Modeling Ground-Air Wireless Connectivity: Continuous Connection Probability Analysis

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Abstract— In this paper, we consider three ground-aircraft communication issues, namely, ground-centric connectivity, aircraft-centric connectivity, and link capacity analysis. Particularly, we propose the continuous connection probability over at least \( \tau \) seconds, termed as \( \tau \)-connectivity, for the ground-centric link and aircraft-centric link respectively, which are derived in the expression of several key parameters such as the aircraft/ground station density, the aircraft velocity, and the connectable elevation angle threshold. Theoretical tradeoff between ergodic outage capacity and the number of connected aircraft is analyzed for low and high transmit power regimes. The hypotheses in the ground-centric \( \tau \)-connectivity model are tested, and we observe a good match between the theoretic probability and the real world data-based probability although some real world aircraft statistic characters do not match the ideal assumption perfectly. Next, the aircraft-centric \( \tau \)-connectivity is analyzed theoretically. Approximate yet practically accurate closed form aircraft-centric connection probabilities are derived and verified with Monte Carlo simulations. Furthermore, considering atmospheric gas and cloud attenuation, we compare the link budget and capacity between the ground-centric ground-to-aircraft link and the ground-to-satellite link under the \( \tau \)-connectivity and demonstrate the advantage of ground-aircraft communication.

Index Terms— Aeronautic communication, non-terrestrial network, coverage probability, atmospheric attenuation.

I. INTRODUCTION

The prosperity of information and communication technology industry has nourished various emerging applications, with billions of devices which are generating a tremendous amount of data that is needed to be transmitted. To fulfill the rapidly increasing data transmission demands, terrestrial networks tend to embrace heterogeneous and denser base station (BS) deployments [1], [2] in populous areas. However, on the one hand, there is a limitation of the network densification [3], i.e., the network performance starts to decrease when the cell density exceeds a threshold. On the other hand, the network coverage in the rural area is very limited due to the high cost of terrestrial network deployment. Moreover, the diverse emerging wireless applications require different key performance indicators (KPIs), such as throughput, outage probability, energy efficiency, fairness, latency, mobility, etc., which are difficult to be fulfilled with a single terrestrial network [4]–[6]. Therefore, constructing non-terrestrial networks to enhance and support wireless communications is gaining more and more interest in that it can mitigate the burden of terrestrial networks in populous areas, provide network coverage to the rural areas, and fulfill some KPIs that are difficult to achieve by terrestrial networks.

According to the altitude of the communication infrastructure, the non-terrestrial communications can be categorized into 1) unmanned aerial vehicle (UAV, up to 500 m) communication [7], [8], 2) aircraft communication, which consists of commercial and personal aircraft at altitudes around 5 - 13 km [9], 3) high-altitude platform station (HAPS) systems at altitudes around 20–50 km [10], [11], and 4) satellite communications which include low-earth orbit (LEO) satellites at altitudes 400–2000 km, medium-earth orbit (MEO) at altitudes 2000–35786 km, and geostationary equatorial orbit (GEO) satellites at an altitude of 35786 km [12]–[15]. Operating at the lowest altitude among the non-terrestrial communication platforms, UAV is more flexible and cost-effective to deploy and enhance the terrestrial networks [16]. But due to the battery and altitude limitation, a UAV can only provide limited coverage in a limited duration. The early effort of constructing non-geostationary orbit (NGSO) satellite-based commercial communication networks such as Iridium were made decades ago. Recently, thanks to the maturity of low-cost and high-capacity commercial spacecraft launching and satellite communication, substantial efforts are made to launch thousands of NGSO satellites, such as Starlink, OneWeb, Telesat, Iridiumnext and Kuiper [17], [18]. These satellites will form space networks to support the data transmission for ground users. The idea of HAPS came out more than two decades ago, which aims to construct radio stations operating in the stratosphere to provide wireless connections to ground users in rural areas such as Project Loon [19] or conduct remote sensing missions. Comparing with NGSO satellite communication, the loon-based HAPS has smaller coverage but much lower building and launching costs. Since the altitude of HAPS is lower than the altitude of NGSO satellite, the radio propagation encounters less
attenuation in the ground-HAPS link than the ground-NGSO link.

Due to the complexity and interdisciplinary nature of aircraft communication, there are different research focuses in ground to aircraft communications. For instance, the non-payload aircraft communication mainly focuses on transmitting/detecting aircraft control information, of which the goal is to increase the reliability and security of the control information for aircraft communication [20]–[22]. For payload aircraft communication, the scenario can be either to provide in-flight WiFis service to passengers [23], [24] or to serve as radio stations for ground terminals [9], [25]. Both aforementioned payload communication scenarios have the same goal to establish a high-throughput reliable ground-air connection. Aiming to achieve this goal, a 40 Gbps bidirectional mmWave ground-air link and a 100 Gbps bidirectional free space optics (FSO) link are realized in [26] and [27], respectively. We also note that the recently proposed large-scale NGSO satellites based integrated communication and radio astronomy system [12], [13] would need to transport huge amount of radio astronomical observation data from the satellites to the ground. For such a system, the aircraft based communication infrastructure can serve as a relay system to provide better energy and spectrum efficiencies.

As the feasibility of high-rate link for single ground-aircraft link is analyzed and proved by more and more research projects [28]–[30], in this paper, we investigate two different continuous connection problems between ground equipment (GE) and aircraft, namely, 1) ground-centric connectivity, and 2) aircraft-centric connectivity. We use the term GE to represent all kinds of equipment that are located on (or near) the ground and are capable to transmit/receive signal to/from an aircraft, e.g., specific devices or stations that need to communicate with the aircraft. Particularly, ground-centric means the GE triggers the communication with aircraft, i.e., this is a case that aircraft serve the GE and that the GE cares how stable the connection can be. Aircraft-centric, on the other hand, means the aircraft triggers the communication with GEs, i.e., this is a case that GEs serve the aircraft and that the aircraft cares how stable the connection can be. Definitely, both the ground-centric and the aircraft-centric communication can be bi-directional, and there are in total four scenarios, 1) the GE wants to transmit data, which corresponds to the ground-centric single-directional ground-to-aircraft (G2A) link, 2) the aircraft wants to transmit data, which corresponds to the aircraft-centric single-directional aircraft-to-ground (A2G) link, 3) the GE wants to receive data, which corresponds to the ground-centric single-directional A2G link, and 4) the aircraft wants to receive data, which corresponds to the aircraft-centric single-directional G2A link. It can be interpreted that, with respect to connection probability, scenario-1 and scenario-3 are the same, scenario-2 and scenario-4 are the same. Therefore, without loss of generality, we only consider scenario-1 and scenario-2 in this paper.

In the following of this paper, we simply use the acronyms G2A and A2G to represent the ground-centric scenario-1 and the aircraft-centric scenario-2, respectively. To make the two scenarios more distinct, we further use the ground unit (GU) to represent the GE in the G2A link, and use the ground station (GS) to represent the GE in the A2G link. The motivations for our investigation are as follows.

The ground-aircraft communications are opportunistic since the aircraft is not always within the connectable region to a fixed GU, and there is not always a GS connectable to a certain aircraft either. Thus, for a GU (or an aircraft), taking the best effort to connect to as many aircraft (or GSs) as possible is preferable in some sense. On the other hand, as the aircraft moves very fast and the minimum connectable elevation angle is lower bounded, the ground-air connection duration is very limited. Thus, the GU’s strategy of connecting to as many aircraft as possible leads to frequent handovers between different aircraft, which causes problems not only to the physical and medium access control layers, but also the network layer since the IP address changes [31]. Therefore, we define the $\tau$-connectivity (the probability of connection over at least $\tau$ seconds) for the ground-centric G2A link in the following section and analyze with real world aircraft data. From the perspective of an aircraft, knowing how long the aircraft-ground connection can be maintained is also important for a lot of outage-sensitive applications. Therefore, the $\tau$-connectivity for the aircraft-centric A2G link is also defined in next section.

Moreover, as the radio wave propagates through a distant slant path in the atmosphere in G2A and A2G communications, the atmospheric gas and cloud attenuations are also analyzed.

