Analyzing Assessors and Products in Sorting Tasks: DISTATIS, Theory and Applications

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Abstract

In this paper we present a new method called DISTATIS that can be applied to the analysis of sorting data. DISTATIS is a generalization of classical multidimensional scaling which allows one to analyze 3-ways distance tables. When used for analyzing sorting tasks, DISTATIS takes into account individual sorting data. Specifically, when DISTATIS is used to analyze the results of an experiment in which several assessors sort a set of products, we obtain two types of maps: One for the assessors and one for the products. In these maps, the proximity between two points reflects their similarity, and therefore these maps can be read using the same rules as standard metric multidimensional scaling methods or principal component analysis. Technically, DISTATIS starts by transforming the individual sorting data into cross-product matrices as in classical MDS and evaluating the similarity between these matrices (using Escoufier's R_V coefficient). Then it computes a compromise matrix which is the best aggregate (in the least square sense, as STATIS does) of the individual cross-product matrices and analyzes it with PCA. The individual matrices are then projected onto the compromise space. In this paper, we present a short tutorial, and we illustrate how to use DISTATIS with a sorting task in which ten assessors evaluated eight beers. We also provide some insights into how DISTATIS evaluates the similarity between assessors.

Key words: 3-way analysis, multidimensional scaling, DISTATIS, RV-coefficient, Rand coefficient, sorting task, STATIS.

1. Introduction

A sorting task is a simple method for collecting similarity data that has been very popular in Psychology since the 70's (*cf. e.g.*, Healy & Miller, 1970; Coxon, 1999). In this type of tasks, assessors are asked to sort together stimuli based on their perceived similarity. The sorting task has the advantage of being less tedious and time consuming than other methods such as direct pairwise similarity evaluation. Sorting is particularly suitable when the number of stimuli

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to investigate is large. Even though sorting imposes little demand on participants' attention, its results are often comparable with more demanding methods. For example, Rao and Katz (1971) found a good correlation between sorting tasks and pairwise similarity scaling. Likewise, MacRae, Howgate, and Geelhoed (1990) found that sorting tasks and triadic similarity comparisons gave similar patterns of results. More recently, Cartier *et al.* (2006), showed that trained panelists provided equivalent data when they used quantitative ratings or a sorting task. These authors also indicate that a sorting task performed by novice participants provided information similar to the information obtained from the trained panelists.

In the sensory domain, sorting tasks were first used to investigate the perceptual structure of odor quality (Chrea *et al.*, 2005; Lawless, 1989; Lawless & Glatter, 1990; MacRae *et al.*, 1992; Stevens & O'Connell, 1996). Later, sorting tasks were applied to a large variety of complex products going from food products such as vanilla beans (Heymann, 1994), cheeses (Lawless, Sheng & Knoops, 1995), drinking waters (Falahee & MacRae, 1995; Falahee & MacRae, 1997), grape jellies (Tang & Heymann, 1999), beers (Chollet & Valentin, 2001), wines (Piombino *et al.*, 2004; Ballester *et al.*, 2005), and yoghurts (Saint Eve, Paçi Kora, & Martin, 2004) to non-food products such as automotive fabrics (Giboreau *et al.*, 2001; Picard *et al.*, 2003), cloth fabrics (Souflet, Calonniera, & Dacremont, 2004), and plastic pieces (Faye *et al.*, 2004). Generally, both food and non-food product studies indicate that the sorting task is an efficient and economical way of obtaining information about sensory differences among products. Also, sorting tasks are suitable for use with untrained assessors and with a large number of samples. Moreover, patterns of results obtained from sorting tasks are comparable with those obtained with descriptive methods such as conventional or free profiling (Heymann, 1994; Tang & Heymann, 1999; Faye *et al.*, 2004; Saint Eve *et al.*, 2004, Faye *et al.*, 2006) and seem to be reproducible (Falahee & MacRae, 1997).

Experiments using sorting tasks are generally analyzed using multidimensional scaling (MDS) or sometimes multiple correspondence analysis or variations of these methods (see, *e.g.*, Takane, 1980, 1982; see van der Kloot & van Herk, 1991 for a comparison between some of these approaches; but see Abdi, 1990; and Corter, 1996; for alternative approaches such as additive trees). As a preliminary step for the analysis, a similarity matrix is generated by computing the number of times each pair of stimuli had been sorted in the same group. When MDS analyzes such a matrix, it produces a map of the stimuli. In this map, the stimuli are represented by points which are positioned such that the distances between pairs of points reflect as well as possible the similarities between the pairs of stimuli: Two stimuli which have been often sorted together are close on this representation and two stimuli which have rarely been sorted together are far apart. This representation of the stimuli as points on a map may give some insights into the dimensions underlying stimulus similarities and differences. In order to interpret the dimensions from MDS they are often correlated with additional attributes of the stimuli (Schiffman, Reynolds & Young, 1981; Kruskal & Wish, 1978).

As pointed out by Lawless *et al.* (1995), the only major drawback of the MDS analysis of sorting tasks is that information on participants is lost because the individual data are pooled in order to obtain a similarity matrix. As a consequence, individual differences are hidden and the average representation may bear little resemblance to each of the individual assessor representations. Moreover, there is no easy way to evaluate the agreement among assessors or to visualize the repeatability of individual assessors when repetitions are used. A possible solution would be to use individual difference scaling (INDSCAL, Carroll & Chang, 1970; a close method, PARAFAC, was also developed independently at the same time by Harshman, see Harshman & Lundy, 1994). INDSCAL allows for the simultaneous analysis of several distance matrices. It provides a single spatial representation along with the weights of each assessor on the dimensions of this representation (see Husson & Pagès, 2006, for a geometric interpretation of INDSCAL). Another reasonable alternative could be general procrustean analysis (GPA, see Gower & Dijksterhuis, 2004; see also Meyners, Kunert, & Qanari, 2000; and Meyners, 2003; for comparisons between GPA and alternative approaches). GPA is an iterative method which tries to find the closest matrix to a set of matrices. It is, in general, used with factor scores matrices, but could be adapted to deal with distance matrices.

INDSCAL, PARAFAC, and GPA are often used to integrate several matrices, but they are iterative methods and therefore can necessitate a large number of iterations (especially for badly conditioned matrices such as the binary individual matrices obtained in sorting tasks, see Kiers, 1998) and may be sensitive to the existence of local minima. Therefore a method with similar goals but using a non-iterative approach would be of interest. In this paper we describe such a new method, called DISTATIS (Abdi, Valentin, O'Toole, & Edelman, 2005; Abdi & Valentin, 2007a). The input of DISTATIS is a set of distance matrices obtained on the same set of stimuli, and therefore DISTATIS is a rather versatile tool which can be applied to a variety of situations. In this paper, though, we will restrict our presentation to the specific case of the sorting task for which each distance matrix corresponds to the sorting of a given assessor. DISTATIS combines classical MDS (see *e.g.*, Togerson 1958; Kruskal, 1977; Borg & Groenen, 2000; Abdi, 2007a) and STATIS (Escoufier, 1980; Lavit, 1988; Schlich, 1996; Abdi & Valentin 2007b), and it has the advantage of taking into account individual data. Recall that classical MDS transforms a distance

matrix into a set of coordinates such that the Euclidean distances, derived from these coordinates, approximate as well as possible the original distances. The basic idea of classical MDS is to transform a distance matrix into a cross-product matrix which is then submitted to an eigendecomposition (this gives a PCA of the cross-product matrix). Traditionally, in sensory evaluation, sorting tasks are analyzed using *non metric* MDS procedures such as the Alternating Least-Square sCALing procedure (ALSCAL, Young, Lewyckyj, & Takane 1986; Young & Hamer 1987) or KYST (Kruskal & Wish, 1978). These algorithms make only a rank order assumption to fit the spatial representation to the original data (*i.e.*, only the *order* of the similarities is used to derive the spatial representation). However, classical MDS can also be applied because the similarity matrix derived from a sorting task can easily be transformed into a sorting *distance* matrix (Miller 1969; Abdi, 2007b).

When the results of a sorting task are analyzed with DISTATIS, we obtain two types of maps: One for the assessors and one for the products. In these maps, the proximity between two points reflects their similarity, and therefore these maps can be analyzed using the same rules as standard metric MDS or PCA. The assessors' map is obtained by analyzing the similarity between the distance matrices representing the assessors' sorts. The product map is obtained from the analysis of a between product matrix called the *compromise* matrix which is obtained by combining all the assessors' distance matrices. On this map, we can also project the configuration of the products given by each assessor.

2. General principles

In this section we present the equations of DISTATIS and we illustrate them by showing their effect in pictures with a small fabricated example in which $T = 4$ assessors are evaluating a set of $I = 6$ beers. Each assessor sorts the six beers with the constraint that he or she uses more than 1 group and less than I groups (this constraint is imposed mostly for convenience). Mathematically, this means that each assessor provides a partition of the set of the beers (Hubert & Arabie, 1985). The result of this sort can be first represented by an indicator matrix in which one row represents a beer and one column represents a group. A value of 1 in this matrix means that the beer represented by the row was put in the group represented by the column. For assessor t, this matrix will be denoted $\mathbf{L}_{[t]}$. The number of column of $\mathbf{L}_{[t]}$ gives the number of groups used by assessor t. Each $\mathbf{L}_{[t]}$ matrix can be transformed into a beer by beer co-occurrence matrix (denoted $\mathbf{R}_{[t]}$) by multiplication by its transpose:

$$\mathbf{R}_{[t]} = \mathbf{L}_{[t]}\mathbf{L}_{[t]}^{\mathsf{T}} \ . \tag{1}$$

In a co-occurrence matrix, the rows and the columns represent beers and a value of 1 between a row and a column indicates that the assessor put the beers together, whereas a value of 0 indicates that the beers were not put together (*cf.* Figure 1).

The co-occurrence matrix is transformed then into a distance matrix where the rows and the columns are beers and where a value of 0 between a row and a column indicates that the assessor put the beers together, whereas a value of 1 indicates that the beers were not put together (*cf.* Figure 1). The transformation from a co-occurrence matrix to a distance matrix is obtained by subtracting the co-occurrence matrix from a conformable matrix filled with 1's.

$$\mathbf{D}_{[t]} = \mathbf{1} - \mathbf{R}_{[t]} \ . \tag{2}$$

So we have $T = 4$ distance matrices to analyze, and these matrices are denoted $\mathbf{D}_{[1]}$ to $\mathbf{D}_{[T]}$.

