An Investigation Into Training Signal Design For Frequency Offset Estimation

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Abstract

This paper addresses training signal design for frequency offset estimation. Based on the CRB, the best training signal structure is obtained with constraints on the total energy and peak-to-average energy ratio (PAR) of the training signal. The frequency offset estimation method for the proposed training signal is also presented which can handle normalized frequency offset less than approximately 0.5. To handle larger normalized frequency offsets, modified training structures and corresponding estimation methods are presented. Depending on the PAR constraint, the proposed training signals and corresponding methods can achieve 3 to 5 dB SNR advantage over the reference methods which use training signal consisting of multiple identical parts generated by pseudo-noise sequence in the frequency-domain or time-domain. The proposed training signals can reduce the estimation complexity and increase the system capacity as well.

I. Introduction

Training signals are commonly used in communications systems for timing synchronization, frequency synchronization and channel estimation. In [1]- [4] (and references therein), training signal design for channel estimation in single carrier systems was discussed. In [5], training signal design for channel estimation in OFDM systems was described. In [6] [7] [8], some training signal designs for timing synchronization in OFDM systems were presented. For frequency synchronization in OFDM systems, [9] used a training signal consisting of two identical parts ($L = 2$) and [10] [11] used a training signal composed of $L > 2$ identical parts. These training signals are generated by IFFT of pseudo-noise sequence on every L^{th} subcarrier in the frequency-domain. The above timing and frequency synchronization methods are based on time-domain processing and hence, also applicable to single carrier systems with the same training signal. Alternatively, for easier training generation in single carrier systems, the training signal with L identical parts can be constructed such that each part is of a time-domain pseudo-noise sequence.

In this paper, we investigate training signal design for frequency offset estimation in AWGN channel. The Cramer-Rao lower bound (CRB) for frequency offset estimation is first derived and then the best training signal with peak energy constraint is sought for based on the CRB. Next, frequency offset estimation method for the proposed training signal is presented. In order to handle larger frequency offsets, modified training signals are proposed. Frequency offset estimation methods suitable for these training signals are also derived based on the best linear unbiased estimation (BLUE) principle.

II. THE CRAMER-RAO LOWER BOUND (CRB)

Frequency offset is unavoidable at the receiver due to the oscillators' inaccuracies and the Doppler shift of the mobile wireless channel. In this paper, we consider frequency offset estimation based on the training signal vector $\boldsymbol{s} = [s(0),\ s(1),\ \ldots,\ s(N-1)]^T$ where the superscript T denotes transpose. Let us consider a signal model given by

$$\boldsymbol{r} = e^{j\psi}\ \boldsymbol{W}(v)\boldsymbol{s} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{r}$ is the received training vector of length N, $\boldsymbol{W}(v)$ is a diagonal matrix with diagonal elements $\{\ exp(j2\pi 0v/N),\ exp(j2\pi 1v/N),\ exp(j2\pi 2v/N),\ \ldots,\ exp(j2\pi(N-1)v/N)\}$ corresponding to the normalized frequency offset v (normalized by $1/(NT)$ where T is the sample duration), $\boldsymbol{n}$ is the complex additive white Gaussian noise (AWGN) vector with a covariance matrix $\boldsymbol{C_n} = \sigma_n^2\boldsymbol{I}$ where $\boldsymbol{I}$ is an identity matrix, and ψ represents an arbitrary phase.

For the parameter vector $\boldsymbol{\alpha} = [v,\ \psi]^T$, the received vector $\boldsymbol{r}$ has a complex Gaussian pdf, $p(\boldsymbol{r};\boldsymbol{\alpha})$, with the mean vector and the covariance matrix respectively given by

$$\boldsymbol{\mu_r} = e^{j\psi}\ \boldsymbol{W}(v)\boldsymbol{s},\ \boldsymbol{C_r} = \boldsymbol{C_n} = \sigma_n^2\boldsymbol{I}. \tag{2}$$

