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ASSOCIATIVE CONFUSION EFFECT IN COGNITIVE ARITHMETIC: EVIDENCE FOR PARTIALLY AUTONOMOUS PROCESSES

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Abstract. *When adults are asked to verify simple arithmetic problems (e.g., 8 + 4 = 12, True? False?), confusion problems (e.g., 8 + 4 = 32) produce slower rejection times than other incorrect problems (e.g., 8 + 4 = 13). This confusion effect has been interpreted as the result of memory associative confusions between addition and multiplication problems. The present experiments aimed at determining whether the confusion effect would remain with delay. Arithmetic problems with different answers were presented to adults and children (9 and 10 year-old) either simultaneously or with variable delay (i.e., 100, 300, and 500 ms). In adults, confusion problems produced significantly slower reaction times than other incorrect problems both at 0 and 100 ms delays, but not at 300 and 500 ms delays. For 10 year-old children, the confusion effects disappeared at 300 and 500 ms delays. For 9 year-old children, the confusion effects only disappeared at 500 ms delay. These results are congruent with Zbrodoff and Longan's (1986) hypothesis that processes underlying simple arithmetic are partially autonomous.*

Key words: Cognitive arithmetic, associative confusions, automatic spreading activation.
Mots clés : Arithmétique cognitive, confusions associatives, activation automatique.

When adults are asked to judge simple arithmetic problems (e.g., 8 + 4 = 12, True? False?), confusion problems (e.g., 8 + 4 = 32) produce slower reaction times than other incorrect problems (e.g., 8 + 4 = 13). The main purpose of the present paper was to investigate further the confusion effect observed in previous works (Findlay, 1978; Stazyk, Ashcraft, & Hamann, 1982; Winkelman & Schmidt, 1974; Zbrodoff, 1979; Zbrodoff & Logan, 1986). This very important phenomenon supports models that emphasize associative learning rather than computational learning. Although some theorists have argued that adults' performance on basic arithmetic problems relies extensively on the use of rule-based procedures (e.g., Baroody, 1985, 1987; Svenson, 1985), other problems have claimed that the procedural methods used during initial acquisition are largely replaced by direct retrieval from semantic memory (Abdi, 1986; Ashcraft, 1982, 1987; Campbell & Graham, 1985; Koshmider & Ashcraft, 1991; Siegler & Shrager, 1984; see Cornet, Seron, Deloche, & Lories, 1988; Fayol, 1990; Lemaire & Bernoussi, in press; McCloskey, Harley, & Sokol, 1991, for recent syntheses).

The role of retrieval processes in elementary mathematics is clearly demonstrated by evidence that performance on the basic combinations shows a substantial influence of associative confusions (Campbell, 1987 a & b; Campbell & Graham, 1985; Hamann & Ashcraft, 1985; Siegler, 1988; Siegler & Shrager, 1984; Stazyk, Ashcraft, & Hamann, 1982; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). For example, Campbell and Graham (1985) found that most errors in simple multiplications made by children and adults (89% for adults) involved tabled answers that are correct products for other simple multiplication problems, usually within the same multiplication tables as the correct answer (e.g., 3 x 7 = 24 or 3 x 7 = 28). Campbell (1985) found a comparable pattern in the errors made by adults on simple division problems. These systematic patterns suggest that simple arithmetic errors often result fron incorrect retrieval or associatively related answers. In Campbell and Graham's (1985) model, the error patterns reflect an associative-network structure. In this network, each problem is linked to a few candidate answers and each answer is linked to many different problems.

An associative model is also proposed by Winkelman and Schmidt (1974) to account for associative confusions between addition problems and multiplication problems. The authors assume that there exist associations between pairs of digits and both their sums and products. For example, the digit pair (3,4) would have associations with both 7 and 12. The obvious prediction is that stimuli of the type 3 + 4 = 12 and 3 x 4 = 7 will produce an increase in reaction times. To test this prediction, Winkelman and Schmidt asked their subjects to judge addition and multiplication problems. The problems were presented either with corrector incorrect responses. The incorrect responses were non-confusion responses (e.g., 3 + 5 = 25) or correct responses to the other operation (e.g., 3 + 5 = 15). The authors reported that confusion problems produced slower reaction times and more errors than non-confusion problems. This confusion effect was observed when subjects were presented with arithmetic problems both in a "mixed condition" (i.e., addition and multiplication problems are randomly presented in the same test) and in a "pure condition" (in which subjects were presented with either addition or multiplication problems in one test). These results suggest that competing associations between additive and multiplicative facts would exist in long-term memory and that these competing associations would be activated. To state their response, subjects would have to inhibit the inappropriate responses. Therefore, reaction times and the number of errors would increase.

Stazyk, Aschraft, and Hamann (1982) proposed an alternative explanation of this confusion effect. Reaction times may have been elevated in the confusion condition not because of competing memory associations but, instead, because of perceptual or encoding confusions. That is, subjects may have simply misperceived the arithmetic operator sign on these confusion problems, and thus giving spurious longer reaction time. Therefore, the authors investigated the confusion effect within multiplication only. In their experiments, confusion products were answers that were multiples of one problem's multiplier. They also reported a confusion effect, and interpreted this result as a dismissal of the "perceptual confusion" hypothesis.

