The STATIS Method

Hervé Abdi[1] & Dominique Valentin

1 Overview

1.1 Origin and goal of the method

STATIS is a generalization of principal component analysis (PCA) whose goal is to analyze several sets of variables collected on the same set of observations. It is attributed to Escouffier (1980) and L'Hermier des Plantes (1976, see also Lavit *et al.,* 1994), a related approach is known as *procrustes matching by congruence coefficients* in the English speaking community (Korth & Tucker, 1976). The goal of STATIS is 1) to compare and analyze the relationship between the different data sets, 2) to combine them into a common structure called a *compromise* which is then analyzed via PCA to reveal the common structure between the observations, and finally 3) to project each of the original data sets onto the compromise to analyze communalities and discrepancies. STATIS is used in very different domains such as sensory evaluation (*e.g.,* Qannari *et al.,* 1995; Schlich, 1996; Chaya *et al.,* 2003), molecular imaging (Coquet *et al.,* 1996) brain imaging (*e.g.,* Kherif *et al.,* 2003), ecology (Thioulouse *et al.,*), and chemometrics (Stanimirova *et al.,* 2004).

[1]In: Neil Salkind (Ed.) (2007). *Encyclopedia of Measurement and Statistics.* Thousand Oaks (CA): Sage.
Address correspondence to: Hervé Abdi
Program in Cognition and Neurosciences, MS: Gr.4.1,
The University of Texas at Dallas,
Richardson, TX 75083–0688, USA
E-mail: herve@utdallas.edu http://www.utd.edu/~herve

Its relationships with other methods are discussed in Abdi (2003), and Meyners *et al.,* (2000).

1.2 When to use it

The number and/or nature of the variables used to describe the observations can vary from one data set to the other, but the observations should be the same in all the data sets.

For example, the data sets can be measurements taken on the same observations (individuals or objects) at different occasions. In this case, The first data set corresponds to the data collected at time $t = 1$, the second one to the data collected at time $t = 2$ and so on. The goal of the analysis, then is to evaluate if the position of the observations is stable over time.

As another example, the data sets can be measurements taken on the same observations by different subjects or groups of subjects. In this case, The first data set corresponds to the first subject, the second one to the second subject and so on. The goal of the analysis, then, is to evaluate if there is an agreement between the subjects or groups of subjects.

1.3 The main idea

The general idea behind STATIS is to analyze the structure of the individual data sets (*i.e.,* the relation between the individual data sets) and to derive from this structure an optimal set of weights for computing a common representation of the observations called the compromise. The weights used to compute the compromise are chosen to make it as representative of all the data sets as possible. The PCA of the compromise gives then the position of the observations in the compromise space. The position of the observations for each data set can be represented in the compromise space as supplementary points. Finally, as a byproduct of the weight computation, the data sets can be represented as points in a multidimensional space.

2 An example

A typical example of using STATIS is the description of a set of products by a group of experts. This type of data can be analyzed using a standard PCA after having averaged the data across experts. However, this approach neglects the inter-experts differences. STATIS has the advantages of providing a compromise space for the products as well as evaluating the differences between experts. We illustrate the method with an example from wine testing.

Red wines often spend several months in oak barrels before being bottled because oak adds interesting components to the wine. However, only certain species of oaks seem to work well. Suppose that we want to evaluate the effect of the oak species on barrel-aged red Burgundy wines. First, we selected six wines coming from the same harvest of Pinot Noir and aged in six different barrels made with one of two different types of oak. Wines 1, 5 and 6 were aged with the first type of oak, whereas wines 2, 3 and 4 were aged with the second. Next, we asked each of three wine experts to choose from two to five variables to describe the six wines. For each wine, the expert was asked to rate the intensity of the variables on a 9-point scale. The results are presented in Table 1. The goal of the analysis is twofold. First we want to obtain a typology of the wines and second we want to know if there is an agreement between the experts.