The $[i,j]$ element of the Fisher information matrix is given by

$$J_{i,j}{=}E\left[\frac{\partial\ ln\ p(\boldsymbol{r};\boldsymbol{\alpha})}{\partial\alpha_i}\ \frac{\partial\ ln\ p(\boldsymbol{r};\boldsymbol{\alpha})}{\partial\alpha_j}\right] \tag{3}$$

$$={tr}\left[\boldsymbol{C_r}^{-1}\ \frac{\partial\boldsymbol{C_r}}{\partial\alpha_i}\ \boldsymbol{C_r}^{-1}\ \frac{\partial\boldsymbol{C_r}}{\partial\alpha_j}\right] + 2\ \Re\left[\frac{\partial\boldsymbol{m_r}^H}{\partial\alpha_i}\ \boldsymbol{C_r}^{-1}\ \frac{\partial\boldsymbol{m_r}}{\partial\alpha_j}\right] \tag{4}$$

where α_i is the i^{th} element of $\boldsymbol{\alpha}$. We have then

$$J_{1,1}{=}\frac{8\pi^2}{N^2\ \sigma_n^2}\ \boldsymbol{s}^H\Lambda^2\boldsymbol{s} \tag{5}$$

$$J_{1,2}{=}J_{2,1} = \frac{4\pi}{N\sigma_n^2}\ \boldsymbol{s}^H\Lambda\boldsymbol{s} \tag{6}$$

$$J_{2,2}{=}\frac{2}{\sigma_n^2}\boldsymbol{s}^H\ \boldsymbol{s} \tag{7}$$

where Λ is a diagonal matrix with diagonal elements $\{0,\, 1,\, \ldots,\ N-1\}$. The CRB of the frequency offset estimation is given by

$$\text{CRB}{=}[J^{-1}]_{1,1} = (J_{1,1} - J_{1,2}\ J_{2,2}^{-1}\ J_{2,1})^{-1} \tag{8}$$

$$=\frac{N^2\sigma_n^2}{8\pi^2}\ \frac{\boldsymbol{s}^H\boldsymbol{s}}{\boldsymbol{s}^H\boldsymbol{s}\boldsymbol{s}^H\Lambda^2\boldsymbol{s} - (\boldsymbol{s}^H\Lambda\boldsymbol{s})^2}. \tag{9}$$

III. TRAINING SIGNAL DESIGN

In this section, we investigate the training signal design that minimizes the CRB. The best training vector $\boldsymbol{s}_o$ is given by

$$\boldsymbol{s}_o = arg\ min_{\boldsymbol{s}}\ \{\text{CRB}\}\ \text{, constraint: }\ \boldsymbol{s}^H\boldsymbol{s} = E_s \tag{10}$$

where E_s is the total energy of the training vector. From (10) and after some manipulation, we have the best training signal given by

$$\boldsymbol{s}_o = arg\ min_{\boldsymbol{s}} \left((\boldsymbol{s}^H \Lambda \boldsymbol{s})^2 - \boldsymbol{s}^H \boldsymbol{s}\boldsymbol{s}^H \Lambda^2 \boldsymbol{s} \right), \text{constraint: } \boldsymbol{s}^H \boldsymbol{s} = E_s \tag{11}$$

$$= arg\ min_{\boldsymbol{s}} \left(\sum_{k=0}^{N-1} \sum_{n=0}^{N-1} |s(k)|^2 |s(n)|^2 (nk - n^2) \right), \text{constraint: } \boldsymbol{s}^H \boldsymbol{s} = E_s. \tag{12}$$

From (12), it can be observed that the phases of the training samples have no effect on the CRB. It is only the training energy allocation among the training samples that has effect on the CRB and that has to be found. Define the following:

$$\boldsymbol{y} = [|s(0)|^2,\ |s(1)|^2,\ \ldots, |s(N-1)|^2]^T \tag{13}$$

$$G_{k,n} = n(k-n)\ ,\ 0 \le k \le (N-1),\ 0 \le n \le (N-1) \tag{14}$$

where $G_{k,n}$ is the $[k, n]$ element of an $N \times N$ matrix G. Note that the matrix G is singular and not symmetric. $\boldsymbol{y}$ can be viewed as the training energy vector or the training energy allocation. Then the best training vector is determined by the best training energy allocation given by

$$\boldsymbol{y}_o = arg\ min_{\boldsymbol{y}} \left\{ \boldsymbol{y}^T G \boldsymbol{y} \right\}$$