Distance matrices need first to be transformed into cross-product matrices to be analyzed by MDS. To obtain a cross-product matrix from a distance matrix we need first to square each element of the distance matrix and center this squared distance matrix so that the origin of the distances is at the center of gravity of the dimensions of the matrix (*cf.* Figure 2). This step is necessary because distances are invariant by any change of origin and thus several cross-product matrices can generate the same distance matrix. Centering is obtained by transforming each element of the squared distance matrix into a similarity by removing the grand mean effect as well as the row and column effects (this is also called double centering, see Kruskal, 1977, pp 309*ff* for more details). To describe this transformation, first denote by $d_{i,j}$ the element which is at the intersection of row i and column j of a squared distance matrix $\mathbf{D}$. This element will be transformed into $\widetilde{s}_{i,j}$ which is the corresponding element of the cross-product matrix as

$$\begin{aligned}\widetilde{s}_{i,j} &= -\frac{1}{2}\left\{\left(d_{i,j} - \overline{d}_{+,+}\right) - \left(\overline{d}_{i,+} - \overline{d}_{+,+}\right) - \left(\overline{d}_{j,+} - \overline{d}_{+,+}\right)\right\} \\ &= -\frac{1}{2}\left(d_{i,j} - \overline{d}_{i,+} - \overline{d}_{j,+} + \overline{d}_{+,+}\right) \end{aligned} \tag{3}$$

with $\overline{d}_{i,+}$ (respectively $\overline{d}_{j,+}$) being the mean of the squared distances for the ith (respectively j-th) row, and where $\overline{d}_{+,+}$ is the grand mean of $\mathbf{D}$. Strictly speaking, this transformation from distance to cross-product assumes that $\mathbf{D}$ is the matrix of a squared Euclidean distance (or equivalently that the matrices $\widetilde{\mathbf{S}}_{[t]}$ defined in Equation 5 are positive semi definite, this is shown in Appendix B).

The transformation from (squared) distance to cross-product can also be expressed using matrix operations. In this case, we start by defining a centering matrix:

$$\underset{I \times I}{\mathbf{\Xi}} = \underset{I \times I}{\mathbf{I}} - \underset{I \times 1}{\mathbf{1}}\underset{1 \times I}{\mathbf{m}^{T}} \ , \tag{4}$$

(where $\mathbf{m}$ is a vector of mass composed of positive numbers

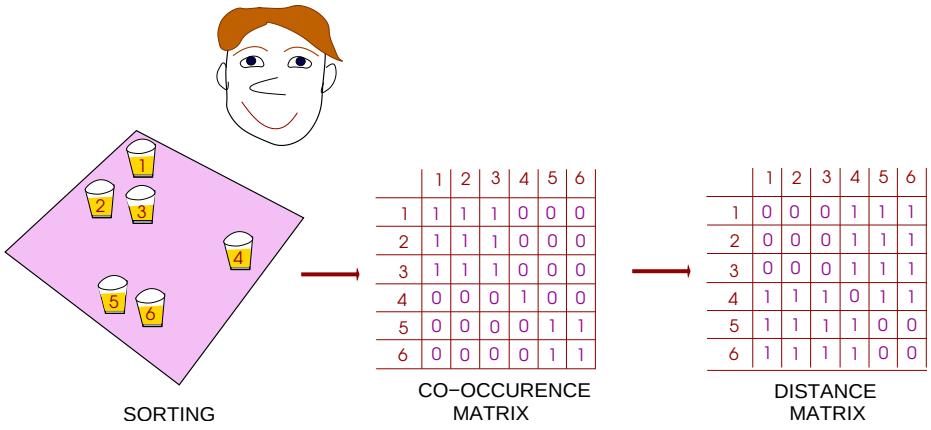

Fig. 1. The partition given by an assessor is converted first into a co-occurrence matrix which is then transformed into a distance matrix.

whose sum is equal to 1, in most cases, all observations are of equal importance and each element of $\mathbf{m}$ is equal to $m_i = \frac{1}{I}$). A cross-product matrix denoted by $\widetilde{\mathbf{S}}_{[t]}$ is then obtained as

$$\widetilde{\mathbf{S}}_{[t]} = -\tfrac{1}{2}\mathbf{\Xi}\mathbf{D}_{[t]}\mathbf{\Xi}^{\mathsf{T}} \ . \tag{5}$$

In order to compare the cross-product matrices, we need to *normalize* them (an idea akin to multiple factor analysis, *cf.* Escofier & Pagès, 1998; Abdi & Valentin, 2007c). This is done by dividing all the elements of each individual cross-product matrix by its first eigenvalue. The normalized cross-product matrices are denoted $\mathbf{S}_{[t]}$ (*cf.* Figure 2). By construction, a normalized matrix has a first eigenvalue equal to 1.

Then, these individual cross-product matrices are aggregated in order to create a compromise cross-product matrix which represents their consensus. This compromise matrix is obtained as a weighted average of the individual cross-product matrices. The weights used to compute the compromise are chosen to make it as representative as possible of all the assessors. In other words, we want to chose the weights so that assessors agreeing the most with other assessors will have the larger weights. So, in order to find these weights, we need first to evaluate the similarity between the assessors. This is done by computing an R_V coefficient between all pairs of assessors. The R_V coefficient was introduced by Escoufier (1973, see also Robert & Escoufier, 1976; Abdi, 2007c; Holmes, in press) as a measure of similarity between squared symmetric matrices (specifically: positive semi-definite matrices [2]). It is sometimes called *matrix* correlation coefficient although this is a potentially misleading appellation since these coefficients are *not* correlation coefficients because, contrary to the correlation coefficient, the mean of the observations is not subtracted prior to the computation. The R_V coefficient between two individual cross-product matrices $\mathbf{S}_{[t]}$ and $\mathbf{S}_{[t']}$ (representing the sort of assessors t and t') is defined as

$$R_V = \left[c_{t,t'}\right] = \frac{\text{trace}\left\{\mathbf{S}_{[t]}^{\mathsf{T}}\mathbf{S}_{[t']}\right\}}{\sqrt{\text{trace}\left\{\mathbf{S}_{[t]}^{\mathsf{T}}\mathbf{S}_{[t]}\right\} \times \text{trace}\left\{\mathbf{S}_{[t']}^{\mathsf{T}}\mathbf{S}_{[t']}\right\}}} \tag{6}$$

(this is not an efficient computational expression though, see Abdi, 2007c, for alternative versions).

The R_V coefficient takes on values between 0 and $+1$ (this is a consequence of the cross-product matrices being positive semi-definite matrices). A value of $+1$ for the R_V coefficient between two assessors means that the beers are sorted identically by these two assessors. A value of 0 means that the two assessors evaluate the beers into completely different ways (*i.e.*, they do not agree on any pair of beers). As detailed in Appendix A, the R_V coefficient reflects the agreement between assessors as a function of the number of beers that are either put together by both assessors or not put together by both assessors.

The R_V coefficients are compiled in a between-assessor similarity matrix denoted $\mathbf{C}$. The eigendecomposition (*i.e.*, PCA) of this matrix gives the weights used for computing the compromise. It is obtained as :

$$\mathbf{C} = \mathbf{P}\mathbf{\Theta}\mathbf{P}^{\mathsf{T}} \text{ with } \mathbf{P}^{\mathsf{T}}\mathbf{P} = \mathbf{I} \ , \tag{7}$$

where $\mathbf{P}$ is the matrix of eigenvectors and $\mathbf{\Theta}$ is the diagonal matrix of the eigenvalues of $\mathbf{C}$. This eigendecomposition reveals the structure of the assessors' space (see Figure 3). Because the matrix $\mathbf{C}$ is not centered, its first eigenvector (denoted $\mathbf{p}_1$) represents what is common to the assessors. Also because all the elements of $\mathbf{C}$ are positive, all the elements of the first eigenvector will have the same sign which is, as a convention, considered positive. The more similar an assessor is to the other assessors, the more it will contribute to this eigenvector. In other words, assessors with

[2] A matrix is *positive semi-definite* when it can be obtained as the product of a matrix by its transpose. As a consequence of this definition, a positive semi-definite matrix is square, symmetric, and its diagonal elements are always larger or equal to zero.

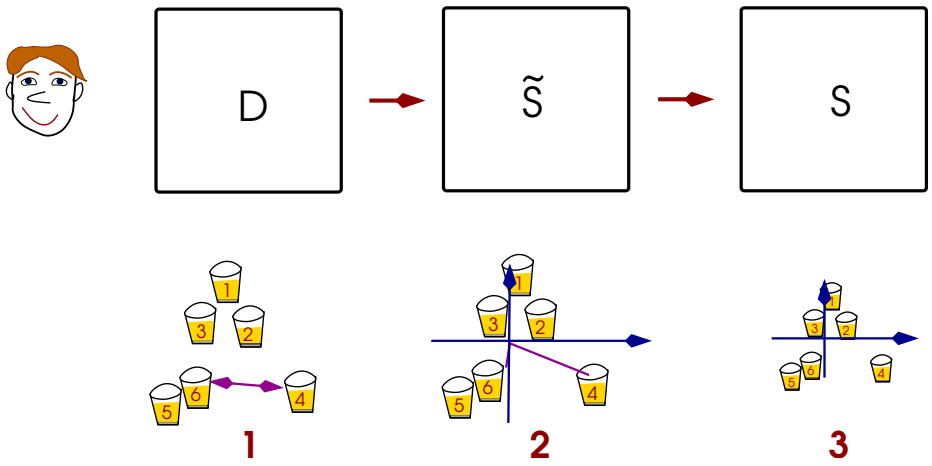

Fig. 2. The distance matrices are transformed into cross-product matrices which are then normalized. The normalization gives a first eigenvalue equal to one for each cross-product matrix.