B. Contributions

The contributions of this paper can be described in three parts.

i) Ground-centric connectivity:

- We propose the $\tau$-connectivity, which is derived analytically and verified by Monte Carlo simulation and real world aircraft data. The $\tau$-connectivity describes the ground-centric G2A continuous connection probability with four parameters, the connection duration $\tau$, the projected aircraft velocity, the aircraft density, and the radius of connection region (or reference height and elevation angle threshold equivalently).
- With the analytic expression and several simple estimated aircraft statistics such as density, altitudes and velocity in a certain area, the achievable continuous connection probability can be estimated before planning certain applications.
- Theoretical tradeoff between ergodic outage capacity and the number of connected aircraft is analyzed for low and high transmit power regimes.
- A method to restrict the three dimensional analysis to two dimensional analysis with real world data is proposed, where all aircraft are projected to a reference plane.
- We capture the real world aircraft data and analyze the key statistics (e.g., density, speed, altitude, position) in four different suburban/rural areas.
- A detailed hypothesis test for the ground-centric G2A $\tau$-connectivity model is made, where the Chi-square goodness-of-fit test shows the acceptance of Poisson distribution hypothesis of the number of aircraft. We then
observe a good match between the analytic probability and the real world data-based probability although some real world aircraft statistic characters do not match the ideal assumption perfectly. The factors that may cause the gap between the two probabilities are analyzed with Monte Carlo simulation.

ii) Aircraft-centric connectivity: The $\tau$-connectivity of the aircraft-centric A2G link is proposed and its closed form approximations are derived in terms of four parameters, the connection duration $\tau$, the aircraft velocity, the GS density, and the radius of the aircraft’s footprint, which can provide a guidance in planning GSs to support aircraft-centric communications. The analytic aircraft-centric A2G $\tau$-connectivity is then verified by Monte Carlo simulation.

iii) Ground-aircraft link capacity: We compare the link budget and capacity between the G2A link and the OneWeb’s ground-to-satellite (G2S) link under the $\tau$-connectivity, considering atmospheric gas and cloud attenuation, where we use the real world aircraft data and cloud attenuation data sheet published by International Telecommunication Union (ITU).

This paper is organized as follows. In Section II, we introduce the system models, define the $\tau$-connectivity of ground-centric G2A and aircraft-centric A2G links, and describe different choices of ground-aircraft carrier frequency and antenna systems. The theoretic G2A connectivity is derived and analyzed in Section III. Next, the hypotheses in the G2A connectivity are tested in Section IV, where different aircraft statistics, theoretic-based, Monte Carlo-based, and real world data-based G2A connection probabilities are presented. In Section V, we derive and analyze the $\tau$-connectivity for the A2G link. In Section VI, the link capacity of ground-aircraft links are analyzed. Finally, Section VII concludes this paper.

**Notations:** The following notations are used in this paper. $x$ is a scalar. $x$ is a vector. $B(c, R)$ represents a ball centered at coordinate $c$, with a radius of $R$. $\Phi$ represents a PPP and $\Phi(A)$ represents the number of points in region $A$.

**II. SYSTEM MODEL**

**A. Geometrical Connectivity Models**

We consider a broad-area communication paradigm using aircraft as relays or carriers of data packets, where connection probabilities of two links, the ground-centric G2A link, and the aircraft-centric A2G link, are investigated. In the G2A communication scenario, there is one typical GU and multiple aircraft, where the GU can connect to any aircraft above a certain elevation angle $\theta_{ele}$ shown in Fig. 1. In the A2G communication scenario, there is one typical aircraft and multiple GSs, where the aircraft can connect to any GS within its footprint which corresponds to a circular region on the ground shown in Fig. 2.

We consider $\tau$-second seamless connection probabilities for both G2A and A2G links. Particularly, from an aircraft’s perspective, the locations of GSs form a Poisson point process (PPP) with a constant density $\lambda_G$ within the considered region. From a GU’s perspective, there can be either zero or at least one aircraft flying over its connectable region. However, the arrival and departure of these aircraft are random or quasi random, making the connection unstable. Moreover, most aircraft move with high velocity such that the maximum continuous connection duration to a particular GU/GS can be very limited. Thus, we propose a $\tau$-connectivity metric to evaluate the connection quality of the ground-centric G2A link and the aircraft-centric A2G link in definition 1 and 2, respectively.$^1$

Definition 1: The $\tau$-connectivity for the ground-centric G2A link is the probability that a GU can seamlessly connect to at least $M$ aircraft for at least $\tau$ seconds for each aircraft, represented as $P_{\text{G2A}}(M)$, where $M \geq 1$. (Note that if $M > 1$, each of the $M$ aircraft should always be connected to the GU within the $\tau$-second duration.)

Definition 2: Assuming ideal handover, the $\tau$-connectivity for the aircraft-centric A2G link is the probability that an aircraft can always have connections to one or more GSs for at least $\tau$ seconds, represented as $P_{\text{A2G}}$. (Note that this aircraft-centric $\tau$-connectivity is achieved if and only if there is always at least one GS located in the aircraft’s footprint within the $\tau$-second duration.)

The analysis for G2A connectivity involves multiple aircraft in a certain region. Normally, different aircraft fly at

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$^1$As mentioned in the introduction, Definition 1 and Definition 2 can be easily extended to scenario-3 and scenario-4, respectively.
different altitudes and speeds within certain ranges. For the convenience of analysis, the locations of aircraft are projected to the same altitude $h_{\text{ref}}$, making it possible to analyze in a two-dimensional plane instead of three-dimensional space. The velocities of aircraft are also scaled according to the altitudes. For example, if an aircraft is at a three-dimensional Cartesian coordinate $(x_0, y_0, h_0)$ with $v_0^{\text{(real)}}$ velocity, the projected two-dimensional coordinate is $(x_0 h_{\text{ref}}/h_0, y_0 h_{\text{ref}}/h_0)$, and the projected velocity is $v_0^{\text{(real)}} h_{\text{ref}}/h_0$. Without loss of generality, we assume the GU is located at the origin when analyzing $G2A$ connection probabilities. Furthermore, we assume the velocities of the considered aircraft are the same, which can be approximated by the average velocity from the captured aircraft data, e.g., the velocity information broadcast by the Automatic dependent surveillance-broadcast (ADS-B) system from different flights in a representative time duration. We will show that simply using the average velocity fits the proposed model well in Section IV. Moreover, to provide more clear understanding, we draw a diagram to show the procedures of three approaches to compute the ground-centric $G2A$ connection probabilities in Fig. 3, where the real-world data based simulation (i.e., Fig. 11, and Figs. 14 - 17) verifies the accuracy of the theoretic $G2A$ connection probability, and the Monte-Carlo simulation (i.e., Fig. 13) shows the effect of non-ideal real-world data statistics.

B. Frequency Selection and Antenna Models

Different from cellular wireless communications, the ground-air communications involve distant end nodes, resulting in much higher opportunities of line-of-sight (LOS) channels. Propagating through atmosphere, radio waves encounter different attenuation at different frequencies. Thus, proper frequency bands need to be selected for $A2G/G2A$ communication. To enable tens-of-gigabytes high data rate connections between high-altitude and ground units, current ground-aircraft communications mainly consider two solutions, millimeter wave (mmWave) and free space optics (FSO) [9], [26], [27]. However, FSO suffer from significant attenuation in cloudy, foggy, rainy and snowy weathers, which makes several specific mmWave bands such as 71-86 GHz to be more suitable for high-rate ground-aircraft communication in all weather conditions. Therefore, for link capacity analysis, we consider mmWave frequency bands for ground-aircraft communication links.

An aircraft is moving with a high angular speed from the view of a GU/GS. Meanwhile, a very sharp beam is needed to enable high data rate connections between a GU/GS and an aircraft. Thus, the antenna system for ground-air communication should have decent tracking ability. Two types of antenna systems can fulfill these requirements, 1) a physically switchable ellipse antenna with a tracking system [26], and 2) a large-size antenna array with tracking ability [32]. In our link capacity analysis, we select the first option since its feasibility has been proved in different projects.

III. GROUND-CENTRIC G2A CONNECTIVITY

According to the 3GPP report [33], a ground-to-HAPS link’s LOS probability is over 91.9% for a 30-degree elevation angle in suburban and rural scenarios. Since the LOS probability is very high, we assume an aircraft can be successfully connected to a GU if the elevation angle between them is larger than a threshold.

From the view of a GU, the visibility of aircraft depends on multiple parameters, such as the relative locations of the GU and the aircraft, and the speed and trajectory of the aircraft. On the one hand, if the aircraft traffic in a certain region come from and aim to multiple airports (e.g., in populated areas), the traffics are more likely to be uncorrelated [30]. On the other hand, if the considered region is near an airport, the trajectories of different aircraft can be quite correlated.

