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The Role of Working Memory Resources in Simple Cognitive Arithmetic

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Two experiments tested the hypothesis that simple arithmetic requires working memory resources. Subjects were presented a simple verification task (e.g., $8 + 4 = 12$. True? False?) with (or without) secondary tasks. We varied the difficulty of the problems (i.e., easy *vs* hard problems) and the potential for inducing associative confusion (e.g., $8 \times 4 = 12$). Secondary tasks were chosen so as to overload the phonological loop and the central executive of the working memory system. We found greater disruption of performance on true problems when both the phonological loop and the central executive were overloaded, and greater disruption of performance on false problems when the central executive system was overloaded. This pattern of results is consistent with the working-memory resource hypothesis and suggests that the central executive is a critical system involved in simple arithmetic. Finally, the results of the present study on both true and false problems and their implications for cognitive arithmetic theories are discussed.

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1. Introduction

The ability to correctly solve simple arithmetic problems such as $8+4=?$ is one of the basic skills acquired during elementary school. A number of studies have investigated the representations of arithmetic facts in memory and the cognitive processes that underlie aspects of mental arithmetic. Two major determinants of subjects' performance have been advanced, namely the memory organization of simple arithmetic facts and working-memory resources. The role of the representation of arithmetic facts in long-term memory (LTM) has been largely documented in both adults and children. Much less is known, however, about the role of working-memory resources in simple arithmetic. The purpose of the present paper is to further investigate the role of working-memory resources in simple arithmetic.

First, we briefly review the main assumptions of attentional resource theories and current models of simple arithmetic. Then, we report two experiments that examined whether or not simple arithmetic is affected by varying memory demands. The experimental results provide further insight on the way in which working memory resources play a critical role in simple mental arithmetic. Finally, we discuss the implications of these results for formal models of simple arithmetic.

1.1. *Attentional resource theories.*

1.1.1. *Single-capacity versus multiple-resource theories.* Over the past two decades, cognitive psychologists have made the distinction between automatic and control processes (e.g., Logan, 1985, 1988; Navon & Gopher, 1979; Posner & Snyder, 1975; Shiffrin & Schneider, 1977). Briefly, automatic processes are fast, effortless, autonomous, and do not interfere with other processes, whereas controlled processes are effortful, slow, and rely on attentional resources. The nature of attentional resources subserving cognitive processes has been the subject of a long debate.

Two types of theories have been proposed regarding the number of attentional capacities involved in cognitive tasks, that is single-capacity and multiple-resource theories. The strengths and weaknesses of each of these approaches have been extensively discussed in the literature (see Just & Carpenter, 1992; Logan, 1979, 1980, 1985; Navon, 1984; Navon & Gopher, 1979, for extensive discussions). Therefore, we recall briefly the main assumptions of each perspective here. The idea behind single-resource theories (e.g., McLeod, 1977a and b; Posner & Boies, 1971; Posner & Klein, 1973; Kahneman, 1973; Kahneman & Treisman, 1984) is that performance in general depends on the availability of resources: the more information to be processed in a task, the more resources are needed. In situations where subjects are faced with a concurrent task, the amount of resources is reduced and performance decreases, as shown by an increase in reaction time (RT). This single-capacity approach, however, has been seriously challenged in recent years (e.g., Allport, 1980; Navon, 1984; Navon & Gopher, 1979; Wickens, 1980, 1984) and most resource theorists have adopted a multiple-resource perspective. They argue that performance is limited by several different resources, as evidenced by the effect of input, output, and central processing modalities rather than by the amount of information to be processed. One problem with mutiple-resource approaches, however, has been to characterize the nature of the resources a particular task is expected to use, a burden that is possible only with a theoretical framework that enables this level of specification. We believe that Baddeley's theory of working memory is an example of such a framework.

1.1.2. *An example of multiple-resource theories.* Baddeley's theory of working memory can be viewed as a particular instance of a multiple-resource approach. Working memory refers to the temporary storage and processing of information in a variety of cognitive tasks (Baddeley, 1986, 1990; Baddeley & Hitch, 1974) and is thought to consist of a limited-capacity central executive and subsidiary slave systems. One of the functions of the central executive system would be to monitor the allocation of attentional resources during cognitive activities. Two slave systems have been suggested: a phonological-articulatory loop that stores and manipulates speech-based material and a visuo-spatial sketch pad that is responsible for storage and manipulation of visuo-spatial material. These characteristics make it a very useful conceptual framework within which to study attentional processes that coordinate and control performance in cognitive tasks like simple arithmetic (e.g., Logie & Baddeley, 1987; Hitch, 1978 a and b). This framework also helps in characterizing futher the nature of the resources involved in subjects' performance and was adopted here to pintpoint what particular working-memory system, if any, is involved in simple arithmetic.

1.2. *A brief review of simple arithmetic.*

1.2.1. *Associative models of simple arithmetic.* The consensus appears to be that arithmetic facts are stored in an interrelated network in LTM (Ashcraft, 1982, 1983, 1987; Ashcraft & Fierman, 1982; Campbell, 1987 a and b; Campbell & Graham, 1985; Lemaire & Siegler, 1995; Miller, Perlmuter, & Keating, 1984; Siegler, 1988 a and b; Siegler & Shipley, 1995; Siegler & Shrager, 1984; Stazyk, Ashcraft, & Hamann, 1982; for recent syntheses see Ashcraft, 1992; Cornet, Seron, Deloche, & Lories, 1988; Lemaire & Bernoussi, 1991;

McCloskey, Harley, & Sokol, 1991). According to associative models, the presentation of an arithmetic problem (e.g., $3 + 4$) results in activation of the number nodes specified in the problem (e.g., 3 and 4). Activation spreads from these nodes along associative links so that related nodes, such as for the sum or product, are also activated.

Subsequent research has tested predictions of associative models in simple cognitive arithmetic with mainly two types of task, that is production (e.g., $8 + 4 =?$) and verification (e.g., $8 + 4 = 12$. True? False?). In both type of tasks, subjects retrieve a set of candidate answers via a spreading activation mechanism (Anderson, 1983; Collins & Loftus, 1975), select the appropriate answer candidate and/or compare the answer selected with the proposed answer. However, it has been shown that the effects of some factors reflect processes of retrieval or computation. For example, the size of the problem determines the difficulty of retrieving the appropriate answer in the memory network (e.g., Ashcraft & Battaglia, 1978; Campbell & Graham, 1985; Groen & Parkman, 1972; Stazyk et al., 1982). Here, difficult problems (e.g., $9 \times 8 = 72$) produce longer RTs and more errors than easy problems (e.g., $3 \times 4 = 12$). This problem difficulty effect typically relies on memory retrieval processes and reflects the time to access the solution into the memory network (see LeFevre, Sadesky, & Bisanz, in press, for a recent thorough discusion on this point).

Also consistent with predictions of associative models, several post-retrieval/computation effects have been reported in the cognitive arithmetic literature (e.g., Ashcraft, 1982, 1983, 1987, 1992; Ashcraft & Fierman, 1982; Campbell, 1987 a and b; Campbell & Graham, 1985; Lemaire & Fayol, 1995; Miller & Paredes, 1990; Miller, Perlmuter, & Keating, 1984). One of these effects is called the "associative confusion" effect or the "interference" effect and has been observed in both production and verification tasks. It consists in observing longer latencies and/or more errors for problems in which the proposed answer matches a correct answer to another problem or is correct under another operation (e.g., $8 \times 4 = 12$; $8 + 4 = 32$). For example, Campbell and Graham (1985) found that most errors in simple multiplication problems made by children and adults (89% for adults) involved table-related answers (see also Lemaire & Siegler, 1995). These are usually correct products to be found within the same multiplication table as correct answers for other simple multiplication problems (e.g., $3 \times 7 = 24$ or $3 \times 7 = 28$). Furthermore, Winkelman and Schmidt (1974) reported that associative confusion problems (e.g., $3 + 5 = 15$) produced slower rejection times than non-confusion problems (e.g., $3 + 5 = 25$), in a verification task. This confusion effect has been reported with fourth- and fifth- grade children by Lemaire, Fayol, and Abdi (1991) using a verification task as well as a number-matching task in both adults and children (LeFevre, Bisanz, & Mrkonijc, 1988; LeFevre, Kulak, & Bisanz, 1991; Lemaire, Barrett, Fayol, & Abdi, 1994). Typically, the associative confusion effect is interpreted as the effect of the automatic activation of interference problems on subjects' true/false decisions.

1.2.2. *Attentional resources in mental arithmetic.* Associative models of simple cognitive arithmetic have emphasized long-term memory structures and processes. However, it has recently pointed out that simple arithmetic may also involve some attentional resources (Ashcraft, Donley, Halas, & Vakali, 1992; Brainerd, 1983; Hitch, 1978 a and b; Kaye, 1986; Kaye, Bonnefil, & Chen, 1983; Kaye, deWinstanley, Chen, & Bonnefil, 1989; Logie, Gilhooly, & Wynn, 1994). For example, Hitch (1978a) reported four experiments which explored the role of information storage in working memory when one performs mental arithmetic. Hitch showed that (1) When solving aurally presented multi-digit problems such as 434 + 87, subjects reported breaking down the problems into series of elementary stages; (2) when subjects were asked to delay the output of individual partial results on calculation, interim information was forgotten if not utilized immediately; (3) forgetting the initial information was a source of errors and the forgetting increased as a function of the number of calculation stages. In addition, Brainerd (1983) showed that, in children, short-term memory encoding and retrieval developed more rapidly than the arithmetic processing.