Additionally, Zbrodoff and Logan (1986) noted two important points in Winkelman and Schmidt's (1974) investigation. First, a restricted series of stimuli was used (i.e., 3 + 3; 4 + 3; 3 + 5; 5 + 5 and 3 x 3; 4 x 3; 3 x 5; 5 x 5). Second, a within-subject comparison was performed between pure and mixed conditions. Zbrodoff and Logan hypothesized that the confusion effect might have been due to the subjects being surprised upon seeing a confusion problem and not due to the time needed to inhibit incorrectly activated answers. Zbrodoff and Logan explored the possibility that arithmetic involved autonomous processes. They defined a process as autonomous "... if it can (a) begin without intention, triggered by the presence of a relevant stimulus in the task environment, and (b) run on to completion ballistically once it begins, whether or not it is intended" (Zbrodoff & Logan, 1986, p. 118). In a series of experiments (Experiments 1 to 4), they tested the associative confusion effect. They used all pairs of digits from 1 to 9 (excepted 2 and 2) as stimuli. To avoid encoding perception bias, they also considered "mixed-pure presentation" as a between-subjects factor. Additionally, they manipulated the salience of "confusion", knowing that, in Stroop-like paradigms, such a factor has an important effect (Logan, 1980; Logan & Zbrodoff, 1979). For half of the subjects, 80% of false items contained a confusion answer. For the other group of subjects, only 20%

of the false answers were confusion answers. These different manipulations enabled them to observe two important phenomena. First, when subjects were presented with problems in a mixed condition, the confusion effect was strong, but when subjects were presented with problems in a pure condition, the confusion effect was weaker. Second, the confusion effect was more important with multiplication problems than with addition problems. Finally, on one third of the trials in a simple arithmetic equation verification task, subjects were asked to inhibit their response to the equation at an auditory stop signal (Experiments 5 and 6). On the average, subjects responded on 30.5% of the stop signal trials and inhibited their responses on 69.5%. As subjects were able to inhibit their overt responses to the arithmetic verification task on 69.5% of the stop signal trials, the hypothesis that the underlying processes are completely autonomous was disconfirmed. But the fact that subjects showed an associative confusion effect ruled out the possibility that arithmetic processes are not at all autonomous. Therefore, Zbrodoff and Logan (1986) concluded that the processes underlying simple arithmetic can be partially autonomous.

The partially autonomous perspective makes a number of predictions, some of which are tested in the present paper. Specifically, the hypothesis that the processes underlying simple arithmetic are partially autonomous predicts that, if the presentation of arguments (e.g., 8 + 4 =) and the presentation of the answer (e.g., 32) are sufficiently delayed), the confusion effect should disappear. Indeed, subjects would have the time to select the correct answer and not confuse it with a related answer. Therefore, it should be possible to show an inhibition of the confusion effect.

EXPERIMENT 1

Experiment 1 was suggested by Zbrodoff and Logan's (1986) hypothesis that the processes underlying simple arithmetic are partially autonomous. If this is so, then it should be possible to show an inhibition of the confusion effect. Specifically, if the presentation of the arguments (e.g., 8 + 4) and the presentation of the answer (e.g., 32) are sufficiently delayed, the confusion effect should disappear because subjects would have the time to select the correct answer.

Method

Subjects. Thirty-six introductory psychology students (15 males and 21 females) received a class credit to participate in the experiment. Mean age of the subjects was 21.2 years, ranging from 18.3 to 29.2.

Stimuli and apparatus. Stimuli were generated and presented using a SAMSUNG PC-compatible computer. Subjects' responses were recorded by assigning one key on the keyboard as "true" (`*M*' key) and one key as false (`*Q*' key). The stimuli were equations representing all operations from 2 + 3 to 9 + 9. "Zero-time" and "one-time" problems were not tested because these problems are probably solved by retrieval of rules, such as $N \times 0 = 0$, $N \times 1 = N$ (e.g., Ashcraft, 1982; Baroody, 1985) as opposed to direct retrieval of an answer. When operand order is ignored (i.e., when for example, 3 x 4 and 4 x 3 are considered as the same problem), there are 35 arithmetic problems. For both addition and multiplication problems, 35 true problems were formed, such that *c* was the sum of *a* and *b* (for addition problems) or the product of a and b (for multiplication problems). True problems were presented twice so that the number of true answers and false answers was the same. Thirty-five confusion problems were formed such that *c* was the product of *a* and *b* for addition problems (e.g., 3 + 5 = 15) and the sum for multiplication problems (e.g., 3 x 5 = 8). Finally, seventy non-confusion problems were formed, such that *c* was neither the sum nor the product of a and b (e.g., 3 + 4 = 8, 3 x 4 = 11). Following Zbrodoff and Logan (1986), in both addition and multiplication problems, the *c* term for the false problems was chosen to match the difference or split between the left and right sides of the confusion problems. This was necessary because some confusion problems had large splits (e.g., 9 + 9 = 81, 9 x 9 = 18), and the split is known to affect reaction times in arithmetic inequality judgments (Moyer & Landauer, 1967; Restle, 1970) as well as in false responses in arithmetic verification tasks (Ashcraft & Battaglia, 1978). By equating the mean split in the set of non-confusion problems with the mean split in the confusion problems, a potential confound has been removed.