3 Notations

The raw data consist in T data sets. For convenience, we will refer to each data set as a study. Each study is an $I \times J_{[t]}$ rectangular data matrix denoted $\mathbf{Y}_{[t]}$, where I is the number of observations and $J_{[t]}$ the number of variables collected on the observations for the t-th study. Each data matrix is, in general, preprocessed (*e.g.,* centered, normalized) and the preprocessed data matrices actually used in the analysis are denoted $\mathbf{X}_{[t]}$.

For our example, the data consist in $T = 3$ studies. The data were centered by column (*i.e.,* the mean of each column is zero)

Table 1: Raw data for the barrel-aged red burgundy wines example

		Expert 1			Expert 2				Expert 3		
wines	Oak-type	fruity	woody	coffee	red fruit	roasted	vanillin	woody	fruity	butter	woody
wine_1	1	1	6	7	2	5	7	6	3	6	7
wine_2	2	5	3	2	4	4	4	2	4	4	3
wine_3	2	6	1	1	5	2	1	1	7	1	1
wine_4	2	7	1	2	7	2	1	2	2	2	2
wine_5	1	2	5	4	3	5	6	5	2	6	6
wine_6	1	3	4	4	3	5	4	5	1	7	5

and the starting point of the analysis consists in three matrices $\mathbf{X}_{[t]}$:

$$\mathbf{X}_{[1]} = \begin{bmatrix} -3.00 & 2.67 & 3.67 \\ 1.00 & -0.33 & -1.33 \\ 2.00 & -2.33 & -2.33 \\ 3.00 & -2.33 & -1.33 \\ -2.00 & 1.67 & 0.67 \\ -1.00 & 0.67 & 0.67 \end{bmatrix}, \mathbf{X}_{[2]} = \begin{bmatrix} -2.00 & 1.17 & 3.17 & 2.50 \\ 0.00 & 0.17 & 0.17 & -1.50 \\ 1.00 & -1.83 & -2.83 & -2.50 \\ 3.00 & -1.83 & -2.83 & -1.50 \\ -1.00 & 1.17 & 2.17 & 1.50 \\ -1.00 & 1.17 & 0.17 & 1.50 \end{bmatrix} \quad (1)$$

$$\text{and} \quad \mathbf{X}_{[3]} = \begin{bmatrix} -0.17 & 1.67 & 3.00 \\ 0.83 & -0.33 & -1.00 \\ 3.83 & -3.33 & -3.00 \\ -1.17 & -2.33 & -2.00 \\ -1.17 & 1.67 & 2.00 \\ -2.17 & 2.67 & 1.00 \end{bmatrix}$$

Each of the $\mathbf{X}_{[t]}$ matrix is then transformed into an $I \times I$ scalar product matrix denoted $\mathbf{S}_{[t]}$ and computed as

$$\mathbf{S}_{[t]} = \mathbf{X}_{[t]}\mathbf{X}_{[t]}^{\mathsf{T}} . \quad (2)$$

For example, the 6×6 between wine scalar product matrix for the first wine expert is denoted $\mathbf{S}_{[1]}$. It is obtained as:

$$\mathbf{S}_{[1]} = \mathbf{X}_{[1]}\mathbf{X}_{[1]}^{\mathsf{T}} = \begin{bmatrix} 29.56 & -8.78 & -20.78 & -20.11 & 12.89 & 7.22 \\ -8.78 & 2.89 & 5.89 & 5.56 & -3.44 & -2.11 \\ -20.78 & 5.89 & 14.89 & 14.56 & -9.44 & -5.11 \\ -20.11 & 5.56 & 14.56 & 16.22 & -10.78 & -5.44 \\ 12.89 & -3.44 & -9.44 & -10.78 & 7.22 & 3.56 \\ 7.22 & -2.11 & -5.11 & -5.44 & 3.56 & 1.89 \end{bmatrix} .$$