Using a dual-task paradigm, two other studies, reported evidence that working memory is critical for simple arithmetic performance. Kaye et. al (1983; 1989) attempted to measure the attentional capacity left free during the different stages of verifying a two-term addition problem: the encoding of the addends, the computation or retrieval of the actual sum, the encoding of the stated answer, and finally the decision and response stages. Children in grades 2, 4, and 6, as well as college students were tested on a two-digit addition problem verification task. The concurrent task was the simple detection of an auditory probe interpolated at different intervals during the visual presentation of the addition problems. The authors found that differences in ability among middle elementary school children were correlated with attentional efficiency during the encoding and computation phases, whereas differences in ability among college students were correlated with attentional and response efficiency at the comparison and decision stages. These results are consistent with the idea that working memory resources are required to perform a simple arithmetic problem verification task. The dual-task paradigm has also been used by Logie and co-workers to show that articulatory suppression and unattended speech affect subjects' performance in simple counting tasks (Logie & Baddeley, 1987; see also Ashcraft et al., 1992; Healy & Nairne, 1985; Nairne & Healy, 1983) and in complex arithmetic tasks (Logie et al., 1994).

Finally, a recent report by Ashcraft et al. (1992) addressed directly the issue of the involvement of working memory resources in simple arithmetic. They discussed the possibility that retrieval of basic arithmetic facts consumes working memory resources. In an exploratory investigation Ashcraft et al. asked their subjects to verify simple and complex addition problems such as "$8 + 4 = 12$. True? False?" or "$24 + 23 = 47$. True? False?" Each of the problems appeared with its correct answer and also with an incorrect answer in three experimental conditions. In one condition, subjects saw a letter repeated four times on the screen. They were asked to repeat that letter out loud through the entire trial, at a rapid and constant rate (Repeat condition). In another condition, the subjects saw the same letter as in the repeat condition, but were required to generate words beginning with that letter, naming them out loud throughout the duration of the trial ("word generation" condition). Finally, in a third condition, the subjects saw four consonants, had to name them out loud, and then name them in correct alphabetical order ("alphabetization" condition). Ashcraft et al. also manipulated the problem difficulty variable, using problems considered low, medium, and high in term of difficulty. For simple problems, Ashcraft and co-workers' results showed an effect of working memory load conditions on latencies. Indeed, RTs in the repeat condition (1353 msec) were faster than either the alphabetization (1763 msec) or the word-generation (1798 msec) conditions. Although not significant, the memory load $\times$ problem difficulty interaction tended to show a greater effect of the secondary task on difficult problem latencies than on easy problem latencies. Ashcraft et al. discussed the lack of power of their working memory load manipulation. For example, they noted that subjects experienced difficulties in maintaining the rate of overt verbalization. In effect, subjects' verbalisations slowed noticeably during the word generation and alphabetisation tasks, a factor that was carefully controlled in the present experiments. Overall, the trends reported by Ashcraft et al. suggest that the retrieval of simple arithmetic facts involves attentional resources in working memory.

The approach adopted by Ashcraft et al. as well as by Logie et al. is interesting because it attempts to investigate directly the involvement of working memory resources in the area of simple cognitive arithmetic and to test what kind of attentional resources, if any, are relevant for arithmetic processing. Furthermore, this approach suggests that the use of the classical dual-task paradigm reported in Baddeley and Hitch (1974; see also Hitch, 1978 a and b;

Logie & Baddeley, 1987; Logie et al., 1994) may be very fruitful in investigating the role of working memory resources in simple mental arithmetic. Finally, Ashcraft et al.'s work suggests that the involvement of attentional resources may be tracked by the nature of the disruption of the arithmetic performance as a function of concurrent task. We adopted this approach in the present study.

1.3. *The present study.* The main purpose of the present experiments was to test the hypothesis that simple arithmetic requires working memory resources. Consistent with the logic underlying concurrent tasks (Logan, 1979; Ogden, Levine, & Eisner, 1980), we generally assume that the pool of working memory resources that may be devoted to an arithmetic task is limited and that competition for working memory resources from a secondary task will degrade performance to the extent that (1) the primary task requires working memory resources to be completed, (2) the total amount of working memory resources is limited and (3) the two tasks require more resources than are available within the working memory system.

In simple arithmetic verification tasks, subjects are presented with simple equations such as $8 + 4 = 12$ and must respond "true" or "false" by pressing appropriate keys. Typically, in this task, it is assumed that the operand encoding processes result in the activation of candidate answer (Ashcraft, 1987, 1992; Campbell, 1987 a and b; Lemaire & Fayol, 1995; Lemaire et al., 1991; 1994; Zbrodoff & Logan, 1986, 1990). Because true problems involve stronger operand-correct answer associations, RTs for these true problems are typically shorter than RTs for any false problems. Small true and false problems (e.g., $3 \times 4 = 12$) are more quickly verified than large problems (e.g., $9 \times 8 = 72$) and hence are seen as easier problems. These longer latencies for difficult problems can be interpreted by assuming that difficult problem/answer associations take longer to activate in LTM because the trace in memory of these associations is weaker than the small problem/answer associations. As suggested by the trends reported by Ashcraft et al. (1992), in dual-task conditions, the secondary task is expected to (1) increase latencies, and (2) interfere more strongly with difficult problems than with easy problems. It is clear that what should be disrupted by secondary tasks is the rate of processing involved in the primary task. That is, secondary tasks should add different amounts of time in easy and hard problems. A simple change in intercept would not evidence the working-memory hypothesis.

Within the false problems, associative confusion problems are correct under other operations (e.g., $8 + 4 = 32$). Therefore, compared with non-associative confusion problems (e.g., $8 + 4 = 13$), associative confusion problems require additional processing to be rejected. Indeed, confusion problems are judged plausible correct problems because of associative relations between operands and confusion answers. Subsequently, the cognitive system needs to process these activated confusion problems furthern in order to reject them. This further processing takes time and accounts for the fact that confusion problems take longer to be rejected than comparable false problems (Lemaire et al., 1991; 1994; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). In the working memory overload condition, if the dual task prevents the further processing of associative candidates, then subjects must still think that these associative candidates are true. Consequently, they should respond "true" and produce more errors than in a standard condition. However, this effect may not be reflected in RT since an increase in RT generally follows overlapping-task experiments, and an increase in RT may absorb any confusion/non-confusion latency differences.

In summary, the present experiments investigated the effect of concurrent tasks on arithmetic problems verification. The working memory demand hypothesis predicts that (1) the impact of the secondary task would be greater on difficult problems, and (2) the associative confusion effect should be strengthened in working memory overload situations, at least in accuracy measures. These two predictions were tested because of the possibility that the retrieval or decision processes (or both) in cognitive arithmetic require attentional resources in working memory. Therefore, by showing that the difference between easy/difficult problems varies with memory load and/or that the associative confusion effect can be increased by working memory overload would demonstrate the role of working memory in simple cognitive arithmetic.

2. **Experiment 1**

In the present experiments, we tested the effect of problem difficulty and associative confusion in different memory load conditions with both addition and multiplication problems. Undergraduates verified simple addition and multiplication problems that were presented in mixed blocks of trials. We chose to intermix these problems because Zbrodoff and Logan's (1986; see also Lemaire et al., 1991; 1994) findings suggested that the associative confusion effect is heightened under these conditions.

Furthermore, to investigate the possibility that one of the working memory components is very important in the verification of simple arithmetic problems, we employed four experimental conditions. In the control condition, subjects were presented a standard verification task with no concurrent task. In the first "articulatory suppression" condition, subjects were required to repeatedly say "the" (*le* in French), while performing the verification task. In the second articulatory suppression task ("canonical letters"), subjects were asked to verbally repeated "abcdef". Finally, in the last experimental condition ("random letters"), subjects were asked to generate without stopping a random series of letters within the "abcdef" set. The two articulatory suppression conditions were used to determine whether articulatory suppression has an effect on the verification of simple arithmetic problems, just as it has an effect on counting (Logie & Baddeley, 1987; Healy & Nairne, 1985; Nairne & Healy, 1983) and on complex arithmetic (Hitch, 1978a and b; Logie et al., 1994). Moreover, the canonical letter condition was used to control the fact that any disruption of the associative confusion effect in the random letter condition would not be due simply to the fact that subjects had to produce six different items. Alternatively generating random series of six items was expected to overload the central executive system in working memory (Baddeley, 1986).

