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Automatic Activation of Addition and Multiplication Facts in Elementary School Children

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When adults are asked to indicate whether a probe had been present in a previously viewed number pair, probes that are the sum of the pair take more time to reject than unrelated numbers. In Experiment 1, we explore how factors such as the size of the numbers and the delay between the pair and probe influence this effect in adults. In Experiment 2, we present evidence that this interference effect is also present in elementary school children although it varies with the size of the numbers in the pair and the age of the child. In our third experiment, we

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explore a related interference effect known as the associative confusion effect which arises for adults in arithmetic verification tasks when multiplicative answers are given to addition problems and additive answers are given to multiplication problems. The results of our third experiment suggest that children in grades 3 through 5 also exhibit associative confusion effects but that again these effects vary with the size of the numbers. We argue that the processes that underlie mental arithmetic are partially autonomous in elementary school children and that although integer size has a more pronounced effect on the performance of younger children, the temporal parameters that govern these effects are similar for children and adults.

How do children and adults solve simple arithmetic problems? Most researchers believe adults solve these problems by retrieving the solutions directly from semantic memory (Abdi, 1986; Ashcraft, 1982, 1983, 1987, 1992; Campbell & Graham, 1985; Siegler & Shrager, 1984; but see Baroody, 1985, for an opposing view). Advocates of this view argue that knowledge about arithmetic is represented and accessed in the same fashion as other forms of long-term memory knowledge. More specifically, this information is presumably accessed by a process of spreading activation in which separate sources of activation spread though a network and intersect most strongly at the point of information retrieval.

Research on children's mental arithmetic skills has focused on a number of developmental questions that have arisen in connection with these associative models. These questions include whether children, like adults, rely primarily on direct retrieval to solve these problems or whether reconstructive processes dominate early in development and later give way to retrieval-based algorithms. Current developmental models (e.g., Ashcraft, 1982, 1983, 1987, 1992; Fayol, 1990; Siegler & Shrager, 1984) suggest that even though counting-based algorithms play an important role in young children's mental arithmetic, kindergartners and first-graders may at times use direct retrieval to solve small highly practiced addition problems.

Although it is likely that memory retrieval plays an increasingly important role in mental arithmetic during the elementary school years, it is less clear whether retrieval processes operate in the same way in the child as they do in the adult. According to associative network models, presentation of an arithmetic problem (e.g., 3 + 4) results in activation of the number nodes specified in the problem (e.g., 3 and 4). Activation spreads from these presented nodes along associative links so that related number nodes, including the sum, are activated. An important question in the adult literature, and one with developmental implications that the present study addresses, is whether this activation process is autonomous or under intentional control.

Zbrodoff and Logan (1986) have suggested that a process be classified as autonomous "if it can (a) begin without intention, triggered by the presence of a relevant stimulus in the task environment; and (b) run on to completion ballistically once it begins, whether or not it is intended" (p. 118). In a series of experiments, Zbrodoff and Logan explored the possibility that the processes underlying simple mental arithmetic are autonomous in adults. They asked subjects to verify simple arithmetic equations of the form $a + b = c$ and $a \times b = c$. Half of the equations were false, and, more importantly, half of these false equations contained c terms that would have been correct if the subjects performed a different operation (e.g., $3 + 4 = 12$; $3 \times 4 = 7$). Zbrodoff and Logan called these distractors *associative lures.* The remaining false problems contained *nonassociative lures,* that is, answers that would not be true for any conventional arithmetic operation (e.g., $3 + 4 = 8$). Zbrodoff and Logan reasoned that if the processes underlying mental arithmetic are autonomous, once the two arguments (3 and 4) are encoded, activation should spread to both the sum and product. This activation would increase the amount of time needed to reject the associative as opposed to nonassociative lures, a pattern of results referred to as the *associative confusion effect* (see also Winkelman & Schmidt, 1974). Obviously, one prerequisite for this effect is that the facts for the relevant operations, here, multiplication and addition, must be stored in the same network.

Zbrodoff and Logan (1986) attempted to distinguish between completely and partially autonomous processes by manipulating the extent to which subjects intended to perform the irrelevant operation within a block of trials. The logic was that if a process is completely autonomous, it should not be affected by intention nor should it be possible to inhibit it once it has begun. They found that the associative confusion effect is modulated by intention and that the processes that give rise to this effect can be inhibited within certain temporal parameters. The general conclusions they draw from their studies are first, that there are direct associations between single digits and their sums and products which are activated without intention and that this activation gives rise to the associative confusion effect; and second, that this initial activation may be supplemented with deliberate activation to achieve the required threshold and that this supplemental activation might result from memory search strategies or counting algorithms.

In a subsequent study, LeFevre, Bisanz, and Mrkonjic (1988) examined the extent to which this activation is obligatory by asking undergraduates to perform a number-matching task. Subjects were presented with number pairs separated by a space or an addition sign. In the latter case, the number pairs were presented in both digit and verbal form (i.e., 5 + 1, five + one). On each trial, the subject's task was the same: He or she had to indicate whether a subsequent probe matched one of the numbers on the first slide. Two types of distractors were included on the false trials. On half of the false trials, the probe was the sum of the first two numbers and on the remaining "neutral" trials, there was no relationship between the first two numbers and the probe. LeFevre et al. found that the sum probes were rejected more slowly than the neutral probes, and this effect did not depend on the format of the presentation. What did affect the results, however, was the amount of time between the appearance of the pair and probe. The sum probes resulted in significant interference at 120-ms stimulus onset asynchronies (SOAs) but this effect disappeared at 180-ms SOAs. Interference effects with sum probes were also obtained at 60-ms SOAs, but only in the blank condition, suggesting that the spread of activation may reach a critical threshold earlier if there is one less symbol to encode. More importantly, it suggests that within 180-ms of exposure, adults are able to inhibit the obligatory activation of arithmetic facts when arithmetic operations are not relevant for the task.

Taken together, LeFevre et al.'s (1988) and Zbrodoff and Logan's (1986) results suggest that in adulthood the processes underlying simple mental arithmetic can be initiated without intention, and that for addition, this activation can occur even when the primary goal of the task does not involve arithmetic operations. These processes, however, appear to be only partially autonomous; given sufficient time, adults are able to inhibit operations that are not relevant for the task. The confusion effects that have arisen in these studies suggest that adults' knowledge of simple arithmetic facts can be captured by a network model in which information about different arithmetic operations is highly integrated and activation spreads throughout the network. As processing proceeds, the information relevant for the target task is activated more strongly whereas the irrelevant information decreases in strength. In the present paper, we explore how this network might develop during the elementary school years.

The interference effects reported by LeFevre et al. (1988) and Zbrodoff and Logan (1986) presuppose strong associations between a number pair and its sum or product, but it is not clear whether these associations are strong enough in elementary school children to lead to automatic activation. In adults, interference effects in number-matching tasks are not only a product of sum activation; *distance effects* have also been observed. For example, LeFevre et al. (1988) found that numbers close in magnitude to the original number pair produce more interference than distant numbers. These distance effects are presumably rooted in number-line associations that have been built up through counting experiences. For adults, sum activation is a more potent source of interference effects than number-line associations but number-line associations may be the primary source of interference effects in young elementary school children (see LeFevre, Kulak, & Bisanz, 1991; Siegler & Shrager, 1984).

LeFevre et al. (1991) used a number-matching task to evaluate the relative strength of sum activation and number-line associations in children. Sum activation only produced minimal interference effects in the third-, fourth-, and fifth-graders who participated in their study, but these children did exhibit substantial interference effects as a function of the distance between the distractor and the original number pair. Based on these findings and the results of an earlier study (LeFevre & Bisanz, 1987), LeFevre et al. concluded that whereas number-line associations remain a major source of interference effects up through fifth grade, sum activation does not produce substantial interference until sixth grade. Similarly, Hamann & Ashcraft (1985) only found interference effects due to sum activation in their older subjects. In their study, interference effects due to cross-operation activation of sums and products did not emerge prior to tenth grade.

Although these studies suggest that sums and products are not automatically activated in elementary school children, a recent study by Lemaire, Fayol, and Abdi (1991) calls this conclusion into question. In that study, fourth- and fifth-graders exhibited associative-based confusion effects in an arithmetic verification task: Children were slow to reject distractors that would have been correct if a different operation were performed. These findings suggest that sums and products are automatically activated when children solve arithmetic equations, but that the extent to which interference effects are observed may depend on specific aspects of the task situation. The present set of experiments examine in more detail the developmental time-course of these two types of interference effects. In the first two experiments, we focus on interference effects resulting from the obligatory activation of sums in a number-matching task, and in the third experiment, we examine confusion effects resulting from obligatory activation of sums and products in an arithmetic verification task.