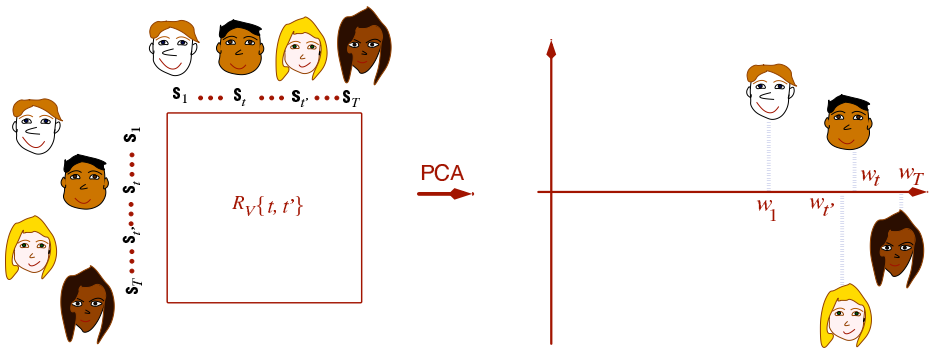

Fig. 3. The similarity between assessors' cross-product matrices is evaluated with the R_V coefficient and stored in the matrix $\mathbf{C}$. The eigen-analysis of matrix $\mathbf{C}$ shows the similarity structure between the assessors.

larger projections on the first eigenvector are more similar to the other assessors than assessors with smaller projections. Thus the elements of the first eigenvector give the optimal weights to compute the compromise matrix. Practically, the optimal weights can be obtained by re-scaling these values such that their sum is equal to one. So the weights are obtained by dividing each element of $\mathbf{p}_1$ by the sum of the elements of $\mathbf{p}_1$. The vector containing these weights is denoted $\boldsymbol{\alpha}$, and is computed as:

$$\boldsymbol{\alpha} = \left(\mathbf{1}^{\mathsf{T}}\mathbf{p}_1\right)^{-1} \times \mathbf{p}_1 \ . \tag{8}$$

Using these optimum weights, the compromise is then obtained as the weighted average of the assessors' cross-product matrices (see Figure 4). Specifically, with α_t denoting the weight for the t-th assessor, the compromise matrix, denoted $\mathbf{S}_{[+]}$, is computed as:

$$\mathbf{S}_{[+]} = \sum_{t}^{T} \alpha_t \mathbf{S}_{[t]} \ . \tag{9}$$

The compromise matrix is itself a cross-product matrix, and therefore its eigendecomposition amounts to a PCA. From this analysis, we can explore the structure of the set of beers. The eigendecomposition of the compromise is:

$$\mathbf{S}_{[+]} = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{\mathsf{T}} \tag{10}$$

and the compromise factor scores for the observations are computed as:

$$\mathbf{F} = \mathbf{V}\boldsymbol{\Lambda}^{\frac{1}{2}} \ . \tag{11}$$

The compromise matrix defines a common space. In addition to this common space, we want to see how each assessor "interprets" or distorts this space. This can be achieved by projecting the assessors' cross-product matrices onto the common space (see Figure 5). The mechanics of this projection can be deduced from Equations 10 and 11. Specifically, from

$$\mathbf{S}_{[+]} = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{\mathsf{T}} \text{ and } \mathbf{F} = \mathbf{V}\boldsymbol{\Lambda}^{\frac{1}{2}} \ , \tag{12}$$

and from

$\mathbf{V}^{\mathsf{T}}\mathbf{V} = \mathbf{I}$ (because the columns of $\mathbf{V}$ are normalized),

we get that

$$\mathbf{F} = \mathbf{S}_{[+]}\mathbf{V}\boldsymbol{\Lambda}^{-\frac{1}{2}} \ . \tag{13}$$

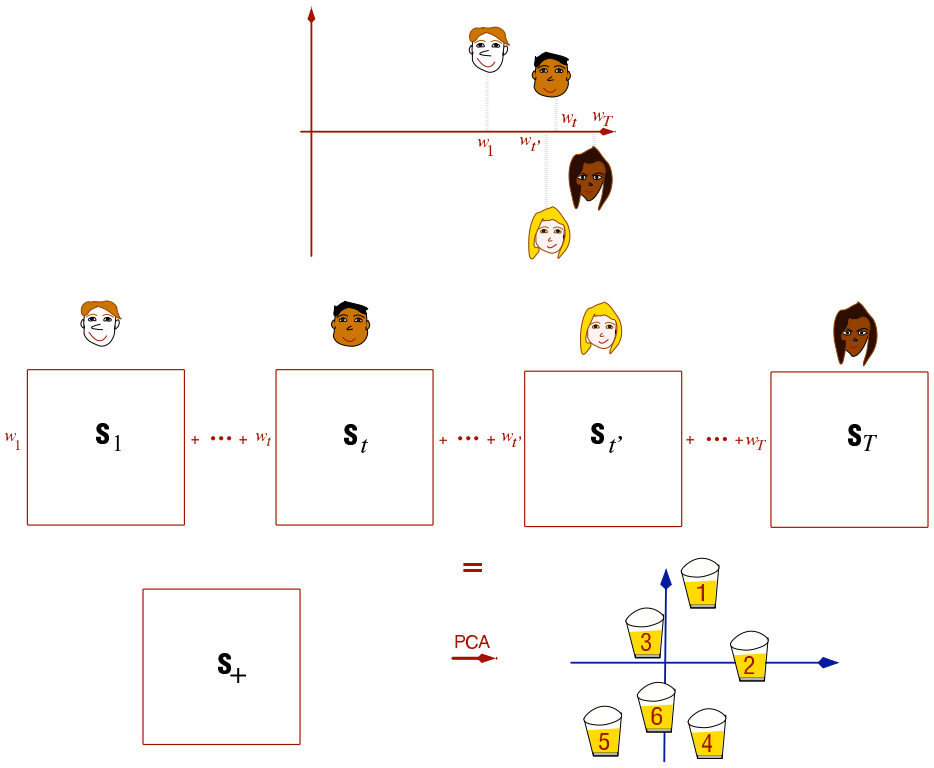

Fig. 4. The optimal weights are obtained from the projection of the assessors onto the first eigenvector of the between-assessor similarity matrix $\mathbf{C}$. The optimal weights are then used to compute the compromise which is the weighted average of the assessors' cross-product matrices. The eigendecomposition of the compromise shows the structure of the products.

Therefore, the matrix $\left(\mathbf{V}\mathbf{\Lambda}^{-\frac{1}{2}}\right)$ transforms the cross-product matrix into its projections onto the compromise space (*i.e.*, the matrix $\mathbf{V}\mathbf{\Lambda}^{-\frac{1}{2}}$ is a projection *operator*). We can also use this matrix to project (as supplementary elements) the assessors cross-product matrices into the common space. Specifically, the factor scores for the t-th assessor are computed as

$$\mathbf{F}_{[t]} = \mathbf{S}_{[t]}\left(\mathbf{V}\mathbf{\Lambda}^{-\frac{1}{2}}\right) \quad . \tag{14}$$

To sum-up, the different steps involved in DISTATIS are:

(i) Transform each distance matrix into a normalized between-stimuli cross-product matrix (*cf.* Figure 2).
(ii) Analyze the structure of the similarity between the cross-product matrices (as a PCA of the R_V coefficient matrix) (*cf.* Figure 3).
(iii) Derive an optimal set of weights for computing the compromise (*cf.* Figure 4).
(iv) Compute the compromise as a weighted sum of the individual cross-product matrices.
(v) Compute the PCA (*i.e.*, eigendecomposition) of the compromise matrix and plot the projections of the stimuli (*cf.* Figure 5).
(vi) Project the original distance matrices as supplementary points in the compromise space (*cf.* Figure 5).

3.An example: sorting beers

In this section we present a small example illustrating the steps of DISTATIS. In this example, ten assessors sorted eight beers. As additional information we collected hedonic ratings on the beers (from these assessors) and also asked the assessors to describe each beer. We also recorded the alcohol content of the beers.

3.1.*Material and Method*

3.1.1. *Assessors*

Ten volunteers from the University of Bourgogne participated in the study (4 women and 6 men ranging in age from 22 to 39 years). They were beer consumers but did not have any formal training in sensory evaluation of beers.

3.1.2. *Beers*

Eight commercial beers were used (see Table 1). All beers were presented in brown plastic tumblers and served at 8°C. Assessors were asked to carry opaque glasses to mask any visual information (color or froth differences).

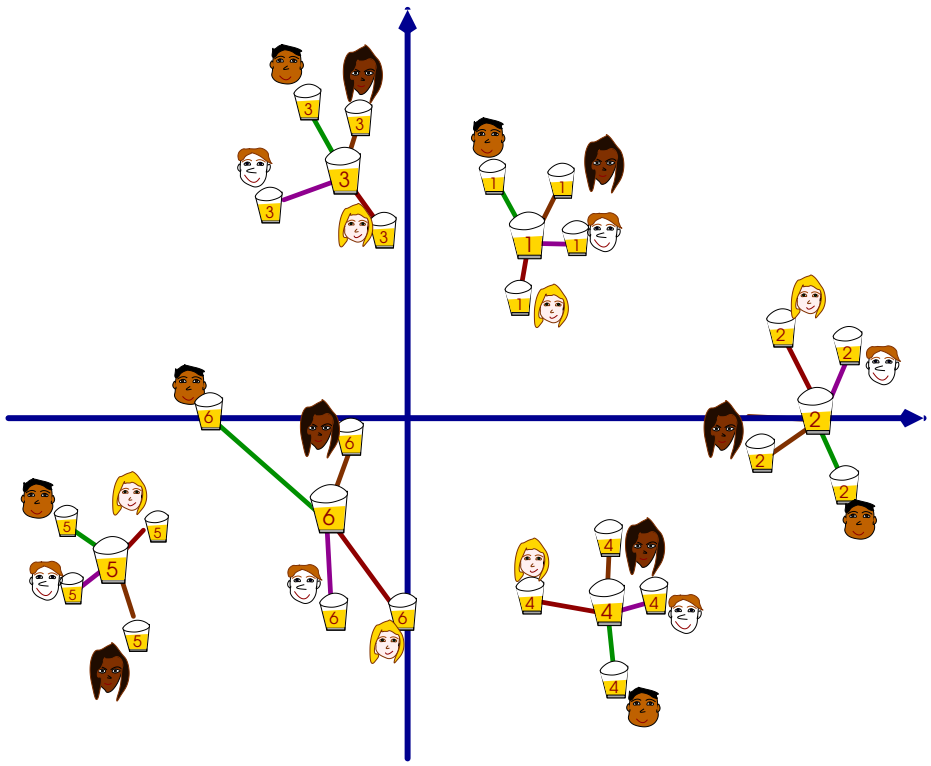

Fig. 5. Compromise map of the beers plus each assessor's map. The cross-product matrices representing the result of the sort of each assessor are projected as supplementary elements. This is shown on the map by representing each beer as evaluated by a given assessor as a glass with the face of the assessor next to the glass. The compromise position is the barycenter of all the assessors. A line segment links the position of the beer for a given assessor to the compromise position for this beer.