PPP is a widely used model to analyze real-world random spatial patterns [34]. In wireless communication research, the PPP-based model usually provides a lower bound of the coverage probability of a transmitter [35]. Thus, we adopt PPP as the aircraft locations in a snapshot. And we simply assume in the theoretic analysis that the aircraft density remains the same in the considered region during the considered time for all snapshots. For the convenience of analysis, we assume that the aircraft’s moving directions, represented by speed azimuth angles, are independent and uniformly distributed in $[0^\circ, 360^\circ]$, and that all aircraft in the considered region
Theorem 1: Given the aircraft locations form PPP within \( B(0, R) \) at time \( t \), and \( v \) is the projected velocity of all aircraft, the \( \tau \)-connectivity probability of the G2A link, i.e., there are at least \( M \) aircrafts in \( B(0, R) \) for at least \( \tau \) seconds, is

\[
P_{G2A}(M) = \sum_{n=M}^{\infty} \sum_{m=M}^{n} \binom{n}{m} p_c^m (1 - p_c)^{n-m} \frac{\lambda_A \pi R^2}{n!} e^{-\lambda_A \pi R^2},
\]

where we define \( p_0 \equiv \int_{d - R}^{d} \frac{2r}{\pi R^2} \arccos \left( \frac{d^2 + r^2 - R^2}{2dr} \right) dr \), \( d \equiv \frac{v \tau}{w} \) and

\[
p_c = \begin{cases} 
  p_0 + \left( 1 - \frac{d}{R} \right)^2, & 0 \leq d \leq R \\
  p_0, & R < d \leq 2R \\
  0, & 2R < d.
\end{cases}
\]

Proof: Please see Appendix A.

Furthermore, from Theorem 1, we have the following corollary.

Corollary 1: For \( M = 1 \), we have

\[
P_{G2A}(M = 1) = 1 - \exp(-\lambda_A \pi R^2 p_c).
\]

For \( \tau \to 0 \), we have

\[
\lim_{\tau \to 0} P_{G2A}(M) = 1 - \sum_{n=0}^{M-1} \frac{\lambda_A \pi R^2}{n!} e^{-\lambda_A \pi R^2}.
\]

For \( M = 1 \) and \( \tau \to 0 \), we have

\[
\lim_{\tau \to 0} P_{G2A}(M = 1) = 1 - \exp(-\lambda_A \pi R^2),
\]

which is the non-empty probability of a PPP with density \( \lambda_A \) over \( B(0, R) \).

It can be interpreted that the strictly sufficient conditions of Theorem 1 and Corollary 1 are as follows.

1) The number of aircraft in \( B(0, R) \) is Poisson distributed with parameter \( \lambda_A \pi R^2 \), and this parameter is fixed, i.e., aircraft density is constant, during at least \( \tau \) second.

2) In a polar coordinate \((\rho, \theta)\), the aircraft distance \( \rho \) to the origin has a PDF of \( f(\rho) = 2\rho R^2, \rho \leq R \) and zero otherwise, and the position angle \( \theta \) is uniformly distributed in \([0, 2\pi]\). In other words, the aircraft are uniformly distributed in \( B(0, R) \).

3) On the projected plane at altitude \( h_{\text{ref}} \), all aircraft fly at the same constant velocity.

4) The aircraft moving direction is uniformly distributed in \([0, 2\pi]\), and each aircraft’s trajectory within \( B(0, R) \) is a straight line.

In next section, we will discuss how much the aircraft data fit to these idealized conditions, and how much theoretical and real world data-based results differ.

Theorem 1 and Corollary 1 show that the \( \tau \)-connection probability can be simply computed with three parameters such as the aircraft density \( \lambda_A \), the projected aircraft velocity \( v \), and the radius \( R \) (or reference height \( h_{\text{ref}} \) and elevation angle threshold \( \theta_{\text{ele}} \) equivalently). In other words, with estimated aircraft density and projected velocity values for a certain region and time, we are able to obtain the theoretical G2A \( \tau \)-connectivity without conducting real-world data based simulation. And given a target \( \tau \)-connectivity, we are able to compute the required variable values such as aircraft density and elevation angle threshold numerically. The effect of \( \theta_{\text{ele}} \),
λ_A and v are shown in Fig. 5 to Fig. 7, where the numbers on contours represent the connection probabilities and the observation angle, i.e., the zenith angle, is \((90° - \theta_{ele})\). Fig. 5 indicates that to achieve a 0.99 connection probability for at least 10 seconds needs the aircraft density and the observation angle to be no less than \(2 \times 10^{-3} \text{ km}^{-2}\) and 57°, respectively, when the average projected aircraft velocity is 200 m/s. Furthermore, Fig. 5 shows when aircraft density is low, increasing the observation angle achieves slow growth of connection probability, vice versa. Similarly, Fig. 6 and Fig. 7 also show that as the aircraft density grows, the connection probability becomes more sensitive to the variation of velocity and duration.

Moreover, we have the following corollary for the choice of reference height \(h_{\text{ref}}\).

**Corollary 2:** If the values of aircraft density \(\lambda_A\) and the projected velocity \(v\) are chosen according to the average (more generally, linear combination) of real-world aircraft density and the average (more generally, linear combination) of projected velocity in a certain duration, the choice of reference height \(h_{\text{ref}}\) does not affect the G2A \(\tau\)-connectivity probability.

**Proof:** Please see Appendix B.

## IV. HYPOTHESIS TESTING FOR THE G2A MODEL

In this subsection, we justify our proposed aircraft distribution model and the theoretic connection probability results with real-time aircraft data broadcast from the ADS-B systems. The G2A connection probability analysis is based on the assumption of PPP modeling of the aircraft locations, constant aircraft velocity, and uniformly distributed moving directions within a circular region. Thus, given the aircraft data, we discuss the following questions. First, how accurate are the aircraft distribution and movement models? Second, how accurate is the theoretic G2A connection probability based on these assumptions?

### A. Aircraft Data Acquisition

ADS-B system is a widely used surveillance system, with which an aircraft is able to receive the position information from Global Navigation Satellite System (GNSS) and then broadcast its position and other information, e.g., altitude, velocity, and call sign, to any other aircraft and ground stations. Typically, up to 6.2 messages can be transmitted per second at a 1090 MHz carrier frequency [36]. By gathering and decoding the broadcast messages with ADS-B receivers, real-time commercial aircraft information can be known at the ground station side. Furthermore, there are several websites and services that gather world-wide ADS-B messages and provide the real-time aircraft data on a map. We use Flightradar24 website as our data source.

In this paper, the aircraft data is sampled every 5 seconds, and the connection duration of \(\tau = 5k\) seconds, where \(k = 2,3,\ldots\), are tested. Particularly, we consecutively capture the figures from flightradar24 website, and process them with pattern recognition methods to extract aircraft information, within four selected regions inside the United States. A screenshot example is shown in Fig. 8, where the yellow object represents the location of an aircraft, the three lines of words indicate the call sign, velocity and altitude respectively.\(^2\)

To test the feasibility of the proposed model, we build four aircraft data sets which are described in Table I. Particularly, data sets 1 to 4 correspond to rural and suburban areas in Mojave National Preserve, Riverside city in CA, Kinnelon borough in NJ, and Joshua Tree National Park, respectively.

### B. Aircraft Traffic, Position, and Speed Distributions

In this subsection, we present and analyze the basic statistics of the aircraft data. First of all, the one-week aircraft densities

\(^2\)Based on our estimation, the error rate of recognizing call sign, velocity and altitude is about 5%, if no overlapped aircraft shown in the sampled picture.
Table I

<table>
<thead>
<tr>
<th>Data set</th>
<th>Location</th>
<th>Starting time/Duration</th>
<th>Avg/max/min density ($\times 10^{-4} \text{km}^{-2}$)</th>
<th>Avg altitude</th>
<th>Avg proj. velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34.89°N, -115.32°W</td>
<td>12/06/2019 3:30pm / 1 week</td>
<td>4.09 / 11.8 / 0</td>
<td>9.96 km</td>
<td>212 m/s</td>
</tr>
<tr>
<td>2</td>
<td>34.00°N, -117.28°W</td>
<td>01/09/2020 10:41am / 5 hours</td>
<td>6.28 / 7.46 / 2.15</td>
<td>2.70 km</td>
<td>335 m/s</td>
</tr>
<tr>
<td>3</td>
<td>41.01°N, -74.41°W</td>
<td>01/09/2020 13:39pm / 24 hours</td>
<td>3.69 / 9.70 / 0</td>
<td>9.82 km</td>
<td>226 m/s</td>
</tr>
<tr>
<td>4</td>
<td>33.90°N, -115.9°W</td>
<td>02/04/2020 7:00pm / 24 hours</td>
<td>7.93 / 16.8 / 0.160</td>
<td>8.03 km</td>
<td>235 m/s</td>
</tr>
</tbody>
</table>

Fig. 9. Aircraft trajectories in 24 hours (Data set-1).