$$\text{constraint: } \boldsymbol{y}^T \mathbf{1} = E_s \text{ and } 0 \le y(i) \le P \tag{15}$$

where $\mathbf{1}$ is an all-one vector of length N, P is a design value representing the allowable peak training sample energy. The reason for the introduction of the upper limit P is to avoid non-linear distortion of the training signal at the transmit amplifier. The optimization (15) containing equality and non-equality constraints can be solved by quadratic programming (QP) such as MATLAB function $quadprog.m$. The general result can be given by

$$y_o(k) = \begin{cases} min\{P, E_s/2\} & \text{if } k = 0 \text{ or } N-1 \\ min\left\{P, E_s/2 - \sum_{l=0}^{m-1} y_o(l)\right\} & \text{if } k = m \text{ or } k = N-1-m;\ 1 \le m \le N/2-1 \\ E_s - 2\ \sum_{l=0}^{k-1} y_o(l) & \text{if } k = (N-1)/2 \text{ and } N \text{ is an odd integer.} \end{cases} \tag{16}$$

In the following, we present an alternative approach which gives the same result as QP. Let f_d represent the optimization objective function of (15) with the subscript d indicating that the training energy is allocated to d samples only. We investigate f_d, which is to be minimized, for different values of d, their locations (indexes of the d samples) and the energy allocation among the d samples.

(i) If $y(i) = E_s \delta[i-n], i = 0,\ 1,\ \ldots,\ N-1$ for any n ($0 \le n \le N-1$), then we have

$$f_1 = y^2(n) G_{n,n} = 0. \tag{17}$$

(ii) If $y(i) = y(k)\delta[i-k] + y(n)\delta[i-n],\ i = 0,\ 1,\ \ldots,\ N-1$ for any k and n such that $0 \le k < n \le N-1$ and $y(k) \ne 0$, $y(n) \ne 0$, then after some manipulation, we have

$$f_2 = -y(k)y(n)(n-k)^2. \tag{18}$$

Since $f_2 < f_1$, allocating the total training energy on two samples is better than allocating on one sample. Now we focus on minimizing the CRB. $k = 0$ and $n = N-1$ will maximize $(n-k)^2$

and with the constraint $y(k) + y(n) = E_s$, $y(k) = y(n) = E_s/2$ will maximize $y(k)y(n)$. The corresponding minimum value of f_2 is

$$(f_2)_{min} = -(E_s/2)^2(N-1)^2. \tag{19}$$

(iii) If $y(i) = y(k)\delta[i-k] + y(p)\delta[i-p] + y(n)\delta[i-n]$, $i = 0,\ 1,\ \ldots,\ N-1$ for any k, p and n such that $0 \le k < p < n \le N-1$ and $y(k) \ne 0$, $y(p) \ne 0$, $y(n) \ne 0$, then after some manipulation, we have

$$f_3 = -y(k)y(p)(p-k)^2 - y(k)y(n)(n-k)^2 - y(p)y(n)(n-p)^2. \tag{20}$$

It can be observed that $k = 0$ and $n = N-1$ will minimize f_3. $y(k) = y(n)$ will minimize the dominant term $-y(k)y(n)(n-k)^2$. With $k = 0$, $n = N-1$ and $y(k) = y(n)$, f_3 will be minimized if $p = 1$ or $p = N-2$. Then we have

$$(f_3)_{min} = -(E_s/2)^2(N-1)^2 + E_s y(p)(N^2/2 - 3/2) + y^2(p)(N^2/4 - 3N/2 + 9/4). \tag{21}$$

Since $(f_2)_{min} < (f_3)_{min}$, allocating training energy on two samples is better than allocating on three samples. Furthermore, (21) indicates that if there is a peak sample energy constraint of P where $E_s/2 > P \ge E_s/3$, then the best energy allocation is $y(0) = y(N-1) = P$ and $y(p) = E_s - 2P$. If $P \ge E_s/2$, then the best allocation is $y(0) = y(N-1) = E_s/2$ and $y(p) = 0$ which becomes allocation on two samples only.