It could be argued that our secondary tasks were not only qualitatively different (i.e., tapping different components of working memory) but also possessed different degrees of difficulty. However, as pointed out by Logie et al., (1994), (1) there is no *a priori* reason to suspect that these tasks vary systematically in difficulty; (2) the estimated difficulty of these tasks has been shown to be a poor predictors of subjects' performance in dual-task conditions (e.g., Wickens & Weigartner, 1985); and (3) these tasks have been shown to have independent effects. As we wished to pinpoint what particular working memory sub-system is critical, we needed to use secondary tasks that are known to affect different working memory sub-systems.

Because basic arithmetic facts are first learned by means of oral repetitions, it could be contended that the phonological loop is a privileged pathway to the solution of a simple problem in the memory network. This view predicts that (l) the problem difficulty effect (i.e., easy-difficult problem difference) will be greater in conditions where the phonological system is overloaded and (2) the associative confusion effect will also be enhanced in the phonological overload condition. An alternative view would state that most cognitive processes involved in cognitive arithmetic take place in the central executive system. This view predicts that (l) the problem difficulty effect will be greater in the central executive overload condition (i.e, in the random letters condition) and (2) the associative confusion effect will also be larger in the central executive overload condition. These two predictions were tested in Experiment 1 using repeated measures on the operation factor. That is, subjects were presented with both addition and multiplication problems.

2.1. *Method.*

2.1.1. *Subjects.* Twenty introductory psychology students (8 males and 12 females) at the *Université de Bourgogne à Dijon* received a class credit for participating in the experiment. The mean age of the subjects was 21.2 (range from 18.1–26.7). All subjects had normal or corrected-to-normal vision.

2.1.2. *Stimuli and apparatus.* The stimuli were addition and multiplication equations presented in standard form (i.e., $a+b=c$ or $a\times b=c$). The equations represented all combinations of digits from 2–3 through 9–9. One and zero were not included because it is generally believed that subjects do not solve these problems by retrieving the solution directly from memory but instead by retrieving rules (e.g., $N \times 0 = 0$) that guide their solution (see Ashcraft, 1982; Baroody, 1985). The equations $2+2$ and 2×2 were not included because the sum and product of these operands are identical. The resulting set consisted of 63 true addition problems and 63 true multiplication problems. A set of false problems was also created. For each equation, both a confusion and a non-confusion problem were generated. When the equation involved addition, the c term for the confusion problem was the product of the two operands ($3+5=15$), and for the multiplication problems, c was the sum ($3\times 5=8$). Non-confusion problems were also generated such that c was neither the sum nor the product of a and b (e.g., $3+5=7$). The c terms for the non-confusion problems were chosen so that the difference between the correct answer to the equation and the c term (i.e., the split) matched the corresponding split in the confusion condition (see Zbrodoff & Logan, 1986; Lemaire et al., 1991; 1994). This was necessary because some confusion problems have large splits (e.g., $9+9=81$) and the split is known to affect reaction time in arithmetic verification tasks (Ashcraft & Battaglia, 1978; Zbrodoff & Logan, 1990). To equate the splits, the answer to the non-confusion problem was equal to the answer to the corresponding confusion problem plus or minus one. Because the correct answers and the answers to the confusion problems for additions and multiplications were the inverse of one another, the split did not vary across operations. Each of the 126 true equations appeared twice, so that the subjects saw 252 true equations and 252 false equations (half of which were confusion problems). Both true and false problems were categorized as involving easy and difficult problems. The arithmetic problems have been described in the literature as being easy or difficult for adults based on difficulty rating scales (Ashcraft's index; see Hamann & Ashcraft, 1985; Koshmider & Ashcraft, 1991). This measure has been described in the literature as one of the best predictors of problem difficulty. To determine whether a problem was easy or difficult, we first computed the median Ashcraft's difficulty index. Then, problems with smaller difficulty values were categorized as easy problems ($N =$ 252) and problems with higher values were categorized as difficult problems

($N = 252$). Thus, the mean difficulty was .69 versus .81 for easy versus difficult additions and .34 versus .69 for easy versus difficult multiplications.

2.1.3. *Procedure.* The stimuli were presented horizontally in the centre of a computer screen (IBM PS/2). Each trial began with a 750-msec ready signal (a line of five a's) which appeared in the centre of the screen. The equations were in the form "$a + b = c$" or "$a \times b = c$." The symbols and digits were separated by spaces equal to one-half of the width of each character. The equations remained on until the subject responded. Subjects were instructed to respond "true" or "false" by pressing the appropriate key on the keyboard. The "m" and "q" keys were designated as true and false. All subjects were instructed to use their left and right index fingers to press these keys; for half of the subjects the "m" key was designated as true and for the other half it was designated as false.

Each subject was randomly assigned to one of the four memory load conditions. The control condition mimicked the standard verification tasks in that no concurrent task was required; the subjects merely verified arithmetic problems. In the articulatory suppression condition, the subjects were required to repeatedly say the word "the". This concurrent task was supposed to disturb the operation of the phonological loop. In the canonical letter condition, the subjects were required to repeatedly say the standard series of "abcdef" letters. The random letter condition required the subjects to generate without stopping a random series of letters from the set of the six 'abcdef' letters. The articulatory suppression and canonical letter conditions were run to determine whether articulatory suppression results in impaired performance. Moreover, the canonical letter condition was adopted to control for the fact that any disruption of the confusion effect in the random letter condition would be due to the subjects having to generate at random series of letters. For the three concurrent task conditions, subjects were required to say one item every 2 sec.

In each experiment, all subjects were presented with the same problems. These problems were randomly ordered for each subject with the restriction that no more than four consecutive trials could require the same response. The stimuli were presented in four 126 problem trial blocks; each block was composed of 63 true and 63 false problems. The subjects were permitted a 5-min rest period between blocks.

Before the experimental trials, the subjects in the control condition were given two sets of 20 practice trials in order to familiarise themselves with the apparatus, procedure, and stimulus display. For the other experimental conditions, the subjects were first required to practise the concurrent task with a metronome until they were able to say one item every 2 sec. Then, they practiced only the cognitive arithmetic verification task with one set of 20 practice trials. Finally, they practiced the two concurrent tasks with another set of 20 practice trials until they felt comfortable and showed no apparent problems. The subjects were told their mean latency and error rate after the practice trials and after each block. Instructions emphasised speed and accuracy equally. At the end of each block, subjects were given a brief rest period. Each experimental session lasted approximately 30–45 min.

3. Results and Discussion.

For latency performance, trials on which subjects made errors were dropped from the analyses. Error rates were very low with true problems (mean: 2.1%) and correlated significantly with latencies ($r = .91$). Errors rates were higher with false problems (mean: 4.7%) and were critical for the associative confusion effects tested here. Therefore, we report analyses on latencies for true and false problems separately, and on errors on false problems (the analyses of errors on true problems being redundant with the corresponding analyses of latencies).

3.1. *Latencies for True Problems.* The mean median correct response times for the true problems were analysed using a 4 (memory load: control, articulatory suppression, canonical letters, random letters) × 2 (operation: addition *vs* multiplication) × 2 (problem difficulty: easy *vs* difficult) analysis of variance (ANOVA) with repeated measures on the last two factors. Latencies varied significantly with memory load, [$F(3, 16) = 67.73$, $p< .001$, $MS_e= 413430$]. The mean median latencies were 1106, 1111, 1074, and 1842 msec, for the control, articulatory suppression, canonical letter, and random letter conditions, respectively. Planned comparisons revealed significant differences between the random letter condition and each of the other memory load condition and no other comparison was significant. This memory load variable interacted with operation, [$F(3, 16) = 7.04$, $p< .01$, $MS_e= 208893$], showing longer latencies with addition problems in the control and articulatory suppression conditions and with multiplication problems in the other two conditions. Furthermore, easy problems (1169 msec) were verified more quickly than difficult problems (1398 msec), [$F(1, 16) = 134.55$, $p< .0001$, $MS_e= 8562$]. As Table 1 shows, the difference between easy and difficult problems tended to vary with memory load, the difference between the control and the random letter conditions being the greatest (mean respective differences between easy and difficulty problems: 146, 222, 234, and 3l4 msec, for the control, articulatory suppression, canonical letter, and random letter conditions, respectively), although the memory load × difficulty interaction was only marginally significant, [$F(3, 16) = 2.8$,

	Control	*Articulatory Suppression*	*Canonical Letters*	*Random Letters*
Addition Problems				
Easy problems	1203 (1.1)	1270 (1.4)	962 (1.5)	1331 (1.2)
Difficult problems	1306 (1.7)	1487 (2.2)	1177 (2.1)	1645 (4.6)
Difference	103 (0.6)	208 (0.8)	215 (0.7)	314 (3.4)
Multiplication Problems				
Easy problems	863 (1.0)	720 (1.5)	953 (1.5)	2039 (1.0)
Difficult problems	1051 (1.8)	957 (2.3)	1205 (2.9)	2353 (4.8)
Difference	188 (0.8)	237 (0.8)	252 (1.4)	314 (3.8)

TABLE 1. Mean Median latencies (msec) and Percentage Errors (in Parentheses) for the True Problems: Experiment 1.

$p = .07$, $MS_e = 8562$]. Planned comparisons showed significant difficulty effects for each of the memory load × operation conditions.