EXPERIMENT 1

In the first experiment, we focused on whether sum activation occurs in number-matching tasks that do not in any way implicate addition and whether these effects vary as a function of the size of the integers. In studies of mental arithmetic, problems with large answers have consistently been associated with long response times (see Ashcraft, 1992, for a review). The initial research on the parameters surrounding this problem size effect involved children's and adults' performance on mental addition tasks (Groen & Parkman, 1972). Differences in the shape and temporal characteristics of the response curves suggested that mental counting is widely used in childhood but serves only as a back-up strategy for adults. Although mental counting may be responsible for some of the problem size effects obtained in addition tasks, similar effects have emerged in production and verification tasks involving subtraction, multiplication, and division (e.g., Siegler, 1987b; Campbell, 1987c). This suggests that to some extent problem size effects are a function of how arithmetic knowledge is retrieved and stored in long-term memory. If the problem size effect reflects how arithmetic facts are stored in long-term memory, then the size of the numbers may affect performance even when arithmetic operations are not relevant for the task. In the first experiment, we explored whether the size of the initial integers affects the magnitude of sum activation for adults in a number-matching task. This question is especially important for understanding the developmental time-course of obligatory activation of arithmetic facts because it is likely that interference effects appear first with small numbers. Experiment 1 focused on whether the size of the integers affects adult performance in a number-matching task modeled after LeFevre et al.'s (1988) study. In Experiment 2, we explore these same issues with elementary school children.

In Experiment 1, undergraduates were shown number pairs and asked to indicate whether a subsequent probe had been present in the pair. Our task was similar to LeFevre et al.'s (1988) with one exception: In their study, subjects viewed number pairs with and without addition signs in the same session whereas our subjects never saw addition signs. Addition was not mentioned during our sessions so we could evaluate whether sum activation occurs even when arithmetic operations are not in any way implicated in the task. Like LeFevre et al., we varied the interval between the presentation of the number pair and probe, and based on their findings, we expected to find interference effects due to sum activation provided the probe appeared less than 180 ms after the original pair. The stimuli in our study were drawn from three size categories defined by the size of the initial pair. If the problem size effects that have been found in other arithmetic tasks are also present in simple number-matching tasks, then interference effects should be strongest with the small problems.

Method

Subjects. Twenty introductory students at the University of Bourgogne at Dijon, France (10 males and 10 females) participated in partial fulfillment of class requirements. The mean age of the students was 19 years, 2 months (the range was 18.0 to 26.1).

Stimuli. Each trial consisted of a pair of numbers separated by a space followed by a probe. The same integer never appeared twice within a pair. Twelve different number pairs were presented; four pairs of integers were selected for each of the three size conditions. In the *small integer* condition, both integers were 5 or smaller. In the *medium integer* condition, one integer was 5 or smaller and the other integer was between 6 and 9. In the *large integer* condition, both integers were between 6 and 9. The complete set of number pairs is presented in Table 1.

Three types of problems were presented. For *true* problems, the problem matched one of the members of the original pair. To equate the number of true and false responses, 2 true problems were created for

TABLE 1
STIMULUS ITEMS FOR EXPERIMENTS 1 AND 2

		Probe	
Number pair	Sum	Neutral	True
Small integers			
1 3	4	5	1,3
2 4	6	7	2,4
3 5	8	7	3,5
4 5	9	8	4,5
Medium integers			
2 6	8	9	2,6
3 6	9	8	3,6
3 7	10	11	3,7
4 8	12	11	4,8
Large integers			
6 7	13	12	6,7
6 8	14	15	6,8
6 9	15	14	6,9
7 9	16	17	7,9

each pair by presenting each of the initial integers in the probe position. Thus, subjects were presented with a total of 24 true problems and 24 false problems. Two types of false problems were created. For *sum* problems, the probe was equal to the sum of the original pair, and for *neutral* pairs, the probe was equal to the sum plus or minus one and was never the product of the original numbers. Because additional time is required to reject numbers that are close to the original pair (LeFevre et al., 1991), the mean distance for each false problem was computed by calculating the average distance between the probe and each member of the number pair. Although the mean distance necessarily increased with the size of the initial number pair (the respective mean values were 3.38, 4.88, and 7.25 for small, medium, and large problems), more importantly, the mean distance was the same for the sum and neutral problems within each condition.

Although only single digits appeared in the original pairs, false answers for all of the large integer problems and half of the medium integer problems were two-digit numbers. Consequently, for these problems, it would be possible to reject probes based on the presence of two digits without comparing the probe to the original pair. To eliminate this confound, all single digits were preceded by a zero (e.g., 08).

Procedure. The integers were presented in the center of a computer screen (IBM PS/2). Each trial began when a 750-ms ready signal (a line of five "a"s) appeared in the center of the screen. The number pair then appeared in the center of the screen separated by a double space. Following a variable delay (80, 150, 240, or 480 ms), the number pair disappeared and the probe was presented in the center of the screen one line below the position of the number pair. The probe remained on the screen until the subject responded "yes" or "no" by pressing the appropriate key on the keyboard with his or her index finger. The two middle keys on the bottom row were designated for responses. Subjects were instructed to rest their finger on the space bar in between trials. The position of the yes key was counterbalanced across subjects. After the subject responded, there was a 1500-ms delay before the next trial began.

Each of the 48 problems in Table 1 were presented at each SOA, resulting in 192 test trials per subject. Problems were randomized separately for each subject with the constraint that no more than three consecutive true or false answers appear in a row. SOA, integer size, and type of probe varied across trials. After each set of 50 problems, subjects were given a brief rest period. Before the experimental trials, subjects were given one block of 40 practice trials to familiarize them with the apparatus, procedure, and stimulus display. Practice trials included items that were similar to (but not identical to) the test items. Twenty true, 10 sum, and 10 neutral problems were included in the practice set. After the practice trials, subjects were told their median latency and error rate and encouraged to respond as quickly as possible without making mistakes. The entire session lasted 20–30 min.

Results

Latencies. Median correct latencies for the false items that were correctly rejected were analyzed in a 4 (SOA: 80, 150, 240, or 480 ms) × 3 (Size: small, medium, or large) × 2 (Probe: sum, neutral) repeated measures analysis of variance. There was a main effect of SOA; response times decreased as SOA increased, $F(3, 57) = 10.84$; $p < .001$, and this effect was more pronounced with large and medium integers than with small integers, $F(6, 114) = 4.25$, $p < .001$. Presumably, the significant decline in latencies with SOA reflects the fact that at short SOAs the response latencies include encoding and preprocessing time for the initial pair. With longer SOAs, much or all of this processing can be completed during the delay, and so the initial processing time has only a minimal effect on response time.

The most interesting effects, however, concern differences between the sum and neutral probes. As predicted, sum probes were rejected more slowly than neutral probes (814 ms vs 766 ms), $F(1, 19) = 11.90$, $p < .01$, and there was a significant interaction between probe and SOA, $F(3, 57) = 10.64$, $p < .001$. The difference between sum and neutral probes was only significant at 80-ms SOAs, $F(1, 57) = 22.78$, $p < .001$, and 150-ms SOAs, $F(1, 57) = 39.29$, $p < .001$. These interference effects are

TABLE 2
ADULTS' MEAN MEDIAN LATENCIES (IN ms) AND PERCENTAGE ERRORS FOR THE FALSE ITEMS (SUM/NEUTRAL PROBES) IN EXPERIMENT 1

		SOA			
Integers		80	150	240	480
Small	*M*	908/787***	897/740***	781/775	758/682**
	E(%)	18.8/3.8***	17.5/5.0***	7.5/7.5	7.5/2.5***
Medium	*M*	859/788**	863/782**	774/803	693/744
	E(%)	16.2/6.2***	15.0/5.0***	7.5/6.2	3.8/3.8
Large	*M*	934/878*	837/749***	735/721	727/747
	E(%)	15.0/3.8***	15.0/6.2***	6.2/6.2	4.0/3.8

* Marginal interference effect $p < .10$.
** Significant interference effect $p < .05$.
*** Significant interference effect $p < .01$.

consistent with LeFevre et al.'s (1988) finding that obligatory activation of arithmetic facts is most pronounced at short SOAs.

The difference between sum and neutral probes interacted with the size of the initial pair, $F(2, 38) = 3.48$, $p < .05$. The difference between sum and neutral probes only proved significant with small integers, $F(1, 38) = 4.26$, $p < .05$. Although the size by probe by SOA interaction was not significant, given the significant probe by SOA interaction and the need to know whether the effects of obligatory activation are present with all integer sizes at short SOAs, we used the error term for the three-way interaction to evaluate the size by probe interaction separately at each SOA. The relevant means are presented in Table 2. At 80-ms SOAs, the difference between sum and neutral probes proved significant with small integers, $F(1, 114) = 12.39$, $p < .001$, with medium integers, $F(1, 114) = 4.26$, $p < .05$, and marginally significant with large integers, $F(1, 114) = 2.65$, $p < .10$.[1] At 150-ms SOAs, the interference effect proved significant with small integers, $F(1, 114) = 20.85$, $p < .01$, medium integers, $F(1, 114) = 5.55$, $p < .05$, and large integers, $F(1, 114) = 6.55$, $p < .01$. At 240 ms, there was no evidence of significant interference, but at 480 ms, the interference effect proved significant with the small integers, $F(1, 114) = 4.89$, $p < .05$. LeFevre et al. (1991) also found that interference effects resurfaced at 500 ms but only for their low-skill subjects; their low-skill subjects also failed to experience interference at 80-ms SOAs. LeFevre et al. attribute the interference experienced at longer

[1] In the process of equating the splits for the sum and neutral probes, we inadvertently included a neutral probe ("17") that contained one of the digits in the original pair. This may have elevated the amount of interference obtained with the neutral probes, thereby minimizing the interference effect for the large digits.

delays to either more persistent or less efficient suppression of sum activation in these subjects.