(iv) If $y(i) = y(k)\delta[i-k] + y(l)\delta[i-l] + y(m)\delta[i-m] + y(n)\delta[i-n]$, $i = 0,\ 1,\ \ldots,\ N-1$ for any k, l, m and n such that $0 \le k < l < m < n \le N-1$ and $y(k) \ne 0$, $y(l) \ne 0$, $y(m) \ne 0$, $y(n) \ne 0$, then after some manipulation, we have

$$\begin{aligned} f_4 = & -y(k)y(l)(l-k)^2 - y(k)y(m)(m-k)^2 - y(k)y(n)(n-k)^2 \\ & -y(l)y(m)(m-l)^2 - y(l)y(n)(n-l)^2 - y(m)y(n)(n-m)^2. \end{aligned} \tag{22}$$

Similar in (iii), $k = 0$ and $n = N-1$ will minimize f_4. $y(k) = y(n) = [E_s - y(l) - y(m)]/2$ will minimize the dominant term $-y(k)y(n)(n-k)^2$. With these parameter values, f_4 achieves a minimum at $l = 1$, $m = N-2$, and $y(l) = y(m)$. The corresponding minimum f_4 is

$$(f_4)_{min} = -(E_s/2)^2(N-1)^2 + E_s y(l)(2N-4). \tag{23}$$

Since $(f_2)_{min} < (f_4)_{min} < (f_3)_{min}$, allocating training energy on four samples is better than allocating on three samples. Allocating training energy on two samples is better than allocating on four samples. Furthermore, (23) indicates that for a peak sample energy constraint of P where $E_s/2 > P \ge E_s/4$, the best energy allocation is $y(0) = y(N-1) = P$ and $y(1) = y(N-2) = (E_s - 2P)/2$. If $P \ge E_s/2$, then the best allocation is $y(0) = y(N-1) = E_s/2$ and $y(l) = y(m) = 0$ which becomes allocation on two samples only.

(v) Similarly we can obtain f_d for $d > 4$, the corresponding sample locations, and the energy allocation. The general result is that $(f_{2m})_{min} < (f_{2l})_{min}$ for $1 \le m < l \le N/2$ and $(f_{2m})_{min} < (f_{2m+1})_{min}$. Allocating on even number of samples is better than allocating on odd number of samples. For the allocation on even number of samples ($2m$ samples), the sample indexes are $\{0, 1, \ldots, m-1, N-m,\ N-m+1, \ldots, N-1\}$ and the allocation is such that $y(k) = y(N-1-k)$. If there is a peak sample energy constraint of P where $E_s/(2(m-1)) > P \ge E_s/(2m)$, then the best energy allocation is $y(k) = y(N-1-k) = P$ for $k = 0,\ 1, \ldots, m-2$ and $y(m-1) = y(N-m) = (E_s - 2(m-1)P)/2$. If $P \ge E_s/(2(m-1))$ and $E_s/(2(n-1)) > P \ge E_s/(2n)$, then the best allocation is $y(k) = y(N-1-k) = P$ for $k = 0,\ 1, \ldots, n-2$ and $y(n-1) = y(N-n) = (E_s - 2(n-1)P)/2$ which becomes allocation on $2n$ samples only. These results altogether coincide with the result (16) obtained from QP.

IV. FREQUENCY OFFSET ESTIMATION

In this section we propose a few frequency offset estimation methods based on the designed training signal. According to the proposed training design, let us consider the training signal of length N samples where only the first K and the last K samples are nonzero. For simplicity and illustration purpose, let us assume that $E_s/(2K) = P$ and the energy of each non-zero training sample is P where P is the maximum training sample energy allowable without any non-linear distortion at the transmit amplifier.

Considering the AWGN channel, the first step in frequency estimation is noise suppression by means of neglecting the received samples corresponding to the zero training samples. The second step is to perform frequency offset estimation. Assuming that the first K samples and the last K samples are identical, the frequency offset estimate can simply be obtained as

$$\hat{v} = \frac{N}{2\pi(N-K)} arg\{\sum_{k=0}^{K-1} r(k)r^*(k+N-K)\}. \tag{24}$$

This estimator assumes that the normalized frequency offset v has to satisfy the condition $|v| < \frac{N}{2(N-K)}$. For the situations where v does not satisfy this condition, we propose the following approaches.