Fig. 10. Aircraft traffic in a week (Data set-1).

Fig. 11. Void probabilities and p-values of aircraft numbers.

for data set-1 are shown in Fig. 10, where the density of each point is computed from the average of 1440 samples in 2 hours. Fig. 10 shows that the aircraft densities fluctuate daily, and that the maximum density is achieved on Thursday 11:30 am. It also shows that the daily bottoms appear from 3 am to 7 am, while the daily peaks are achieved either around noon or in the afternoon. The fluctuating aircraft density leads to different connection probabilities at different times. Thus, both the long term (e.g., one-week) and the short term (e.g., 2-hour) connection probabilities are tested.

Fig. 9 shows the first 24 hours’ accumulative trajectories of all aircraft within the observation region, i.e., above the elevation angle threshold $\theta_{\text{ele}} = 34.5^\circ$, where the small dots represent the aircraft positions and the straight lines represent the trajectories. Then, the 2-hour (51st-53rd hours) and 1-week statistics of aircraft altitude, velocity, moving direction (speed azimuth angle), distance to the center, and the relative position angle to the center (location azimuth angle) from data set-1, are shown in Fig. 12. As shown in Fig. 9, the aircraft fly towards different directions. And there are several common trajectories that are covered many times in the day, which correspond to the major speed azimuth angles and location azimuth angles in Fig. 12. Specifically, comparing the two columns in Fig. 12, we can see that the short term and long term statistics have some differences. For instance, the majority of altitude is around 1.1 km based on the 1-week data. But there are two peaks in the 2-hour based figure, around 0.8 km and 1.1 km. This indicates there is a relatively higher percentage of low altitude aircraft during the 51st to 53rd hour. Next, we can see most velocities are between 150 m/s and 250 m/s, and that the distance distribution in one week roughly fits to the PDF mentioned in the second strictly sufficient conditions of Theorem 1 and Corollary 1. On the other hand, the speed and location azimuth angle distributions are not similar to the idealized uniform distribution.

Next, the testing results, e.g., the p values and the void probabilities, for the number of aircraft within different observation angles are shown in Fig. 11, using the 51st to 53rd hour data in data set-1. In particular, we conduct a Chi-square goodness-of-fit test with the null hypothesis to be that the number of aircraft within the observation angle is Poisson distributed with the density to be the average real world aircraft number. Fig. 11 shows that the minimum p value is 0.046, which indicates that the null hypothesis can be accepted at a significance level no less than 0.046. On the other hand, the void probability curves also show the similarity between real-world aircraft number distribution and the Poisson distribution. These testing results indicate that the first strictly sufficient condition of Theorem 1 and Corollary 1 is achievable in practice.

C. G2A Connection Rates

In this subsection, we show that even though some idealized assumptions made in Theorem 1 cannot strictly fit the aircraft data, the proposed theorem is robust enough for real-world scenarios and yields just a small difference to the real world results.
First of all, we consider six cases of Monte-Carlo simulation to analyze the effects of nonideal aircraft location and speed distributions, i.e.,

- Case-0: all settings are idealized, (i.e., the same as in the theoretic analysis),
- Case-1: the velocity distribution is extracted from the real-world data; other settings are idealized,
- Case-2: the speed direction is extracted from the real-world data; other settings are idealized,
- Case-3: the distance distribution between an aircraft and the origin is from the real-world data; other settings are idealized,
- Case-4: the azimuth angle distribution of aircraft is extracted from the real-world data; other settings are idealized,
- Case-5: the distributions of the velocity, speed direction, distance between an aircraft and the origin, and the azimuth angle of aircraft are extracted from the real-world data.

Recall that examples of aforementioned data distributions are shown in Fig. 12. When extracting distributions from the real-world data, we use Gaussian kernel density estimation with optimal bandwidth under Gaussian density assumption [37].

Fig. 13 shows the G2A connection probabilities for different cases, where 10000 trials are averaged in Monte-Carlo simulations. We use the data in the 51st-53rd hour of data set-1, when the average aircraft density is $\lambda_A = 7.9 \times 10^{-4} \text{km}^{-2}$. As shown in Fig. 13, the theoretical G2A connection probabilities match the case-0, case-1, case-2, and case-4 well. Case-3 matches case-5 well, but their connection probabilities are different from the theoretical results. This indicates the non-idealized distance distribution between aircraft and the origin is a major factor that causes the gaps between theoretical and real G2A connection probabilities for the analyzed data.

Fig. 14 to Fig. 17 show the real-world data based and the theoretic G2A connection probabilities computed from the four data sets, where both short term (2-hour) and long term data are analyzed. We have the following observations. First, although the four data sets correspond to different aircraft altitude, speed, and location distributions, these figures show a good match between the theoretic and real G2A connection probabilities. This indicates that our proposed model is applicable to different real world scenarios. Second, the gap between theoretic connection probabilities and the real connection probabilities narrows down as more data are analyzed. For instance, in Fig. 14 to Fig. 17, the 2-hour data based probabilities have gaps around 0.05 for $M = 1$, whereas the gaps are very small for the 1-week and 24-hour data based probabilities.
Summary: In section IV-B, we have tested the distributions of real world data and showed that the first strict sufficient condition of Theorem 1 and Corollary 1 is achieved, and that the last three conditions are partially attained. We have also shown that the choice of reference height does not affect the computation of G2A $\tau$-connectivity. Then, section IV-C has shown that even though the strict sufficient conditions are not fully satisfied, the theoretic G2A connection probabilities match the practical data based probabilities well, especially when the data set is sufficient. The existing suburban/rural area aircraft data statistics have shown a good potential of using aircraft to support high data-rate delay-tolerant applications and to enhance existing wireless communication system.

V. AIRCRAFT-CENTRIC A2G CONNECTIVITY

In the analysis of ground-centric G2A connection probability, we focus on the data transmission from a GU to one or more aircraft, where the aircraft are flying through independent or quasi-independent trajectories at different speeds and altitudes. While in this section, we consider the scenario where data are transmitted from a moving aircraft to one or more ground stations (GSs), which is shown in Fig. 2. Note that in the G2A scenario, the relative locations among the aircraft are random, and their movement directions with respect to the GU are random, leading to different PPP realizations in different snapshots. In contrast, in the analysis of aircraft-centric A2G connection probability, the locations of GSs are fixed, resulting in the same PPP realization in the whole ground region all the time (but a gradually changing PPP realization in the aircraft’s footprint as the aircraft moves). Besides, the definitions of $\tau$-connectivity of the ground-centric link and the aircraft-centric link are different. For the ground-centric scenario, the GU always connects to the same airplane(s) within the $\tau$-second duration. However, for the aircraft-centric scenario, the condition of the aircraft always connecting to the same GS(s) within the $\tau$-second duration is not guaranteed. Thus, although we assume PPP for the location distributions of the aircraft in the G2A scenario and of the GSs in the A2G scenario respectively, these scenarios are not reciprocal and need to be considered separately.

In this section, we assume the aircraft is moving with a constant velocity $v$ towards a fixed direction at an altitude $h$ from time $t = t_0$ to $t = t_0 + \tau$. The locations of GSs form a PPP within the connectable range of the aircraft with a constant density $\lambda_G$. Furthermore, we assume the aircraft is able to connect to any GS if the GS is within the footprint of the aircraft, i.e., the elevation angle is above the threshold $\theta_{ele}$, where $h/R = \tan(\theta_{ele})$ and $R$ is the radius of the aircraft footprint. The A2G $\tau$-connection probability approximation is derived in Theorem 2.