The larger problem difficulty effect seen with increasing memory load is consistent with the working memory resource hypothesis. The working memory resource hypothesis accounts for this trend by assuming that difficult problems take longer to retrieve in long-term memory and are affected to a greater degree by a secondary task (Ashcraft et al., 1992), suggesting that some major component of mental arithmetic is sensitive to an attentional overload situation.

3.1.1. *Latencies for False Problems.* The mean median correct response times for the false problems were analyzed using a 4 (memory load: control, articulatory suppression, canonical letters, random letters) × 2 (operation: addition *vs* multiplication) × 2 (problem difficulty: easy *vs* difficult) × 2 (answer: non-confusion *vs* confusion) ANOVA with repeated measures on the last three factors. As can be seen in Table 2, the major experimental manipulations were successful. The mean median latencies for the control, articulatory suppression, canonical letter, and random letter conditions were 1132, 1189, 1140, and 1596 msec, respectively, indicating a significant memory load condition effect, [$F(3, 32) = 6.29$, $p < .05$, $MS_e = 750289$]. The only significant planned comparisons indicated that latencies in the random letter condition were significantly longer than in any other memory load condition. The difference between easy (1222 msec) and difficult (1307 msec) problems was significant, [$F(1, 16) = 32.01$, $p < .001$, $MS_e = 17084$]. The global 80-msec associative confusion effect was significant, [$F(1, 16) = 16.68$, $p < .01$, $MS_e = 19023$].

	Easy Problems			Difficult Problems		
	NCP	CP	CE	NCP	CP	CE
Addition Problems						
Control						
Mean	874	1095	221[a]	1063	1115	52[a]
Errors (%)	1.8	2.1	0.3	2.1	2.2	0.1
Articulatory Suppression						
Mean	845	1135	290[a]	1054	1130	76[a]
Errors (%)	3.9	4.1	0.2	4.1	4.5	0.4
Canonical Letters						
Mean	1101	1232	131[a]	1173	1158	−1.5
Errors (%)	3.7	4.0	0.3	4.2	4.8	0.6
Random Letters						
Mean	1436	1331	−105	1604	1491	−113
Errors (%)	4.5	8.7	3.2[a]	7.9	11.7	3.6[a]
Multiplication Problems						
Control						
Mean	1102	1187	85[a]	1318	1305	−13
Errors (%)	1.9	2.3	0.4	2.1	2.4	0.3
Articulatory Suppression						
Mean	1209	1336	127[a]	1375	1428	53[a]
Errors (%)	3.2	3.7	0.5	4.2	4.8	0.6
Canonical Letters						
Mean	954	1295	341[a]	1046	1164	119[a]
Errors (%)	3.8	4.2	0.4	4.0	4.4	0.4
Random Letters						
Mean	1699	1739	30	1731	1749	18
Errors (%)	4.3	8.9	3.4[a]	8.3	11.3	3.0[a]

Abbreviations: NCP, non-confusion problems; *CP*, confusion problems; *CE*, non-confusion effect.

[a] Significant confusion effect.

TABLE 2. Mean Median Latencies (msec) and Percentage Errors for the False Problems (Confusion and Non-confusion Problems): Experiment 1.

More importantly, the two critical interactions were significant. The memory load × difficulty interaction was significant, [$F(3,\ 16) = 3.85$, $p< .05$, $MS_e=$ 17084:] difficulty effects tended to be smaller in the canonical letter (−10 msec) and random letter (95 msec) conditions than in the other two conditions (136 and 116 msec for the control and articulatory suppression conditions, respectively). Furthermore, the memory load × answer interaction was significant, [$F(3,\ 16) = 3.12$, $p= .05$, $MS_e=$ 19026]. Planned comparisons revealed significant associative confusion effects in the control condition, [$F(1,\ 16) = 4.44$, $p< .05$], the articulatory suppression condition, [$F(1,\ 16) = 9.72$, $p< .01$], and the canonical letter condition, [$F(1,\ 16) = 10.75$, $p< .01$], but not under the random letter condition ($F < 1$). No other simple or interaction effects were significant.

3.1.2. *Errors for False Problems.* As can be seen in Table 2, errors were low, averaging 4.7% across subjects, and accordingly, differences in accuracy must be interpreted with caution. Errors were analyzed like latencies using a 4 (memory load) × 2 (operation) × 2 (difficulty) × 2 (answer) mixed-design ANOVA. Error rates varied with memory load, [$F(3,\ 16) = 16.75$,$p< .01$, $MS_e=$ 46.89]. Planned comparisons revealed significant differences between the random letter condition and each of the three other memory load conditions, and no other comparisons were significant. Easy problems produced fewer errors than difficult problems, [$F(1,\ 16) = 8.6$,$p< .01$, $MS_e=$ 4.22]. Although no other main or interaction effects were significant, we used planned comparisons to test the associative confusion effects in each memory load × operation × difficulty condition ($MS_e=$ 2.78). As can be seen from Table 2, significant associative confusion effects were observed only in the random letter conditions with both addition and multiplication problems.

In summary, results of Experiment 1 are consistent with the hypothesis that simple mental arithmetic requires working memory resources. The difference between easy and hard problems latency was greatest in the random letter condition; the difference between confusion and non-confusion problems error rates increased in the random letter condition. This suggests that the central executive sub-system is the critical component of working memory involved in simple mental arithmetic.

However, before we accept this conclusion, we must address one relevant issue about research on the associative confusion effect, which is whether perceptual factors are responsible for this effect. That is, subjects may perform poorly with confusion problems because after seeing an answer that would have been correct under a different operation, they check to see if they have "misperceived" the sign. Several pieces of data have ruled out this perceptual artefact possibility (e.g., Koshmider & Ashcraft, 1991; Lemaire et al., 1991;

1994; Lemaire & Siegler, 1995; Zbrodoff & Logan, 1986) for the control condition. However, in our dual-task conditions, there remains the possibility that the secondary task may trigger different strategies and lead subjects to double-check that they have encoded the operation sign correctly. This strategic change in overloading conditions would result in adding spurious time and error rates on confusion problems that would have nothing to do with a real working memory load effect on the retrieval-and-compare processes involved in simple arithmetic. Therefore, to control for the possibility that the conclusions of Experiment 1 with regard to working memory load effect were not an artifact of our within-subject design, we used the operation factor as a between-subject factor in Experiment 2.

4. Experiment 2

4.1. *Method.*

4.1.1. *Subjects.* Forty introductory psychology students (19 males and 21 females) at the *Université de Bourgogne à Dijon*, received a class credit for participating in the experiment. The mean age of the subjects was 26.4 years (range 19.3–27.3 years).

4.1.2. *Stimuli.* The stimuli were identical to those used in Experiment 1.

4.1.3. *Procedure.* The procedure was identical to that used in Experiment 1 with one exception: The subjects were shown either addition or multiplication problems, and were randomly assigned to one of the operation × load conditions. The stimuli were presented in two blocks of 126 problems each (63 true and 63 false problems). The subjects were permitted a five minute rest period between each block. Each experimental session lasted approximately 20–30 min.

4.2. *Results and Discussion.* For both latencies and error rates, data were analyzed as in Experiment 1, that is for true and false problems separately. Latency trials on which subjects made errors were dropped from the analyses. Error rates were low with true problems (mean: 1.5%) and correlated significantly with latencies ($r = .93$). As in Experiment 1, error rates were higher with false problems (mean: 4.7%).

4.2.1. *Latencies for True Problems.* The mean median correct response times for the true problems were analyzed as in Experiment 1 using a 4 (memory load: control, articulatory suppression, canonical letters, random letters) × 2 (operation: addition *vs* multiplication) × 3 (problem difficulty: easy *vs* difficult) analysis of variance (ANOVA) with repeated measures on the last factor.

	Control	*Articulatory Suppression*	*Canonical Letters*	*Random Letters*
Addition Problems				
Easy problems	1050 (0.9)	1193 (1.3)	785 (0.9)	1145 (1.7)
Difficult problems	1181 (1.3)	1370 (1.3)	985 (1.9)	1533 (2.7)
Difference	131 (0.2)	177 (0.0)	200 (1.0)	388 (1.0)
Multiplication Problems				
Easy problems	724 (1.0)	638 (1.0)	797 (1.2)	1726 (1.6)
Difficult problems	865 (1.2)	847 (1.6)	963 (1.6)	1940 (2.8)
Difference	141 (0.2)	209 (0.6)	186 (0.6)	214 (1.2)

TABLE 3. Mean Median latencies (msec) and Percentage Errors (in Parentheses) for the True Problems: Experiment 2.

Latencies varied significantly with memory load, [$F(3, 32) = 5.79$, $p < .01$, $MS_e = 357007$]. The mean median latencies were 955, 1012, 888, 1586 msec, for the control, articulatory suppression, canonical letter, and random letter conditions, respectively. Planned comparisons revealed significant differences between the random letter condition and each of the other memory load conditions; no other comparisons were significant. Furthermore, easy problems (1007 msec) were verified more quickly than difficult problems (1213 msec) [$F(1, 32) = 80.80$, $p < .0001$, $MS_e = 10557$]. As in Experiment 1, the problem difficulty effects tended to differ with memory load, the difference between the control and the random letter conditions being the greatest (mean differences: 136, 193, 194, and 301 msec, for the control, articulatory suppression, canonical letter, and random letter conditions, respectively), although the memory load × difficulty interaction was only marginally significant [$F(3, 32) = 2.4$, p= .08, $MS_e = 10557$]. Planned comparisons showed significant difficulty effects for each of the memory load conditions. This effect was observed for both addition and multiplication problems. No other simple or interaction effect proved to be significant.