Let the non-zero training samples contains three parts $\{s(0), \ldots, s(K-1)\}$, $\{s(M), \ldots, s(M+K-1)\}$, and $\{s(N-K), \ldots, s(N-1)\}$ where $M \geq K$ and M should be chosen to satisfy the condition $|v| < \frac{N}{2M}$. Each of these samples has energy $P = E_s/(3K)$. The first step is noise suppression by means of neglecting the received samples corresponding to the zero training samples. The second step is to calculate the frequency offset estimate. We consider an approach using correlation between the three parts where the phases of $s(k)$, $s(k+M)$, and $s(k+N-K)$, $k = 0, 1, \ldots, K-1$, from three parts have to be the same for lower complexity. Let us assume for simplicity that the three parts are identical. Then we have three estimates $\{\theta_k\}$ based on three correlation terms as:

$$\theta_0 = \frac{N}{2\pi M} arg\{\sum_{k=0}^{K-1} r(k)r^*(k+M)\} \tag{25}$$

$$\beta_1 = \sum_{k=0}^{K-1} r(M+k)r^*(N-K+k) \tag{26}$$

$$\beta_2 = \sum_{k=0}^{K-1} r(k)r^*(N-K+k) \tag{27}$$

$$v_c = \frac{N}{2\pi M} arg\{\beta_1^* \beta_2\} \tag{28}$$

$$\theta_1 = v_c + \frac{N}{2\pi(N-M-K)} arg\{\beta_1 e^{-j\frac{2\pi v_c(N-M-K)}{N}}\} \tag{29}$$

$$\theta_2 = v_c + \frac{N}{2\pi(N-K)} arg\{\beta_2 e^{-j\frac{2\pi v_c(N-K)}{N}}\} \tag{30}$$

where the use of v_c is to avoid possible ambiguity in frequency offset estimation based on correlation terms with larger correlation distances. The frequency offset estimate based on BLUE can then be given by

$$\hat{v} = \sum_{k=0}^{2} \theta_k w_k \tag{31}$$

where the weighting values $\{w_k\}$ are the elements of the weighting vector $\boldsymbol{w}$ which is calculated [13] as follows:

$$\boldsymbol{w} = \frac{\boldsymbol{C}_\theta^{-1}\,\mathbf{1}}{\mathbf{1}^T\,\boldsymbol{C}_\theta^{-1}\,\mathbf{1}}. \tag{32}$$

Note that for simplicity and illustration purpose, we have assumed that energy of each non-zero training sample has $P = E_s/(3K)$. Following Method B of [11], the covariance matrix of $\{\theta_k\}$ can be given by

$$\boldsymbol{C}_{\boldsymbol{\theta}} = \frac{3N}{8\pi^2\mathrm{SNR}} \begin{bmatrix} \frac{2+\frac{3K}{N\cdot\mathrm{SNR}}}{M^2} & \frac{-1}{M(N-K-M)} & \frac{1}{M(N-K)} \\ \frac{-1}{M(N-K-M)} & \frac{2+\frac{3K}{N\cdot\mathrm{SNR}}}{(N-K-M)^2} & \frac{1}{(N-K-M)(N-K)} \\ \frac{1}{M(N-K)} & \frac{1}{(N-K-M)(N-K)} & \frac{2+\frac{3K}{N\cdot\mathrm{SNR}}}{(N-K)^2} \end{bmatrix} \tag{33}$$

where $\mathrm{SNR} = E_s/(N\sigma_n^2)$. The required inverse covariance matrix is then given by

$$\boldsymbol{C}_{\boldsymbol{\theta}}^{-1} = \rho \begin{bmatrix} M^2(1+\frac{3K}{N\cdot\mathrm{SNR}}), & M(N-K-M), & -M(N-K) \\ M(N-K-M), & (N-K-M)^2(1+\frac{3K}{N\cdot\mathrm{SNR}}), & -(N-K-M)(N-K) \\ -M(N-K), & -(N-K-M)(N-K), & (N-K)^2(1+\frac{3K}{N\cdot\mathrm{SNR}}) \end{bmatrix} \tag{34}$$

where $\rho = 8\pi^2\mathrm{SNR}^2/[27K(1+K/(N\cdot\mathrm{SNR}))]$.