Accuracy. As can be seen in Table 2, errors were low, averaging 8.1% across subjects, and accordingly, differences in accuracy must be interpreted with caution. Errors were analyzed using a 4 (SOA) × 3 (Size) × 2 (Probe) repeated measures analysis of variance. Error rates decreased with SOA, $F(3, 57) = 5.28$, $p < .01$, and sum probes were associated with higher error rates than neutral probes, $F(1, 19) = 22.50$, $p < .001$. Once again, the probe by SOA interaction proved significant, $F(3, 57) = 6.81$, $p < .05$, with sum probes resulting in significantly more errors at 80-ms SOAs, $F(1, 57) = 6.81$, $p < .01$, and at 150-ms SOAs, $F(1, 57) = 5.05$, $p < .01$.

There was no main effect of integer size on accuracy nor was integer size a factor in any significant interaction. Following the analyses performed on the latency data, the probe by size interaction was evaluated separately at each SOA. Once again, at 80-ms SOAs, the difference between sum and neutral probes was significant with small integers, $F(1, 114) = 13.90$, $p < .001$, with medium integers, $F(1, 114) = 6.18$, $p < .05$, and with large integers, $F(1, 114) = 7.82$, $p < .01$. At 150-ms SOAs, the interference effect was again significant with all integer sizes, $F(1, 114) = 9.85$, $p < .01$ for small integers, $F(1, 114) = 6.18$, $p < .05$ for medium integers, and $F(1, 114) = 4.73$, $p < .05$ for large integers.

Discussion

The results of the first experiment were consistent with LeFevre et al.'s (1988, 1991) findings concerning obligatory activation of sums in a number-matching task. In our experiment, adults were never presented with equations, and single-digit numbers were presented in a less familiar form (i.e., preceded by a zero), and yet, our findings were remarkably similar to LeFevre et al.'s. At 80- and 150-ms SOAs, probes that were the sum of the initial number pair were rejected more slowly than other false items. Thus our findings support LeFevre et al.'s argument that obligatory activation of sums occurs at delays of less than 180 ms and persist even when less familiar formats are used. Like LeFevre et al.'s findings, our results suggest that when the delay exceeds 240 ms, subjects are able to inhibit incorrectly activated sums. The pattern of results in our experiment is consistent with Zbrodoff and Logan's (1986) hypothesis that the processes underlying simple arithmetic are partially autonomous. There appears to be a direct association between single digits and their sums which is activated immediately without intention and produces interference effects in adults. These effects, however, are not long-lasting and within 240 ms of processing sum-based associations dissipate or are inhibited.

Interference effects persisted with all three integer sizes in our number-matching task. According to network distance models, interference effects should have been strongest with small number pairs because these operands are most closely associated with the correct answer node in memory (e.g., Ashcraft, 1982, 1987; Siegler & Jenkins, 1989; Siegler & Shipley, in press). Our findings were consistent with this prediction: The difference in response time for sum and neutral probes was greater for small problems than for large problems in the 80-ms SOA and 150-ms SOA conditions. In Experiment 2, we explore sum-based interference effects in children. Our prediction was that the size of the problem would prove to be an important determinant of sum-based interference effects in elementary school children.

EXPERIMENT 2

The interference effects we obtained with adults presuppose strong associations between a number pair and its sum. Although Siegler and Shrager (1984) offer evidence that a few of these associations are present in 4-year-olds, it is not clear whether these sum-based associations are strong enough to lead to automatic activation in elementary school children. In the second experiment, we explore whether children in grades 2 through 5 experience interference effects in a number-matching task as a consequence of obligatory activation of addition facts and whether these effects vary with the size of the numbers. LeFevre et al.'s (1991) work suggests that, at most, sum activation only plays a minimal role in producing confusion effects prior to sixth grade. It is possible, however, that sum-based confusion effects are present in younger children, but that initially these effects are limited to small numbers because only the associations between small addends and their sums are of sufficient strength to produce interference effects in the early elementary school years. Evidence that these associations are built up early in development comes from findings that 4- and 5-year-olds use direct retrieval to solve small sum addition problems (Siegler & Robinson, 1982; Siegler & Shrager, 1984). Size-related differences in the strength of association between the operands and answers are expected to be strongest early in development, and presumably, these differences are related to the order in which small and large number problems are learned and the frequency with which they occur in children's textbooks (Hamann & Ashcraft, 1985).

In Experiment 2, in addition to exploring the effects of problem size, we will also consider the temporal parameters that govern interference effects at different points in development. It is quite possible that the time-course of interference effects differs for children and adults. LeFevre et al. (1991) found that adults who are more practiced in basic arithmetic experience interference effects at shorter SOAs and that interference effects persist at longer delays for adults with lower skill levels. This suggests that longer SOAs may be required to produce interference effects in children and that sum-based associations may take more time to inhibit. In Experiment 2, SOA was varied so this possibility could be evaluated.

Method

Subjects. Eighty elementary school children, 20 each from grades 2, 3, 4, and 5, were randomly selected from a French upper class urban public school to take part in the study. The respective mean ages for each grade were 7.1 (range, 6.11 to 7.2), 8.2 (range, 8.0 to 8.7), 9.0 (range, 8.10 to 9.5), and 10.4 (range, 10.0 to 10.11).

Stimuli. The stimuli were identical to those used in the first experiment (see Table 1).

Procedure. The procedure was identical to that used in the first experiment with one exception. In pilot work, second- and third-graders' error rates exceeded 30% at 80-ms SOAs, and consequently, we opted to use only the 150-, 240-, and 480-ms SOAs in this experiment. In Experiment 2, children were presented with 144 trials in a session lasting approximately 30 to 45 min. Like the adults, children were told their median latencies and error rates for the practice trials and were encouraged to respond as quickly as possible without making mistakes. Children had been in school a few months before they participated in the experiment; the study was conducted in November.

Results

Latencies. Mean median correct latencies for false problems were analyzed in a 4 (Grade: second, third, fourth, or fifth) × 3 (SOA: 150, 240, or 480 ms) × 3 (Size: small, medium, or large) × 2 (Probe: sum or neutral) ANOVA with repeated measures on the last three factors. The main effect of grade was significant, $F(3, 76) = 11.28$, $p < .001$; children in higher grades responded more quickly than children in lower grades. Latencies declined with increasing SOA, $F(2, 152) = 15.88$, $p < .001$, presumably because longer SOAs afforded more encoding and preprocessing time for the initial pair. Response time decreased as the size of the initial pair increased, $F(2, 152) = 3.37$, $p < .05$. Although the main effect of probe was not significant, the probe by SOA interaction was $F(2, 152) = 17.37$, $p < .001$. More specifically, sum probes only resulted in significant interference at 150-ms SOAs, $F(1, 152) = 7.77$, $p < .001$. The probe by size interaction also proved significant, $F(2, 152) = 3.27$, $p < .05$; interference effects were obtained with small integers, $F(1, 152) = 5.19$, $p < .01$, but not with medium or large integers. For the sum probes, response time decreased as integer size increased, $F(2, 38) = 8.02$, $p < .01$; however, integer size did not have an effect on trials involving neutral probes. More importantly, the Grade × SOA × Size interaction was significant, $F(12, 304) = 2.48$, $p < .001$. The nature of this interaction is discussed below. No other effects proved to be significant in the overall analysis.

Given our interest in more specific age effects and the differences in variability across grades, mean median correct rejection latencies were analyzed separately for each grade level using a 3 (SOA) × 3 (Size) × 2 (Probe) repeated measures ANOVA. The data from both the fourth- and fifth-graders mirror our earlier findings with adults. More specifically, for the fifth-graders, there was a significant effect of SOA, $F(2, 38) = 19.73$, $p < .001$, due to a decrease in response time with increasing SOA[2]. Although there was no main effect of probe, there were significant two-way interactions between probe and SOA, $F(2, 38) = 9.33$, $p < .001$, and probe and size, $F(2, 38) = 3.35$, $p < .05$. The probe by SOA interaction reflected the fact that the interference effect was only significant at 150-ms SOAs, $F(1, 38) = 15.7$, $p < .001$, and the probe by size interaction resulted from significant overall interference effects being obtained with the small integers, $F(1, 38) = 6.38$, $p < .01$, but not with medium or large integers. The more important findings, however, concern the relation among SOA, integer size, and type of probe. Although the three-way interaction did not prove significant, planned comparisons were used to test the difference between sum and neutral probes separately for small, medium, and large integers at each SOA. The pattern of results was similar to our earlier findings with adults; significant interference was obtained with small, $F(1, 76) = 8.67$, $p < .01$, medium, $F(1, 76) = 5.10$, $p < .05$, and large, $F(1, 76) = 6.89$, $p < .01$, integers but only at 150-ms SOAs (see Table 3). Unlike the adults, however, with the large integers, fifth-graders rejected the sum probes more quickly than the neutral probes at 480-ms SOAs, $F(1, 76) = 6.05$, $p < .05$. As we report below, a similar facilitation effect appeared in the second-graders' data for the large integers at 480-ms SOAs. We do not have any explanation for these negative effects but given they only appear with our oldest and youngest subjects, and in each case only with one size category, it is probably best not to attach too much weight to these occurrences.

Our findings with fourth-graders were quite similar. The SOA factor again proved significant, $F(2, 38) = 9.48$, $p < .001$, and once again, there was a decrease in response time with increasing SOA. SOA also interacted with the probe factor, $F(2, 38) = 3.30$, $p < .05$, showing a significant interference effect at 150-ms SOAs, $F(1, 38) = 5.23$, $p < .05$, but not at longer SOAs. Although the SOA × Size × Probe interaction was not significant, planned comparisons revealed that at 150-ms SOAs, the sum probes were responded to more slowly than the neutral probes regardless

[2] Polynomial trend analyses (Abdi, 1987; Keppel, 1982) were used to assess the differences among the three means associated with significant F values. In all cases, the results of the trend analyses proved significant.