3.1.3. *Procedure*

The 10 beer samples were presented in front of the assessors in a different order for each assessor. The assessors first had to smell and taste each beer sample. Then they had to sort the samples on the basis of their perceptual similarity. Assessors could make as many groups as they wanted and each group could contain as many beers as they wished. Assessors were allowed to take as much time as they needed and to smell/taste the samples as often as they wished. Mineral water was available for assessors to rinse between samples.

After completion of the sorting, assessors were asked to provide a few words to describe the beers and to indicate their liking of each beer on a seven-point scale going from "I don't like at all" to "I like it very much."

3.2.*Results*

The original data are given in Table 1. They can also be found in `www.utdallas.edu/~herve` along with a MATLAB DISTATIS routine (`distatis.m`) and the MATLAB script (`biere.m`) used to analyze these data. An `R` program written by Guillaume Blancher is also available from this address.

Step 0: Transform each partition into an indicator matrix and compute each cross-product and distance matrices

Each assessor gives a partition of the beers. For example, Assessor 1 gives the following partition with five classes

$$\begin{aligned}&\{\text{Affligen, St Landelin, Fruit Défendu}\},\\ &\{\text{Buckler Highland}\},\\ &\{\text{Buckler Blonde}\},\\ &\{\text{Budweiser, Killian, }\},\\ &\{\text{EKU28}\}\ .\end{aligned} \tag{15}$$

We start by deriving, for each assessor, an indicator matrix in which the rows represent the beers and the columns represent the groups. A value of 1 means that the beer represented by the row was put into the group represented by the column (note, incidentally, that the order of the column is arbitrary). In an indicator matrix representing a sorting task, there is one and only one 1 per row (this indicates that a beer belongs to one and only one group). For example, the indicator matrix for Assessor 1 is equal to

Table 1
Beer Sorting Task. Results. Ten assessors sorted eight beers. For each assessor, beers with the same number were sorted together.

Beers	Alcohol content	Hedonic score	Most frequent descriptor	Assessors									
				1	2	3	4	5	6	7	8	9	10
			Sex	F	M	F	F	M	M	M	M	F	M
Affligen	6.5	5.5	Fruity/Bitter	1	4	3	4	1	1	2	2	1	3
Budweiser	5.0	2.5	Insipid/Bitter	4	5	2	5	2	3	1	1	4	3
Buckler Blonde	< 1.2	1.8	Insipid/Bitter	3	1	2	3	2	4	3	1	1	2
Killian	6.5	4.3	Fruity/Coffee/Bitter	4	2	3	3	1	1	1	2	1	4
St. Landelin	6.5	4.7	Fruity/Bitter/Fade	1	5	3	5	2	1	1	2	1	3
Buckler Highland	< 1.2	1.0	Coffee/Caramel	2	3	1	1	3	5	4	4	3	1
Fruit Défendu	8.8	6.2	Fruity/Sweet	1	4	3	4	1	1	2	2	2	4
EKU28	11.0	1.8	Caramel/Sweet/Alcohol	5	2	4	2	4	2	5	3	4	5

$$\mathbf{L}_{[1]} = \begin{bmatrix} 1&0&0&0&0\\ 0&0&0&1&0\\ 0&0&1&0&0\\ 0&0&0&1&0\\ 1&0&0&0&0\\ 0&1&0&0&0\\ 1&0&0&0&0\\ 0&0&0&0&1 \end{bmatrix}. \quad (16)$$

From each indicator matrix, we derive a co-occurrence matrix in which a value of 1 means that the beer represented by the row and the beer represented by the column were put in the same group and a value of 0 means that these two beers were not put together (*cf.* Figure 1). For example, the co-occurrence matrix for Assessor 1 is equal to:

$$\mathbf{R}_{[1]} = \mathbf{L}_{[1]}\mathbf{L}_{[1]}^{\mathsf{T}} = \begin{bmatrix} 1&0&0&0&1&0&1&0\\ 0&1&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&1&0&1&0&0&0&0\\ 1&0&0&0&1&0&1&0\\ 0&0&0&0&0&1&0&0\\ 1&0&0&0&1&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix}. \quad (17)$$

From each co-occurrence matrix, we derive a between-beer distance matrix in which a value of 0 means that the beer represented by the row and the beer represented by the column were put in the same group and a value of 1 means that these two beers were not put together (*cf.* Figure 1). For example, the distance matrix for Assessor 1 is equal to:

$$\mathbf{D}_{[1]} = \mathbf{1} - \mathbf{R}_{[1]} = \begin{bmatrix} 0&1&1&1&0&1&0&1\\ 1&0&1&0&1&1&1&1\\ 1&1&0&1&1&1&1&1\\ 1&0&1&0&1&1&1&1\\ 0&1&1&1&0&1&0&1\\ 1&1&1&1&1&0&1&1\\ 0&1&1&1&0&1&0&1\\ 1&1&1&1&1&1&1&0 \end{bmatrix}. \quad (18)$$

Step 1: Transform each distance matrix into a normalized between-stimuli cross-product matrix

The distance matrices are then transformed into cross-product matrices (see Equation 5) and normalized by their eigenvalues. For example, Assessor 1 gives the following cross-product matrix:

$$\widetilde{\mathbf{S}}_{[1]} = \begin{bmatrix} 0.25 & -0.19 & -0.12 & -0.19 & 0.25 & -0.12 & 0.25 & -0.12 \\ -0.19 & 0.38 & -0.06 & 0.38 & -0.19 & -0.06 & -0.19 & -0.06 \\ -0.12 & -0.06 & 0.50 & -0.06 & -0.12 & 0 & -0.12 & 0 \\ -0.19 & 0.38 & -0.06 & 0.38 & -0.19 & -0.06 & -0.19 & -0.06 \\ 0.25 & -0.19 & -0.12 & -0.19 & 0.25 & -0.12 & 0.25 & -0.12 \\ -0.12 & -0.06 & 0 & -0.06 & -0.12 & 0.50 & -0.12 & 0 \\ 0.25 & -0.19 & -0.12 & -0.19 & 0.25 & -0.12 & 0.25 & -0.12 \\ -0.12 & -0.06 & 0 & -0.06 & -0.12 & 0 & -0.12 & 0.50 \end{bmatrix}. \quad (19)$$

The first eigenvalue of this cross-product matrix is equal to 1.25. And therefore the normalized cross-product matrix for this assessor is obtained by dividing all the elements of $\widetilde{\mathbf{S}}_{[1]}$ by 1.25. This gives:

$$\mathbf{S}_{[1]} = \begin{bmatrix} 0.20 & -0.15 & -0.10 & -0.15 & 0.20 & -0.10 & 0.20 & -0.10 \\ -0.15 & 0.30 & -0.05 & 0.30 & -0.15 & -0.05 & -0.15 & -0.05 \\ -0.10 & -0.05 & 0.40 & -0.05 & -0.10 & 0 & -0.10 & 0 \\ -0.15 & 0.30 & -0.05 & 0.30 & -0.15 & -0.05 & -0.15 & -0.05 \\ 0.20 & -0.15 & -0.10 & -0.15 & 0.20 & -0.10 & 0.20 & -0.10 \\ -0.10 & -0.05 & 0 & -0.05 & -0.10 & 0.40 & -0.10 & 0 \\ 0.20 & -0.15 & -0.10 & -0.15 & 0.20 & -0.10 & 0.20 & -0.10 \\ -0.10 & -0.05 & 0 & -0.05 & -0.10 & 0 & -0.10 & 0.40 \end{bmatrix}. \quad (20)$$

Step 2: Analyze the structure of the assessors cross-product matrices

Using the R_V coefficient (see Equation 6) to evaluate the similarity between normalized assessor cross-product matrices gives the following 10×10 between-assessor similarity matrix (see Equation 7):

$$\mathbf{C} = \begin{bmatrix} 1.00 & 0.57 & 0.60 & 0.57 & 0.35 & 0.68 & 0.67 & 0.60 & 0.37 & 0.45 \\ 0.57 & 1.00 & 0.42 & 0.83 & 0.65 & 0.49 & 0.76 & 0.42 & 0.28 & 0.57 \\ 0.60 & 0.42 & 1.00 & 0.40 & 0.63 & 0.93 & 0.46 & 1.00 & 0.44 & 0.46 \\ 0.57 & 0.83 & 0.40 & 1.00 & 0.60 & 0.49 & 0.76 & 0.40 & 0.42 & 0.57 \\ 0.35 & 0.65 & 0.63 & 0.60 & 1.00 & 0.59 & 0.53 & 0.63 & 0.33 & 0.53 \\ 0.68 & 0.49 & 0.93 & 0.49 & 0.59 & 1.00 & 0.56 & 0.93 & 0.56 & 0.56 \\ 0.67 & 0.76 & 0.46 & 0.76 & 0.53 & 0.56 & 1.00 & 0.46 & 0.35 & 0.45 \\ 0.60 & 0.42 & 1.00 & 0.40 & 0.63 & 0.93 & 0.46 & 1.00 & 0.44 & 0.46 \\ 0.37 & 0.28 & 0.44 & 0.42 & 0.33 & 0.56 & 0.35 & 0.44 & 1.00 & 0.35 \\ 0.45 & 0.57 & 0.46 & 0.57 & 0.53 & 0.56 & 0.45 & 0.46 & 0.35 & 1.00 \end{bmatrix}. \quad (21)$$

How to interpret this matrix? We can see from the R_V matrix that some assessors agree strongly in their way of sorting the beers (*e.g.*, Assessors 3 and 8, 3 and 6, 4 and 2 with R_V of respectively 1, .93, and .83). Other assessors are more original. In particular, Assessor 9 seems to behave differently from the other assessors with R_V coefficients going from .28 to .56.