**Theorem 2:** Given the aircraft moves with a constant speed $\vec{v}$ towards a fixed direction at an altitude $h$ from time $t = t_0$ to $t = t_0 + \tau$. The locations of GSs form a PPP within the connectable range of the aircraft with a constant density $\lambda_G$. Furthermore, we assume the aircraft is able to connect to any GS if the GS is within the footprint of the aircraft, i.e., the elevation angle is above the threshold $\theta_{ele}$, where $h/R = \tan(\theta_{ele})$ and $R$ is the radius of the aircraft footprint. The A2G $\tau$-connection probability approximation is derived in Theorem 2.

$$P_{A2G} \approx \tilde{P}_{A2G} = \left(1 - e^{-\pi R^2 \lambda_G}\right) \exp\left(\frac{2\sqrt{\pi R \lambda_G}}{e^{\pi R^2 \lambda_G} - 1}\right),$$

where $v = |\vec{v}|$ is the velocity of aircraft, $R = h/\tan(\theta_{ele})$.

Since there is only one aircraft considered in the aircraft-centric connectivity analysis, the parameter $v$ in this section represents the real-world aircraft velocity.
Proof: Please see Appendix C.

Although approximation is involved in (6), we observe that the theoretic $\tilde{P}_{A2G}$ matches the Monte-Carlo simulation results well when the connection probability is high. However, the approximation in (6) is less accurate in the low connection probability region. A more accurate approximation based on heuristic compensation is shown in Proposition 1 as follows.

Proposition 1: The $A2G$ $\tau$-connectivity can be accurately described by the product of a compensation multiplier $\zeta$ and $\tilde{P}_{A2G}$, where

$$\zeta = e^{\frac{-v^2 \tau^2}{2R^2} e^{-2\pi R^2 \lambda_G}}. \quad (7)$$

Thus, we use

$$\hat{P}_{A2G} = \zeta \tilde{P}_{A2G} \quad (8)$$

to represent the compensated $A2G$ $\tau$-connectivity.

The derivation of Proposition 1 is shown in Appendix D.

Fig. 18 shows the theoretic and Monte-Carlo simulation based $A2G$ connection probabilities under several settings. We can see that $\tilde{P}_{A2G}$ matches the Monte-Carlo simulation well when the duration is small or the connection probability is high (i.e., the footprint radius $R$ and the GS density $\lambda_G$ are large), and that with the compensation multiplier $\zeta$, $\hat{P}_{A2G}$ matches the Monte-Carlo simulation based probability very well.

Next, Fig. 19 shows the $A2G$ connection probability versus velocity under different GS density and connection duration. For high GS density (e.g., $\lambda_G = 4 \times 10^{-3}$ km$^{-2}$) or short duration (e.g., $\tau = 30$ s), it is observed that 1) both the approximate probability $\hat{P}_{A2G}$ and the compensated probability $\hat{P}_{A2G}$ match the Monte-Carlo simulation well, 2) the variation of $A2G$ connection probability is small when velocity changes. When the GS density is low, the connection duration is long, and the velocity is high, the gap between $\hat{P}_{A2G}$ and Monte-Carlo simulation becomes large, but the compensation parameter $\zeta$ reduces the gap. Thus, Fig. 18 and Fig. 19 indicate $\hat{P}_{A2G}$ is a practically accurate $\tau$-connection probability expression for the $A2G$ link.

Furthermore, the contours of $A2G$ connection probability for different settings of GS density and footprint radius are shown in Fig. 20. It can be observed that as the radius $R$ grows, i.e., as the elevation angle decreases, the $A2G$ connection probability is influenced by the GS density more significantly. In particular, a 60-second connection probability higher than 0.95 with $v = 250$ m/s cannot be achieved if the GS density is less than $4.1 \times 10^{-3}$ per km$^2$ or the footprint radius is less than 15.2 km.

VI. CASE STUDIES

As the key theorems and corollary for the ground-aircraft connectivity are derived and verified in Section III to V, we investigate two relevant issues in this section, namely, 1) the tradeoff analysis between $\tau$-connectivity and outage capacity for the ground-centric G2A link, and 2) the link capacity analysis and comparison. In particular, discussing the first issue can provide more insight about how aircraft serve the GU in a ground-centric scenario and how to optimize the number of connectable aircraft given that the GU’s power is limited and the number of aircraft is sufficient. The analysis of the second issue proposes a potential data relaying scheme using the ground-aircraft communication paradigm and shows the advantage over a pure large-scale LEO satellite system.

A. Case-1: Ground-Centric G2A Tradeoff Analysis

After deriving the $\tau$-connectivity for the G2A link in Theorem 1, a following question is that how many aircraft a GU should connect to achieve maximum capacity. On the
one hand, increasing the number of connected aircraft $M$ decreases the connection probability. On the other hand, if there are multiple connectable aircraft, spatial multiplexing can be applied to achieve higher G2A capacity. Thus, in this subsection, we analyze the theoretical tradeoff between the connection probability and the outage capacity for G2A links. Without loss of generality, we assume the GU uses spatial multiplexing scheme and is capable to track and connect to multiple aircraft with a highly directional antenna system. For the $m$th G2A link, we represent the integrated channel gain, path loss, and antenna gain as $g_m(t)$, $\tau$, and $\gamma_m(t)$, respectively, and define the total transmit power $P$ where $P$ is the total transmit power. Then, assuming each G2A link has a bandwidth $W$ and that the noise power is $\sigma^2$, the ergodic (in terms of the time duration $\tau$) outage capacity for $M$ connected aircraft in $\tau$ seconds is

$$C(M, \tau) = \frac{P_{G2A}(M)}{\tau} \int_{t=0}^{\tau} \sum_{m=1}^{M} \log_2(1 + \gamma_m(t)) \, dt,$$

where $\gamma_m(t) = g_m(t)P_m(t)/\sigma^2$, and $\sum_{m=1}^{M} P_m(t) = P$.

Particularly, in low SNR regime, we have

$$\lim_{\{\gamma_m(t)\} \to 0} C(M, \tau) = \log_2(e)P_{G2A}(M)\frac{W}{\tau} \int_{t=0}^{\tau} \sum_{m=1}^{M} \gamma_m(t) \, dt.$$  

Since $P_{G2A}(M)$ decreases with the growth of $M$ and $\gamma_m(t)$ approaches to zero when $P \to 0$, the optimal connecting strategy for the low-SNR case is only maintaining the best G2A link, which maximizes not only the capacity but also the $\tau$-connection probability. In high SNR regime, equal transmit power allocation achieves the optimal water-filling capacity asymptotically. Thus, assuming equal power allocation, the outage capacity is

$$\lim_{\{\gamma_m(t)\} \to -\infty} C(M, \tau) = P_{G2A}(M)\frac{W}{\tau} \int_{t=0}^{\tau} \sum_{m=1}^{M} \log_2\left(\frac{g_m(t)P}{\sigma^2M}\right) \, dt.$$  

Theoretically, to achieve the maximum capacity at the high-SNR regime, the optimal $M$ can be derived from equation (11).

In the numerical simulation, we simply assume all links have the same gain and bandwidth, i.e., $g_m(t) = g, \forall m, t$, $W = 100$MHz, and define $\Gamma = \frac{\Gamma}{\sigma^2}$ to be the total receive SNR. Fig. 21 and Fig. 22 show the theoretic G2A connection probability $P_{G2A}(M)$ and the outage capacities under different aircraft density, respectively. Fig. 21 and Fig. 22 indicate that increasing the number of connected aircraft $M$ can reduce the connection probability and either increase or decrease the outage capacity depending on other system settings. In particular, higher aircraft density leads to the peaks of outage capacity to be achieved at a large $M$.

B. Case-2: Link Capacity Analysis

In this subsection, we analyze the ground-to-air communication system from the perspectives of average capacity and its advantages by integrating with low earth orbit (LEO) satellite communication system (LEO-SCS). Specifically, we consider a power and battery limited GU with non-terrestrial wireless access. With LEO-SCS alone, the GU uses the LEO satellite service such that the data are sent from the GU to a satellite and then relayed to a GS connected to the Internet, i.e., the data route is G-S-G. In an aircraft-aided LEO-SCS, as aircraft are available to relay the GU’s data packets, two additional route options are G-A-G and G-A-S-G. The first route can be selected if the $\tau$-connectivity requirements of both G2A and A2G links are satisfied, and otherwise, the second route should be selected. The satellite system has a full coverage of the considered area such that the connectivity of G2S, S2G and A2S links are always satisfied. Compared with the G-S-G route, both the G-A-G and G-A-S-G routes have the advantages of saving the GU’s transmit power and increasing the link capacity since the G2A distance is much shorter than the G2S distance. In addition, sending through the G-A-G route also saves the transmit power of the serving satellite. In the following of this subsection, we compare the transmit power and achievable capacity of both G2A and G2S links, where the OneWeb LEO satellite constellation [17] is used.