Procedure. Stimuli were presented in a line form (e.g., $a \times b = c$ or $a + b = c$) in the center of the computer screen. A total of 280 trials was required to complete the design. The order of operations and answers was randomized separately for each subject, with the restriction that no more than four trials requiring the same response occurred consecutively. Each trial began with a 750 ms ready signal (a line of five `a' letters) that appeared in the center of the screen. Then a problem and a solution appeared, arranged so that the entire equation would be centered on the screen. The equation was displayed horizontally. Each equation included the two arguments, the operation symbol (+ or x), an equal sign, and the answer. The argument display included one argument, a space, the operation symbol, a space, the second argument, a space, the equal symbol, a space, and the answer. Each equation occupied seven or eight character spaces on the screen, depending on whether the answer involved one or two digits.

We used four different delays between arguments and answer: 0, 100, 300, and 500 ms. The 0 ms delay mimicked standard verification tasks in that arguments and answer appeared simultaneously. The 500 ms delay was intended to provide the subjects with enough time to retrieve the true answer before the presentation of the putative answer. We chose two short delays (0 and 100 ms) and two long delays (300 and 500 ms), because LeFevre, Bisanz, and Mrkonijc (1988) reported that two digits facilitated the activation of their sum at short delays (less than 180 ms).

Subjects were individually tested. The instructions began by describing the events in a trial. In the 100, 300, and 500 ms delay condition groups, subjects were told that the arguments would appear before the answer. Subjects were instructed to respond "true" or "false" as quickly and accurately as possible after the answer appeared. Half the subjects pressed the "true" answer key with their left hand and the "false" answer key with their right hand and the other half of the subjects did the opposite. Subjects were told to rest the index fingers of their right and left hands on the keys throughout the experiment in order to be able to respond as quickly as possible. Each session lasted approximately twenty to thirty minutes.

Before the experimental trials, subjects were given 20 practice problems to familiarize them with the apparatus and procedure. Subjects were reminded of the instructions after their practice trials.

Results and discussion

Trials on which the subjects made errors were dropped from analyses. Median correct latencies for the no items were analyzed in a 4 (delay: 0, 100, 300, and 500 ms) x 2 (operation: addition and multiplication) x 2 (answer: non-confusion, confusion)[1] analysis of variance (ANOVA) with repeated measures on the last two factors. Error rates were too low (4.2%) to be analyzed statistically but showed no evidence of speed-accuracy trade-off that would compromise the interpretation of the results.

As can be seen in Figure 1, the major experimental manipulations were successful. The mean median latencies for delays of 0, 100, 300, and 500 ms were 1241, 1278, 720, and 573 ms, respectively, indicating a significant effect [$F(3,32) = 34.47$, $p < .01$, $MSe = 1890838$]. Addition problems showed a significant shorter latency (950 ms) than multiplication problems (976 ms) [$F(1,32)$

1. As expected, in each of the present experiments, latencies for the true conditions were faster than for the false conditions. Because data from these control conditions do not bear on the hypotheses in any other way, they are not discussed further.

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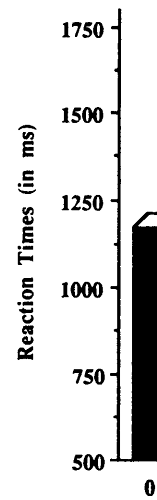

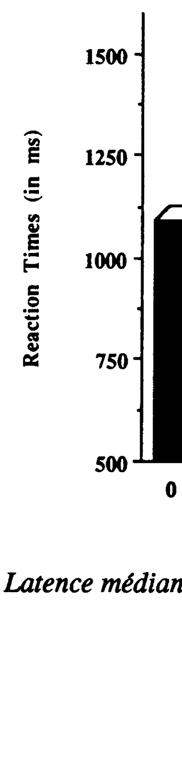

Figure 1. Latence médian

= 8.96, $p<.01$, MSe = 2682]. Non-confusion problems were verified 77 ms faster on the average than confusion problems [$F(1,32)$ = 35.36, $p<.01$, MSe = 6462], showing the standard confusion effect.

Figure 1. Mean median latencies (in ms) as a function of answer and delay, in adults.