The scalar product matrices for the second and third wine experts are equal to:

$$\mathbf{S}_{[2]} = \begin{bmatrix} 21.64 & -3.03 & -19.36 & -20.86 & 13.97 & 7.24 \\ -3.03 & 2.31 & 2.97 & 1.47 & -1.69 & -2.03 \\ -19.36 & 2.97 & 18.64 & 18.14 & -13.03 & -7.36 \\ -20.86 & 1.47 & 18.14 & 22.64 & -13.53 & -7.86 \\ 13.97 & -1.69 & -13.03 & -13.53 & 9.31 & 4.97 \\ 7.64 & -2.03 & -7.36 & -7.86 & 4.97 & 4.64 \end{bmatrix}$$

and

$$\mathbf{S}_{[3]} = \begin{bmatrix} 11.81 & -3.69 & -15.19 & -9.69 & 8.97 & 7.81 \\ -3.69 & 1.81 & 7.31 & 1.81 & -3.53 & -3.69 \\ -15.19 & 7.31 & 34.81 & 9.31 & -16.03 & -20.19 \\ -9.69 & 1.81 & 9.31 & 10.81 & -6.53 & -5.69 \\ 8.97 & -3.53 & -16.03 & -6.53 & 8.14 & 8.97 \\ 7.81 & -3.69 & -20.19 & -5.69 & 8.97 & 12.81 \end{bmatrix} .$$

4 Computing the compromise matrix

The *compromise matrix* is a scalar product matrix that gives the best compromise (hence its name) of the scalar product matrices representing each study. It is obtained as a weighted average of the study scalar product matrices. The weights are chosen so that studies agreeing the most with other studies will have the larger weights. To find these weights we need to analyze the relationships between the studies.

4.1 Comparing the studies

To analyze the similarity structure of the studies we start by creating a *between study cosine matrix* denoted $\mathbf{C}$. This is a $T \times T$ matrix whose generic term $c_{t,t'}$ gives the cosine between studies. This cosine, also known as the R_V-coefficient (see the corresponding entry for more details on this coefficient), is defined as

$$R_V = \left[c_{t,t'}\right] = \frac{\operatorname{trace}\left\{\mathbf{S}_{[t]}^{\mathsf{T}}\mathbf{S}_{[t']}\right\}}{\sqrt{\operatorname{trace}\left\{\mathbf{S}_{[t]}^{\mathsf{T}}\mathbf{S}_{[t]}\right\} \times \operatorname{trace}\left\{\mathbf{S}_{[t']}^{\mathsf{T}}\mathbf{S}_{[t']}\right\}}} . \tag{3}$$

Using this formula we get the following matrix $\mathbf{C}$:

$$\mathbf{C} = \begin{bmatrix} 1.00 & .95 & .77 \\ .95 & 1.00 & .82 \\ .77 & .82 & 1.00 \end{bmatrix} . \tag{4}$$

4.2 PCA of the cosine matrix

The eigendecomposition of the cosine matrix reveals the structure between the studies. This amounts to performing a *non centered* PCA of **C**. Formally, this matrix has the following eigendecomposition

$$\mathbf{C} = \mathbf{P}\boldsymbol{\Theta}\mathbf{P}^{\top} \text{ with } \mathbf{P}^{\top}\mathbf{P} = \mathbf{I}\,, \tag{5}$$

where **P** is the matrix of eigenvectors of **C** and $\boldsymbol{\Theta}$ the diagonal matrix of eigenvalues. An element of a given eigenvector represents the projection of one study on this eigenvector. Thus the studies can be represented as points in the eigenspace and their similarities visually analyzed. In this case, the projections are computed as

$$\mathbf{G} = \mathbf{P}\boldsymbol{\Theta}^{\frac{1}{2}}\,. \tag{6}$$

For our example, we find that

$$\mathbf{P} = \begin{bmatrix} 0.58 & -0.49 & 0.65 \\ 0.59 & -0.29 & -0.75 \\ 0.55 & 0.82 & 0.12 \end{bmatrix} \text{ and } \boldsymbol{\Theta} = \begin{bmatrix} 2.70 & 0.00 & 0.00 \\ 0.00 & 0.26 & 0.00 \\ 0.00 & 0.00 & 0.04 \end{bmatrix}, \tag{7}$$

and

$$\mathbf{G} = \begin{bmatrix} 0.96 & 0.25 & 0.14 \\ 0.98 & 0.14 & -0.16 \\ 0.91 & -0.42 & 0.03 \end{bmatrix}. \tag{8}$$

As an illustration, Figure 1 displays the projections of the experts onto the first and second components. It shows that the three studies are positively correlated with the first component (this is due to all the elements of the cosine matrix being positive).

