Lecture #6: Small-step Operational Semantics

CS 4301/6371: Advanced Programming Languages

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1 Syntax

commands	$c ::= \texttt{skip} \mid c_1; c_2 \mid v := a \mid \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \mid \texttt{while}\ b\ \texttt{do}\ c$
boolean expressions	$b ::= \texttt{true} \mid \texttt{false} \mid a_1 \texttt{<=} a_2 \mid b_1 \texttt{\&\&} b_2 \mid b_1 \texttt{||} b_2 \mid \texttt{!}b$
arithmetic expressions	$a ::= n \mid v \mid a_1 \texttt{+} a_2 \mid a_1 \texttt{-} a_2 \mid a_1 \texttt{*} a_2$
variable names	v
integer constants	n

2 Small-step Operational Semantics

2.1 Commands

$$\frac{\langle c_1, \sigma\rangle \to_1 \langle c_1', \sigma'\rangle}{\langle c_1; c_2, \sigma\rangle \to_1 \langle c_1'; c_2, \sigma'\rangle} \tag{1}$$

$$\langle \texttt{skip}; c_2, \sigma\rangle \to_1 \langle c_2, \sigma\rangle \tag{2}$$

$$\frac{\langle a, \sigma\rangle \to_1 \langle a', \sigma'\rangle}{\langle v\texttt{:=}a, \sigma\rangle \to_1 \langle v\texttt{:=}a', \sigma'\rangle} \tag{3}$$

$$\langle v\texttt{:=}n, \sigma\rangle \to_1 \langle \texttt{skip}, \sigma[v \mapsto n]\rangle \tag{4}$$

$$\frac{\langle b, \sigma\rangle \to_1 \langle b', \sigma'\rangle}{\langle \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2, \sigma\rangle \to_1 \langle \texttt{if}\ b'\ \texttt{then}\ c_1\ \texttt{else}\ c_2, \sigma'\rangle} \tag{5}$$

$$\langle \texttt{if true then}\ c_1\ \texttt{else}\ c_2, \sigma\rangle \to_1 \langle c_1, \sigma\rangle \tag{6}$$

$$\langle \texttt{if false then}\ c_1\ \texttt{else}\ c_2, \sigma\rangle \to_1 \langle c_2, \sigma\rangle \tag{7}$$

$$\langle \texttt{while}\ b\ \texttt{do}\ c, \sigma\rangle \to_1 \langle \texttt{if}\ b\ \texttt{then}\ (c; \texttt{while}\ b\ \texttt{do}\ c)\ \texttt{else skip}, \sigma\rangle \tag{8}$$

2.2 Boolean Expressions

$$\frac{\langle a_1, \sigma\rangle \to_1 \langle a_1', \sigma'\rangle}{\langle a_1 \texttt{<=} a_2, \sigma\rangle \to_1 \langle a_1' \texttt{<=} a_2, \sigma'\rangle} \tag{9}$$

$$\frac{\langle a_2, \sigma\rangle \to_1 \langle a_2', \sigma'\rangle}{\langle n_1 \texttt{<=} a_2, \sigma\rangle \to_1 \langle n_1 \texttt{<=} a_2', \sigma'\rangle} \tag{10}$$

$$\frac{n_1 \leq n_2}{\langle n_1 \texttt{<=} n_2, \sigma\rangle \to_1 \langle \texttt{true}, \sigma\rangle} \tag{11}$$

$$\frac{n_1 > n_2}{\langle n_1 \texttt{<=} n_2, \sigma\rangle \to_1 \langle \texttt{false}, \sigma\rangle} \tag{12}$$

$$\frac{\langle b_1, \sigma\rangle \to_1 \langle b_1', \sigma'\rangle \quad op \in \{\,\texttt{\&\&}\,,\ \texttt{||}\,\}}{\langle b_1\ op\ b_2, \sigma\rangle \to_1 \langle b_1'\ op\ b_2, \sigma'\rangle} \tag{13}$$

$$\langle \texttt{true} \texttt{\&\&} b_2, \sigma\rangle \to_1 \langle b_2, \sigma\rangle \tag{14}$$

$$\langle \texttt{false} \texttt{\&\&} b_2, \sigma\rangle \to_1 \langle \texttt{false}, \sigma\rangle \tag{15}$$

$$\langle \texttt{true} \texttt{||} b_2, \sigma\rangle \to_1 \langle \texttt{true}, \sigma\rangle \tag{16}$$

$$\langle \texttt{false} \texttt{||} b_2, \sigma\rangle \to_1 \langle b_2, \sigma\rangle \tag{17}$$

$$\frac{\langle b, \sigma\rangle \to_1 \langle b', \sigma'\rangle}{\langle \texttt{!}b, \sigma\rangle \to_1 \langle \texttt{!}b', \sigma'\rangle} \tag{18}$$

$$\langle \texttt{!true}, \sigma\rangle \to_1 \langle \texttt{false}, \sigma\rangle \tag{19}$$

$$\langle \texttt{!false}, \sigma\rangle \to_1 \langle \texttt{true}, \sigma\rangle \tag{20}$$

2.3 Arithmetic Expressions

$$\langle v, \sigma\rangle \to_1 \langle \sigma(v), \sigma\rangle \tag{21}$$

$$\frac{\langle a_1, \sigma\rangle \to_1 \langle a_1', \sigma'\rangle \qquad op \in \{\,\texttt{+}\,,\ \texttt{-}\,,\ \texttt{*}\,\}}{\langle a_1\ op\ a_2, \sigma\rangle \to_1 \langle a_1'\ op\ a_2, \sigma'\rangle} \tag{22}$$

$$\frac{\langle a_2, \sigma\rangle \to_1 \langle a_2', \sigma'\rangle \qquad op \in \{\,\texttt{+}\,,\ \texttt{-}\,,\ \texttt{*}\,\}}{\langle n_1\ op\ a_2, \sigma\rangle \to_1 \langle n_1\ op\ a_2', \sigma'\rangle} \tag{23}$$

$$\langle n_1 \texttt{+} n_2, \sigma\rangle \to_1 \langle n_1 + n_2, \sigma\rangle \tag{24}$$

$$\langle n_1 \texttt{-} n_2, \sigma\rangle \to_1 \langle n_1 - n_2, \sigma\rangle \tag{25}$$

$$\langle n_1 \texttt{*} n_2, \sigma\rangle \to_1 \langle n_1 n_2, \sigma\rangle \tag{26}$$