For ground-aircraft/ground-satellite communications, the cloud attenuation affects the propagation loss significantly and higher frequency corresponds to severer cloud attenuation. Thus, we consider different carrier frequencies for the two links: 1) 14.25 GHz, which is approved by Federal Communications Commission (FCC) for the use of mobile-satellite service [38], 2) 82 GHz, which is a tested frequency for high-rate G2A/A2G communications [26], 3) 26 GHz, 38 GHz, 46 GHz and 68 GHz, which are identified 5G frequencies by ITU [39]. Each frequency corresponds to different vertical
cloud attenuation $\rho_C$ and vertical atmospheric attenuation $\rho_A$.

The bandwidth is assumed to be 100 MHz for all transmissions. Other system settings are shown in the Appendix E.

We define the $\tau$-connected capacity $C_{con}$ to be the average data rate in $\tau$ seconds when the connection is guaranteed, and the effective outage capacity $C_{eff}$ to be the average $\tau$-connection data rate in a certain period, e.g., several hours or days, as

$$C_{con} = \frac{B_0}{\tau N_{c,\tau}} \sum_{i=1}^{N_{c,\tau}} \int_{t_i}^{t_i+\tau} \log_2 (1 + \gamma_R(t)) \, dt,$$

$$C_{eff} = \frac{N_{c,\tau} C_{con}}{N_{c}}$$

where $t_i$ is the start time of the $i$th $\tau$-connectable duration, $\gamma_R(t)$ is the instantaneous receiving SNR at time $t$, $N_c$ is the total number of durations in the considered period, among which $N_{c,\tau}$ is the total number of durations that are $\tau$-connectable. Thus, for G2A links, $C_{eff} = P_{G2A} C_{con}$, while for G2S links, the two data rates are the same since we assume the satellites have full coverage on the lands.

In the numerical simulation, we select the 51st to 53rd hours aircraft data in data set-1. For both G2A and G2S scenarios, we set $\tau = 10$ s. If the GU is connectable to multiple satellites for $\tau$ seconds, we selected the nearest satellite to connect. Different cloud attenuation levels $\rho_C$ are considered, where corresponding percentage indicates the portion of time of a year when the real cloud attenuation is larger than $\rho_C$.

Fig. 24 shows the capacity versus transmit power curves for the G2S link with 14.25 GHz carrier frequency (FCC approved mobile satellite band) and the G2A links at carrier frequencies 14.25 GHz, 68 GHz (identified 5G frequency) and 82 GHz (practically tested band for G2A link in the literature). The $\tau$-connected capacities of the G2A link are higher than the capacities of the G2S link for all considered conditions. In terms of the effective achievable capacity, if the two links use the same band of 14.25 GHz, the G2A link clearly outperforms the G2S link. If we consider the G2S link at 14.25 GHz versus the G2A link at 68 GHz and 82 GHz (unfair comparisons since the cloud attenuations are much severer at 68 GHz and 82 GHz), the effective outage capacities of the G2A link are still better than the G2S capacities in the low transmit power regime around 10 – 33 dBm for 68 GHz and 10 – 27 dBm for 82 GHz, but worse in the high transmit power regime above 33 dBm and 27 dBm. (This is mainly because the $\tau$-connected capacity is scaled by the less-than-one connection probability for the G2A link.) Even in these unfair cases, the results indicate that transmitting through the G2A link rather than the G2S link is preferable for power and energy limited application scenarios.

As the cloud condition significantly affects the propagation attenuation, we compare the G2A and G2S $\tau$-connected capacity under different total columnar water vapour contents and frequencies, in Fig. 25. The exceeding probabilities of different total columnar water vapour content values are also shown in the figure. We can see that the capacities decrease with the increase of the total columnar water vapour content since the cloud attenuation caused by the water vapour increases. The cloud attenuation is severer at higher frequencies than

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4 According to [40], the cloud attenuation of a particular location can be calculated based on total-columnar-water-vapour-content at that place. ITU document [41] offers a data sheet with annual total-columnar-water-vapour-content of different locations for different exceeding probabilities. For example, a total-columnar-water-vapour-content value $x$ with exceeding probability $p$ indicates the total-columnar-water-vapour-content of the considered place can exceed $x$ in the 100$p$ percent of the year.

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at lower frequencies, resulting that the link capacities of different frequencies decrease with different slopes. Clearly, when using 14.25 GHz and 26 GHz carrier frequencies, the G2A \( \tau \)-connected capacities outperform the G2S capacity. Increasing the G2A carrier frequency leads to more probability that the 14.25 GHz G2S capacity outperforms the G2A capacity. With the G2A carrier frequency at the high end (82 GHz), the G2A \( \tau \)-connected capacity outperforms the G2S capacity when the total columnar water vapour content is lower than 9 kg/m². Moreover, for the given transmit power, the probability that the 82-GHz G2A \( \tau \)-connected capacity is larger than the G2S capacity is about 48% in a year.

VII.结论

In this paper, we studied the ground-aircraft wireless connectivity and link capacity. We proposed a method to model the ground-centric connection, derived the \( \tau \)-connectivity for the ground-centric link, and verified the theoretical ground-centric connection probability with real world aircraft data. Next, we proposed and derived the aircraft-centric \( \tau \)-connectivity in closed-form expression. With the ground-centric and aircraft-centric \( \tau \)-connectivity, we can efficiently analyze the performance of continuous ground-aircraft connection with a small number of parameters, such as the aircraft/GS density, the aircraft velocity, and the connectable elevation angle threshold. Finally, with practical antenna parameters and atmospheric gas and cloud attenuation data, we compared link capacities between the G2A link and the G2S link and demonstrated the advantage of ground-aircraft communication.

APPENDIX

A. Proof of Theorem 1

The \( \tau \)-second connectivity of each aircraft is independent, since the aircraft locations are PPP distributed. Denoting the \( \tau \)-second connection probability for one aircraft to be \( p_c \), and given \( \Phi(B(0,R)) = N \), i.e., there are \( N \) aircraft in \( B(0,R) \), we know that the number of aircraft that have \( \tau \)-second connectivity, \( M \) is Binomial distributed, i.e., \( M \sim \text{Binomial}(N, p_c) \). Thus, given the existence of \( N \) aircraft, the \( \tau \)-second connection probability for at least \( M \) aircraft is

\[
P_{G2A}(M, \Phi(B(0,R)) = N) = \sum_{m=M}^{N} \binom{N}{m} p_c^m (1 - p_c)^{N-m}.
\]

Furthermore, \( N \) is a Poisson random variable with parameter \( \lambda_A \pi R^2 \) since the aircraft locations are PPP distributed with density \( \lambda_A \). Therefore, we derive (1) as

\[
P_{G2A}(M) = \sum_{n=M}^{\infty} P_{G2A}(M, \Phi(B(0,R)) = n) \frac{(\lambda_A \pi R^2)^n}{n!} e^{-\lambda_A \pi R^2} = \sum_{n=M}^{\infty} \sum_{m=M}^{n} \binom{n}{m} p_c^m (1 - p_c)^{n-m} \frac{(\lambda_A \pi R^2)^n}{n!} e^{-\lambda_A \pi R^2}.
\]

(15)

Next, defining \( d = v \tau \) to be the distance that an aircraft travels during \( \tau \) seconds, \( p_c \) can be derived for three scenarios according to distance \( d \).

First, for \( R \leq d \leq 2R \), given an aircraft located at a distance \( r \) from the origin and that its moving direction is uniformly distributed within \([0, 2\pi]\), the \( \tau \)-second connection probability is

\[
p_c(r) = \begin{cases} \frac{\theta(r)}{\pi}, & d - R \leq r \leq R \\ 0, & \text{otherwise} \end{cases}
\]

(16)

where \( \theta(r) = \arccos \left( \frac{R^2 + r^2 - d^2}{2rd} \right) \) is the angle corresponding to the distance \( r \), as shown in the LHS of Fig. 26. Specifically, the geometrical relationship shown in the LHS of Fig. 26 indicates that, for \( R \leq d \leq 2R \), an aircraft located at \( r \) distance can still be within region \( B(0,R) \) after it has moved a distance \( d \) towards a uniformly distributed random direction, only if it is moving to the shadow region in Fig. 26, which corresponds to a probability of \( \theta(r)/\pi \) and leads to (16). Note that if \( 0 \leq r < d - R \), the aircraft cannot be within \( B(0,R) \) after \( \tau \) seconds no matter what direction it has moved.