4.3 Computing the compromise

The weights used for computing the compromise are obtained from the PCA of the cosine matrix. Because this matrix is not centered, the first eigenvector of **C** represents what is common to the different studies. Thus studies with larger values on the first eigenvector are more similar to the other studies and therefore will have a larger weight. Practically, the weights are obtained by re-scaling

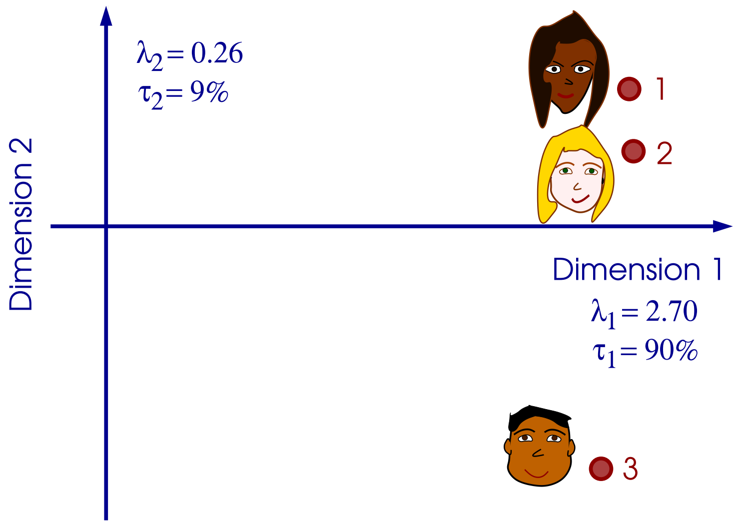

Figure 1: The expert space.

the elements of the first eigenvector of $\mathbf{C}$ so that their sum is equal to one. We call the weight vector $\boldsymbol{\alpha}$. For our example we find that

$$\boldsymbol{\alpha} = \begin{bmatrix} .337 & .344 & .319 \end{bmatrix}^{\mathsf{T}} . \tag{9}$$

So with α_t denoting the weight for the t-th study, the compromise matrix, denoted $\mathbf{S}_{[+]}$, is computed as:

$$\mathbf{S}_{[+]} = \sum_{t}^{T} \alpha_t \mathbf{S}_{[t]} . \tag{10}$$

In our example, the compromise is obtained as:

$$\mathbf{S}_{[+]} = \begin{bmatrix} 22.15 & -5.42 & -19.37 & -17.83 & 12.57 & 7.90 \\ -5.42 & 2.45 & 5.59 & 3.09 & -3.01 & -2.71 \\ -19.37 & 5.59 & 23.61 & 14.76 & -13.38 & -11.21 \\ -17.83 & 3.09 & 14.76 & 17.47 & -10.84 & -6.65 \\ 12.57 & -3.01 & -13.38 & -10.84 & 8.61 & 6.05 \\ 7.90 & -2.71 & -11.21 & -6.65 & 6.05 & 6.62 \end{bmatrix} \tag{11}$$

4.4 How Representative is the compromise?

The compromise is the best aggregate of the original scalar product matrices. But how *good* is this "best?" An index of the quality of the compromise is given by the ratio of the first eigenvalue of **C** to the sum of the eigenvalues of **C**:

$$\text{Quality of compromise} = \frac{\vartheta_1}{\sum_\ell \vartheta_\ell} = \frac{\vartheta_1}{\operatorname{trace}\{\boldsymbol{\Theta}\}} . \tag{12}$$

For our example, the quality of the compromise is evaluated as $\frac{2.74}{3} \approx .91$. So we can say that the compromise "explains" 91% of the inertia of the original set of data tables.