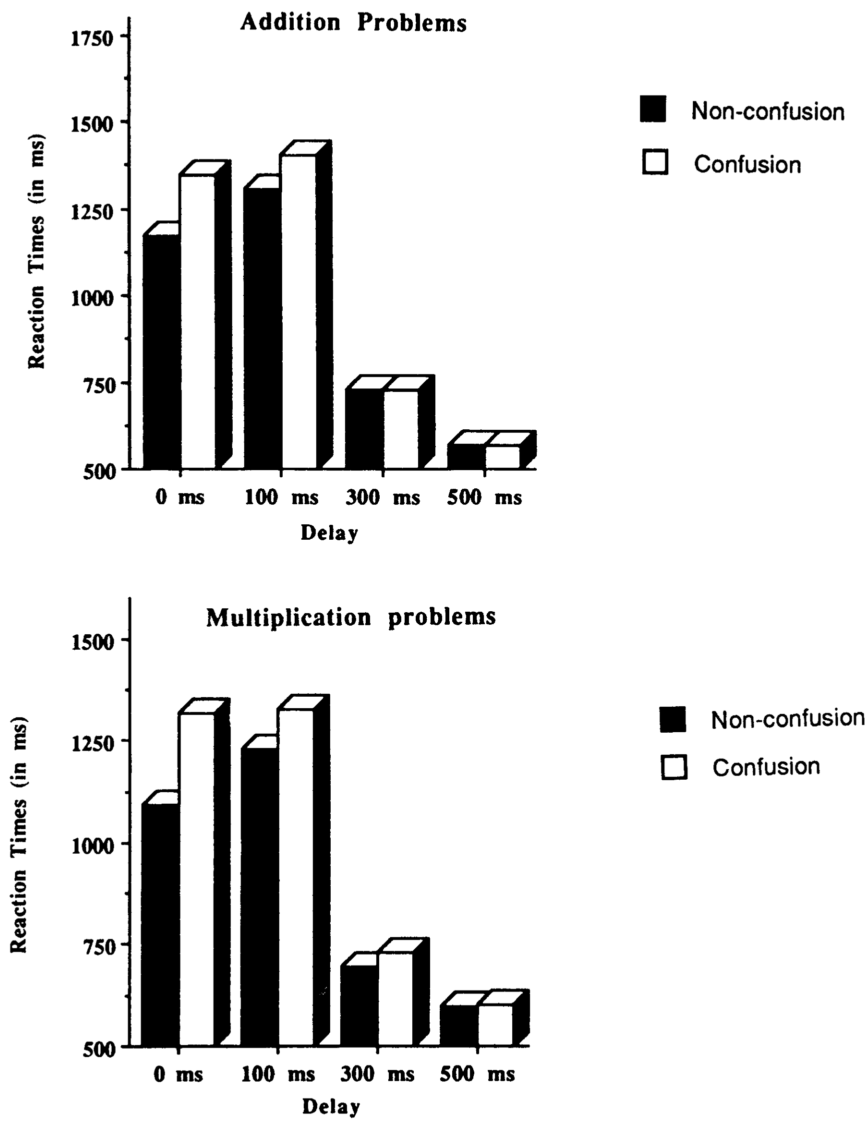

Figure 1. Latence médiane en fonction des réponses et des délais chez les adultes.

More importantly, several interactions among factors were significant. The delay x operation interaction effect [$F(3,32) = 7.68$, $p<.01$, $MSe = 2682$], and the delay x answer interaction effect [$F(3,32) = 11.96$, $p<.01$, $MSe = 3132$] were further explored in separate ANOVAS performed for each delay, involving a 2 (operation) x 2 (answer) design with repeated measures on each factor.

For 0 ms delay, adults showed a significant difference in mean median latencies between addition problems (1214 ms) and multiplication problems (1267 ms) [$F(1,8) = 11.20$, $p<.02$, $MSe = 2233$], as well as a significant difference between non-confusion problems (1149 ms) and confusion problems (1332 ms) [$F(1,8) = 31.82$, $p<.01$, $MSe = 3336$]. For 100 ms delay, the operation effect [$F(1,8) = 15.83$, $p<.01$, and the confusion effect [$F(1,8) = 9.91$, $p<.02$] proved to be significant. For 300 ms delay, no operation or answer effect proved to be significant.

Consistent with our hypothesis, the present results showed a confusion effect with both addition and multiplication problems for 0 ms and 100 ms delays. The confusion effect was not observed with 300 or 500 ms delays. The fact that latencies decreased with delay suggests that, at longer delays, subjects had the time to retrieve the answer of the problems. When the putative answer appeared, they only had to compare the retrieved answer with the presented answer, and to press the appropriate key on the keyboard. However, latencies at 0 ms delay are shorter than latencies at 100 ms delay, although the difference is not significant ($F<1$). These shorter latencies may be due to the fact that subjects waited to see the answer before retrieving the solution, instead of trying to retrieve the solution as soon as they were presented with the arguments, yielding a spurious long latency.

These results are consistent with LeFevre, Bisanz, and Mrkonijc's (1988) results that showed a largest confusion effect at stimulus onset asynchronies of less than 180 ms between the pair of number and the probe, in a single matching task. In a verification task, it is important that the confusion effect disappeared at longer delays. Indeed, when presented with arguments with a sufficient long delay before the answer, subjects could inhibit incorrect activated candidates, select the correct answer, and did not show any confusion effect. The pattern of results in our experiment is congruent with Zbrodoff and Logan's (1986) hypothesis that processes underlying simple arithmetic are partially autonomous. However, Zbrodoff and Logan also concluded that autonomy should be construed as a continuous dimension rather than a dichotomous one. Because studies on children's addition and multiplication reported different rates of the retrieval of arithmetic facts across ages (Ashcraft & Fierman, 1982; Hamann & Ashcraft, 1985; Koshmider & Ashcraft, 1991), testing children should bring further evidence to the "continuous dimension hypothesis". Indeed, do children

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show inhibition of the confusion effect? If they do, with which delay? This issue is addressed in Experiment 2.