4.2.2. *Latencies for False Problems.* The mean median correct response times for the false problems were analyzed using a 4 (memory load: control, articulatory suppression, canonical letters, random letters) × 2 (operation: addition, multiplication) × 3 (problem difficulty: easy *vs* difficult) × 2 (answer: non-confusion *vs*confusion) ANOVA with repeated measures on the last two factors. The mean median latencies are presented in Table 4 and showed a significant memory load effect [$F(3, 32) = 3.55$, $p< .05$, $MS_e= 558862$]. The mean median latencies were 968, 1104, 1120, and 1739 msec, for the control, articulatory suppression, canonical letter, and random letter conditions, respectively. As in Experiment 1, the only significant planned comparisons indicated that latencies in the random letter condition were significantly longer than in any other memory load condition. The difference between easy (1173 msec) and difficult (1293 msec) problems was significant, [$F(1, 32) = 16.4$, $p< .001$, $MS_e= 17103$]. The global 86-msec associative confusion effect was significant [$F(1, 16) = 12.05$, $p< .01$, $MS_e= 21195$].

The memory load × difficulty interaction was marginally significant [$F(3, 32) = 2.47$, p= .08, $MS_e= 17103$], showing significantly larger difficulty effects in the canonical letter (119 msec) and random letter (224 msec) conditions than in the control (96 msec) or the articulatory suppression (44 msec) conditions. Finally, the memory load × answer interaction was significant [$F(3, 32) = 3.50$, $p< .05$, $MS_e= 21195$]. Planned comparisons revealed significant associative confusion effects in the control condition [$F(1, 32) = 5.92$, $p< .05$], in the articulatory suppression condition [$F(1, 32) = 5.01$, $p< .01$], and in the canonical letter condition [$F(1, 32) = 10.90$, $p< .01$], but not in the random letter condition ($F < 1$). The disruption of the confusion effect in the random letter condition and not in any of the other memory load conditions replicates the disjunctive disruption of the associative confusion effect observed in Experiment 1. No other simple or interaction effect proved to be significant.

4.2.3. *Errors for False Problems.* As in Experiment 1, errors were low, averaging 3.5% across subjects, and accordingly, differences in accuracy must be interpreted with caution (see Table 4). Errors were analysed in the same way as latencies using a 4 (memory load) × 2 (operation) × 2 (difficulty) × 2 (answer) mixed-design ANOVA. The only significant effect was the memory load effect [$F(3, 16) = 25.75$, p < .01, $MS_e= 25.88$]. Planned comparisons revealed significant differences between the random letter condition and each of the other three memory load condition, and no other comparisons were significant. Although no other main or interaction effects were significant, we used planned comparisons to test the associative confusion effects in each memory load × operation × difficulty condition ($MS_e= 2.31$). As can be seen from Table 4, significant associative confusion effects were observed only under the random letter conditions with both addition and multiplication problems.

In sum, the results of both Experiments 1 and 2 clearly demonstrate working memory resource demands in the verification of simple arithmetic problems. When subjects had to generate random series of letters within a predefined set of six letters "abcdef" while verifying problems, they showed larger difficulty effects and no longer showed any confusion effects in RTs, and a significant

	Easy Problems			*Difficult Problems*		
	NCP	*CP*	*CE*	*NCP*	*CP*	*CE*
Addition Problems						
Control						
Mean	857	1041	184[a]	1000	1055	55[a]
Errors (%)	1.7	2.2	0.5	2.1	2.2	0.1
Articulatory Suppression						
Mean	1010	1209	199[a]	1176	1183	7
Errors (%)	3.5	3.6	0.1	3.6	3.9	0.3
Canonical Letters						
Mean	893	1163	270[a]	943	1105	162[a]
Errors (%)	3.7	3.9	0.2	4.1	4.2	0.1
Random Letters						
Mean	1433	1348	−85	1783	1710	−73
Errors (%)	2.9	5.7	2.8[a]	6.1	8.5	2.4[a]
Multiplication Problems						
Control						
Mean	808	979	171[a]	978	1031	53[a]
Errors (%)	3.5	3.6	0.1	3.4	4.1	0.7
Articulatory Suppression						
Mean	974	1135	161[a]	1050	1095	45[a]
Errors (%)	3.5	3.6	0.1	3.4	4.1	0.7
Canonical Letters						
Mean	1029	1156	127[a]	1311	1358	47[a]
Errors (%)	4.1	4.1	0.0	3.9	4.0	0.0.1
Random Letters						
Mean	1846	1880	34	1940	1970	30
Errors (%)	3.1	5.5	2.4[a]	6.0	8.7	2.7[a]

Abbreviations: NCP, non-confusion problems; *CP*, confusion problems; *CE*, non-confusion effect.

[a] Significant confusion effect.

TABLE 4. Mean Median Latencies (msec) and Percentage Errors for the False Problems (confusion and Non-Confusion Problems): Experiment 2.

confusion effect in errors. In conditions where they had to perform another concurrent task involving minimum load on the central executive system, subjects still showed confusion effects in RTs, no confusion effect in errors, and standard difficulty effects. Having tested the potential effect of the overload of one of the slave systems, namely the phonological loop, the present data suggest that the central executive plays a critical role in simple cognitive arithmetic. Moreover, consistent with Lemaire et al. (1994) and as discussed in that paper, confusion effects were also disrupted by problem difficulty (specifically, in their Expreiment 1, Lemaire et al. reported 56-msec interference effects with large problems and 121-msec interference effects with small problems). Finally, the disruption of the associative confusion effect in RTs and its increase in errors by the central executive overload was observed with both addition and multiplication problems when presented together within the same test or in different tests, in the random letter condition only.

5. General Discussion

The goal of this study was to determine what kind of attentional resources, if any, are relevant for simple arithmetic processing. We used the dual-task method and looked at the disruption of subjects' performance when the phonological loop and the central executive of working memory were or were not overloaded during problem verification. We manipulated the difficulty of problems and the potential for inducing associative confusion. The present data allow us to delineate what kinds of working memory resources are critical for verifying simple problems like $8 \times 4 = 32$. Given that this has been considered as a fairly automatic activity in adults, the present results have important implications for future arithmetic investigations. In the rest of this section, we discuss the kind of attentional resources that are required when solving arithmetic problems, the potentials for future investigations, and, more generally, the issue of working memory resources and cognitive activities.

5.1. *Attentional demands in mental arithmetic?* The results of Experiments 1 and 2 are consistent with the working memory hypothesis tested here. When subjects verified true problems, the size of the problem difficulty effect increased in the secondary task conditions compared to the control condition. The predicted trend for a larger problem difficulty effect was found whether the working memory sub-system involved was the phonological loop or the central executive systems. It suggests that to verify true problems, attentional resources in both the phonological and central executive systems are involved. The effect of the concurrent task was greater on difficult problems, as predicted by the working memory hypothesis, because these problems are harder to retrieve in memory and require more memory resources than easy problems. Therefore, in a situation where part of the resources had to be devoted to a secondary task, difficult problems became even harder to retrieve. What is important is that the effect of working memory load conditions was not constant across problem difficulty: The secondary task did not add a constant amount of processing (i.e., change in intercept) but added different amounts of processing (i.e., changes in slopes) in easy and hard problems. In sum, performance on these hard problems interferred the most with the secondary task as the rate of processing of difficult problems was slowed down more than that of easier problems under working memory overload condition.

With false problems, no associative confusion effects were observed in RTs under the random letter condition. However, this associative confusion effect was larger in accuracy under the random letter condition than under any of the other conditions. These results regarding latency and accuracy may seem inconsistent, although predicted by the working memory resource hypothesis. Indeed, this hypothesis predicts that latency should increase because the dual task reduces the amount resources available, and the fewer the resources devoted to the primary task, the longer the latency. One way this could happen is suggested by divided attention models that tried to characterise our limits on managing simultaneous cognitive processes. One class of these models assume an attentional bottelneck that would limit our capacity to execute two tasks concurrently (e.g., Pashler, 1984, 1994; Pashler & Johnston, 1989). Due to this bottleneck, certain stages of processing cannot be performed simultaneously; these stages have to be performed in sequence. In the case of an arithmetic problem verification task, as in this study, it is possible that the response-execution stage is delayed so that, when subjects compared the retrieved answer with the proposed answer, the interference answer may no longer be activated and therefore no longer affects the subject's latencies. In other words, the increase in latency in the dual-task condition absorbs the differences between non-associative and associative confusion problems. This raises the interesting issue for cognitive arithmetic researchers of which stages in arithmetic problem solving require attentional resources and which do not. Investigation of this question would tell us if, like in other cognitive domains, within the same task, some of the processes are automatic (e.g., perceptual encoding) and some require attentional resources (e.g., response execution). This would also help us to characterise the exact nature of each process within cognitive arithmetic models and make some specific prediction regarding factors that can affect particular stages.