5 Analyzing the compromise

The compromise matrix is a scalar product matrix, and therefore its eigen-decomposition amounts to a PCA. From this analysis we can explore the structure of the set of observations. The eigen-decomposition of the compromise gives:

$$\mathbf{S}_{[+]} = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^T \tag{13}$$

with

$$\mathbf{Q} = \begin{bmatrix} 0.54 & -0.34 & -0.57 & -0.03 & 0.31 \\ -0.14 & -0.15 & 0.49 & 0.54 & 0.51 \\ -0.55 & -0.57 & 0.00 & -0.40 & -0.23 \\ -0.45 & 0.59 & -0.49 & 0.19 & 0.01 \\ 0.34 & 0.03 & 0.17 & 0.34 & -0.75 \\ 0.25 & 0.43 & 0.39 & -0.63 & 0.16 \end{bmatrix} \tag{14}$$

and

$$\operatorname{diag}\{\boldsymbol{\Lambda}\} = \begin{bmatrix} 69.70 \\ 7.35 \\ 2.52 \\ 1.03 \\ 0.32 \end{bmatrix} . \tag{15}$$

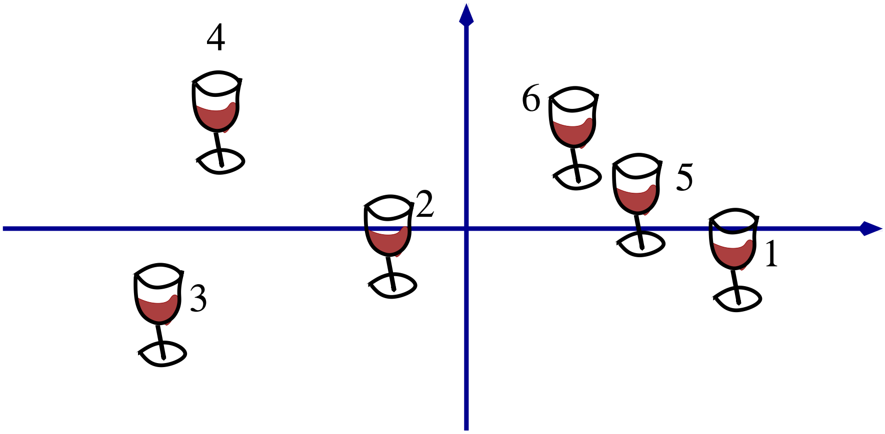

Figure 2: Analysis of the compromise: Plot of the wines on the first two principal components.

From Equations 14 and 15 we can compute the compromise factor scores for the wines as:

$$\mathbf{F} = \mathbf{Q}\mathbf{\Lambda}^{\frac{1}{2}} \tag{16}$$

$$= \begin{bmatrix} 4.52 & -0.92 & -0.91 & -0.03 & 0.17 \\ -1.13 & -0.42 & 0.78 & 0.54 & 0.29 \\ -4.59 & -1.53 & 0.01 & -0.40 & -0.13 \\ -3.77 & 1.61 & -0.78 & 0.19 & 0.01 \\ 2.87 & 0.08 & 0.27 & 0.34 & -0.43 \\ 2.10 & 1.18 & 0.63 & -0.64 & 0.09 \end{bmatrix} . \tag{17}$$

In the **F** matrix, each row represents an observation (*i.e.,* a wine) and each column is a component. Figure 2 displays the wines in the space of the first two principal components. The first component has an eigenvalue equal to $\lambda_1 = 69.70$, which corresponds to 85% of the inertia ($\frac{69.70}{69.70+7.35+2.52+1.03+0.32} = \frac{69.70}{80.91} \approx .85$.) The second component, with an eigenvalue of 7.35, explains almost 10% of the inertia. The first component is easily interpreted as the opposition of the wines aged with the first type of oak (wines 1, 5, and 6) to the wines aged with the second type of oak (wines 2, 3, and 4).