Substituting (34) into (32) yields the BLUE weighting vector

$$\boldsymbol{w} = \frac{[M^2,\ (N-K-M)^2,\ (N-K)^2]^T}{(M^2+(N-K-M)^2+(N-K)^2)}. \tag{35}$$

Note that although $\boldsymbol{C}_{\boldsymbol{\theta}}^{-1}$ depends on SNR values, $\boldsymbol{w}$ does not.

The results from Section III may be applied to the modified training signal by extending the sample-wise results to the part-wise results. For example, the modified training signal consisting of four parts may be better than that consisting of three parts. In order to investigate this, in the following, we derive a BLUE frequency offset estimation method for the modified training signal consisting of four non-zero parts $\{s(0),\ \ldots, s(K-1)\}$, $\{s(M),\ \ldots, s(M+K-1)\}$, $\{s(N-M-K),\ \ldots, s(N-M-1)\}$ and $\{s(N-K),\ \ldots,\ s(N-1)\}$ where $M \geq K$ and M should be chosen to satisfy the condition $|v| < \frac{N}{2M}$. For simplicity, we assume that these four parts have identical phases. To keep the same PAR as in three part training signal, each sample of the first and fourth parts has energy $E_s/(3K)$ and that of the second and third parts has energy $E_s/(6K)$. Define the following:

$$R_0 = \sum_{k=0}^{K-1}\{r^*(k)r(k+M) + r^*(N-M-K+k)r(N-K+k)\} \tag{36}$$

$$R_1 = \sum_{k=0}^{K-1} r^*(M+k)r(N-M-K+k) \tag{37}$$

$$R_2 = \sum_{k=0}^{K-1}\{r^*(k)r(N-M-K+k) + r^*(M+k)r(N-K+k)\} \tag{38}$$

$$R_3 = \sum_{k=0}^{K-1} r^*(k)r(N-K+k). \tag{39}$$

Then we have four estimates $\{\theta_k : 0 \leq k \leq 3\}$ as follows:

$$\theta_0 = \frac{N}{2\pi M} arg\{R_0\} \tag{40}$$

$$\theta_1 = \beta + \frac{N}{2\pi(N-2M-K)} arg\{R_1 e^{-j2\pi\beta(N-2M-K)/N}\} \tag{41}$$

$$\theta_2 = \beta + \frac{N}{2\pi(N-M-K)} arg\{R_2 e^{-j2\pi\beta(N-M-K)/N}\} \tag{42}$$

$$\theta_3 = \beta + \frac{N}{2\pi(N-K)} arg\{R_3 e^{-j2\pi\beta(N-K)/N}\} \tag{43}$$

$$\beta = \frac{N}{2\pi M} arg\{R_2^* R_3\}. \tag{44}$$

The frequency offset estimate based on BLUE can then be given by

$$\hat{v} = \sum_{k=0}^{3} \theta_k w_k \tag{45}$$

where $\{w_k\}$ are determined by (32) with the covariance matrix given by

$$\boldsymbol{C_\theta} = \frac{N}{4\pi^2 \text{SNR}} \begin{bmatrix} \frac{\frac{9}{2}(\frac{1}{2}+\frac{K}{N\text{SNR}})}{M^2}, & \frac{-3}{M(N-2M-K)}, & \frac{-3/4}{M(N-M-K)}, & \frac{3/2}{M(N-K)} \\ \frac{-3}{M(N-2M-K)}, & \frac{18(\frac{1}{3}+\frac{K}{N\text{SNR}})}{(N-2M-K)^2}, & \frac{3}{(N-M-K)(N-2M-K)}, & 0 \\ \frac{-3/4}{M(N-M-K)}, & \frac{3}{(N-M-K)(N-2M-K)}, & \frac{\frac{9}{2}(\frac{1}{2}+\frac{K}{N\text{SNR}})}{(N-M-K)^2}, & \frac{3/2}{(N-M-K)(N-K)} \\ \frac{3/2}{M(N-K)}, & 0, & \frac{3/2}{(N-M-K)(N-K)}, & \frac{\frac{9}{2}(\frac{2}{3}+\frac{K}{N\text{SNR}})}{(N-K)^2} \end{bmatrix} \tag{46}$$

Note that $\boldsymbol{C_\theta}$ and $\boldsymbol{w}$ are functions of SNR. However, it can easily be checked that $\boldsymbol{w}$ is not sensitive to SNR value changes.