The situation is different regarding accuracy. In a standard condition, when subjects have to verify an associative confusion problem, the problem first appears to be a plausible correct equation, and further processing is necessary to reject this equation. As a concurrent task uses up part of the resources normally devoted to this further processing, the present data on accuracy suggest that subjects still think that these associative problems are true. Consequently, they respond "true" more often and make more errors, in this working memory overload condition.

As mentionned earlier, it could be argued that the data reported here are the results of the inherent difficulty of our secondary tasks rather than of their tapping different working memory components. This point has been raised several times in studies using the dual-task method and has, most recently, been discussed at length by Logie et al. (1994) in the context of arithmetic. The same basic argument used by Logie et al. can be used here, namely that the concept of difficulty risks the danger of circularity (i.e., these tasks are more difficulty because they result in greater disruption; different amounts of disruption result in different degrees of difficulty) and is not a powerful explanatory construct. The empirical evidence mostly cited shows that the difficulty of secondary tasks is a poor predictor of subjects' performance in dual tasks (e.g., Logie, Baddeley, Mane, Donchin, & Sheptak, 1989; Wickens, & Weigartner, 1985; Yeh & Wickens, 1988). Also, in the context of the present experiments, the difficulty hypothesis does not explain how secondary tasks differentially affect problems of different sizes or problems with different types of responses.

In sum, the present data on both true and false problems are consistent with the hypothesis that verifying simple arithmetic problems requires attentional resources. The working memory hypothesis has received support here with a verification task that was appropriate for testing the possibility that both retrieval and comparison/decision processes in cognitive arithmetic require attentional resources. For the sake of generality, it would however be desirable to replicate these findings using another, more natural task, like a production task (e.g., $7 \times 9 =?$). This would also enable us to probe more precisely the role of working memory resources in arithmetic fact retrieval processes. The problem with the verification task itself is that the proposed true answers in true problem verification have a priming status (see Ashcraft, 1992; Ashcraft, Fierman, & Bartolotta, 1984; Campbell, 1987 a and b; Lemaire et al., 1991; 1994; Miller, Perlmutter, & Keating, 1984; Zbrodoff & Logan, 1990). That is, the main effect of the proposed true answers is mostly to speed up the retrieval process. By contrast, the associative confusion answers are typically assumed to slow down comparison/decision processes. Even though we cannot say that retrieval processes and comparison/decision processes occur sequentially (see Ashcraft, 1992; Zbrodoff & Logan, 1990), the fact that the two processes may be differently affected by a secondary task remains a possibility. This suggests that the retrieval and comparison/decision processes may involve different amounts of attentional resources or may involve resources in a different sub-system of the working memory system (see Kaye et al., 1983, for a similar argument). Consistent with this, we found that overloading the phonological loop and the central executive systems increased the difference between easy and difficult true problems. The fact that the effect of the secondary task on true problems was also observed in the phonological overload conditions could be accounted for by assuming that the phonological pathway is a privileged route to the true problems in memory, especially because subjects learn true arithmetic facts by means of oral repetitions.

5.2. *Arithmetic and working memory resources: Future investigations.* The present data are clear regarding the involvement of working memory resources in the central executive system with both true and false problems. However, concerning the role of the phonological system (in particular in verifying false problems), the evidence for working memory resources is weaker. It is possible that this is a true fact; that is, phonological working memory resources are required to verify only true problems and not false problems[1]. This is an *a priori* possibility given that basic arithmetic facts (i.e., correct problems) are first learned by means of oral repetitions. However, it is also possible that our experimental manipulation was not strong enough to detect a possible effect. Indeed, the subject performed the concurrent task at the rate of one item every 2 sec. This rate was high enough to make the random-generation task demanding. However, it may not have overloaded the phonological system to a great enough extent.

Another possibility is that there was some trade-off between the primary and secondary tasks. Such a trade-off between task performance has already been reported in the literature. For example, Ashcraft et al. (1992) observed that subjects had difficulties maintaining the rate of overt verbalizations and slowed down when they had to generate words or random series of letters, as a secondary task. In the present experiments, we tried to control for this possibility by training our subjects to produce one item every 2 sec and by running the experiment only when they were able to do so. However, the

[1]Note that this is an issue that can be raised only if we adopt a particular multiple-resource theory of attentional capacity like Baddeley's. Such issue of which particular kind of attentional resources, if any, are involved in arithmetic, could not be addressed within a view of working memory as a single flexible resource that provides both storage and processing functions.

possibility that subjects slowed their rate of verbalisation—in particular, on the most difficult problems—remains. Futher investigations need to clarify this point. As a desirable feature, these investigations will not only have to try to control for the maintenance of rehearsal rate, but will also have to have measures of secondary-task performance (e.g., rate of responding, degree of randomness). Such measures of secondary tasks performance have become been more and more common, as they allow the investigation of possible trade-off effects between primary and secondary task performance.

The potential insights gained by using more realistic arithmetic tasks like production tasks as well as by the collection of separate single- and dual-task performance measures can be illustrated by a study recently reported by Logie et al. (1994). They also used the dual-task paradigm to investigate the possibility that mental arithmetic requires working memory resources. They used a calculation task in which subjects had to add a series of two-digit numbers that were auditorily or visually presented. The secondary tasks were aimed at disrupting each component of the working memory system, as conceived by Baddeley (i.e., the phonological loop, the central executive, the visual sketchpad, and the spatial systems). Interestingly, results showed that: (1) calculation performance was disrupted when the central executive system was overloaded, and, to a lesser extent, when the phonological loop system was overloaded; (2) their subjects made approximately the same number of errors in single- and dual-task conditions, but the errors in the dual-task condition tended to be close to the correct answer. These patterns of results enabled Logie et al. to suggest that (1) the role of the phonological loop is most likely to keep track of running totals and to maintain accuracy in calculation, and (2) the role of the central executive is to estimate the correct totals and to select and implement the appropriate heuristics when the solution to a problem is not directly available through retrieval.

Logie and co-workers' (1994) study has interesting implications regarding the present study as well as potential future investigations. In the context of Logie and co-workers' view of the respective roles of working memory components in arithmetic, the present data suggest that verification paradigms require very little on-line maintenance of running totals. One potential reason is that the problems used here have all, at one point or another, been studied individually and can be solved via a single-step memory retrieval mechanism, making redundant the on-line maintenance of running totals. In this perspective, this verification paradigm is expected to show little if no effect of concurrent articulatory suppression, which is what was found here for false arithmetic problem verification. For these false problems, however, because subjects have to retrieve correct answers, compare these retrieved correct answers with the proposed answers, and state their response, it is understandable that concurrent random letter generation showed the greatest disruption on subjects' performance.

One important implication of Logie and co-workers' suggestion regarding the role of each working memory component in arithmetic concerns the possible different involvement of working memory resources in the verification of close false problems (e.g., $8 + 4 = 13$) compared with the verification of distant close problems (e.g., $8 + 4 = 17$). It is typically observed that subjects take significantly more time to verify close results than distant ones, an effect that has been termed the "split effect" (e.g., Ashcraft & Battaglia, 1978; Dehaene & Cohen, 1991; Zbrodoff & Logan, 1990). This effect is interesting because it suggests that subjects use two different types of strategy—a retrieval strategy used to verify close products and plausibility strategy to verify distant products (Lemaire & Fayol, 1995). Using the retrieval strategy, subjects first retrieve the correct solution, compare the retrieved correct solution, and state their response. Using the plausibility strategy, the whole verification process is not run to completion, but would be short-circuited by a "fast-no" decision, given that the proposed answer is too remote from the correct answer to be plausible. It would be interesting to determine whether these two kinds of strategy are impaired by overloading the same working memory component and to the same extent in a dual-task situation.

As a final point, the data reported in the present paper have further implications for the domain of cognitive arithmetic: They provide an impetus for formal models of simple arithmetic to account for the involvement of mental resources at different points in the course of simple arithmetic problem verification. Thus far, none of these models makes any commitment regarding the role of mental resources in simple arithmetic. Indeed, the models assume that cognitive arithmetic involves automatic processes that consume very few working memory resources if any at all. Because of our subjects' high speed and accuracy, it was difficult to show that simple cognitive arithmetic involves some mental resources in the working memory system. The dual-task paradigm however has been fruitful in showing that verifying simple arithmetic problems requires attentional resources. The results presented in this paper are consistent with those showing that cognitive arithmetic could be influenced by some central mechanism (e.g., Ashcraft et al., 1992; Brainerd, 1983; Hitch, 1978 a and b; Kaye, 1986; Kaye et al., 1983; 1989) or involve some partially autonomous processes (Lemaire et al., 1991; 1994; Lemaire & Fayol, 1995; Zbrodoff & Logan, 1986). It further suggests that some of the processes involved in the verification task may require different amounts of working memory resources and thus may have reached different levels of automaticity.