6 Projecting the studies into the compromise space

The analysis of the compromise reveals the structure of the wine space common to the experts. In addition, we also want to see how each expert “interprets" this space. This is achieved by projecting the scalar product matrix of each expert onto the compromise. This operation is performed by computing a projection matrix which transforms the scalar product matrix into loadings. The projection matrix is deduced from the combination of Equations 13 and 16 which gives

$$\mathbf{F} = \mathbf{S}_{[+]}\mathbf{Q}\boldsymbol{\Lambda}^{-\frac{1}{2}} \, . \tag{18}$$

This shows that the projection matrix is equal to $\left(\mathbf{Q}\boldsymbol{\Lambda}^{-\frac{1}{2}}\right)$. It is used to project the scalar product matrix of each expert onto the common space. For example, the coordinates of the projections for the first expert are obtained by first computing the matrix

$$\mathbf{Q}\boldsymbol{\Lambda}^{-\frac{1}{2}} = \begin{bmatrix} 0.06 & -0.12 & -0.36 & -0.03 & 0.55 \\ -0.02 & -0.06 & 0.31 & 0.53 & 0.90 \\ -0.07 & -0.21 & 0.00 & -0.39 & -0.41 \\ -0.05 & 0.22 & -0.31 & 0.18 & 0.02 \\ 0.04 & 0.01 & 0.11 & 0.34 & -1.34 \\ 0.03 & 0.16 & 0.25 & -0.63 & 0.28 \end{bmatrix} , \tag{19}$$

and then using this matrix to obtain the coordinates as:

$$\mathbf{F}_{[1]} = \mathbf{S}_{[1]}\left(\mathbf{Q}\boldsymbol{\Lambda}^{-\frac{1}{2}}\right) \tag{20}$$

$$= \begin{bmatrix} 5.27 & -1.96 & -4.07 & -1.40 & 1.11 \\ -1.51 & 0.54 & 1.48 & 0.71 & -0.47 \\ -3.76 & 1.42 & 2.60 & 0.69 & -0.64 \\ -3.84 & 1.72 & 1.51 & 0.69 & 1.29 \\ 2.50 & -1.15 & -0.76 & -0.34 & -1.08 \\ 1.34 & -0.57 & -0.75 & -0.35 & -0.20 \end{bmatrix} . \tag{21}$$

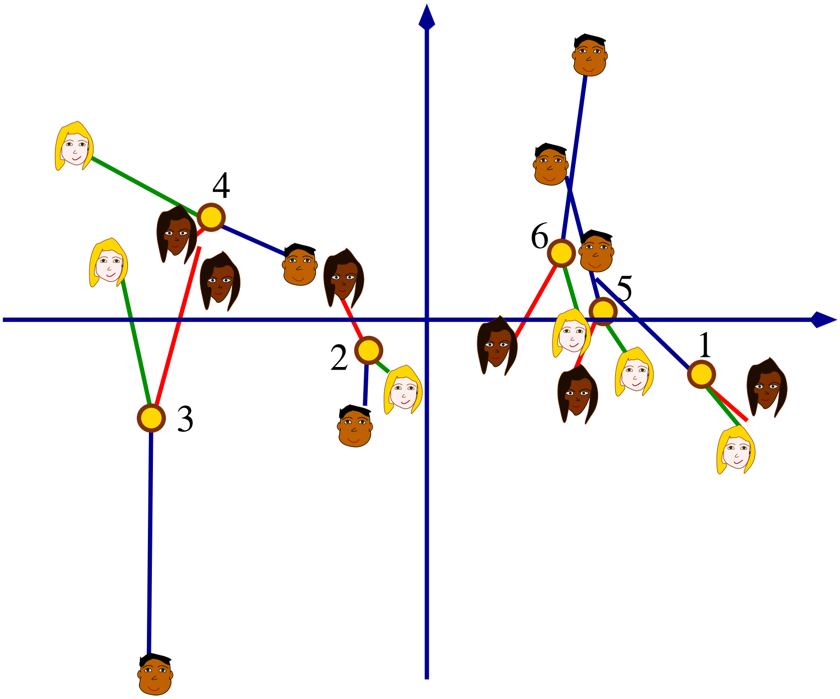

Figure 3: The Compromise: Projection of the expert matrices. Experts are represented by their faces. A line segment links the position of the wine for a given expert to the compromise position for this wine.