EXPERIMENT 2

Experiment 2 tested the hypothesis that autonomy is a continuous dimension. This hypothesis predicts a different inhibition of the confusion effect in children of different ages and at different delays. Indeed, as children of different ages have a different mastery of arithmetic facts (Ashcraft, 1987; Ashcraft & Fierman, 1982; Cooney, Swanson, & Ladd, 1988; Svenson & Sjoberg, 1982, 1983), a different inhibition of the confusion effect should be observed at different delays.

To test this prediction, fifth and fourth grade children were presented with simple arithmetic problems to verify as in Experiment 1. Fourth- and fifth-graders served as subjects in our experiment, because third-graders did not show any confusion effect in Experiment 1. Each subject was randomly presented with simple arithmetic problems with three delay conditions (0, 300, and 500 ms). The hypothesis that autonomy is a continuous dimension predicts a larger inhibition of the confusion effect with older children. Indeed, as children of different ages have a different mastery of arithmetic facts (Ashcraft, 1987; Ashcraft & Fierman, 1982; Cooney, Ladd, & Swanson, 1988; Svenson & Sjoberg, 1982, 1983), a different inhibition of the confusion effect should be observed at different delays.

Method

Subjects. Forty-two elementary school students, 21 each from grades 4 and 5, were randomly selected in a French upper-class urban public school (respective mean ages: 10.2 and 9.3 years old). All children attended a regular-class.

Stimuli and apparatus. The apparatus was the same as used in Experiment 1. However, as subjects were presented with the three delay conditions (0, 300, and 500 ms), the session would have been too long for children if all combinations of digits 1 through 9 would have been presented. Therefore, problems were chosen within tables from 1 to 9. These problems were also chosen to control a possible confounding size effect consistent of an increase in latency with the size of problems (Ashcraft & Stazyk, 1981; Campbell, 1987 a & b; Campbell & Graham, 1985; Geary, Widaman, & Little, 1986; Groen & Parkman, 1972; Miller, Perlmutter, & Keating, 1984; Zbrodoff & Logan, 1986). Following Zbrodoff and Logan (1986), argument magnitude was controlled by choosing problems into three different levels: four problems in which both arguments were 5 or smaller, four problems in which both arguments were 6 or larger, and four problems in which one argument was 5 or smaller and the other was 6 or larger. Problems in which the arguments are identical (e.g., 2 + 2, 3 x 3) were excluded, because they often show no any problem size effect (Ashcraft & Battaglia, 1978; Groen & Parkman, 1972).

We chose three delay conditions (0, 300, and 500 ms). The delay factor was a within-factor. Each subject was randomly presented with the three delay conditions. In a verification task, it is typically assumed that upon presentation of the arguments, a candidate set of nodes is activated that includes the two numbers from the arguments and related correct or incorrect nodes, such as the sum or the product (Ashcraft, 1982, 1983, 1987; LeFevre et al., 1988). Moreover, Experiment 1 did not show any difference in the confusion effects with 0 and 100 ms delays. Therefore, a 0 ms delay was chosen for the control condition in children, thus mimicking standard verification task.

Procedure. As with adults, the stimuli were presented in a line form (i.e., $a \times b = c$ or $a + b = c$). Each subject was presented with two operations (addi-tion, multiplication), true and false problems, and three delays (0, 300, and 500 ms), giving a total of 278 equations to verify. Stimuli were randomly ordered with the restriction that no more than four trials requiring the same response occurred consecutively. True problems were presented twice for the number of true answers and that of false answers to be the same.

Results and discussion

Trials on which the subjects made errors were dropped from analyses. Median correct latencies for the false items were analyzed in a 2 (grade: fifth, fourth) x 3 (delay: 0, 300, and 500 ms) x 2 (operation: addition, multiplication) x 2 (answer: non-confusion, confusion) ANOVA with repeated measures on the last three factors. Error rates were too low (5.8%) to be analyzed statistically but showed no evidence of a speed-accuracy trade-off that would compromise the interpretation of the results.

Several significant main and interaction effects were apparent in the analysis. Specifically, fifth graders (1617 ms) were faster to reject false equations than fourth graders (1881 ms) [$F(1,40) = 28.75$, $p < .01$, $MSe = 1890838$]. This grade effect reflects the fact that older children encode problems and retrieve answers in memory more quickly than younger children. Like in adults, latencies decreased with delays [$F(2,80) = 196.71$, $p < .01$, $MSe = 420880$].

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Probably like in adults, children performed some part of the processing between the presentation of the argument and the presentation of the answer. When the putative answer was presented, they only had to compare the retrieved answer with the presented answer. Non-confusion problems (1679 ms) were more quickly rejected than confusion problems (1749 ms) [$F(1,40) = 18.85$, $p<.01$, $MSe = 41587$]. Moreover, interaction effects were observed with grade x delay [$F(2,80) = 44.93$, $p<.01$, $MSe = 420880$], with delay x answer [$F(2,80) = 10.47$, $p<.01$, $MSe = 52728$], with grade x delay x answer [$F(2,80) = 5.94$, $p<.01$, $MSe = 52728$], and finally with grade x delay x operation x answer [$F(2,80) = 3.21$, $p<.05$, $MSe = 59747$]. These interaction effects are further analyzed in separate ANOVAS performed in each grade. For both fourth graders and fifth graders, median correct latencies for the false items were analyzed with a 3 (delay) x 2 (operation) x 2 (answer) ANOVA with repeated measures on each factor.