A PCA (see Equation 7) of matrix $\mathbf{C}$ provides an easier way to examine the relationships between the assessors (see Figure 6). It can be used to reveal clusters of assessors (such as Assessors 3, 8, and 6 or 2 and 4), and to detect atypical assessors such as Assessor 9 which has the smallest projection onto the first dimension. This PCA can also be used—as illustrated in Figure 6 with gender effects—to explore the effect of assessor characteristics (*e.g.*, gender, age) on their evaluation of the beers. Here we see that there is no gender effect because the barycenter of the women assessors is very close to the barycenter of the men assessors.

Step 3: Derive an optimal set of weights for computing the compromise

The weights used to compute the compromise correspond to the rescaled projections of the assessors onto the first dimension shown in Figure 6 (see Equation 8):

$$\boldsymbol{\alpha} = \begin{bmatrix} .100 & .101 & .109 & .101 & .099 & .116 & .101 & .109 & .074 & .090 \end{bmatrix}^{\mathsf{T}}. \quad (22)$$

We can see that Assessor 9 has the smallest weight (.074) which means that she will contribute less to the compromise than, for example, Assessor 6 who has the largest weight (.116).

Step 4: Compute the compromise

The compromise is then computed as the weighted average of all the assessors' cross-product matrices (see Equation 9). It is equal to

$$\mathbf{S}_{[+]} = \begin{bmatrix} 0.21 & -0.11 & -0.10 & -0.00 & 0.03 & -0.09 & 0.16 & -0.10 \\ -0.11 & 0.30 & 0.03 & -0.06 & 0.06 & -0.05 & -0.13 & -0.03 \\ -0.10 & 0.03 & 0.34 & -0.04 & -0.07 & -0.02 & -0.11 & -0.03 \\ -0.00 & -0.06 & -0.04 & 0.22 & -0.00 & -0.08 & 0.02 & -0.04 \\ 0.03 & 0.06 & -0.07 & -0.00 & 0.20 & -0.09 & -0.02 & -0.10 \\ -0.09 & -0.05 & -0.02 & -0.08 & -0.09 & 0.41 & -0.08 & 0.00 \\ 0.16 & -0.13 & -0.11 & 0.02 & -0.02 & -0.08 & 0.24 & -0.08 \\ -0.10 & -0.03 & -0.03 & -0.04 & -0.10 & 0.00 & -0.08 & 0.39 \end{bmatrix}. \quad (23)$$

The quality of the compromise is evaluated from the PCA of the between-assessor similarity matrix. It is equal to the proportion of variance explained by the first component which explained 60% of the variance. This is a relatively small value and this indicates that the assessors differ substantially in their evaluation of the beers. However, because of the nature of the sorting task which uses $0/1$ values only, the estimation of the explained variance is pessimistic and further work is needed to correct for the effect of the task.

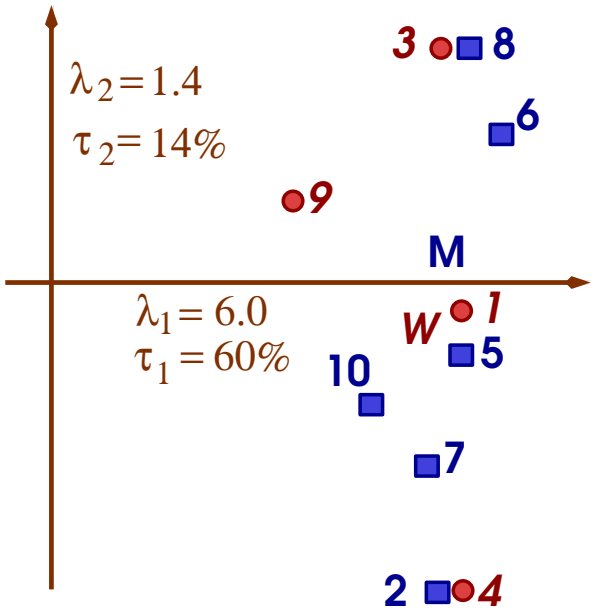

Fig. 6. PCA of the $\mathbf{C}$ matrix. Assessors are identified by their number. Circles represent women assessors, and squares men assessors. The gender barycenters are also plotted (W and M). Because these barycenters are very close to each other, we conclude that gender is not related to systematic differences between the assessors.

Table 2
Beer Sorting Task. Compromise factor scores for the beers.

			Beers							
Axis	λ	τ	1	2	3	4	5	6	7	8
1	.66	29%	−0.39	0.23	0.28	−0.16	−0.15	0.30	−0.40	0.29
2	.49	21%	0.07	−0.37	−0.22	−0.03	−0.24	0.38	0.15	0.26
3	.40	18%	−0.03	−0.02	−0.07	0.09	−0.04	−0.40	−0.01	0.48
4	.34	15%	0.03	−0.25	0.45	0.08	−0.22	−0.10	0.07	−0.06
5	.23	10%	0.16	0.06	0.07	−0.42	−0.01	−0.05	0.11	0.06
6	.13	6%	0.02	−0.20	0.05	−0.05	0.26	0.01	−0.13	0.04
7	.05	2%	0.16	0.02	−0.01	0.03	−0.06	0.00	−0.14	−0.00

Step 5: Compute the PCA of the compromise matrix and plot the projections of the beers

The PCA of the compromise matrix reveals the structure of the beers common to the assessors. The factor scores are given in Table 2 and the projections of the beers on the first four dimensions are shown in Figure 7. Together, these four dimensions explain 84% of the inertia of the compromise matrix, such a large value indicates that the interpretation of the results can be restricted to these 4 dimensions. In order to facilitate the interpretation of Figure 7, we projected as supplementary continuous variables the alcohol content of the beers and their average hedonic scores (*cf.* Table 1). This was done by computing standard loadings (*i.e.*, correlations between the variables and the factor scores) and then re-scaling these loadings by multiplying them by half of the square root of the eigenvalue associated with this dimension (other methods could be used to create biplot maps).

The first dimension, which explains 29% of the inertia of the compromise matrix is highly correlated with the hedonic scores of the beers. It opposes the beers which the assessors like the least (Buckler Highlands, EKU28, Buckler Blonde, and Budweiser) to the beers which assessors liked most (Fruit Défendu and Affligen). The second dimension explains 21% of the inertia. It does not correlate

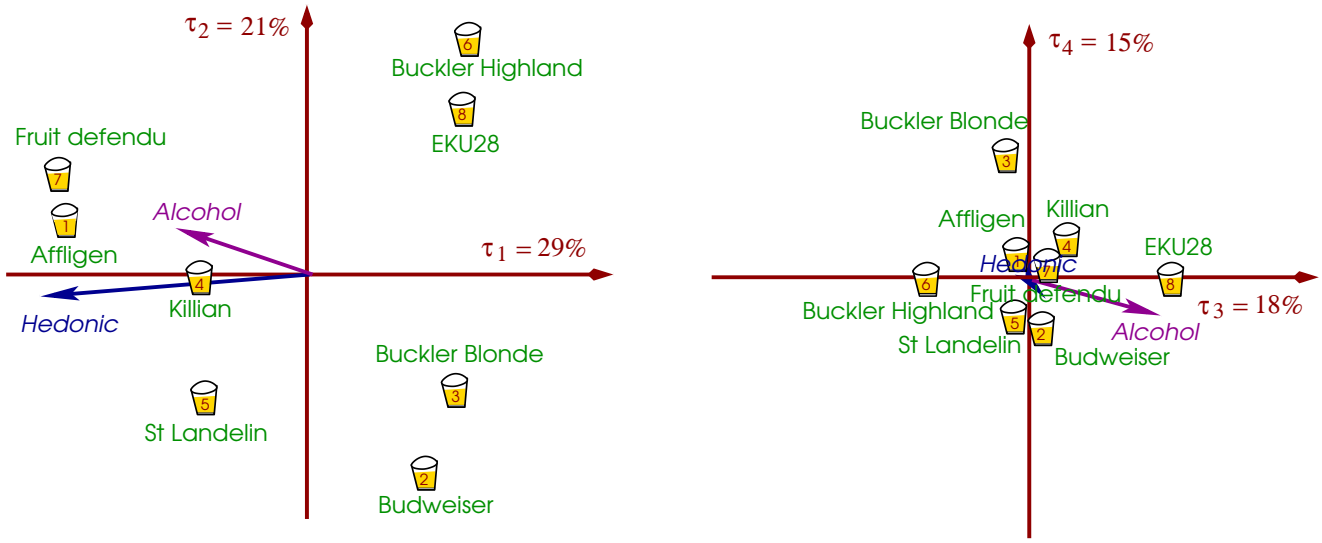

Fig. 7. Analysis of the compromise: Plot of the beers in the plane defined by dimensions 1 and 2 (left side) and 3 and 4 (right side) of the compromise matrix.

with any of the supplementary variables but an examination of the assessors' description indicates that it opposes the beers with the stronger flavor (Buckler Highlands, EKU28, Fruit Défendu) to the beers with the lighter flavor (Buckler Blonde, and Budweiser). The third dimension which explains 18% of the inertia is highly correlated with the alcohol content, and it opposes the beer with the highest alcohol content (EKU28) to the alcohol free beers Buckler Highland and (to a lesser degree) Buckler Blonde. The fourth dimension, which explains 15% of the variance, is somewhat harder to interpret. It seems to isolate the Buckler Blonde beer.