The same procedure is used for Experts 2 and 3 and gives:

$$\mathbf{F}_{[2]} = \begin{bmatrix} 4.66 & -1.68 & 1.04 & 1.33 & 0.04 \\ -0.64 & -0.39 & 0.68 & 1.13 & 0.94 \\ -4.27 & 1.01 & -0.87 & -1.51 & 0.22 \\ -4.59 & 2.29 & -2.37 & -1.08 & -1.14 \\ 3.06 & -0.99 & 0.81 & 1.26 & 0.10 \\ 1.78 & -0.23 & 0.71 & -1.13 & -0.15 \end{bmatrix}, \qquad (22)$$

and

$$\mathbf{F}_{[3]} = \begin{bmatrix} 2.96 & 1.13 & 0.45 & -0.05 & -0.68 \\ -1.10 & -1.40 & 0.06 & -0.33 & 0.35 \\ -5.17 & -7.15 & -1.78 & -0.32 & 0.05 \\ -2.30 & 0.55 & -1.37 & 1.00 & -0.12 \\ 2.65 & 2.52 & 0.75 & 0.04 & -0.24 \\ 2.96 & 4.35 & 1.89 & -0.34 & 0.65 \end{bmatrix} . \tag{23}$$

Figure 3 shows the first two principal components of the compromise space along with the projections of wines for the each expert. Note that, the position of each wine in the compromise is the barycenter of the positions of this wine for the three experts. In order to make this relation clear and also to facilitate the interpretation, we have drawn lines linking the position of each wine for each expert to the compromise position. This picture confirms a conclusion obtained from the analysis of the **C** matrix: Expert 3 tends to be at variance with the other two experts.

7 The original variables and the compromise

The analysis of the compromise reveals the structure of the set of observations, but the original data tables were rectangular tables (*i.e.*, each expert was using several scales to evaluate the wines). And we want to be able to relate these specific scales to the analysis of the compromise.

The original variables can be integrated into the analysis by adapting the standard approach that PCA uses to relate original variables and components: namely by computing loadings (*i.e.*, correlation between the original variables and the factor scores). This approach is illustrated in Table 2 which gives the loadings between the original variables and the factors of the compromise. Figure 4 shows the circle of correlation obtained for each expert (these loadings could have been drawn on the same picture). Here, we see, once again, that Expert 3 differs from the other experts, and is mostly responsible for the second component of the compromise.

Table 2: Original variables and compromise: Loadings (correlation between variables and components).

	Loadings									
	Expert 1			Expert 2				Expert 3		
Axis	fruity	woody	coffee	fruit	roasted	vanillin	woody	fruity	butter	woody
1	−.97	.99	.92	−.90	.95	.96	.97	−.57	.94	.99
2	.19	−.13	−.04	.36	.02	−.19	.13	−.82	.22	.03
3	.00	−.00	−.36	−.22	.29	.01	−.13	.01	.25	−.10
4	.07	.08	−.07	.13	.03	.22	−.13	−.07	−.09	.04
5	.10	.00	.12	−.05	.05	.01	−.07	.00	.05	−.05