Since the aircraft form a PPP within \( B(0,R) \), the PDF of distance \( r \) is

\[
f_r(r) = \frac{2}{R^2}, \quad 0 \leq r \leq R.
\]

(17)

Therefore, we have

\[
p_c = \int_0^R p_c(r) f_r(r) \, dr,
\]

which equals \( p_0 \) in Theorem 1.

Second, for \( d < R \), and if \( r \leq R - d \), the aircraft can still be within \( B(0,R) \) after it has moved a distance \( d \) regardless of its direction, which corresponds to probability \( (R - d)^2/R^2 \). On the other hand, if \( r > R - d \), the aircraft needs to move towards the shadow region shown in the RHS of Fig. 26, which also corresponds to a probability of \( \theta(r)/\pi \). Deconditioning on \( r \),
we derive \( p_0 \) as in Theorem 1. Therefore, the summation of \((R - d)^2/R^2\) and \(p_c\) corresponds to the \(\tau\)-second connection probability \(p_c\) for an aircraft when \(d < R\). Last, if \(d > 2R\), the aircraft has moved out of the connectable region after \(\tau\) seconds, resulting in \(p_c = 0\).

\[ p_c = \begin{cases} 
\frac{p_0}{\pi\cot^2(\theta_{ele})} & 0 \leq \tau \leq \frac{\cot(\theta_{ele})}{b(\cdot)} \\
\frac{\cot(\theta_{ele})}{b(\cdot)} & \frac{\cot(\theta_{ele})}{b(\cdot)} < \tau \leq \frac{2\cot(\theta_{ele})}{b(\cdot)} \\
0 & \frac{2\cot(\theta_{ele})}{b(\cdot)} < \tau.
\end{cases} \]

\( B. \) Proof of Corollary 2

We can estimate \( \lambda_A \) by

\[
\hat{\lambda}_A = \frac{1}{T} \sum_{t=0}^{T} \frac{|N_{\theta_{ele}}[t]|}{\pi R^2}, \tag{18}
\]

where \(N_{\theta_{ele}}[t]\) is the index set of the aircraft that are above the elevation angle threshold at snapshot \(t\), \(t = 0, 1, 2, \ldots, T\), \(|N_{\theta_{ele}}[t]|\) is the volume of \(N_{\theta_{ele}}[t]\), and \(T\) is the total number of snapshots involved in the estimation. Similarly, the velocity is estimated as

\[
\hat{v} = \frac{1}{T} \sum_{t=0}^{T} \frac{1}{|N_{\theta_{ele}}[t]|} \sum_{i \in N_{\theta_{ele}}[t]} v_i[t]
= \frac{1}{T} \sum_{t=0}^{T} \sum_{i \in N_{\theta_{ele}}[t]} \frac{1}{|N_{\theta_{ele}}[t]|} \frac{v_i^{(\text{real})}[t]}{h_i[t]} h_{ref} \tag{19}
\]

where \(v_i[t], v_i^{(\text{real})}[t]\), and \(h_i[t]\) are the projected velocity, the real velocity, and the altitude of the \(i\)th aircraft above the elevation angle threshold, respectively. We further define

\[
a(\theta_{ele}, T, N_{\theta_{ele}}) \triangleq \frac{1}{T} \sum_{t=0}^{T} \frac{|N_{\theta_{ele}}[t]|}{\pi \cot^2(\theta_{ele})} \tag{20}
\]

and

\[
b(\theta_{ele}, T, N_{\theta_{ele}}, \mathcal{V}^{(\text{real})}, \mathcal{H}) \triangleq \frac{1}{T} \sum_{t=0}^{T} \sum_{i \in N_{\theta_{ele}}[t]} \frac{1}{|N_{\theta_{ele}}[t]|} \frac{v_i^{(\text{real})}[t]}{h_i[t]}, \tag{21}
\]

where \(\mathcal{V}^{(\text{real})}\) and \(\mathcal{H}\) are the real-world aircraft velocity and altitude sets, respectively. For simpler expressions, we use \(a(\cdot)\) and \(b(\cdot)\) to represent the left-hand-side of (20) and (21), respectively.

Consequently, we have

\[
\hat{\lambda}_A = a(\cdot) h_{ref}^{-2}, \tag{22}
\]

and

\[
\hat{v} = b(\cdot) h_{ref}. \tag{23}
\]

Substituting (22) and (23) to the equations in Theorem 1, we have

\[
P_{G2A}(M) = \sum_{n=\max(M,1)}^{\infty} \sum_{m=\max(M,1)}^{n} \binom{n}{m} p_c^n(1-p_c)^{n-m} \frac{\pi \cot^2(\theta_{ele}) a(\cdot)^n}{n! e^{\pi \cot^2(\theta_{ele}) a(\cdot)}}, \tag{24}
\]

where we define

\[
p_0 \triangleq \int_{\tau b(\cdot) - \cot(\theta_{ele})}^{\cot(\theta_{ele})} \frac{2x e^{-\frac{\tau b^2(\cdot) - \cot^2(\theta_{ele})}{2\tau b(\cdot)}}}{\pi \cot^2(\theta_{ele})} \times \arccos \left( \frac{\tau^2 b^2(\cdot) - \cot^2(\theta_{ele}) + x^2}{2\tau b(\cdot)} \right) \, dx \tag{25}
\]

Therefore, by using the average density \(\hat{\lambda}_A\) and the average projected velocity \(\hat{v}\) as \(\lambda_A\) and \(v\) in the theoretical G2A probability analysis, we can represent \(P_{G2A}(M)\) with equations (24) to (26). Since the reference height \(h_{ref}\) is not involved in (24) to (26), we can conclude that the reference height is not a factor that affects the ground-centric \(\tau\)-connectivity.

\[ C. \) Proof of Theorem 2

We divide the duration \(\tau\) into \(N\) equal time slots, using \(t_i, i = 0, 1, 2, \ldots, N-1\) to represent the beginning of the \(i\)th time slot, and \(\Delta t\) to be the duration of a slot, where \(\tau = \Delta tN\). Denoting event \(S_i\) to be the successful connection between the aircraft and a GS at time \(t_i\), and event \(F_i\) to be the failure of connection between the aircraft and all GSs at time \(t_i\), the successful connection probability during \(\tau\) seconds is represented as

\[
\lim_{\Delta t \to 0} \mathbb{P}(S_0, S_1, \ldots, S_{N-1}) = \lim_{\Delta t \to 0} \mathbb{P}(S_0) \prod_{i=1}^{N-1} \mathbb{P}(S_i|S_{i-1}, \ldots, S_0). \tag{27}
\]

To derive this probability, we begin with discrete-time analysis and then extend to continual-time results. First, we project the aircraft location to the ground plane such that the connectable GSs at time \(t_i\) are the GSs that are within the circle centered at the aircraft’s location with a radius of \(R = h \cot(\theta_{ele})\), where we call this circular region as the footprint of the aircraft. Thus, we have the following lemma.

**Lemma 1:** As the aircraft moves from time \(t_0\) to \(t_{N-1}\), the \(\tau\)-duration connectivity can be guaranteed if and only if there always exists at least one GS in each of all footprints of the aircraft.

According to Lemma 1, we can further interpret \(S_i\) as and event that there exists at least one GS \(i\) within the footprint of the aircraft at time \(t_i\) and \(F_i\) as an event that there is no GS within the footprint of the aircraft at time \(t_i\). Due to the difficulty in deriving a closed form expression of the conditional probability \(\mathbb{P}(S_i|S_{i-1}, \ldots, S_0)\), we use the following approximation,

\[
\mathbb{P}(S_i|S_{i-1}, \ldots, S_0) \approx \mathbb{P}(S_i|S_{i-1}). \tag{28}
\]

Furthermore, since the GS locations form a PPP, we have probability

\[
\mathbb{P}(S_0) = 1 - \mathbb{P}(F_0) = 1 - e^{-\lambda_G \pi R^2}. \tag{29}
\]
The above analysis indicates that the \( \tau \)-second connection probability for A2G link can be written as

\[
P_{A2G} = \lim_{\Delta t \to 0} \Pr(S_0, S_1, \ldots, S_{N-1}),
\]

and the approximation is

\[
P_{A2G} \approx \tilde{P}_{A2G} = \lim_{\Delta t \to 0} \Pr(S_0) \prod_{i=1}^{N-1} \Pr(S_i | S_{i-1}).
\]

Thus, the next step is to derive \( \Pr(S_i | S_{i-1}) \).

