Languages
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#let rec length =
    function [] -> 0 | _::t -> (length t)+1;;

length : 'a list -> int = <fun>

“function ... -> ...” is an abbreviation for “fun x ->
(match x with ... -> ...)”

#type staff = Programmer | Manager of dept
and dept = Outsourced | Staffed of staff;;

Type dept defined.
Type staff defined.

#Manager (Staffed Programmer);;
- : staff = Manager (Staffed Programmer)

Mutually recursive types are separated by the word “and”. Notice that there is no “;;” before
the “and” and there is no second “type” keyword. You can string as many mutually recursive types
together as you wish with “and”.

#type 'a btree
    = BNull
    | BNode of ('a * 'a btree * 'a btree);;

Type btree defined.

#BNode (3,BNull,BNull);;
-: int btree = BNode (3, BNull, BNull)
#BNode ("foo",BNull,BNull);;
-: string btree = BNode ("foo",BNull,BNull)
#BNode("foo",BNode(3,BNull,BNull),BNull);;

Polymorphic variants define a type constructor
that is parameterized by a type variable.

#let rec staff2str s =
(match s with
    Programmer -> "Peon"
  | Manager d ->
    "Dictator["^(dept2str d)^"]")

and dept2str d =
  (match d with Outsourced -> "Exiled"
    | Staffed s -> staff2str s);

staff2str : staff -> string = <fun>

department2str : dept -> string = <fun>

Mutually recursive functions are also defined with
“and”. The first function in the group begins with
“let rec”. Each subsequent function begins with
“and” (and no “let rec”). The only “;;” appears at the end of the whole group.

#let rec tree2list t =
  (match t with
      BNull -> []
    | BNode (x,t1,t2) ->
        (tree2list t1) @ (x::(tree2list t2));;

  tree2list : 'a btree -> 'a list = <fun>

Here’s an example of a function that converts a
polymorphic binary tree to a polymorphic list
(with list elements given in prefix order). The
“@” operator concatenates two lists. This differs
from the “::” operator, which inserts an element
onto the head of a list.

#let rec fold_left f b l =
  (match l with
    [] -> b
  | h::t -> fold_left f (f b h) t);;

fold_left : ('a -> 'b -> 'a) -> 'a list -> 'a = <fun>

"Fold" is an extremely important list operation in
functional programming. (fold_left f b [w;x;y;z])
computes the formula f(f(f(f(b,w),x),y),z).

#let exists f l =

From “fold” one can derive many useful list
functions, such as existence and forall functions
that check if a given condition holds for any or all
fold_left (fun b x -> b || (f x))
false l;;
exists:('a->bool)->'a list->bool = <fun>
#let for_all f l =
  fold_left (fun b x -> b && (f x))
true l;;
for_all:('a->bool)->'a list->bool = <fun>
#for_all (fun x -> x>2) [1;2;3];;
- : bool = false

#let rec fold_right f l b =
  (match l with
   [] -> b
  | h:t -> f h (fold_right f t b));;
fold_right : ('a -> 'b -> 'b) -> 'a list ->
'b -> 'b = <fun>
#fold_right (fun x y -> x-y) [1;2;3] 0;;
- : int = 2
#fold_left (fun x y -> x-y) 0 [1;2;3];;
- : int = -6

There is another operation called “fold_right” that applies function f starting with the rightmost element. That is, (fold_right f [w;x;y;z] b) computes f(w,f(f(x,f(f(y,f(f(z,b))))))).

A function is “tail recursive” if the value that it returns is the value returned by a direct recursive call to itself. Note that fold_left is tail-recursive but fold_right is not. Try to write tail-recursive functions whenever possible, since these can be optimized much better by functional compilers.

Many of the functions we’ve defined for lists are defined for you in standard libraries, including the ones listed to the left. The “fst” and “snd” functions are also useful for manipulating pairs.

Exceptions are defined like types, except that you use the keyword “exception” in place of “type”. Use the “raise” command to throw an exception.

An expression’s type declares its return type IF the function or expression returns normally. When you raise an exception, you don’t need to satisfy the return type of the enclosing expression because the expression is not returning normally. Warning: If you program using exceptions, you lose many of the benefits of functional programming! I recommend avoiding them.

Catch exceptions with “try ... with ...”. The “with” part is a pattern-match on the exception type. Each value returned by the right side of an arrow must be of the same type that would be returned if no exception was thrown.