TABLE 3
MEAN MEDIAN LATENCIES (IN ms) AND PERCENTAGE ERRORS FOR THE FALSE ITEMS (SUM/NEUTRAL PROBES) IN EXPERIMENT 2

Grade	Integers		SOA 150	240	480
Second					
	Small	*M*	1998/1704*	1808/1631	1637/1577
		E(%)	28.8/8.8	25.0/11.3	21.3/8.8
	Medium	*M*	1712/1598	1645/1671	1448/1543
		E(%)	22.5/6.3	21.3/5.0	20.0/5.0
	Large	*M*	1528/1632	1552/1483	1441/1879⁺
		E(%)	27.5/10.0	25.0/6.3	20.0/6.3
Third					
	Small	*M*	1786/1531*	1711/1667	1686/1816
		E(%)	26.3/8.8	25.0/8.9	16.3/5.0
	Medium	*M*	1820/1567*	1735/1602	1430/1596
		E(%)	17.5/7.5	16.3/2.5	10.0/2.5
	Large	*M*	1762/1637	1634/1696	1444/1511
		E(%)	26.3/5.1	26.3/3.8	16.3/2.5
Fourth					
	Small	*M*	1661/1512*	1467/1457	1363/1486
		E(%)	26.3/8.8	16.3/5.0	13.8/5.0
	Medium	*M*	1725/1559*	1375/1207	1352/1483
		E(%)	20.0/3.8	16.25/3.8	12.5/5.0
	Large	*M*	1647/1510*	1373/1557	1337/1340
		E(%)	22.5/5.1	13.8/5.0	12.5/3.8
Fifth					
	Small	*M*	1157/1011*	955/936	987/970
		E(%)	22.5/7.5	23.8/10.0	12.5/6.3
	Medium	*M*	1167/995*	954/1025	894/1016⁺
		E(%)	17.5/3.8	21.3/3.8	12.5/2.5
	Large	*M*	1159/1029*	926/990	905/942
		E(%)	18.8/5.1	16.3/5.0	8.8/1.3

Note. All differences in error rates for sum and neutral probes were significant at $p < .05$.

* Significant interference effect, $p < .05$.

⁺ Reversal of the interference effect, $p < .05$.

of integer size, $F(1, 76) = 4.24$, $p < .05$ for the small integers, $F(1, 76) = 4.78$, $p < .05$ for the medium integers, and $F(1, 76) = 3.90$, $p < .05$ for the large integers. No other comparisons reached significance.

As predicted, for both the third-graders and second-graders the interference effects depended on both SOA and the size of the integers. For the third-graders, the SOA factor was marginally significant, $F(2, 38) = 2.90$, $p < .10$, and response times decreased with increasing SOA. SOA interacted with integer size, $F(4, 76) = 2.56$, $p < .05$; for the small integers, response time increased as SOA increased, $F(1, 76) = 13.0$, $p < .001$, whereas for the medium and large integers, response times decreased as SOA increased, $F(1, 76) = 5.20$, $p < .05$ and $F(1, 76) = 7.67$, $p < .01$, respectively. Although the reason for this interaction is unclear, it appears to be primarily a function of differences that occurred with the neutral probes. The SOA by probe interaction also proved significant, $F(2, 38) = 8.09$, $p < .01$; again because the interference effect was only significant at 150-ms SOAs, $F(1, 38) = 13.09$, $p < .001$. The three-way interaction involving SOA, size, and probe was not significant, but planned comparisons revealed that the interference effect was only significant at 150-ms SOAs with the small integers, $F(1, 76) = 4.11$, $p < .05$, and the medium integers, $F(1, 76) = 4.05$, $p < .05$ (see Table 3).

For second-graders, the SOA by probe interaction was significant, $F(2, 38) = 4.40$, $p < .05$; the interference effect was marginally significant at 150-ms SOAs, $F(1, 38) = 3.66$, $p < .10$, and did not approach significance at any other SOA. There was a main effect of size, $F(2, 38) = 3.33$, $p < .05$; response times quickened as integer size increased. The size factor also interacted with the probe factor, $F(2, 38) = 4.56$, $p < .05$; significant interference effects were only obtained with small integers, $F(1, 38) = 5.09$, $p < .05$. There was no significant three-way interaction, but planned comparisons revealed that the interference effect was only significant with small integers at 150-ms SOAs, $F(1, 76) = 5.48$, $p < .05$, and that significant facilitation for the sum probes was seen at 480 ms for the large integers, $F(1, 76) = 12.61$, $p < .01$.[3]

[3] One question that might be raised in connection with our findings is whether our results were affected by multiplicative relations that existed between some of the pairs and probes. Although the product of the original digit pair never appeared as a probe, occasionally a member of the pair was a multiple of one of the probes. For the small integers, a multiple of one of the probes appeared on both a sum trial and neutral trial, making any potential effects on this condition negligible. However, for the medium integers, three of the sum probes bore this relation to a member of the digit pair, and for the large integers, one of the neutral probes did. If these multiplicative relations affected performance, sum-based interference effects would have been overestimated with the medium integers and underestimated with the large integers. Although this possibility cannot be ruled out, the interaction between age and integer size that emerged in our experiments argues against it. Older children and adults experienced interference with all three integer sizes although it is possible that this factor may have inflated the interference effect displayed by the third-graders with the medium integers. It is also worth pointing out that in our third experiment, interference effects again depended on the age of the child and the size of the numbers even though different stimulus materials were used. Finally, we point out that although cross-operation answers produce interference in addition tasks and same table errors have proved problematic in multiplication tasks, there is no evidence that multiplication relations, especially those involving a digit that is one possible multiplier of another, disrupt performance in number-matching tasks that do not make explicit mention of arithmetic operations. Thus, although differences in the number of multiples that appeared in our stimulus set

Accuracy. Errors averaged 12.5% across subjects and were analyzed both in an overall analysis and in separate analyses performed at each grade level (see Table 3 for the relevant means). For the overall analysis, error rates were examined in a 4 (Grade) × 3 (SOA) × 3 (Size) × 2 (Probe) ANOVA with repeated measures on the last three factors. All main effects proved significant. Older children made fewer errors, $F(3, 76) = 4.19$, $p < .01$; overall error rates were 15.5, 12.6, 11.0, and 11.0% for grades 2, 3, 4, and 5, respectively. Longer SOAs were associated with fewer errors, $F(2, 152) = 15.07$, $p < .001$ (mean error rates were 14.8% at 150 ms, 13.2% at 240 ms, 9.6% at 480 ms SOAs). Error rates for sum probes were 19.4% and for neutral probes 5.7%, indicating a significant interference effect, $F(1, 76) = 34.53$, $p < .001$. The only significant interaction was between SOA and probe, $F(2, 152) = 6.06$, $p < .01$, which reflected the greater interference obtained at shorter SOAs.

Separate ANOVAs were performed at each grade level. For the fifth-graders, higher error rates were associated with shorter SOAs, $F(2, 38) = 5.18$, $p < .01$, and sum probes resulted in more errors than neutral probes, $F(1, 19) = 71.51$, $p < .001$. Error rates were also lower with larger integers, $F(2, 38) = 4.21$, $p < .05$, but none of the interactions proved significant. For the fourth-graders, error rates were higher at shorter SOAs, $F(2, 38) = 4.42$, $p < .05$, and more errors were made with sum probes than with neutral probes, $F(1, 19) = 62.77$, $p < .001$. No other effects reached significance.

All three main effects proved significant for the third-graders. Shorter SOAs led to more errors, $F(2, 38) = 7.08$, $p < .01$, and error rates were higher with sum probes, $F(1, 19) = 106.06$, $p < .001$. The medium integers resulted in more errors than the small and large integers, $F(1, 38) = 10.42$, $p < .01$. None of the interactions proved significant. For the second-graders, only one effect reached significance; children's error rates were higher with the sum probes than they were with the neutral probes, $F(1, 19) = 110.45$, $p < .001$.

To summarize, at all grade levels, the sum probes were associated with higher error rates, and there was no evidence that differences in response time were a function of a speed–accuracy trade-off. Errors sometimes varied as a function of SOA and integer size, although the interaction effects we observed in the latency data were not always seen in the accuracy data, presumably due to floor effects.

Discussion

Our results show that elementary school children take more time to reject distractors that are equal to the sum of the digits in a number-

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could have affected our results, we do not believe that their presence compromised our findings.

matching task. In our experiment, the prevalence of these latency-based confusion effects depended on the age of the subject and the size of the numbers. Second-graders only evidenced confusion effects when both integers in the original pair were 5 or smaller. Third-graders experienced interference effects with both small problems and medium problems in which one integer in the original pair was 5 or smaller and the other integer was between 6 and 9. Like the adults in Experiment 1, fourth- and fifth-graders experienced these confusion effects with all three integer sizes, including large problems in which both numbers were between 6 and 9. Moreover, the temporal parameters that governed these effects were similar for children and adults. All age groups experienced interference effects at 150-ms SOAs, but these effects were not evident at 240 ms. Our results, then, suggest that associations between integers and their sums are sufficiently built up by second grade to receive obligatory activation in a number-matching task despite the fact that additive operations are not relevant in this context, and that these task-irrelevant associations are inhibited, or begin to dissipate, within 240 ms of processing. Recall that the critical difference between a completely and partially autonomous process is that although both are initiated without intention, only partially autonomous processes can be inhibited once they have begun (see Zbrodoff & Logan, 1986). Thus, our findings suggest that for certain number pairs, the processes that mediate sum activation are partially autonomous in elementary school children, although the extent to which this is true depends on both the age of the child and the size of the integers.