Step 6: Project the original distance matrices as supplementary points in the compromise space

Figure 8 shows the first two principal components of the compromise space along with the projections of the beers for each assessor (*cf.* Equation 14). Note that the position of each beer in the compromise is the barycenter of the positions of this beer for the ten assessors. In order to facilitate the interpretation of the figure, we have drawn the convex envelop of each beer. We can see that if for some beers like "Buckler Highland," "EKU28," "Affligen," or "Fruit Défendu" we have a relatively good between-assessor agrement, this is not the case for all beers. In particular we can note that half the assessors have considered that the "St Landelin" was similar to the "Affligen" and "Fruit Défendu" beers and that the other half of the assessors have considered that it was closer to the "Budweiser" and "Buckler Blonde" beers. The smallest agreement was obtained for the beer "Killian." Interestingly, this is also for this beer that we obtained the widest variety of descriptors going from fruity to coffee, caramel, bitter, up to tasteless. We can also notice that some beers such as "Budweiser" and "Buckler Blonde" are partially overlapping, whereas other beers are clearly separated from each others such as "Budweiser" and "Fruit Défendu" for example. This suggests that some beers share some features because some assessors did put them together, whereas others did not. As a summary we can say that the "Fruit Défendu," "Affligen," "Saint Landelin," and "Killian" share some features. From the descriptions provided by the assessors, it seems that these beers have the common property of having a fruity flavor. Likewise, the "Budweiser," "Saint Landelin," and "Killian" have the common property to taste like a standard lager beer. Only the "Buckler Highland" and the "EKU28" do not seem to share properties with other beers. The first one is described as a beer with a strong flavor of coffee and caramel and isolated by all assessors. The second one is a sweet beer rich in alcohol with a caramel flavor. One assessor sorted it with the "Killian" and another one with the "Budweiser."

4.Discussion and Conclusions

This paper illustrates that, used conjointly with a sorting task, DISTATIS can be a useful tool to evaluate the similarity of a set of products. The advantage of DISTATIS is to provide a compromise space that is comparable with what would be obtained with a nonmetric algorithm such as ALSCAL (see Figure 9 for an illustration) or a standard metric MDS (because the assessors' weights were very similar, a metric MDS gave a solution very similar to the analysis of the compromise). And in addition to this compromise space, DISTATIS gives valuable information on individual assessors and on the consensus between assessors when individual assessor matrices are analyzed. Also, the compu-

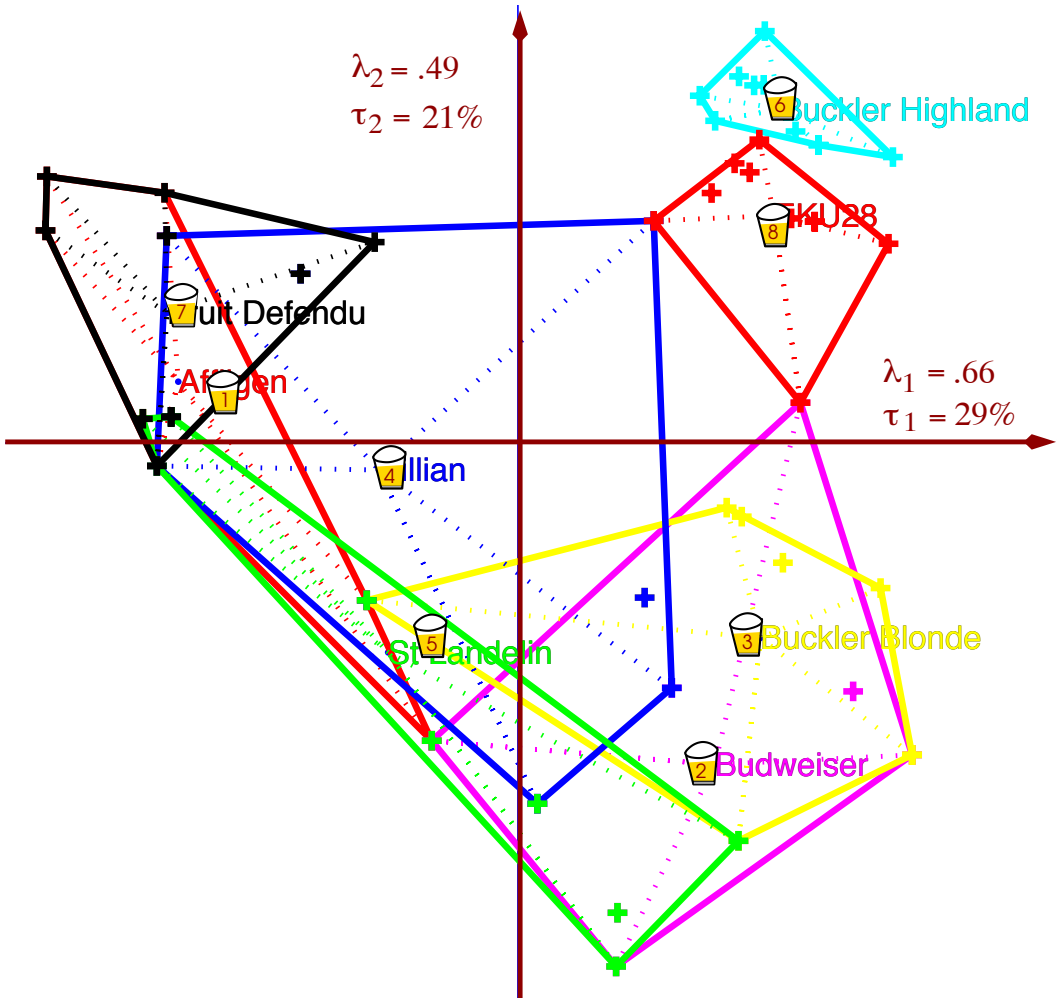

Fig. 8. Compromise map of the beers plus each assessor's map. The compromise position is the barycenter of all the assessors. A convex hull have been drawn for each beer by joining the most extreme points for this beer to indicate the variability of each beer. The larger the convex hull the less the assessors agreed for this beer.

tational core of DISTATIS consists of the eigendecomposition, which is one of the most well studied computational techniques. As a consequence, DISTATIS can be used on large sets of data. It is also possible to develop robust versions of DISTATIS by using robust eigendecomposition algorithms. DISTATIS could also be used to compare groups of assessors such as assessors coming from different cultural environment or with different expertise levels. In this case, the consensus would be evaluated between the groups of assessors rather than between the assessors themselves. One could also imagine using this technique to compare the variability observed between individual assessors of a given group with that observed between groups of assessors. This would provide an ANOVA-like decomposition (see Le Dien & Pagès, 2003; for a similar idea about multiple factor analysis). DISTATIS can, in principle, also accommodate, mixtures of distances (*i.e.*, sorting distances and projective mapping or "tablecloth distances" as described by Risvik, McEwan, Colwill, & Roger, 1994; and Pagès, 2005), but it is not clear, at the moment, if the current normalization used (*i.e.*, division of each scalar product matrix by its first eigenvalue) will be robust enough to eliminate the specific effect of each type of distance. Further empirical work should explore this problem.

The application presented here is a first step into utilizing DISTATIS in combination with sorting tasks. Obviously, some aspects of the method can be improved. First, the analysis of the R_V coefficient matrix gives a PCA of the assessors, and it would be useful to enrich this representation by adding the equivalent of confidence intervals around the point representing each assessor. Possible lines of development could be the use of permutation tests (*cf.* Schlich, 1996; Kazi-Aoual, Hitier, Sabatier & Lebreton, 1995).

DISTATIS relies on the R_V coefficient to evaluate the similarity between assessors, but other coefficients specifically tailored for evaluating the similarity between parti-

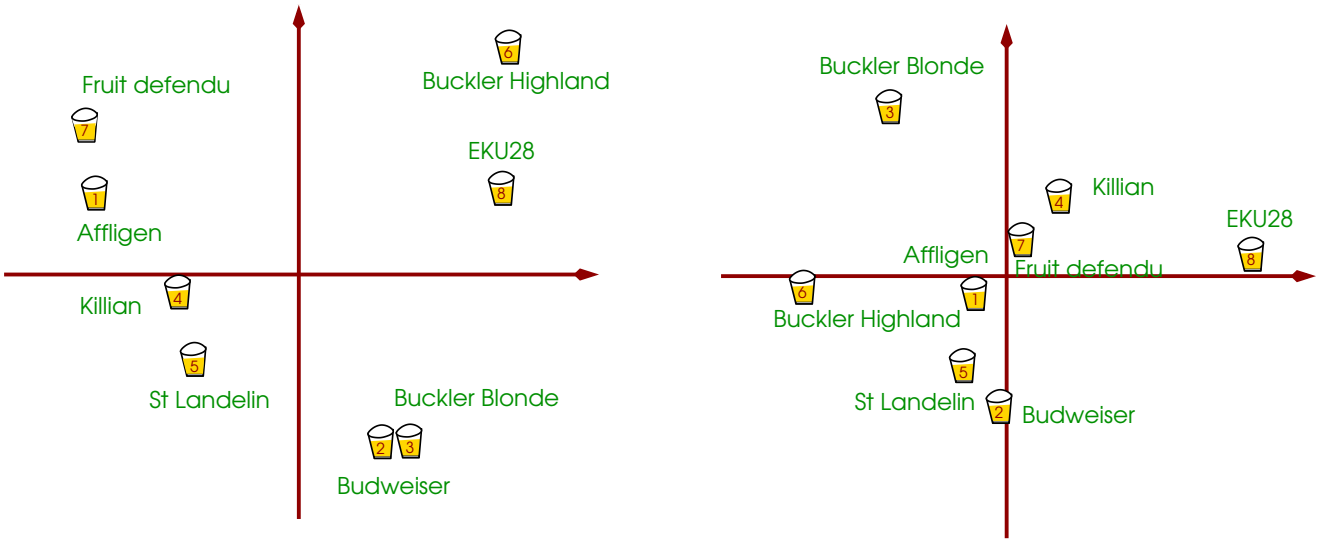

Fig. 9. ALSCAL analysis of the sum of the individual similarity matrices: Plot of the beers in the plane defined by dimensions 1 and 2 (left side) and 3 and 4 (right side) of the 4 dimensional solution (stress=.02). The projections of the beers are quite close to those presented in Figure 7.

tions of a set (see, *e.g.*, Hubert & Arabie, 1985) maybe more adequate than the R_V coefficient. These questions should be the basis for further work as well as the application of DISTATIS to other type of tasks than the plain sorting task (see *e.g.*, Coxon, 1999).

Appendix A: What does the RV coefficient measure in DISTATIS?