Fig. 27 shows the geometrical relationship of two successive footprints at time \( t_{i-1} \) and \( t_i \), where \( \Delta \theta = \Delta t \cdot v \) is the distance between the centers of footprints. \( B_i \) represents the \( i \)th footprint, \( A_i \) represents the intersection area between the \( (i-1) \)th and \( i \)th footprints, and \( A_i = B_i - A_{i-1} \). It can be derived that the areas of \( A_i \) and \( \bar{A}_i \) are

\[
|A_i| = 2\theta R^2 - \Delta \theta dR \sin \theta = \hat{A}, \quad \forall i
\]

\[
|\bar{A}_i| = (\pi - 2\theta)R^2 + \Delta \theta dR \sin \theta = \hat{A}, \quad \forall i
\]

respectively, where \( \theta = \arccos \frac{\Delta \theta}{2\pi R} \). Next, we have

\[
\Pr(S_i | S_{i-1}) = 1 - \Pr(F_i \cap S_{i-1}) = 1 - \Pr(\Phi(\bar{A}_i) = 0)\Pr(\Phi(A_i) = 0)\Pr(\Phi(A_i) = 0) | S_{i-1}, \quad \forall i
\]

where

\[
\Pr(\Phi(A_i) = 0) = e^{-|\hat{A}| \lambda_G}
\]

and

\[
\Pr(\Phi(A_i) = 0) | S_{i-1} = \sum_{k=1}^{\infty} \Pr(\Phi(B_i - A_i) = k, \Phi(B_{i-1}) = k) \Pr(\Phi(B_{i-1}) > 0)
\]

\[
= \sum_{k=1}^{\infty} \Pr(\Phi(A) = k, \Phi(B_{i-1}) = k) \Pr(\Phi(B_{i-1}) > 0)
\]

\[
= \sum_{k=1}^{\infty} \Pr(\Phi(A) = k) \Pr(\Phi(B_{i-1}) = k) \Pr(\Phi(B_{i-1}) > 0)
\]

\[
= \sum_{k=1}^{\infty} \frac{(|\hat{A}|)^k}{\pi R^2} \frac{(\lambda_G \pi R^2)^k}{k!} \frac{e^{-\lambda_G \pi R^2}}{1 - e^{-\lambda_G \pi R^2}}
\]

\[
= \sum_{k=1}^{\infty} \frac{(\lambda_G |\hat{A}|)^k}{k!} \frac{e^{-\lambda_G |\hat{A}|}}{1 - e^{-\lambda_G \pi R^2}}
\]

\[
= \left(1 - e^{-\lambda_G |\hat{A}|} \right) \frac{e^{-\lambda_G |\hat{A}|}}{1 - e^{-\lambda_G \pi R^2}} = \frac{e^{-\lambda_G (\pi R^2 - |\hat{A}|)}}{e^{-\lambda_G \pi R^2} - 1}.
\]

Substituting (35) and (36) into (34), we have

\[
\Pr(S_i | S_{i-1}) = 1 - \frac{1 - e^{-\lambda_G |\hat{A}|}}{e^{-\lambda_G \pi R^2} - 1}.
\]

Next, substituting (29) and (37) into (31), we derive

\[
\tilde{P}_{A2G} = \lim_{\Delta t \to 0} \left(1 - e^{-\lambda_G \pi R^2}\right) \left[1 - \frac{1 - e^{-\lambda_G |\hat{A}|}}{e^{-\lambda_G \pi R^2} - 1}\right]^N.
\]

Note that both \( N \) and \( |\hat{A}| \) are functions of \( \Delta t \). By taking the logarithm of \( \tilde{P}_{A2G} \) and using L’Hôpital’s rule, we have

\[
\ln(\tilde{P}_{A2G}) = \ln \left(1 - e^{-\lambda_G \pi R^2}\right) + \lim_{\Delta t \to 0} \frac{\partial \ln \left[1 - \frac{1 - e^{-\lambda_G |\hat{A}|}}{e^{-\lambda_G \pi R^2} - 1}\right]}{\partial \Delta t / \tau}
\]

\[
= \ln \left(1 - e^{-\lambda_G \pi R^2}\right) - \frac{2\tau R \lambda_G}{e^{-\pi R^2 \lambda_G} - 1}.
\]

Therefore,

\[
\tilde{P}_{A2G} = \left(1 - e^{-\pi R^2 \lambda_G}\right) \exp \left(-\frac{2\tau R \lambda_G}{e^{-\pi R^2 \lambda_G} - 1}\right),
\]

which completes the proof of Theorem 2.

D. Derivation of Proposition 1

The inaccuracy of \( \tilde{P}_{A2G} \) comes from the approximation made in (28). Recognizing that \( \Pr(S_i | S_{i-1}, \ldots, S_0) < \Pr(S_i | S_{i-1}) \) and that a larger \( i \) leads to more difference between \( \Pr(S_i | S_{i-1}, \ldots, S_0) \) and \( \Pr(S_i | S_{i-1}) \), we can introduce a heuristic compensation multiplier

\[
e_i = \exp(-a \Delta d^2 i),
\]

where \( a > 0 \) needs to be estimated, such that \( e_i \Pr(S_i | S_{i-1}) \) is a more accurate approximation to \( \Pr(S_i | S_{i-1}, \ldots, S_0) \). Then, we have

\[
\tilde{P}_{A2G} = \tilde{P}_{A2G} \lim_{\Delta t \to 0} \prod_{i=1}^{N-1} e_i
\]

\[
= \tilde{P}_{A2G} \lim_{\Delta t \to 0} e^{-a \Delta d^2 i \sum_{i=1}^{N-1} i}
\]

\[
= \tilde{P}_{A2G} \lim_{\Delta t \to 0} e^{-a \Delta d^2 \frac{N(N-1)}{2}} = \tilde{P}_{A2G} e^{-a \Delta d^2}. \]

Next, by comparing the theoretic \( \tilde{P}_{A2G} \) and the Monte Carlo simulation results, we estimate \( a = \frac{1}{e \pi R^2 \lambda_G} \). Defining \( \zeta \triangleq e^{-a \Delta d^2} \) and substituting it to (42), we obtain the result shown in Proposition 1.

E. System Settings for Link Capacity Analysis

Since the ambient temperature in the space is lower than that in the atmosphere, we assume the satellite system has a smaller noise temperature, which are 290 K and 190 K, respectively. The other receiver loss is set to be 7.8 dB according to [9], which consists of noise figure and other SNR losses. The GU’s equivalent antenna size and the satellite’s receive antenna size are set to be 0.3 m and 0.5 m, respectively. And the antenna efficiency is assumed to be 65%.

We assume LOS channels and consider the atmospheric gas and cloud attenuation in the link budget model. The instantaneous receiving SNR at time \( t \) is given by

\[
\gamma_R(t) = P_T + G_T + G_R - 20 \log \left(\frac{4\pi f_d(t)}{c}\right) - \rho_A + \rho_C \sin(\theta(t)),
\]

(43)
where $P_T$ is the transmit power, $G_T$ and $G_R$ are the transmit and receive antenna gain in dB respectively, $P_N$ is the noise power in dB, $d(t)$ is the distance between the GU and the receiver of aircraft/satellite at time $t$, $c$ is the speed of light, $\theta(t)$ is the elevation angle from the GU to the airplane/satellite, and $\rho_A$ and $\rho_C$ are the attenuation of atmospheric gas and cloud, respectively in dB. The vertical atmospheric gas attenuation $\rho_A$ is obtained from NASA's handbook [42]. The vertical cloud attenuation $\rho_C$ is computed according to the ITU document [40]. Specifically, $\rho_C$ is the product of cloud liquid water specific attenuation coefficient and total columnar content of liquid water, where the former is computed according to [40] and the latter is computed according to the data sheet specified by [41].

REFERENCES

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