For fifth grade children, results for non-confusion and confusion answers are displayed in Figure 2. Mean median latencies decreased linearly[2] with delays [$F(1,40) = 8.16$, $p<.01$, $MSe = 441849$, percentage of variance explained by the linear trend = .91], and confusion answers (1642 ms) were slower to be rejected than non-confusion answers (1593) [$F(1,20) = 4.04$, $p<.05$, $MSe = 40537$]. However, this confusion effect interacted with the delay factor [$F(2,40) = 16.88$, $p<.01$, $MSe = 39121$]. A second-order interaction effect was portrayed in a significant confusion effect only with addition problems at 0 ms delay [$F(1,40) = 51.16$, $p<.01$, $MSe = 36032$], and a non-significant confusion effect both with 300 ms and 500 ms delays. At 300 ms and 500 ms delays, no confusion effect was observed for either addition or multiplication problems.

Fof fourth grade children, Figure 3 depicts the results for non-confusion and confusion answers. Mean median latencies decreased linearly with delay [$F(1,40) = 18.14$, $p<.01$, $MSe = 361152$, percentage of variance explained by the linear trend = 83%], and confusion answers (1856) were slower to be rejected than non-confusion answers (1765 ms). However, this confusion effect interacted with the delay factor [$F(2,40) = 4.47$, $p = .01$, $MSe = 66614$]. Specifically, this confusion effect was significant only with multiplication problems [$F(2,40) = 4.99$, $p<.05$, $MSe = 79262$], with 0 ms delay and with 300 ms delay [$F(1,40) = 4.83$, $p<.05$, $MSe = 79262$], but not with 500 ms delay.

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2. Trend analyses in each of the present experiments have been assessed by polynomial trend analyses (Abdi, 1987, chap. 9).

Figure 2. Mean median latencies (in ms) as a function of answer and delay in fifth-grade children.

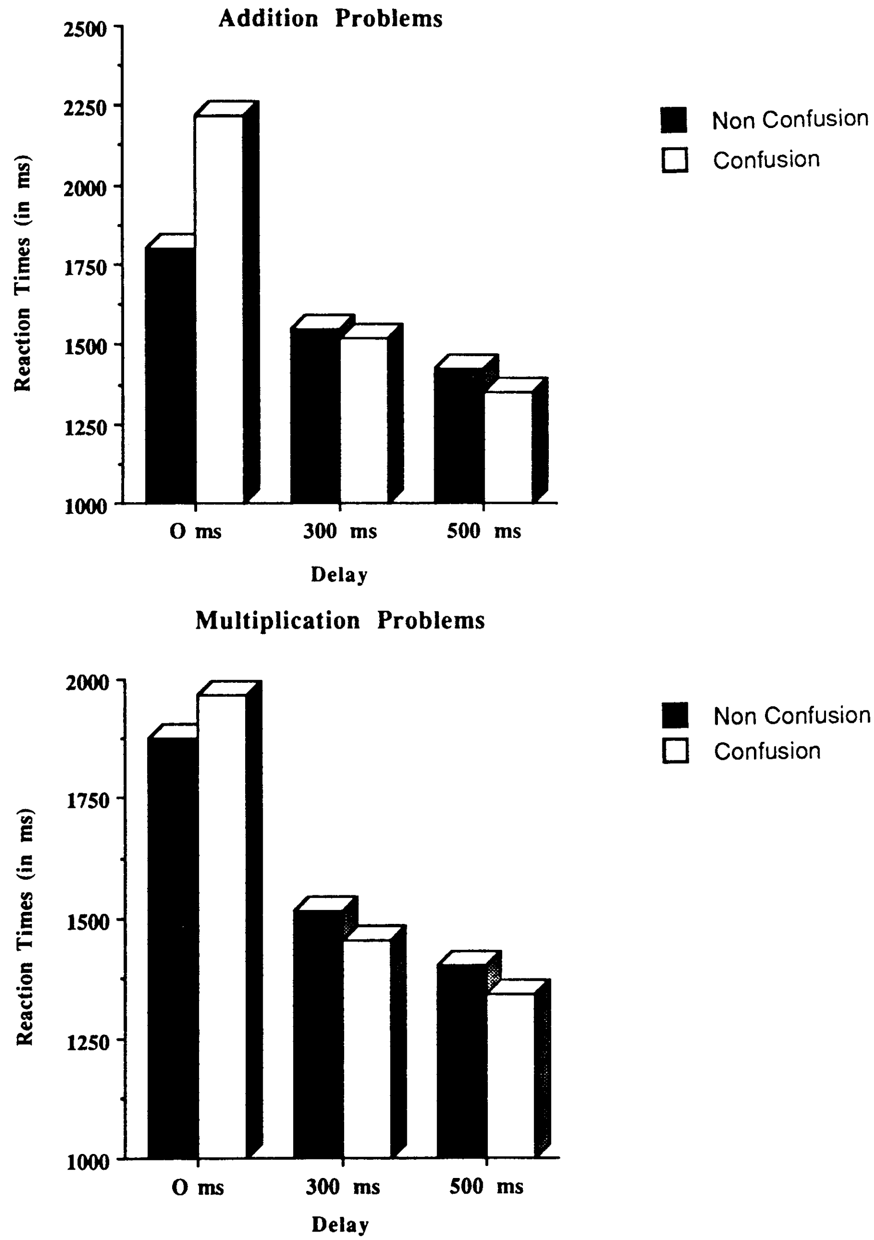

Figure 2. Latence médiane en fonction des réponses et des délais chez les enfants de CM2.