From a developmental perspective, research in simple cognitive arithmetic (and in many other domains) established that performance at any particular age is commonly a mixture of processing strategies (e.g., Ashcraft & Fierman, 1982; Koshmider & Ashcraft, 1991; Lemaire et al., 1994; Lemaire & Siegler, 1995; Siegler, 1987, 1988a; Siegler & Shipley, 1995). The development of simple arithmetic has been characterized by the fact that younger children are more reliant on slower implicit counting strategies, and with development, children retrieve a declarative representation of numerical facts from memory (Ashcraft, 1982, 1983, 1987, 1992; Ashcraft & Fierman, 1982; Campbell & Graham, 1985; Cooney, Ladd, & Swanson, 1988; Siegler, 1988a; Siegler & Shrager, 1984; Svenson & Sjoberg, 1983). The shift from counting strategies to memory retrieval is determined by the strength of fact representation in memory (e.g., Geary & Burlingham-Dubree, 1989; Siegler, 1987; Lemaire et al., 1991; 1994; Lemaire & Siegler, 1995; Siegler & Jenkins, 1989; Siegler & Shrager, 1984). That is, facts stored with high associative strengths are routinely, rapidly and accurately retrieved from memory. Those with lower associative strengths are more prone to solution via counting. The present study suggests that working memory resources could also be an important contributor to the achievement and devolpment of numerical cognition (see Brainerd, 1983; Geary, Brown, & Samaranayake, 1991; Kaye, 1986, for a similar argument). In particular, there may be a greater involvement of working memory resources on more difficult items for children, and this greater involvement of working memory on difficult items might change with age, a prediction that remains to be tested (in simple arithmetic, but in other domains as well).

6. Conclusion

Interestingly, the present data regarding the role of attentional resources in simple arithmetic are consistent with data reported in other domains that were thought to involve mostly automatic processes, like lexical access, problem solving, thinking, information encoding, or spelling (e.g., Britton, 1980; Britton & Tesser, 1982; Fayol, Largy, & Lemaire, 1994; Herdman & Dobbs, 1989; Herdman & Friedman, 1985; Largy, Fayol, & Lemaire, in press; Miyake, Just, & Carpenter, 1994; Ogden, Martin, & Paap, 1980) but which have in fact been shown to involve attentional resources as well. A common dimension of each of these domains is that bringing prior knowledge from the inactive state into an active state to perform ongoing cognitive tasks not only happens very quickly but also uses some mental resources that vary with the difficulty of access of information in LTM. The evidence accumulated in different domains is consistent with general ideas stated within more general cognitive information-processing theories (e.g., Anderson, 1990; Logan, 1988 a and b, 1992) which include automaticity as a key cognitive phenomenon to be accounted for.

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References

[1] Allport, D. A. (1980). Attention and Performance. In G. Claxton (Ed.), *Cognitive psychology* (pp. 112-153). London: Routledge & Kegan Paul.

[2] Anderson, J.R. (1983). A spreading activation theory of memory. *Journal of Verbal Learning and Verbal Behavior, 2,* 261-295.

[3] Anderson, J.R. (1990). *The Adaptive character of human thought.* Hillsdale, NJ: Erlbaum.

[4] Ashcraft, M.H. (1982). The development of mental arithmetic: A chronometric approach. *Developmental Review, 3,* 213-236.

[5] Ashcraft, M.H. (1983). Procedural knowledge versus fact retrieval in mental arithmetic: A reply to Baroody. *Developmental Review, 3,* 231-235.

[6] Ashcraft, M.H. (1987). Children's knowledge of simple arithmetic: A developmental model and simulation. In J.C. Bisanz, C.J. Brainerd, & R. Kail (Eds). *Formal methods in developmental psychology: Progress in cognitive development research.* (pp. 302-338). New York: Springer-Verlag.

[7] Ashcraft, M.H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition, 44,* 75-106.

[8] Ashcraft, M.H., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning & Memory, 4,* 527-538.

[9] Ashcraft, M.H., & Fierman, B.A. (1982). Mental addition in third, fourth, and sixth graders. *Journal of Experimental Child Psychology, 3* , 216-234.

[10] Ashcraft, M.H., Fierman, B.A., & Bartolotta, R. (1984). The production and verification tasks in mental addition: An empirical comparison. *Developmental Review, 4,* 157-170.

[11] Ashcraft, M.H., Donley, R.D., Halas, M.A., & Vakali, H. (1992). Working memory, automaticity, and problem difficulty. In J. Campbell (Ed.), *The Nature and Origins of Mathematical Skills.* Amsterdam: Elsevier Science Publishers B.V.

[12] Baddeley, A.D. (1986). *Working Memory.* Oxford: Oxford University Press.

[13] Baddeley, A.D. (1990). *Human Memory Theory and Practice.* London: Lawrence Erlbaum Associates, Ltd.

[14] Baddeley, A.D., & Hitch, G.J. (1974). Working memory. Ln G. Bower (Ed.), *Recent Advances in Learning and Motivation, VI.* Hillsdale, NJ: Erlbaum.

[15] Baroody, A.J. (1985). Mastery of the basic number combinations: Internalization of relationships or facts? *Journal for Research in Mathematics Education, 16,* 83-98.

[16] Brainerd, C.J. (1983). Young Children's Mental Arithmetic Errors: A working memory analysis. *Child Development, 54,* 812-830.

[17] Britton, B.K. Use of cognitive capacity in reading: Effects of processing information from text for immediate recall and retention. *Journal of Reading Behavior, 12,* 129-137.

[18] Campbell, J.I.D. (1987a). Network interference and mental multiplication. *Journal of Experimental Psycholoy: Leaming, Memory, and Cognition, 13,* 109-123.

[19] Campbell, J.I.D. (1987b). Production, verification, and priming of multiplication facts. *Memory & Cognition, 15,* 349-364.

[20] Campbell, J.I.D., & Graham, D.J. (1985). Mental multiplication skill: structure, process, and acquisition. *Canadian Journal of Psychology 39,* 338-366.

[21] Collins, A.M., & Loftus, E.F. (1975). A spreading-activation theory of semantic processing. *Psychological Review, 82,* 407-428.

[22] Cooney, J., Swanson, H.L., & Ladd, S.F. (1988). Acquisition of mental multiplication skill: Evidence for the transition between counting and retrieval strategies. *Cognition and Instruction, 5,* 323-345.

[23] Cornet, J.A., Seron, X., Deloche, G., & Lories, G. (1988). Cognitive models of simple mental arithmetic: A critical review. *Cahiers de Psychologie Cognitive/European Bulletin of Cognitive Psychology, 8,* 551-571.

[24] Dehaene, S., & Cohen, L. (1991). two mental calculation systems: A case study of severe acalculia with preserved approximation. *Neuropsychologia, 29,* 1045-1074.

[25] Geary, D.C., & Burlingham-Dubree, M. (1989). External validation of the strategy choice model for addition. *Journal of Experimental Child Psychology, 47,* 175-192.

[26] Geary, D.C., Brown, S.C., & Samaranayake, V.A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing difference in normal and mathematically disabled children. *Developmental Review, 5,* 787-797.

[27] Groen, G.J., & Parkman, J.M. (1972). A chronometric analysis of simple addition. *Psvchological Review, 79,* 329-343.

[28] Hamann, M.S., & Ashcraft, M.H. (1985). Simple and complex mental addition across development. *Journal of Experimental Child Psycholoy, 40,* 49-70.

[29] Healy, A.F., & Nairne, J.S. (1985). Short-term memory processes in counting. *Cognitive Psvchology, 17,* 417-444.

[30] Herdman, C.M., & Dobbs, A.R. (1989). Attentional demands of visual word recognition. *Journal of Experimental Psychology: Human Perception and Performance, 15,* 124-132.

[31] Herdman, C.M., & Friedman, A. (1985). Multiple resources in divided attention: A cross-modal test of independence of hemisperic resources. *Journal of Experimental Psychology: Human Perception and Performance, 11,* 40-49.

[32] Hitch, G.J. (1978a). The role of short-term working memory in mental arithmetic. *Cognitive Psvchology, 10,* 302-323.

[33] Hitch, G.J. (1978b). Mental arithmetic: Short-term storage and information processing in a cognitive skill. In A.M. Lesgold, J.W. Pellegrino, S.D. Fokkema, & R. Glaser (Eds.), *Cognitive psychology and instruction* (pp. 331-338). New-York: Plenum.

[34] Just, M.A., & Carpenter, P.A. (1992). A capacity theory of comprehension: Individual differences in working memory. *Psychological Review, 99,* 122-149.

[35] Kahneman, D. (1973). *Attention and effort.* Englewood Cliffs, NJ: Prentice-Hall.

[36] Kahneman, D., & Treisman, A. M. (1984). Changing views of attention and automaticity. In R. Parasuraman and R. Daves (Eds.), *Varieties of attention* (pp. 29-61). New York: Academic Press.

[37] Kaye, D.B.(1986). The development of mathematical cognition. *Cognitive Development, 1,* 157-170.