In LeFevre et al.'s (1991) study, the only suggestion that elementary school children might experience additive-based confusion effects came from error rates. Children made more errors when the probe was the sum of the original pair and this effect was strongest when the sum was also relatively close to the original pair. In their study, distance was operationalized as the average difference between each number in the pair and the probe. LeFevre et al. suggest that upon presentation of the original pair, both sums and numbers relatively close to the original digits become active and that it may take activation from both these sources to produce substantial interference in number-matching tasks. There was no evidence in LeFevre et al.'s study that sum activation had an effect on response time either by itself or in conjunction with the distance manipulation. In contrast, the distance effects were rather pronounced: Children responded more slowly to probes that were close to the original numbers. LeFevre et al. offer these findings as evidence that count-string associations remain strong throughout the elementary school years and that fact-based associations only gradually reach sufficient strength to produce interference effects.

In our study, the mean distance between the pair and the probe covaried with integer size, and younger children took longer to reject false answers to small problems. This is consistent with LeFevre et al.'s (1991) claim that interference is strongest when the distance between the pair and probe is small. LeFevre et al. argue that the prevalence of counting-based solution strategies in young children contributes to this effect. The strong number-line connections that develop as a consequence of these computational strategies are thought to result in the numbers close in magnitude to the initial pair being more strongly activated than distant numbers. Although we agree that number-line associations can produce interference, these associations are not sufficient for explaining the overall pattern of response times we observed in our study. In our experiment, integer size only had an effect on second- and third-graders' performance with sum probes; distant effects were not evident with the neutral probes. Although this could mean that both number-line associations and sum-based activation are required to produce interference, an alternative explanation is that sum activation is strongest for small numbers because small number facts are learned first and practiced more frequently. However, regardless of which alternative proves correct, it is clear from our findings that obligatory activation of sums occurs as early as grade 2 in simple number-matching tasks despite the fact that addition is not relevant in these contexts. Our work also suggests that these effects follow a similar time-course in children and adults.

EXPERIMENT 3

When adults are asked to verify simple arithmetic problems, false answers that have associative links to the presented problems take more time to reject. The interference effects that arise when multiplicative answers are given to addition problems and additive answers are given to multiplication problems have been labeled associative confusion effects. Presumably, these effects are a result of activation spreading from the arguments in the equation to their sum and product independent of which operation is designated in the equation. In the third experiment, we examine whether the associations between a number pair and its sum or product are strong enough to produce associative interference effects in children. More specifically, we focus on whether children's knowledge of simple arithmetic is represented in an integrated network that can potentially give rise to associative confusion effects.

One reason for suspecting that elementary school children might experience an associative confusion effect is that they frequently make mistakes of this sort when solving simple arithmetic problems. In their analysis of elementary school children's arithmetic errors, Miller and Paredes (Expt 2, 1990) found that cross-operation errors (responding with a multiplication answer, such as 32, to an addition problem, such as $8 + 4 = ?$) accounted for 48% of all addition errors made by fourth-graders, 14% of the addition errors made by the third-graders, and 4% of the addition errors made by the second-graders. It is worth noting that in this particular experiment, the trials were blocked so that children only needed to perform a single operation for each block of trials. Cross-operation errors were even more frequent when addition and multiplication problems were mixed within the same trial block. What is especially interesting about Miller and Paredes' study is that they found a pattern of developmental changes suggesting that learning to multiply interferes with children's ability to solve addition problems. When children first learn to multiply, their solution times for addition problems become slower and they make substantially more cross-operation errors on addition problems than on multiplication problems. Miller and Paredes argue that this temporary disruption in addition skills is a result of multiplication facts being incorporated into knowledge structures that also serve addition.

Although Miller and Paredes' (1990) work suggests that facts about different arithmetic operations are integrated in the early stages of skill acquisition, it is not clear how or when information about irrelevant operations is activated. Recall that adults experience associative confusion effects because information about irrelevant operations is always activated to some extent. If simple arithmetic operations can be activated in the absence of intention (i.e., if they are at least partially autonomous), then children should take more time to reject an incorrect answer if it is an associative lure.

Hamann and Ashcraft (1985) present data suggesting that the associative confusion effect emerges rather late in development. In their study, students in grades 1, 4, 7, and 10 verified simple and complex addition problems. Half of the false problems contained associative lures (i.e., the incorrect answer was the product). In this study, only the tenth-graders produced the slower responses for the associative problems which are indicative of this confusion effect. In subsequent work on latency-based confusion effects, Koshmider and Ashcraft (1991) found that confusion problems containing incorrect answers that were near multiples of the target problem (e.g., $4 \times 8 = 24$) do not slow verification time until ninth grade, suggesting more generally that simple arithmetic processes that are partially autonomous later in development may not be initiated without intention prior to ninth- or tenth-grade.[4]

The results of our second experiment, however, argue against this interpretation. The interference effects we observed suggest that when children see a pair of numbers, a certain amount of activation automatically spreads to their sum. If sum activation also occurs when number pairs are pre-

[4] Koshmider and Ashcraft (1991) also found that error rates were higher when the false answer was a near multiple of the target problem. This difference in error rates was significant beginning in fifth grade. A similar trend emerged in Miller and Parades' (1990) production task. One of the more common errors made by both children and adults involved reporting a product that was off by a factor of one (see also Campbell, 1987a, 1987b).

sented in a multiplication equation, associative confusion effects should be evident for these problems. Similarly, confusion effects due to product activation also seem likely given that many of the errors older elementary children make when solving addition problems involve incorrectly reporting the product of the addends. Additional evidence that both product and sum activation occur in elementary school children comes from Zbrodoff's (1979) and Lemaire et al.'s (1991) research on latency-based confusion effects in equation verification tasks. In contrast to Hamann and Ashcraft's (1985) findings, both studies report evidence of associative confusion effects in elementary school children.

Experiment 3 focuses on the emergence of associative confusion effects in elementary school children. Children were tested at the beginning, middle, and end of the academic year in an attempt to pin down more clearly when in development these effects emerge and whether they emerge simultaneously for addition and multiplication. In Miller and Parades' (1990) study, solution times for addition and multiplication problems varied during the school year. Although children became faster at solving multiplication problems over the course of the school year, solution times for addition problems revealed a different pattern. More specifically, second-graders in advanced math classes and third-graders in regular math classes required more time to solve addition problems as the school year progressed, and significantly, this increase in solution times for addition problems occurred during the time period when children were actively learning multiplication. Changes in error patterns also suggested that learning to multiply temporarily disrupts addition skills: The percentage of cross-operation errors was highest for third- and fourth-graders and reached its peak in the middle of fourth grade. Based on Miller and Parades' findings, we expected that third- and fourth-graders would find false addition equations in which the answer corresponded to the product especially difficult.

In Experiment 3, children were presented with simple equations and had to indicate whether the presented answer was true or false as quickly and as accurately as possible. Half of the problems contained associative lures that would have been correct under a different operation. Longer rejection times for problems with associative lures would be indicative of an associative confusion effect. In the adult literature, associative confusion effects have proven stronger when addition and multiplication problems are presented in the same block, and in our experiment, we presented children with mixed blocks of simple addition and multiplication problems to maximize the likelihood that we would obtain evidence of these effects.

Method

Subjects. Twenty-seven elementary school students, nine each from grades 3, 4, and 5 of a suburban school in France participated in this study. At the beginning of testing, the third-graders ranged in age from

7.9 to 8.2 ($M = 8.0$), the fourth-graders ranged in age from 8.8 to 9.3 ($M = 9.1$), and the fifth-graders ranged in age from 10.0 to 10.9 ($M = 10.1$).

Stimuli. The stimuli were addition and multiplication equations presented in standard form (i.e., $a + b = c$ or $a \times b = c$), where the operands a and b were single digits from 2 to 9. One and zero were not included because it is generally believed that subjects do not solve these problems by retrieving the solution directly from memory but instead retrieve rules (e.g., $n \times 0 = 0$) that guide their solution (see Ashcraft, 1982; Baroody, 1985). The equations $2 + 2$ and 2×2 were not included because the sum and product of these operands are identical. In generating the stimuli, operand order was ignored (i.e., 3×4 and 4×3 were considered to be equivalent problems and only one variant was presented). The resulting set of stimuli consisted of 35 true addition problems and 35 true multiplication problems. Following the procedure outlined in the first experiment, the problems were classified as small, medium, or large based on the size of the arguments. In the small integer condition, both arguments were 5 or smaller. In the medium integer condition, one argument was 5 or smaller and the other argument was between 6 and 9. In the large integer condition, both arguments were between 6 and 9.