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Figure 3. Mean median latencies (in ms) as a function of answer and delay in fourth-grade children.

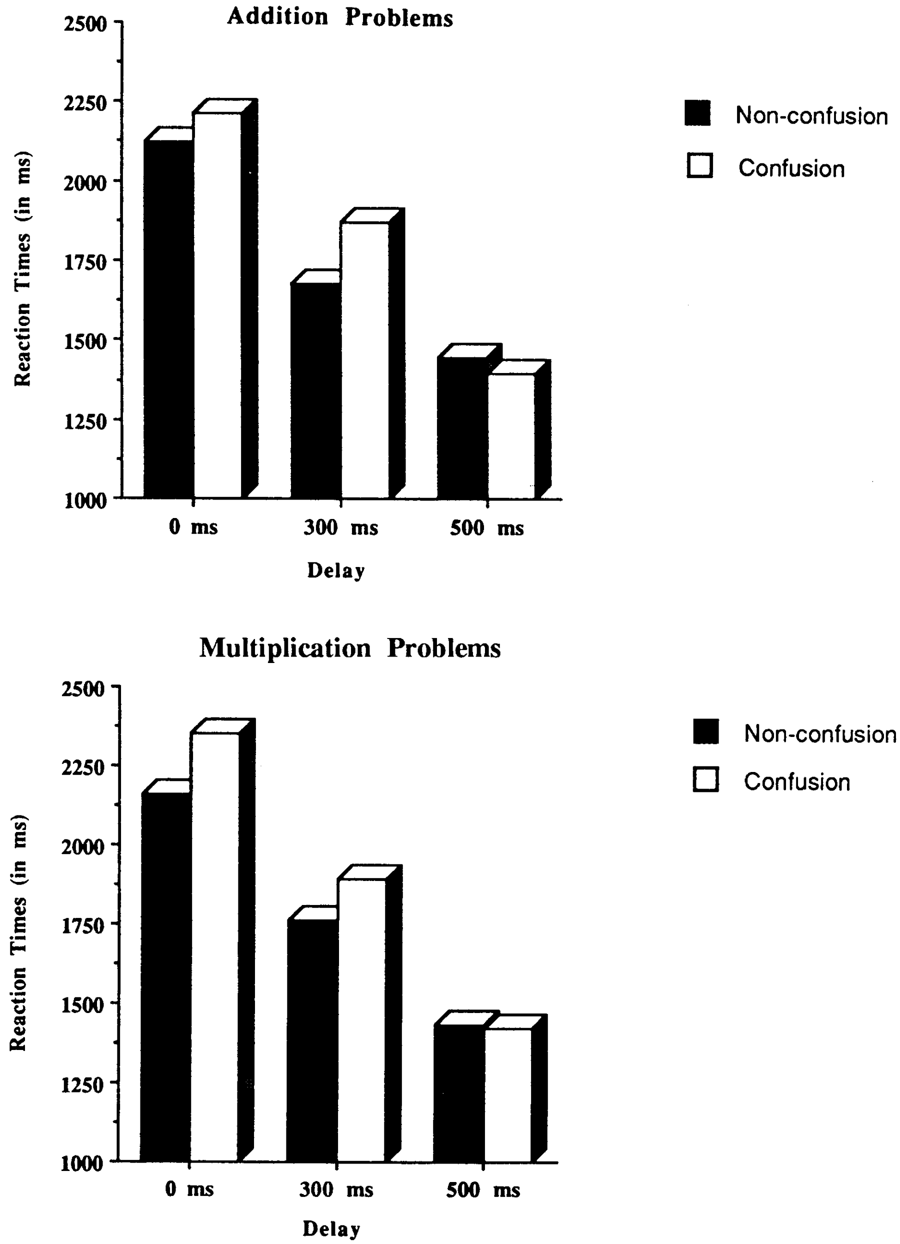

Figure 3. Latence médiane en fonction des réponses et des délais chez les enfants de CM1.

Consistent with our hypothesis, the largest inhibition of the confusion effect was observed with fifth grade as opposed to fourth grade children. The latter showed a confusion effect with one operation or the other with 0 and 300 ms delays. The former showed a confusion effect only with addition problems and only with 0 ms delay.

This inhibition of the confusion effect can be viewed in children exactly in the same fashion as in adults. Indeed, when presented with arguments with a sufficiently long delay before the answer, children could select the correct answer before the presentation of the putative answer, yielding an inhibition of the confusion effect.