[38] Kaye, D.B., Bonnefil, V., & Chen, Q. (1983, November). *The development of arithmetic problem solving efficiency.* Paper presented at the meeting of the Psychonomics Society, San Diego.

[39] Kaye, D.B., de Winstanley, P., Chen, Q., & Bonnefil, V. (1989). Development of efficient arithmetic computation. *Journal of Educational Psychology, 81,* 467-480.

[40] Koshmider, J.W., & Ashcraft, M.H. (1991). The development of children's mental multiplication skills. *Journal of Experimental Child Psychology, 51,* 53-89.

[41] LeFevre, J., Bisanz, J., & Mrkonijc, L. (1988). Cognitive arithmetic: Evidence for obligatory activation of arithmetic facts. *Memory & Cognition, 16,* 45-53.

[42] LeFevre, J., Kulak, & Bisanz, J. (1991). Individual differences and developmental change in the associative relations among numbers. *Journal of Experimental Child Psychology, 52,* 256-274.

[43] LeFevre, J. Sadesky, G. S., & Bisanz, J. (in press). Selection of procedures in mental addition: Reassessing the problem-size effect in adults. *Journal of Experimental Psychology: Learning, Memory, & Cognition.*

[44] Lemaire, P., & Fayol, M. (1995). When plausibility judgments supersede fact retrieval: The example of the odd-even effect on product verification. *Memory & Cognition, 23,* 34-48.

[45] Lemaire, P., & Bernoussi, M. (1991). Arithmétique cognitive: Processus, Dveloppement, et différences individuelles. *L'Année Psychologique, 91,* 419-438.

[46] Lemaire, P., & Siegler, R.S. (1995). Four aspects of strategic change: Contribution to children's learning multiplication. *Journal of Experimental Psychology: General, 124,* 83-97.

[47] Lemaire, P., Barrett, S.E., Fayol, M, & Abdi, H. (1994). Automatic activation of addition and multiplication facts in elementary school children. *Journal of Experimental Child Psychology, 57,* 224-258.

[48] Lemaire, P., Fayol, M., & Abdi, H. (1991). Associative confusion effect in cognitive arithmetic: Evidence for partially autonomous processes. *European Bulletin of Cognitive Psychology, 11,* 587-604.

[49] Logan, G.D. (1979). On the use of a concurrent memory load to measure attention and automaticity. *Journal of Experimental Psychology: Human Perception and Performance, 5,* 189-207.

[50] Logan, G.D. (1980). Attention and automaticity in Stroop and priming tasks: Theory and data. *Cognitive Psychology, 12,* 523-553.

[51] Logan, G.D. (1985). Skill and automaticity: Relations, implications, and future direction. *Canadian Journal of Psychology, 39,* 367-386.

[52] Logan, G.D. (1988a). Automaticity, resources and memory: Theoretical controversies and practical implications. *Human Factors, 30,* 583-598.

[53] Logan, G.D. (1988b). Toward an instance theory of automatization. *Psychological Review, 95,* 492-527.

[54] Logie, R.H., & Baddeley, A.D. (1987). Cognitive Processes in Counting. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 2,* 310-326.

[55] Logie, R. H., Gilhooly, K.J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory & Cognition, 22,* 395-410.

[56] Logie, R.H., Baddeley, A.D., Maine, A., Doncin, E., & Sheptak, R. (1989). Working memory and the analysis of a complex skill by secondary task methodology. *Acta Psychologica, 71,* 53-87.

[57] McCloskey, M., Harley, W., & Sokol, S.C. (1991). Models of arithmetic fact retrieval: An evaluation in light of findings from normal and brain-damaged subjects. *Journal of Experimental Psycholoy: Learning, Memory, and Cognition, 17,* 337-397.

[58] McLeod, P (1977a). A dual task response modality effect: Support for multiprocessor models of attention. *Quarterly Journal of Experimental Psychology, 29,* 651-667.

[59] McLeod, P. (1977b). Parallel processing and the psychological refractory period. *Acta Psychologica, 41,* 381-391.

[60] Miller, K.F., & Paredes, D.R. (1990). Starting to add worse: Effects of learning to multiply on children's addition. *Cognition. 37,* 213-242.

[61] Miller, K.F., Perlmutter, M., & Keating, D. (1984). Cognitive arithmetic: Comparison of operations. *Journal of Experimental Psychology: Leaming, Memory, and Cognition. l0,* 46-60.

[62] Miyake, A., Just, M.A., & Carpenter, P.A. (1994). Working memory constraints on the resolution of lexical ambiguity: maintaining multiple interpretations in neutral contexts. *Journal of Memory and Language, 33,* 175-202.

[63] Nairne, J.S., & Healy, A.F. (1983). Counting backwards produced systematic errors. *Journal of Experimental Psychology: General, 112,* 37 40.

[64] Navon, D. (1984). Resources - A theoretical soup stone? Psychological Review, 91, 216-234.Navon, D., & Gopher, D. (1979). On the economy of the human processing system. *Psychological Review, 86,* 214-255.

[65] Ogden, G.D., Levine, J.M., & Eisner, E.J. (1979). Measurement of workload by secondary tasks. *Human Factors, 7,* 44-64.

[66] Ogden, W.C., Martin, D.W., & Paap, K.R. (1980). Processing demands of encoding: What does secondary task performance reflect? *Journal of Experimental Psychology: Human Perception and Performance, 6,* 355-367.

[67] Pashler, H. (1984). Processing stages in overlapping tasks: Evidence for a central botteleneck. *Journal of Experimental Psychology: Human Perception and Performance, 16,* 1023-1040.

[68] Pashler, H. (1994). Overlapping mental operations in serial performance with preview. *Quarterly Journal of Experimental Psychology, 47A,* 161-191.

[69] Pashler, H., & Johnston, C. (1991). Chronometric evidence for central postponement in temporally overlapping tasks. *Quarterly Journal of Experimental Psychology, 41A,* 19-45.

[70] Posner, M.I., & Boies, S.J. (1971). Components of attention. *Psychological Review, 78,* 391-408.

[71] Posner, M.I., & Snyder, C.R.R. (1975). Facilitation and inhibition in the processing of signals. In P.M.A. Rabbitt & S. Domic (Eds.), *Attention and Performance, V.* (pp. 669-682). New York: Academic press.

[72] Shiffrin, R.M., & Schneider, W. (1977). Controlled and automatic human information processing: II. Perceptual learning, automatic attending and a general theory, *Psvchological Review, 84,* 127-190.

[73] Siegler, R.S. (1987). The perils of averaging data over strategies: An example of children's addition. *Journal of Experimental Psychology: General, 106,* 250-264.

[74] Siegler, R.S. (1988a). Individual differences in strategy choices: Good students, not-so-good students, and perfectionists. *Child Development, 59,* 833-851.

[75] Siegler, R.S. (1988b). Strategy choice procedures and the development of multiplication skills. *Journal of Experimental Psychology: General, 117,* 258-275.

[76] Siegler, R.S., & Jenkins, E. (1989).*How children discover new strategies.* Hillsdale, NJ: Erlbaum.

[77] Siegler, R.S., & Shipley, C. (1995). Variation, selection, and cognitive change. In G. Halford and T. Simon (Eds.), *Developing cognitive competence: New approaches to process modeling.* Hillsdale, NJ: Erlbaum.

[78] Siegler, R.S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills* (pp. 229-293). Hillsdale, NJ: Erlbaum.

[79] Stazyk, E.H., Ashcraft, M.H., & Hamann, M.S. (1982). A network approach to mental multiplication. *Journal of Experimental Psychology: Learning, Memory, & Cognition, 8,* 320-335.

[80] Svenson, 0., & Sjoberg, K. (1983). Evolution of cognitive processes for solving simple addition during the first three school years. *Scandinavian Journal of Psychology, 24,* 117-12.

[81] Wickens, C.D. (1980). The structure of attentional resources. In R.S. Nickerson (Ed.).*Attention and performance, VIII.* Hillsdale, NJ: Erlbaum.

[82] Wickens, C.D. (1984). Processing resources in attention. In R. Parasuraman, R. Davies, & J. Beatty (Eds.), *Varieties of attention* (pp. 63-102). New York: Academic Press.

[83] Wickens, C.D., & Weingartner, A. (1985). Process control monitoring: The effects of spatial and verbal ability and concurrent task demand. In R.E. Eberts & C.G. Eberts (Eds.), *Trends in ergonomics and human factors, Vol. II* (pp. 25-32).

[84] Winkelman, J.H., & Schmidt, J. (1974). Associative confusions in mental arithmetic. *Journal of Experimental Psychology, 102,* 734-736.

[85] Yeh, Y.Y., & Wickens, C.D. (1988). The dissociation of subject measures of mental workoad and performance. *Human Factors, 30,* 111-120.

[86] Zbrodoff, N.J., & Logan, G.D. (1986). On the autonomy of mental processes: A case study of arithmetic. *Journal of Experimental Psychology: General, 115,* 118-130.

[87] Zbrodoff, N.J., & Logan, G.D. (1990). On the relation between production and verification tasks in the psychology of simple arithmetic. *Journal of Experimental Psychology: Leaming, Memory, and Cognition, 16,* 83-97.