In this appendix, we explore the interpretation of the R_V coefficient for DISTATIS. For notational convenience, we will denote by $\mathbf{D}$ and $\mathbf{T}$ two $I \times I$ (squared) distance matrices used to compute the R_V coefficient and we will assume (without loss of generality) that the mass of each observation is equal to $\frac{1}{I}$

The first step of the computation of the R_V coefficient is to transform each distance matrix into a cross-product matrix. Using Equation 3, we obtain $\mathbf{S}$ (respectively $\mathbf{Z}$) which is the cross-product matrix associated to $\mathbf{D}$ (respectively, $\mathbf{T}$). Recall that the i, j term of $\mathbf{S}$ is equal to

$$s_{i,j} = -\frac{1}{2}\left\{d_{i,j} - \overline{d}_{i,+} - \overline{d}_{j,+} + \overline{d}_{+,+}\right\} \qquad (24)$$

with $\overline{d}_{i,+}$ (respectively $\overline{d}_{j,+}$) being the mean of the distances for the ith (respectively j-th) row, and where $\overline{d}_{+,+}$ is the grand mean of $\mathbf{D}$.

Because the denominator of the R_V coefficient is essentially a normalizing factor, we can focus on its numerator to analyze R_V. Also, the normalization of the cross-product matrices by division by their first eigenvalues does not affect the R_V coefficient (because the eigenvalues cancel each other out from the numerator and the denominator). Therefore we will ignore this step in our analysis.

We denote by $\mathcal{N}$ the quantity equal to 4 times the numerator of the R_V coefficient, that is:

$$\mathcal{N} = 4 \times \text{trace}\left\{\mathbf{S}^{\mathsf{T}}\mathbf{Z}\right\} = 4 \times \sum_{i}^{I}\sum_{j}^{I} s_{i,j} z_{i,j} \ . \qquad (25)$$

The value of $\mathcal{N}$ corresponding to the R_V coefficient between the two cross-product matrices obtained from the two distances is equal to (*cf.* Equations 24 and 25):

$$\begin{aligned}\mathcal{N} &= 4 \times \sum_{i}^{I}\sum_{j}^{I} s_{i,j} \times z_{i,j} \\ &= 4 \times \sum_{i,j}\left\{-\frac{1}{2}\left(d_{i,j} - \overline{d}_{i,+} - \overline{d}_{j,+} + \overline{d}_{+,+}\right)\right\} \\ &\quad \times \left\{-\frac{1}{2}\left(t_{i,j} - \overline{t}_{i,+} - \overline{t}_{j,+} + \overline{t}_{+,+}\right)\right\} \qquad (26)\end{aligned}$$

When developed and simplified, Equation 26 reduces to

$$\mathcal{N} = \sum_{i,j} d_{i,j} t_{i,j} - \frac{1}{2I}\left(\sum_{i} d_{i,+} t_{i,+} - \frac{1}{2I} d_{+,+} t_{+,+}\right) \qquad (27)$$

(with the $+$ signs denoting the implicit sum).

RV coefficient for the sorting task

In order to clarify the relationship between the R_V coefficient and other possible measures of the similarity between the performance of two assessors, we will adapt Equation 27 to the specific case of the sorting task. We will denote by $\mathbf{R}$ and $\mathbf{Q}$ the two $I \times I$ co-occurrence matrices correspond-

ing to the distance matrices $\mathbf{D}$ and $\mathbf{T}$ (*i.e.*, $\mathbf{R} = \mathbf{1} - \mathbf{D}$ and $\mathbf{Q} = \mathbf{1} - \mathbf{T}$).

Because the co-occurrence matrices are obtained by adding a constant to the distance matrices, Equation 27 can also be computed directly from the co-occurrence matrices as

$$\mathcal{N} = \sum_{i,j} r_{i,j} q_{i,j} - \frac{1}{2I} \left(\sum_i r_{i,+} q_{i,+} - \frac{1}{2I} r_{+,+} q_{+,+} \right) . \tag{28}$$

Combining Equations 27 and 28, we find that, for sorting tasks, the numerator of the R_V coefficient can be expressed as

$$\begin{aligned} \mathcal{N} = \frac{1}{2} \Bigg\{ & \sum_{i,j} d_{i,j} t_{i,j} + \sum_{i,j} r_{i,j} q_{i,j} \\ & - \frac{1}{2I} \left(\sum_i d_{i,+} t_{i,+} + \sum_i r_{i,+} q_{i,+} \right. \\ & \left. \left. - \frac{1}{2I} d_{+,+} t_{+,+} - \frac{1}{2I} r_{+,+} q_{+,+} \right) \right\} . \end{aligned} \tag{29}$$

The term $\sum d_{i,j} t_{i,j}$ gives the number of objects that are placed in the same categories by both assessors, and the term $\sum r_{i,j} q_{i,j}$ reflects the number of objects that are placed in different categories by both assessors. Therefore the term $(\sum d_{i,j} t_{i,j} + \sum r_{i,j} q_{i,j})$ expresses the *agreement* between the assessors. In the literature concerned with evaluating distances or agreement between partitions, the number

$$A = \left[\left(\sum d_{i,j} t_{i,j} + \sum r_{i,j} q_{i,j} \right) - I \right] \tag{30}$$

is called the number of agreements (we need to subtract I because the diagonal terms of the similarity matrix are always equal to 1 and therefore are irrelevant for evaluating agreement). The index A is used to define several measures of agreement between partitions (see, Hubert & Arabie, 1985; for a review), such as, for example the well known Rand coefficient (Rand, 1971) which is equal to

$$R_{\text{Rand}} = \frac{2A}{I(I-1)} . \tag{31}$$

In Equation 27, the terms other than

$$\left(\sum d_{i,j} t_{i,j} + \sum r_{i,j} q_{i,j} \right)$$

can be interpreted as reflecting the proportion of agreements which could be obtained by chance under the assumption of independence. Therefore we can interpret the R_V coefficient as expressing the proportion of the between assessor agreement after correcting for chance matching. Equations 27 and 31 also show that the R_V coefficient is closely related to standard measures of agreement between partitions when it is used to evaluate the similarity between assessors. When interpreting the analysis of the assessor space (*i.e.*, the PCA of matrix $\mathbf{C}$), the proximity of two assessors reflects the number of objects that they agree to put in the same category as well as the number of objects that they agree *not* to put in the same category. By contrast, the distance between assessors can be interpreted as reflecting the disagreement between the assessors.

Appendix B: With sorting tasks, the matrix S is positive semi definite

Let $\mathbf{L}$ be an $I \times K$ indicator matrix and $\mathbf{m}$ a mass vector of positive numbers such that $\mathbf{m}^\mathsf{T}\mathbf{1} = \mathbf{0}$. Let matrices $\mathbf{R}$, $\mathbf{D}$, and $\mathbf{\Xi}$ defined respectively by Equations 1, 2, and 4.

Theorem ($\mathbf{S}$ is psd)**.** *The matrix* $\mathbf{S}$ *is positive semi definite.*

Proof. Recall that a matrix $\mathbf{X}$ is positive semi definite if and only if there exist a matrix $\mathbf{V}$ such that

$$\mathbf{X} = \mathbf{V}\mathbf{V}^\mathsf{T} . \tag{32}$$

To show that $\mathbf{S}$ is positive semi definite, we just need to plug Equations 1 and 2 into Equation 5, to develop and to simplify

$$\begin{aligned} \mathbf{S} &= -\frac{1}{2} \left\{ \mathbf{\Xi} \mathbf{D} \mathbf{\Xi}^\mathsf{T} \right\} \\ &= -\frac{1}{2} \left\{ \mathbf{\Xi} \left(\mathbf{1} - \mathbf{R} \right) \mathbf{\Xi}^\mathsf{T} \right\} \\ &= -\frac{1}{2} \left\{ \mathbf{\Xi} \left(\mathbf{1} - \mathbf{L}\mathbf{L}^\mathsf{T} \right) \mathbf{\Xi}^\mathsf{T} \right\} \\ &= -\frac{1}{2} \left\{ \mathbf{\Xi}\mathbf{1}\mathbf{\Xi}^\mathsf{T} - \mathbf{\Xi}\mathbf{L}\mathbf{L}^\mathsf{T}\mathbf{\Xi}^\mathsf{T} \right\} \end{aligned} \tag{33}$$

but $\mathbf{\Xi}\mathbf{1}\mathbf{\Xi}^\mathsf{T} = \mathbf{0}$, and therefore

$$\begin{aligned} \mathbf{S} &= \frac{1}{2} \mathbf{\Xi}\mathbf{L}\mathbf{L}^\mathsf{T}\mathbf{\Xi}^\mathsf{T} \\ &= \frac{1}{2} \left(\mathbf{\Xi}\mathbf{L} \right) \left(\mathbf{\Xi}\mathbf{L} \right)^\mathsf{T} \end{aligned} \tag{34}$$

which completes the proof. □

References

[1] Abdi, H. (1990). Additive-tree representations. *Lecture Notes in Biomathematics*, **84**, 43–59.

[2] Abdi, H. (2007a). Metric multidimensional scaling. In N. Salkind (Ed.) *Encyclopedia of measurement and statistics.* Thousand Oaks (CA): Sage.

[3] Abdi, H. (2007b). Distance. In N. Salkind (Ed.) *Encyclopedia of measurement and statistics.* Thousand Oaks (CA): Sage.

[4] Abdi, H. (2007c). The R_V coefficient and the congruence coefficient. In N. Salkind (Ed.) *Encyclopedia of measurement and statistics.* Thousand Oaks (CA): Sage.

[5] Abdi, H., & Valentin, D. (2007a). DISTATIS. In N. Salkind (Ed.) *Encyclopedia of measurement and statistics*. Thousand Oaks (CA): Sage.

[6] Abdi, H., & Valentin, D. (2007b). STATIS. In N. Salkind (Ed.) *Encyclopedia of measurement and statistics*. Thousand Oaks (CA): Sage.

[7] Abdi, H., & Valentin, D. (2007c). Multiple Factor Analysis. In N. Salkind (Ed.) *Encyclopedia of measurement and statistics*. Thousand Oaks (CA): Sage.

[8] Abdi, H., Valentin, D., O'Toole, A.J., & Edelman, B. (2005). DISTATIS: The analysis of multiple distance matrices. *Proceedings of the IEEE Computer Society: International Conference on Computer Vision and Pattern Recognition*. (San Diego, CA, USA). pp. 42–47.