A set of false problems was also created. For each equation, both a *confusion* and *nonconfusion* problem were generated. When the equation involved addition, the c term for the confusion problem was the product of the two operands ($3 + 5 = 15$), and for the multiplication problems, c was the sum ($3 \times 5 = 8$). Nonconfusion problems were also generated such that c was neither the sum nor the product of a and b (e.g., $3 + 4 = 8$). The c terms for the nonconfusion problems were chosen so that the difference between the correct answer to the equation and the c term (the "split") would match the corresponding split in the confusion condition (see Zbrodoff & Logan, 1986). This was necessary because some confusion problems have large splits (e.g., $9 + 9 = 81$) and the split is known to affect reaction time in arithmetic verification tasks (Ashcraft & Battaglia, 1978; Zbrodoff & Logan, 1990). To equate the splits, the answer to the nonconfusion problem was equal to the answer to the corresponding confusion problem plus or minus one. The false problems were also categorized as involving small, medium, or large integers, and the mean splits for the confusion and nonconfusion problems were equated for each problem type. Because the correct answers and the answers to the confusion problems for addition and multiplication were the inverse of one another, the split did not vary across operation. Each of the 70 true equations appeared twice so subjects saw 140 true equations and 140 false equations (half of which were confusion problems).

Procedure. The equations were presented in the center of a computer screen (SAMSUNG SPC 3000V). Each trial began with a 750-ms ready signal (a line of five "a"s) which appeared in the center of the screen. The equation was then displayed horizontally in the center of the screen. The equations were in the form "$a + b = c$" or "$a \times b = c$." The symbols and numbers were separated by spaces equal to one half the width of each character. The equation remained on the screen until the child responded. The subject was instructed to respond "true" or "false" by pressing the appropriate button on the keyboard. The two farthest letter keys on the second row were designated as true and false. All subjects were instructed to use their left and right index fingers to press these keys and the position of the true and false keys was counterbalanced across subjects.

All children were presented with the same 280 problems. These problems were randomly ordered for each subject with the restriction that no more than four consecutive trials could require the same response. The stimuli were presented in two 140-problem trial blocks; each block was composed of 70 true and 70 false problems. Children were permitted a 5-min rest between blocks. Before the experimental trials, children were given one block of 40 practice problems to familiarize them with the apparatus and procedure. After these practice trials, children were reminded of the instructions and were told their median latency and error rate. They were also encouraged to work as fast as they could without making mistakes. Each session lasted approximately 30–45 min.

Children were tested in October, February, and June of the same school year. The same procedure with the same subjects and identical problems was repeated for each session of testing. The first session began after the first 4 weeks in the school year. The final session of testing concluded within 3 weeks of the end of the school term.

Results

Latencies for true problems. The mean median correct response times for the true problems were analyzed using a 3 (Grade: third, fourth, and fifth) $\times$ 3 (Testing session: October, February, and June) $\times$ 3 (Integer size: small, medium, and large) $\times$ 2 (Operation: addition and multiplication) ANOVA with repeated measures on the last three factors. Ties (ie, problems with identical arguments) were excluded from the analysis.[5] Children in higher grades responded more quickly than those in lower grades, $F(2, 24) = 79.01$, $p < .001$, and performance improved across

[5] Although we included ties (i.e., problems with identical arguments) in our design, past work suggests that problem size effects are less likely to be seen with these number pairs. In our preliminary analyses, we found that integer size did not have a significant effect on either the true problems involving ties or the false problems involving ties. Given the importance of integer size in our work and that ties have been eliminated from a number of other studies, we only report the results of the analyses from which ties were excluded although a similar pattern of results emerged when the ties were included in the data set.

sessions, $F(2, 48) = 8.41$, $p < .001$. Verification times also decreased as integer size decreased, $F(2, 48) = 54.94$, $p < .001$. No other effects reach significance.

Given the difference in variability across grades and our interest in more specific age effects, separate ANOVAs were performed on the data for each grade level. The factors wee Testing session, Integer size, and Operation. The fifth-graders performed similarly across sessions. However, their performance did vary as a function of both integer size, $F(2, 16) = 24.68$, $p < .001$, and operation, $F(1, 8) = 14.02$, $p < .01$. The interaction between these two factors was also significant, $F(2, 16) = 16.83$, $p < .001$. Pairwise comparisons using Tukey's honestly significant difference (HSD) test revealed that the multiplication equations were verified more quickly than the addition equations, but that this was only true for problems involving small and medium numbers. For problems with large numbers, although the means were in the same direction, the difference was not significant.

The fourth-graders did improve across testing sessions, $F(2, 16) = 6.44$, $p < .01$, and the interaction between testing session and operation proved significant, $F(2, 16) = 33.17$, $p < .001$. True multiplication facts were verified more quickly than the addition facts but this difference was only significant in June. Children's verification times also decreased as integer size decreased, $F(2, 16) = 59.88$, $p < .001$.

The third-graders also improved across testing sessions, $F(2, 16) = 25.50$, $p < .001$, and their verification times decreased as integer size decreased, $F(2, 16) = 55.37$, $p < .001$. There was a main effect of operation, $F(1, 8) = 15.85$, $p < .01$. Multiplication problems were faster than addition problems, but this advantage was only evident at the February and June testing sessions and was only seen with the small and medium integers. These effects were reflected in the significant session by operation interaction, $F(2, 16) = 49.98$, $p < .001$, and size by operation interaction, $F(2, 16) = 82.46$, $p < .001$.

Latencies for false problems. The mean median reaction times for false problems that were correctly rejected (excluding the ties) were analyzed in a 3 (Grade: third, fourth, and fifth) × 3 (Testing session: first, second, and third) × 3 (Integer size: small, medium, and large) × 2 (Operation: addition and multiplication) × 2 (Probe: confusion, nonconfusion) design with repeated measures on the last four factors.

The grade effect proved significant, $F(2, 24) = 57.93$, $p < .01$: Older children responded more quickly than younger children. Mean median rejection times were 1595 ms for the fifth-graders, 2443 ms for the fourth-graders, and 3035 ms for the third-graders. Overall response times decreased as the school year progressed, $F(2, 48) = 14.36$, $p < .001$, and the grade by testing session interaction also reached significance, $F(4, 48) = 8.25$, $p < .001$. Separate analyses revealed that both the third-graders and fourth-graders rejected the false problems more quickly as the school year progressed [$F(2, 16) = 15.51$, $p < .001$; $F(2, 16) = 5.77$, $p < .05$, respectively]. For fifth-graders, the testing session effect was not significant. More importantly, the confusion problems were rejected more slowly than the nonconfusion problems, $F(1, 24) = 167.60$, $p < .001$, and as the size of the problem increased, so did the time needed to reject the false answers, $F(2, 48) = 83.77$, $p < .001$. The size by probe interaction proved significant, $F(2, 48) = 145.32$, $p < .001$, as did the size by grade and probe by grade interactions [$F(4, 48) = 27.65$, $p < .001$; $F(2, 16) = 93.49$, $p < .001$, respectively]. Finally, the four-way interaction involving grade, size, probe, and testing session was significant, $F(8, 96) = 18.68$, $p < .001$. The nature of this interaction was explored further in separate analyses performed on the data for each grade. Mean correct rejection latencies were analyzed using an ANOVA for a 3 (Testing session) × 3 (Size) × 2 (Operation) × 2 (Probe) within-subjects design. The mean latencies for each age group are presented separately in Table 4.

For fifth-graders, the confusion effect proved significant, $F(1, 8) = 8.80$, $p < .05$, as did the effect of integer size, $F(2, 16) = 44.81$, $p < .001$; rejection time increased as integer size increased. Although the operation factor was not implicated in any interactions in the overall analysis, the fifth-graders exhibited both a significant operation by size effect, $F(2, 16) = 15.43$, $p < .001$, and a significant operation by size by probe effect, $F(2, 16) = 27.21$, $p < .001$. For each grade and each session, planned comparisons were used to evaluate the difference between confusion and nonconfusion problems for each size condition, and the data are summarized in Table 4. With the multiplication problems, fifth-graders evidenced an associative confusion effect for the small and medium integers at each testing session. With the addition problems, confusion effects were not evident in October but were seen in February and June with both the small and medium integers. Confusion effects were not obtained with the large integers at any testing session, and in fact, the fifth-graders were faster at rejecting answers to multiplication problems that were the sum of the arguments at the last two testing sessions, although the reason for this advantage is not known. The more important finding to note from the fifth-grade data is that the associative confusion effect was seen with both small and medium numbers.