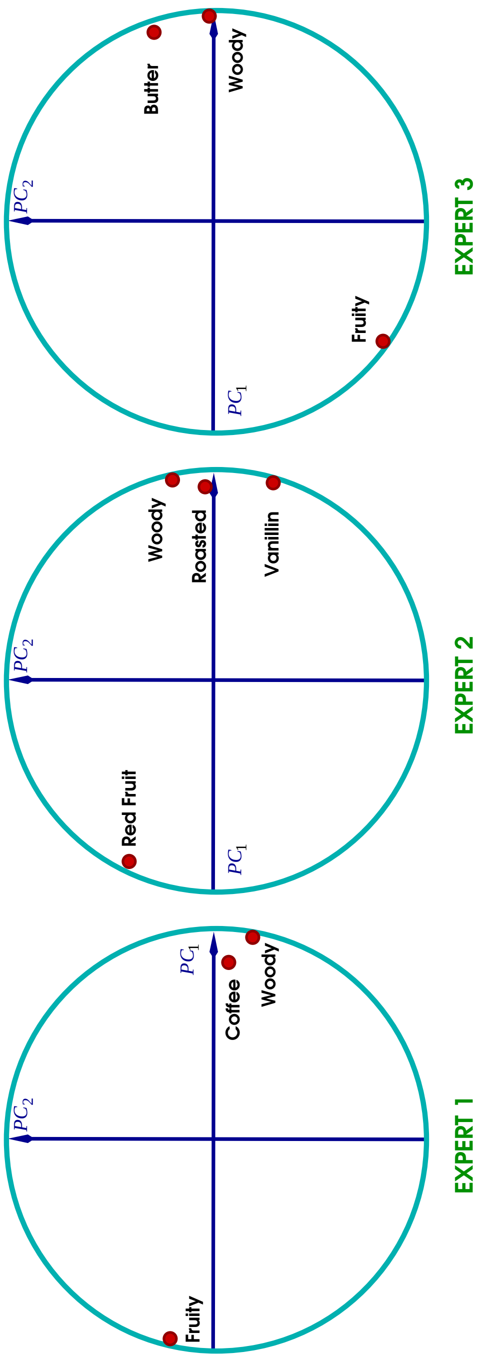

Figure 4: Original variables and compromise: The circle of correlations for each expert.

References

[1] Abdi, H. (2003b). Multivariate analysis. In M. Lewis-Beck, A. Bryman, & T. Futing (Eds): *Encyclopedia for research methods for the social sciences*. Thousand Oaks (CA): Sage.

[2] Chaya, C., Perez-Hugalde, C., Judez, L., Wee, C.S., & Guinard, J.X. (2003). Use of the STATIS method to analyze time-intensity profiling data. *Food Quality and Preference*, **15**, 3–12.

[3] Coquet, R., Troxler, L., & Wipff G. (1996). The STATIS method: Characterization of conformational states of flexible molecules from molecular dymanics simulation in solution. *Journal of Molecular Graphics*, **14**, 206–212.

[4] Escoufier, Y. (1980). L'analyse conjointe de plusieurs matrices de données. In M. Jolivet (Ed.), *Biométrie et Temps*. Paris: Société Française de Biométrie. 59–76.

[5] Kherif, F., Poline, J.-P., Mériaux, S., Benali, H., Flandin, G., & Brett M., (2003) Group analysis in functional neuroimaging: selecting subjects using similarity measures. *NeuroImage*, **20**, 2197–2208.

[6] Korth B., & Tucker L.R., (1976). Procustes matching by congruence coefficients. *Psychometrika*, **41**, 531–535.

[7] Lavit, C., Escoufier Y., Sabatier, R., & Traissac, P., (1994). The ACT (STATIS) method. *Computational Statistics & Data Analysis*, **18**, 97–119.

[8] L'Hermier des Plantes H. (1976). *Structuration des tableaux à trois indices de la statistique*. Thèse de troisième cycle. Université de Montpellier.

[9] Meyners M., Kunert, J., & Qannari E.M. (2000). Comparing generalized procrustes analysis and statis. *Food Quality and Preference*, **11**, 77–83.

[10] Qannari E.M., Wakeling, I., & MacFie , J.H. (1995). A hierachy of models for analyzing sensory data. *Food Quality and Preference*, **6**, 309–314.

[11] Robert, P. & Escoufier, Y. (1976) A unifying tool for linear multivariate statistical methods: the RV-coefficient. *Applied Statistics*, **25**, 257–265.

[12] Schlich, P. (1996). Defining and validating assessor compromises about product distances and attribute correlations. In T,

Næs, & E. Risvik (Eds.), *Multivariate analysis of data in sensory sciences.* New York: Elsevier. 229–306.

[13] Stanimirova, I., Walczak, B., Massart, D.L., Simeonovc, V., & Sabyd, C.A., di Crescenzo, E. (2004). STATIS, a three-way method for data analysis. Application to environmental data. *Chemometrics and Intelligent Laboratory Systems,* **73**, 219–233.

[14] Thioulouse, J., Simier, M., & Chessel D. (2004). Simultaneous Analysis of a sequence of paired ecological tables, *Ecology,* **85**, 272–283.