[9] Ballester J., Dacremont, C., Le Fur, Y., & Etiévant, P. (2005). The role of olfaction in the elaboration and use of the Chardonnay wine concept. *Food Quality and Preference*, **16**, 351–359.

[10] Borg, I., & Groenen, P.J.E. (2000). *Modern multidimensional scaling: Theory and applications*. New York: Springer Verlag.

[11] Carroll, J.D., & Chang, J.J. (1970). Analysis of Individual Differences in Multidimensional Scaling via an N-way Generalization of the "Eckart-Young" Decomposition. *Psychometrika*, **35**, 283–319.

[12] Cartier, R., Rytz A., Lecomte A., Poblete F., Krystlik J., Belin E., & Martin, N. (2006). Sorting procedure as an alternative to quantitative descriptive analysis to obtain a product sensory map. *Food Quality and Preference*, **17**, 562–571.

[13] Chollet, S., & Valentin, D. (2001). Impact of training on beer flavour perception and description: Are trained and untrained subjects really different. *Journal of Sensory studies*, **16**, 601–618.

[14] Corter, J.E. (1996). *Tree models of similarity and association*. Thousand Oaks: Sage.

[15] Chrea, C., Valentin, D., Sulmont-Rossé, C., Ly, M.H., Nguyen, D., & Abdi, H. (2005). Semantic, typicality and odor representation: A cross-cultural study. *Chemical Senses*, **30**, 37–49.

[16] Coxon, A.P.M. (1999). *Sorting data : Collection and analysis*. Thousand Oaks: Sage.

[17] Escofier, B., & Pagès, J. (1998). *Analyses factorielles simples et multiples*. Paris: Dunod.

[18] Escoufier, Y. (1973). Le traitement des variables vectorielles. *Biometrics*, **29**, 751–760.

[19] Escoufier, Y. (1980). L'analyse conjointe de plusieurs matrices de données. In Jolivet, M. (Ed.) *Biométrie et temps*. Paris: Société Française de Biométrie (pp. 59–76).

[20] Falahee, M., & MacRae, A.W. (1995). Consumer appraisal of drinking water: Multidimensional scaling analysis. *Food Quality and Preference*, **6**, 327–332.

[21] Falahee, M., & MacRae, A.W. (1997). Perceptual variation among drinking waters: The reliability of sorting and ranking data for multidimensional scaling. *Food Quality and Preference*, **8**, 389–394.

[22] Faye, P., Brémaud, D., Durand-Daubin, D., Courcoux, P., Giboreau, A., & Nicod, A. (2004). Perceptive free sorting and verbalization tasks with naive subjects: an alternative to descriptive mappings. *Food Quality and Preference*, **15**, 781–791.

[23] Faye, P., Brémaud, D., Teillet E., Courcoux P., Giboreau A, & Nicod H. (2006). An alternative to external preference mapping based on consumer perceptive mapping. *Food Quality and Preference*, **17**, 604–614.

[24] Giboreau, A., Navarro, S., Faye, P., & Dumortier, J. (2001). Sensory evaluation of automotive fabrics: the contribution of categorization tasks and non-verbal information to set-up a descriptive method of tactile properties. *Food Quality and Preference*, **12**, 311–322

[25] Gower, J.C., & Dijksterhuis, G.B. (2004). *Procrustes problems*. Oxford: Oxford University Press.

[26] Harshman, R.A., & Lundy, M.E., (1994). PARAFAC: Parallel factor analysis. *Computational Statistics and Data Analysis*, **18**, 39–72.

[27] Heymann, H. (1994). A comparison of free choice profiling and multidimentional scaling of vanilla samples. *Journal of Sensory Studies*, **9**, 445–453.

[28] Healy, A., & Miller, G.A. (1970). The verb as the main determinant of the sentence meaning. *Psychonomic Science*, **20**, 372.

[29] Holmes, S. (in press) Multivariate Analysis: The French Way. in *Festschrift for David Freedman*. Stanford: IMS Lecture Notes.

[30] Hubert, L., & Arabie, P. (1985). Comparing partitions. *Journal of Classification*, **2**, 193–218.

[31] Husson, F., & Pagès, J. (2006). INDSCAL model: geometrical interpretation and methodology. *Computational Statistics and Data Analysis*, **50**, 358–378.

[32] Kazi-Aoual, F., Hitier, S., Sabatier, R., & Lebreton, J.-D. (1995). Refined approximations to permutation tests for multivariate inference. *Computational Statistics and Data Analysis*, **20**, 643–656.

[33] Kiers, H. (1998). A three step algorithm for CANDECOM / PARAFAC analysis. of large data sets with multicollinearity. *Journal of Chemometrics*, **12**, 155–171.

[34] Kruskal, J. (1977). Multidimensional scaling and other methods for discovering structure. In K. Enslein, A. Ralston, & H.S. Wilf (Eds.), *Statistical methods for digital computers*. New York: Wiley. pp. 296–339.

[35] Kruskal, J., & Wish, M. (1978). *Multidimensional scaling*. Thousand Oaks: Sage.

[36] Lavit, C. (1988). *Analyse conjointe de tableaux quantitatifs*. Paris: Masson.

[37] Lawless, H.T. (1989). Exploration of fragrances categories and ambiguous odors using multidimensional scaling and cluster analysis. *Chemical Senses*, **14**, 349–360.

[38] Lawless, H.T., Sheng T., & Knoops, S. (1995). Multidimensional scaling of sorting data applied to cheese perception. *Food Quality and Preference*, **6**, 91–98.

[39] Lawless, H.T., & Glatter, S. (1990). Consistency of multidimensional scaling models derived from odor sorting. *Journal of Sensory Studies*, **5**, 217–230.

[40] Le Dien, S., Pagès, J. (2003). Hierachical multiple factor analysis: Application to the comparison of sensory profiles. *Food Quality and Preference*, **14**, 397–403.

[41] MacRae, A.W., Howgate, P., & Geelhoed, E.N. (1990). Assessing the similarity of odours by sorting and by tridiadic comparison. *Chemical Senses*, **15**, 661–699.

[42] MacRae, A.W., Rawcliffe, T., Howgate, P., & Geelhoed, E.N. (1992). Patterns of odour similarity among carbonyls and their mixtures. *Chemical Senses*, **17**, 119–125.

[43] Meyners, M., Kunert, J., & Qanari E.M. (2000). Comparing generalized procrustes analysis and statis. *Food Quality and Preference*, **11**, 77–83.

[44] Meyners, M. (2003). Methods to analyse sensory profiling data—a comparison. *Food Quality and Preference*, **14**, 507–514.

[45] Miller, G.A. (1969). A psychological method to investigate verbal concepts. *Journal of Mathematical Psychology*, **6**, 169–191.

[46] Pagès, J. (2005). Collection and analysis of perceived product inter-distances using multiple factor analysis: Application to the study of 10 white wines from the Loire Valley. *Food Quality and Preference*, **16**, 642–649.

[47] Picard, D., Dacremont, C., Valentin, D., & Giboreau, A. (2003). Perceptual dimensions of tactile textures. *Acta Psychologica*, **114**, 165–84.

[48] Piombino, P., Nicklaus, S., LeFur, Y., Moio, L., & Le Quéré J. (2004). Selection of products presenting given flavor characteristics: An application to wine. *American Journal of Enology and Viticulture*, **55**, 27–34.

[49] Rand W.M. (1971). Objective crieria for the evaluation of clustering methods. *Journal of the American Statistical Association*, **66**, 846–850.

[50] Rao, V.R., & Katz, R. (1971). Alternative multidimensional scaling methods for large stimulus sets. *Journal of Marketing Research*, **8**, 488–494.

[51] Risvik, E., McEwan, J.A., Colwill, J.S., Rogers, R., & Lyon, D.H. (1994). Projective mapping: A tool for sensory analysis and consumer research. *Food Quality and Preference*, **5**, 263–269.

[52] Robert, P., & Escoufier, Y. (1976). A unifying tool for linear multivariate statistical methods: the RV-coefficient. *Applied Statistics*, **25**, 257–265.

[53] Saint-Eve, A., Paài Kora, E., & Martin, N. (2004). Impact of the olfactory quality and chemical complexity of the flavouring agent on the texture of low fat stirred yogurts assessed by three different sensory methodologies. *Food Quality and Preference*, **15**, 655–668.

[54] Schiffman, S., Reynolds, M., & Young, F. (1981). *Introduction to multidimensional scaling*. New York: Academic Press.

[55] Schlich, P. (1996). Defining and validating assessor compromises about product distances and attribute correlations. In T, Næs, & E. Risvik (Eds.), *Multivariate analysis of data in sensory sciences*. New York: Elsevier.

[56] Soufflet, I., Calonniera M., & Dacremont, C. (2004). A comparison between industrial experts' and novices' haptic perceptual organization: a tool to identify descriptors of the handle of fabrics. *Food Quality and Preference*, **15**, 689–699.

[57] Stevens, D.A., & O'Connell, R.J. (1996). Semantic-free scaling of odor quality. *Physiological Behavior*, **60**, 211–1155.

[58] Takane, Y. (1980). Analysis of categorizing behavior by a quantification methods. *Behaviometrika*, **8**, 75–86.

[59] Takane, Y. (1982). IDSORT: An individual differences multidimensional scaling program for sorting data. *Behavior Research methods & Instrumentation*, **14**, 546.

[60] Tang, C., & Heyman, H. (1999). Multidimensional sorting, similarity scaling and free choice profiling of grape jellies. *Journal of Sensory Studies*, **17**, 493–509.

[61] Togerson, W.S. (1958). *Theory and methods of scaling*. New York: Wiley.

[62] van der Kloot, W.A., & van Herk, H. (1991). Multidimensional scaling of sorting data: A comparison of three procedures. *Multivariate Behavioral Research*, **26**, 563–581.

[63] Young, F.W., Lewyckyj, R., & Takane, Y. (1986). The ALSCAL Procedure. In *SUGI Supplemental Library User's Guide, Version 5 Edition*. Cary, NC: SAS Institute Inc.

[64] Young, F. W., & Hamer, R. M. (1987). *Multidimensional scaling: History, theory, and applications*. Hillsdale, NJ: Erlbaum.