For the fourth-graders, significant main effects were found for testing session, $F(2, 16) = 5.77$, $p < .05$, integer size, $F(2, 16) = 75.22$, $p < .001$, and probe, $F(1, 8) = 11.64$, $p < .01$. These effects were similar to those found for the fifth-graders. Children's performance improved over the school year, and as integer size increased so did response time. Once again, confusion problems took longer to reject than nonconfusion problems. These effects were qualified by a number of significant two-way

TABLE 4

MEAN MEDIAN LATENCIES (IN ms) AND PERCENTAGE ERRORS FOR THE FALSE ITEMS (CONFUSION/NONCONFUSION ANSWERS) IN EXPERIMENT 3

	Integers		Session		
			October	February	June
			Grade 3		
Multiplication	Small	*M*	3592/3398**	2740/2488**	2238/1987**
		E(%)	2.0/1.9	1.7/1.8	1.1/1.3
	Medium	*M*	3640/3528	2779/2603*	2277/2102
		E(%)	2.2/2.2	2.0/2.1	1.4/1.6
	Large	*M*	3616/3791	2761/2841	2315/2335
		E(%)	2.4/2.2	2.4/2.3	1.7/1.9
Addition	Small	*M*	3424/3631⁺	2798/2829	2576/2334**
		E(%)	2.2/2.0	2.4/2.6	1.8/1.8
	Medium	*M*	3534/3761⁺	2907/2944	2681/2449
		E(%)	2.0/2.0	2.6/2.4	1.9/1.8
	Large	*M*	3738/4065⁺	3103/3256	2876/2761
		E(%)	2.3/2.0	2.5/2.5	1.9/1.8
			Grade 4		
Multiplication	Small	*M*	2869/2573***	2319/2127*	2269/1983***
		E(%)	1.9/2.0	1.7/2.0	1.0/1.1
	Medium	*M*	2917/2703***	2358/2242	2308/2098
		E(%)	1.8/1.9	1.9/2.0	1.2/1.2
	Large	*M*	2893/2978	2339/2497	2290/2353
		E(%)	2.0/2.0	2.1/2.1	1.4/1.3
Addition	Small	*M*	2758/2623	2321/2073**	2045/1805**
		E(%)	2.0/2.0	1.5/1.5	1.9/1.8
	Medium	*M*	2865/2752	2420/2172**	2151/1913**
		E(%)	2.1/2.3	1.6/1.7	1.1/1.2
	Large	*M*	3079/3077	2613/2521	2347/2243
		E(%)	2.2/2.1	1.4/1.3	1.3/1.3
			Grade 5		
Multiplication	Small	*M*	1544/1254***	1455/1313***	1326/1145***
		E(%)	1.7/1.8	1.6/1.7	1.5/1.8
	Medium	*M*	1669/1382***	1600/1428***	1438/1252***
		E(%)	1.8/2.0	1.8/1.9	1.4/1.8
	Large	*M*	1613/1681	1513/1747⁺	1397/1587⁺
		E(%)	1.9/2.2	2.0/2.1	1.6/1.8
Addition	Small	*M*	1709/1629	1710/1532***	1518/1337***
		E(%)	1.6/1.7	1.9/2.1	1.0/1.0
	Medium	*M*	1828/1769	1839/1651***	1617/1492**
		E(%)	1.8/2.0	1.9/2.2	1.1/1.2
	Large	*M*	2003/2046	2017/1990	1825/1844
		E(%)	2.0/2.2	1.9/2.0	1.1/1.3

* Marginal confusion effect, $p < .10$.

** Significant confusion effect, $p < .05$.

*** Significant confusion effect, $p < .01$.

⁺ Reversal of the confusion effect, $p < .05$.

interactions involving session and probe, $F(2, 16) = 24.77$, $p < .001$, session and operation, $F(2, 16) = 51.39$, $p < .001$, size and operation, $F(2, 16) = 21.95$, $p < .001$, as well as a significant three-way interaction involving size, operation, and probe, $F(2, 16) = 28.02$, $p < .001$, and a marginally significant interaction involving session, operation, and probe, $F(2, 16) = 2.81$, $p < .10$. The four-way interaction involving session, size, operation, and probe was also marginally significant, $F(4, 32) = 2.25$, $p < .10$, and once again, the nature of these effects is most easily understood with reference to the means presented in Table 4. As can be readily seen, the fourth- and fifth-graders performed similarly. With multiplication problems, fourth-graders experienced an associative confusion effect with small integers in October and June, although the confusion effect was only marginally significant for the small integers in February. For the medium multiplication problems, the confusion effect was also significant in October and June but not February. With addition problems, a confusion effect was evident in February and June, and again this was present for both the small and medium integers, suggesting that by February product-based associations are strong enough to lead to automatic activation. Response times for the large integers did not differ for the confusion and nonconfusion problems.

For the third-graders, the overall confusion effect was only marginally significant, $F(1, 8) = 4.51$, $p < .10$. Response times became slower as integer size increased, $F(2, 16) = 73.44$, $p < .001$. A significant testing session effect was observed, $F(2, 16) = 15.51$, $p < .001$; children rejected the false problems more quickly at each session. A number of interactions involving integer size and probe proved significant. These included a size by probe interaction, $F(2, 16) = 24.04$, $p < .001$, a size by probe by session interaction, $F(4, 32) = 23.20$, $p < .001$, a size by probe by operation interaction, $F(2, 16) = 17.71$, $p < .001$, and a size by probe by operation by session interaction, $F(4, 32) = 20.60$, $p < .001$. Table 4 helps to clarify the nature of these effects. An associative confusion effect was observed with the small multiplication problems at all three testing sessions which indicates that sum-based associations compete with third-graders' retrieval of multiplication facts. For the addition problems, an associative confusion effect was obtained with both the small and medium integers in June indicating that as children master multiplication facts, confusion effects due to product activation become more prevalent. In October, children were faster at rejecting answers to addition problems that were the product of the two numbers; the reason for this reversal of the confusion effect is not clear although it is worth noting that interference effects due to product activation were not evident at the first session for any age group.

Accuracy. The overall error rates in our verification task were relatively low (3.1% for the true problems and 1.7% for the false problems) in comparison to the error rates obtained when children are required to generate answers to comparable problems (see Miller & Paredes, 1990), and, consequently, any differences in error rates must be interpreted with caution. The mean error rates are included in Table 4. Errors on the true problems were examined using a Grade × Session × Size × Operation ANOVA with repeated measures on the last three factors. The grade effect was significant, $F(2, 24) = 9.55$, $p < .001$; the mean error rates were 2.2% for the fifth-graders, 3.2% for the fourth-graders, and 4.0% for the third-graders. Children's error rates also varied with integer size, $F(2, 48) = 4.96$, $p < .01$, they erred on 2.7% of the small problems, 3.1% of the medium problems, and 3.5% of the large problems. No other main effects or interactions proved significant.

Errors on the false problems were also examined using an ANOVA; Grade was a between-subjects factor and Session, Size, Operation, and Probe were within subject factors. The grade effect again proved significant, $F(2, 24) = 12.10$, $p < .001$; the mean error rates were 1.7% for the fifth-graders, 1.6% for the fourth-graders, and 1.9% for the third-graders. Error rates also decreased across sessions, $F(2, 48) = 15.16$, $p < .001$. Children erred on 2% of the problems in October, 1.9% in February, and 1.4% in June. No other effects were significant, and as indicated in Table 4, differences in error rates for the confusion problems were never significant. Again, this result is likely to be a function of floor effects. Although it is difficult to draw any strong conclusions from the error data, it is clear that the results obtained with the latency data are not compromised by a speed–accuracy trade-off.

Discussion

The results of the third experiment demonstrate that latency-based associative confusion effects are not limited to adults and high school students. In our study, children in grades three through five were slow to reject answers that would have been correct if a different operation was performed. The prevalence of this effect, however, varied with the age of the child and the size of the numbers. Differences in the prevalence of associative confusion effects across sessions were also observed. Confusion effects due to sum activation affected children's performance with the multiplication problems at all three sessions. In contrast, interference due to product activation became evident later in the school year. These variations most likely reflect differences in how addition and multiplication facts are taught in school. Children in the French school system learn to multiply in second grade, and multiplication facts are mastered through rote memorization. In contrast, little time is spent memorizing addition facts; children become familiar with addition facts in the context of problem solving activities. By the end of the school year, the French third- and fourth-graders in our study verified simple multiplication equations faster than simple addition equations, and for the fifth-graders, this advantage for multiplication problems was evident at all three sessions. Miller and Parades (Experiment 1, 1990) report a similar finding: The affluent students, who also had high math achievement scores, solved multiplication problems faster than addition problems. Presumably, these differences in response time are a function of the greater effort spent developing multiplication skills. In our study, interference effects due to product activation became increasingly evident as the school year progressed. The time children spent practicing their multiplication facts undoubtedly contributed to this effect. Thus while associations between two integers and their sum were strong enough in October to produce interference effects, associations between two integers and their product had not yet achieved sufficient strength to be automatically activated.

Although operation was one important determinant of confusion effects, an even more important factor was the size of the numbers. Confusion effects were never seen with large numbers despite the fact that children were drilled on these problems. Not surprisingly, third-graders were most affected by the difference between small and medium problems. Automatic activation of the sum resulted in significant interference effects with the small integers at all three sessions but these effects were never seen with the medium integers. As the school year progressed and children became more proficient in multiplication, interference effects were seen with the addition problems due to product activation. When interference effects were obtained with addition problems, these effects appeared simultaneously with both the small and medium integers and this may be a reflection of the rote training with these facts that was part of the multiplication curriculum. For the fourth- and fifth-graders, there was little difference between the small and medium problems. At each testing session, fifth-graders displayed the same pattern of confusion effects with both the small and medium integers. For the most part, this was also true for fourth-graders.

GENERAL DISCUSSION

It is clear from these experiments that the associations between a number pair and its sum or product are of sufficient strength during the elementary school years to produce interference effects in a variety of situations. When the present findings are considered in the context of other research on confusion effects, it becomes apparent that the processes that underlie mental arithmetic are partially autonomous in older elementary school children and adults and that the temporal parameters that govern the inhibition of these effects are remarkably similar across development.

In our first experiment, we found interference effects in a task which required subjects to verify whether a target number had appeared in a previous number pair. When the target corresponded to the sum of the original number pair, adults took longer to reject it, suggesting that when a pair of numbers is presented, activation automatically spreads to its sum. If the sum was presented within 150 ms of the target, interference effects were clearly evident but these effects disappeared at longer SOAs, suggesting that activation initially spreads to the sum of the arguments and dissipates or is inhibited within 240 ms of processing. Our findings complement LeFevre et al.'s (1991) results and suggest that sum activation also occurs with large number pairs.

