

Lecture #6: Small-step Operational Semantics

CS 6371: Advanced Programming Languages

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1 Syntax

commands	$c ::= \text{skip} \mid c_1; c_2 \mid v := a \mid \text{if } b \text{ then } c_1 \text{ else } c_2 \mid \text{while } b \text{ do } c$
boolean expressions	$b ::= \text{true} \mid \text{false} \mid a_1 \leq a_2 \mid b_1 \ \&\& \ b_2 \mid b_1 \ \ \ \ b_2 \mid !b$
arithmetic expressions	$a ::= n \mid v \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2$
variable names	v
integer constants	n

2 Small-step Operational Semantics

2.1 Commands

$$\frac{\langle c_1, \sigma \rangle \rightarrow_1 \langle c'_1, \sigma' \rangle}{\langle c_1; c_2, \sigma \rangle \rightarrow_1 \langle c'_1; c_2, \sigma' \rangle} \quad (1)$$

$$\langle \text{skip}; c_2, \sigma \rangle \rightarrow_1 \langle c_2, \sigma \rangle \quad (2)$$

$$\frac{\langle a, \sigma \rangle \rightarrow_1 \langle a', \sigma' \rangle}{\langle v := a, \sigma \rangle \rightarrow_1 \langle v := a', \sigma' \rangle} \quad (3)$$

$$\langle v := n, \sigma \rangle \rightarrow_1 \langle \text{skip}, \sigma[v \mapsto n] \rangle \quad (4)$$

$$\frac{\langle b, \sigma \rangle \rightarrow_1 \langle b', \sigma' \rangle}{\langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma \rangle \rightarrow_1 \langle \text{if } b' \text{ then } c_1 \text{ else } c_2, \sigma' \rangle} \quad (5)$$

$$\langle \text{if true then } c_1 \text{ else } c_2, \sigma \rangle \rightarrow_1 \langle c_1, \sigma \rangle \quad (6)$$

$$\langle \text{if false then } c_1 \text{ else } c_2, \sigma \rangle \rightarrow_1 \langle c_2, \sigma \rangle \quad (7)$$

$$\langle \text{while } b \text{ do } c, \sigma \rangle \rightarrow_1 \langle \text{if } b \text{ then } (c; \text{while } b \text{ do } c) \text{ else skip}, \sigma \rangle \quad (8)$$

2.2 Boolean Expressions

$$\frac{\langle a_1, \sigma \rangle \rightarrow_1 \langle a'_1, \sigma' \rangle}{\langle a_1 \leq a_2, \sigma \rangle \rightarrow_1 \langle a'_1 \leq a_2, \sigma' \rangle} \quad (9)$$

$$\frac{\langle a_2, \sigma \rangle \rightarrow_1 \langle a'_2, \sigma' \rangle}{\langle n_1 \leq a_2, \sigma \rangle \rightarrow_1 \langle n_1 \leq a'_2, \sigma' \rangle} \quad (10)$$

$$\frac{n_1 \leq n_2}{\langle n_1 \leq n_2, \sigma \rangle \rightarrow_1 \langle \text{true}, \sigma \rangle} \quad (11)$$

$$\frac{n_1 > n_2}{\langle n_1 \leq n_2, \sigma \rangle \rightarrow_1 \langle \text{false}, \sigma \rangle} \quad (12)$$

$$\frac{\langle b_1, \sigma \rangle \rightarrow_1 \langle b'_1, \sigma' \rangle \quad op \in \{ \&\&, || \}}{\langle b_1 op b_2, \sigma \rangle \rightarrow_1 \langle b'_1 op b_2, \sigma' \rangle} \quad (13)$$

$$\langle \text{true} \&\& b_2, \sigma \rangle \rightarrow_1 \langle b_2, \sigma \rangle \quad (14)$$

$$\langle \text{false} \&\& b_2, \sigma \rangle \rightarrow_1 \langle \text{false}, \sigma \rangle \quad (15)$$

$$\langle \text{true} || b_2, \sigma \rangle \rightarrow_1 \langle \text{true}, \sigma \rangle \quad (16)$$

$$\langle \text{false} || b_2, \sigma \rangle \rightarrow_1 \langle b_2, \sigma \rangle \quad (17)$$

$$\frac{\langle b, \sigma \rangle \rightarrow_1 \langle b', \sigma' \rangle}{\langle !b, \sigma \rangle \rightarrow_1 \langle !b', \sigma' \rangle} \quad (18)$$

$$\langle !\text{true}, \sigma \rangle \rightarrow_1 \langle \text{false}, \sigma \rangle \quad (19)$$

$$\langle !\text{false}, \sigma \rangle \rightarrow_1 \langle \text{true}, \sigma \rangle \quad (20)$$

2.3 Arithmetic Expressions

$$\langle v, \sigma \rangle \rightarrow_1 \langle \sigma(v), \sigma \rangle \quad (21)$$

$$\frac{\langle a_1, \sigma \rangle \rightarrow_1 \langle a'_1, \sigma' \rangle \quad op \in \{ +, -, * \}}{\langle a_1 op a_2, \sigma \rangle \rightarrow_1 \langle a'_1 op a_2, \sigma' \rangle} \quad (22)$$

$$\frac{\langle a_2, \sigma \rangle \rightarrow_1 \langle a'_2, \sigma' \rangle \quad op \in \{ +, -, * \}}{\langle n_1 op a_2, \sigma \rangle \rightarrow_1 \langle n_1 op a'_2, \sigma' \rangle} \quad (23)$$

$$\langle n_1 + n_2, \sigma \rangle \rightarrow_1 \langle n_1 + n_2, \sigma \rangle \quad (24)$$

$$\langle n_1 - n_2, \sigma \rangle \rightarrow_1 \langle n_1 - n_2, \sigma \rangle \quad (25)$$

$$\langle n_1 * n_2, \sigma \rangle \rightarrow_1 \langle n_1 n_2, \sigma \rangle \quad (26)$$