A final result remains difficult to interpret. For fourth graders, the confusion effect proved to be significant only with addition multiplication problems with 0 ms delay and with addition problems with 300 ms delay. The fact that no confusion effect was significant with addition problems with 0 ms delay could be due either to an early inhibitory process that would have occurred on these basic facts or to a slower activation of the product. For fourth graders, the activation phase for multiplication facts could be longer than that of addition facts. Consequently, 500 ms delay would be required for these incorrect multiplication facts to be inhibited. Following this reasoning, addition facts would be retrieved at 0 ms delay for fourth graders and inhibited at both 300 and 500 ms. The fact that older children, with a degree of mastery of arithmetic knowledge, showed a confusion effect with multiplication facts at 0 ms delay is an argument for this interpretation.

GENERAL DISCUSSION

The experiments were designed to test the hypothesis that the processes underlying simple arithmetic are partially autonomous with a confusion effect. That hypothesis predicts an inhibition of the confusion effect when a delay is imposed between the arguments and answers. The delay should enable subjects to inhibit incorrectly activated answers and select the correct answer. The experiments provided evidence for that hypothesis. In Experiment 1, adults showed an inhibition of the confusion effect at 300 and 500 ms delays. Experiment 2 was designed to test the hypothesis that autonomy is a continuous dimension. That hypothesis predicts a different inhibition of the confusion effect in children of different age and at different delays. The patterns of results in Experiment 2 bring some support for this hypothesis. In fifth graders, an inhibition of the confusion effect with 300 and 500 ms was observed, whereas such an inhibition was observed only with 500 ms delay, in fourth graders.

The results reported in this paper are consistent with the view that arithmetic knowledge is a highly connected network of associations (Ashcraft, 1983, 1987; Campbell, 1987 a & b; Campbell & Graham, 1985). According to associative models of cognitive arithmetic, arithmetic facts are accessed upon presentation of arguments (Ashcraft, 1983, 1987). Activation spreads from the presented numbers to related nodes such as the sum. This retrieval process is known to be the most important in adults and from fourth grade in children. Younger children mainly resort to a counting strategy to solve simple arithmetic problems. The finding that, from fourth graders, children showed a confusion effect with one or both operations is consistent with this view. One consequence of the present results is that they enable us to define the conditions for a confusion effect to appear. These conditions involve at least that (a) the access of memory network is sufficiently automatic; (b) the memory network is sufficiently organized to enable an automatic access of the solution. The present results suggest that fourth grade is a minimum grade for these constraints to be satisfied. Indeed, third grade is a transitional grade with respect to memory structure and memory access (Ashcraft & Fierman, 1982; Cooney et al., 1988; Svenson & Sjoberg, 1982, 1983). Fourth and fifth grades seem to be appropriate grades for arithmetic facts to be sufficiently mastered, and so that a confusion effect to be observed.

The results reported in this paper are also consistent with the view that arithmetic processes are not completely autonomous (Zbrodoff & Logan, 1986) by showing that the confusion effect could be inhibited. Arithmetic processes could be concluded to be partially autonomous. The present results showed that the confusion effect was inhibited at shorter delay in older children. This different inhibition of the confusion effect with respect to grade could be assumed to reflect differences in strength of problem-answer associations in memory, due to differential mastery of basic arithmetic combinations. Children with strong problem-answer associations can select quickly the appropriate answer, and consequently can reject more quickly confusion problems. More generally, it raises the question of interaction of strategies and mental representations (Siegler, 1986, 1987).

Identification of the confusion effect and the time course of its inhibition may prove to be useful for gaining a more complete understanding of how arithmetic knowledge is stored and used. A measure of the confusion effect and of its inhibition may serve to evaluate the strength of association between arguments and candidate answers which in turn are critical for determining selection of strategies on arithmetic tasks (Siegler, 1986, 1988; Siegler & Shrager, 1984). The extent of the confusion effect and its inhibition might prove to be a useful source of individual differences and developmental changes. This means that the measurement of the confusion effect and its inhibition is potentially valuable for determining how arithmetic is used.

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RESUME

Lorsqu'on demande à des adultes de vérifier des problèmes arithmétiques simples (e.g., 8 + 4 = 12. Vrai ? Faux ?), les problèmes interférents (e.g., 8 + 4 = 32) entraînent un temps de réaction plus long que des problèmes faux comparables (e.g., 8 + 4 = 13). Cet effet de confusion a été interprété comme le résultat de confusions associatives en mémoire entre les produits et les sommes. L'objectif des expériences rapportées dans cet article était de déterminer si, en introduisant un délai entre l'opération et la réponse, l'effet de confusion était toujours présent. On a donc présenté des opérations associées à plusieurs solutions à des adultes et à des enfants (9 et 10 ans). Les sujets voyaient les opérations et les solutions soit en même temps, soit séparées par un bref délai (0, 100, 300 et 500 ms). Aux délais de 0 et 100 ms, l'effet de confusion était présent chez les adultes, mais non à 300 et 500 ms. L'effet de confusion disparaît à 300 ms chez les enfants de 10 ans et à 500 ms chez les enfants de 9 ans. Ces résultats sont en accord avec l'hypothèse de Zbrodoff et Logan (1986) selon laquelle les processus impliqués dans le calcul mental simple seraient partiellement autonomes.

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Abstract. *Herman W*
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