When we tested for interference effects in elementary school children, we found that the size of the integers was a critical determinant of whether interference effects were obtained with a specific age group. Even second-graders experienced sum-based interference effects provided both integers in the original number pair were small. Although the range of problems affected by interference effects increased during the elementary school years, it is worth noting that the temporal parameters responsible for these effects remained the same. At all ages, interference effects appeared when sums were presented within 150 ms of the original number pair and disappeared at longer SOAs. One conclusion that we can draw from the present study that is also supported by Lemaire et al. (1991) is that obligatory activation of addition facts occurs in both children and adults and that the temporal parameters governing interference effects are remarkably similar in both groups. It would seem, then, that the processes mediating sum activation are partially autonomous in elementary school children as well as adults.

Research with adults suggests that interference effects are not limited to sum activation. In arithmetic verification tasks, interference effects have been attributed to inappropriate activation of both sums and products. More specifically, associative confusion effects appear when adults are presented with false answers that would have been correct if a different operation was performed (Zbrodoff & Logan, 1986). These associative confusion effects, however, can be inhibited and interference effects are not obtained when the cross-operation answer is presented 300 ms after the equation. This suggests that once adults are presented with an equation, they begin to activate nodes related to the two arguments and that this activation spreads to both the sum and product. If adults are presented with an incorrect answer that corresponds to either the sum or the product of these arguments, they are slow to reject it because the corresponding node has been partially activated, albeit incorrectly. However, the effects of this initial activation are not long lasting, and within 300 ms, irrelevant activation patterns either dissipate or are inhibited, and consequently, associative lures cease to be problematic.

In the third experiment, we examined whether associations between number pairs and their sums and products are of sufficient strength to produce interference effects in elementary school children. Our results suggest that associative confusion effects are similar for elementary school children and adults. When presented with a problem, children in grades three though five activate candidate answers. The longer rejection time associated with the confusion condition is necessary to inhibit incorrectly activated answers and to select the correct answer which is then compared to the proposed answer (Ashcraft, 1982, 1987; Ashcraft & Battaglia, 1978; Hamann & Ashcraft, 1985). In our study, the extent to which sum and product activation disrupted performance depended on whether children were tested early or late in the school year. Sum activation proved to be more robust than product activation and interfered with performance at all three testing sessions. In contrast, interference effects due to product activation emerged later in the school year. This suggests that a certain amount of practice with multiplication facts is required before the associations between operands and products are strong enough to lead to automatic activation.

The associative confusion effects we observed in Experiment 3 depended on both the age of the child and the size of the numbers. Fourth- and fifth-graders only evidenced a confusion effect with the small and medium numbers. These results differ slightly from our findings with these age groups in the number-matching task. In that task, sum probes resulted in more errors than neutral probes for small, medium, and large numbers. For the third-graders, interference effects due to sum activation only proved significant with the small integers in the arithmetic verification task, although in the number-matching task, sum-based interference was a problem with both small and medium numbers. Thus, although integer size tends to have a somewhat more restrictive effect on arithmetic verification tasks, in both tasks, with increasing age, children experience interference effects across a wider range of problems.

It should be noted that the interaction between integer size and the confusion effect that emerged in our tasks was very different from the interaction seen in Koshmider and Ashcraft's (1991) study. In that study, seventh-graders, ninth-graders, and college students only experienced confusion effects with large problems; confusion effects were not seen with small problems (operationalized as problems with products under 20). In Koshmider and Ashcraft's study, subjects only viewed multiplication problems and cross-operation answers were not presented. Instead, false answers in the confusion condition were near multiples obtained by increasing or decreasing one of the problem's multipliers by one. The problem size effect Koshmider and Ashcraft observed probably reflects the associations that have been built up for the times table associated with each multiplier. For large integers, the products tend to be more unique to each table, making the associations between one multiplier and the other entries in the table fairly strong. Large problems are also likely to be less well-known and consequently, the associations between the multiplier and the correct answer and the multiplier and the confusion answer are likely to be of comparable strength. In contrast, with small problems, the correct associations between the multipliers and their product would be strong relative to competing incorrect near multipliers, making interference effects less likely.

The confusion answers we presented in our study bore a different relation to the arguments than those used in Koshmider and Ashcraft's (1991) work. In our arithmetic verification task, the association between the arguments and the false answers in the confusion condition would have been correct if the equation involved a different operation. In the number-matching task, the confusion answers were the sum of the two integers. Thus, our tasks look at the relative strength of correct associations between a pair of numbers and its sum and product. Under these conditions, smaller well-known problems would be expected to produce the strongest confusion effects because of the strong connections between the arguments and the answers associated with them under various operations.

Although we only examined the temporal factors that mediate interference in the number-matching task, in a related study, Lemaire et al. (1991) investigated the time-course of the associative confusion effect in fourth- and fifth-graders. In Lemaire et al.'s experiment, children were presented with simple addition and multiplication problems under three different conditions. In one condition, the answer was presented within the equation, but in the other two conditions the answer was presented either 300 or 500 ms after the equation. In that study, children proved capable of inhibiting associative-based confusion effects: Confusion effects were not seen at 500-ms delays although they were present at shorter delays. The fifth-graders in Lemaire et al.'s study only experienced interference effects with the addition problems; the interference effect for the multiplication problems was not significant although the difference between the means was in the right direction. In contrast, for the fourth-graders, sum activation produced more interference than product activation and interference effects were evident at delays of 300 ms. When we compare the present findings to Lemaire et al.'s results, it is not clear why the relative effects of product and sum activation differ between the two studies. It is worth noting, however, that Miller and Parades (1990) report a similar discrepancy between two of their experiments and suggest that differences in the extent to which multiplication is emphasized in the schools and at home may account for the difference. Environmental factors may also be responsible for the differences we observed. One of the striking similarities, however, between the present findings and Lemaire et al.'s is that in both studies, the processes mediating confusion effects in children and adults appear to be partially autonomous.

One question that can be raised about research on the associative confusion effect is whether perceptual factors are responsible for this effect. For example, subjects may perform poorly with associative lures because after seeing an answer that would have been correct under a different operation, subjects check to see if they have "misperceived" the sign. Although we do not want to dismiss summarily the role perception plays in the phenomenon, we believe that our experiments demonstrate rather clearly that confusion effects are not an artifact of perceptual factors. The strongest argument against this view is our finding that the effects of sum activation are remarkably similar in arithmetic verification tasks and number-matching tasks even though the latter does not involve any arithmetic signs that could be misperceived. It is also true that the interaction between age and integer size is not easily explained by a perceptual account nor are the effects of operation and testing session. Finally, Miller and Paredes' (1990) finding that cross-operation errors are commonly produced by third- and fourth-graders even when students are presented with only addition problems or only multiplication problems also suggests that perceptual factors alone are not responsible for associative confusion effects (see also Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1990).

A variety of tasks have been used to evaluate how arithmetic facts are organized and retrieved in elementary school children. These include number-matching tasks, arithmetic verification tasks, and priming tasks (Koshmider & Ashcraft, 1991). One conclusion that emerges from this work is that by third grade, children's knowledge of some addition and multiplication facts has reached sufficient strength to be routinely retrieved. This is not to deny that there are also age-related changes in performance. With schooling, children become increasingly proficient at retrieving arithmetic facts and the range of facts that are retrieved automatically increases. Our findings suggest that processes involved in solving simple addition and multiplication problems can be initiated without intention, and that sum-based associations can be activated even when the primary goal of the task does not involve arithmetic problem solving.

The results of our experiments are consistent with associative models of mental arithmetic (e.g., Ashcraft, 1987, 1992; Campbell, 1987a, 1987b; Campbell & Graham, 1985; LeFevre et al., 1988, 1991; Siegler, 1988, Siegler & Jenkins, 1989, Siegler & Shipley, in press, Siegler & Shrager, 1984). According to these models, arithmetic knowledge consists of a highly interconnected network of associations that are accessed upon presentations of pairs of numbers via spreading activation (Anderson, 1983; Collins & Loftus, 1975). For example, in Siegler's model, children begin with an initial distribution of varying strengths between each simple arithmetic problem and possible answers. Answers are activated for a given problem as a function of their associations to that problem. Our work suggests that with experience, the structure of associations changes; connections between increasingly larger integers and their sums and products are strengthened and activation of these associations becomes automatic. One consequence of this is that with increasing age, children experience interference effects on a wider range of problems. Thus, whereas early in development, activation patterns are restricted to small integers that are near neighbors on the number-line (see LeFevre et al., 1991) or that involve small sums, as children gain experience, arithmetic knowledge becomes less and less locally activated.

By charting the developmental time course of interference effects, our studies help shed light on how arithmetic facts are accessed in school children. More generally, our results suggest that investigating sum-based and associative confusion effects and their inhibition may lead to a more complete understanding of how arithmetic knowledge is stored and used at different points in development. Given that these effects can be observed in elementary school children, interference effects are likely to prove useful tools for exploring developmental changes in the memory network that encompasses simple arithmetic, and when combined with findings from production tasks, may also provide some insights for understanding how children select strategies for solving arithmetic problems (see Siegler, 1986, 1987a, 1987b, 1988; Siegler & Shrager, 1984).

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