V. Simulation Results and Discussions

Simulations have been carried out to evaluate the estimation performance obtained with the proposed training signal. The system can be either a single carrier system or an OFDM system. In the simulation we use an OFDM system with FFT size $N = 1024$. As comparison, similar BLUE-based methods (Minn [12] and M&M [10]) which use a pseudo-noise sequence in the frequency domain are included. The training signal for [12] and [10] consists of $L = 8$ identical parts as in [12].

The frequency offset estimation performance for the proposed training signal and the proposed frequency offset estimation method is presented in Fig. 1. The normalized frequency offset v is set to 0.4 and the proposed training signal is composed of two non-zero parts, each having K samples. The performance is evaluated for K = 1, 8, 32, 64, and 128 which correspond to the PAR values of 27, 18, 12, 9, and 6 dB, respectively. The training signal used for [12] and [10] has a PAR value of about 6 dB. For an OFDM signal with 1024 subcarriers, the PAR value can occasionally be as high as 30 dB.

The simulation results are in line with the theoretical results. The estimation performance is better for smaller values of K. The performances for $K = 1$ and $K = 128$ have approximately 1 dB SNR difference. The proposed training signal with the proposed estimation method outperforms the reference methods [12] [10]. The proposed training signal with $K = 1$ has about 5 dB SNR advantage over the reference methods while that with $K = 128$ has about 4 dB SNR advantage.

Fig. 2 presents the performance comparison among the proposed estimation method (45) with the modified training signal consisting of four non-zero parts and the reference methods [12] [10]. Each part of the modified training signal has K samples. The performance is evaluated for $K = 1$, 8, 32, 64, and 128 which correspond to the PAR values of 25.33, 16.30, 10.28, 7.27, and 4.26 dB, respectively. The normalized frequency offset v is set to 1.6. $M = 128$ is used for all K values. Same observations as in Fig. 1 are obtained. A slightly different observation from Fig. 1 is that the modified training signal with $K = 1$ has about 4 dB and that with $K = 128$ has about 3 dB SNR advantage over the reference methods.

The performance comparison between the modified training signal with 3 non-zero parts and with 4 non-zero parts, both having the same total training energy and the same PAR, is presented in Fig. 3. The modified training signal with 4 non-zero parts is slightly better than that with 3 non-zero parts. This agrees with the implication of the result in Section III. Note that the slight performance improvement of the training signal with 4 non-zero parts over 3 non-zero parts is associated with some increased complexity. We have also performed simulations for the reference methods using a training signal consisting of 8 identical parts, where each part is a pseudo-noise sequence in time-domain. The performance results (not shown) are almost the same as those obtained with the training signal consisting of 8 identical parts generated by a pseudo-noise sequence in the frequency-domain. Note that there is no restriction on the phases of the training samples within each non-zero part of the proposed training signals. The phases can be designed such that lowest PAR of the analog training signal is obtained. Also note that in place of zero training samples, data signal can be transmitted which would increase the capacity.

VI. Conclusions

A training signal structure with PAR constraint for frequency offset estimation is developed based on the CRB. The proposed training signal consists of two non-zero parts, each having K samples. The first part is at the beginning of the training signal of N samples and the second is at the end. The corresponding proposed frequency offset estimation method can handle normalized frequency offset $|v| < \frac{N}{2(N-K)}$. A modified training signal consisting of 3 or 4 non-zero parts, each having K samples, and the corresponding BLUE methods are presented for handling larger normalized frequency offset $|v| < N/(2M)$. The first part and the last are at the beginning and at the end of the training signal, the third (and the fourth) are spaced M samples away from the beginning (and the end). The proposed training signals and corresponding estimation methods achieve approximately 3 to 5 dB SNR advantage over the reference methods which utilize training signal consisting of several identical parts generated by pseudo-noise sequence in the frequency or time domain. The proposed training signals can not only improve the estimation performance but also reduce the estimation complexity and increase the system capacity as well.

Acknowledgment

This work was supported, in part, by the Clark Foundation Grant, Erik Jonsson School of Engineering and Computer Science, University of Texas at Dallas, U.S.A.

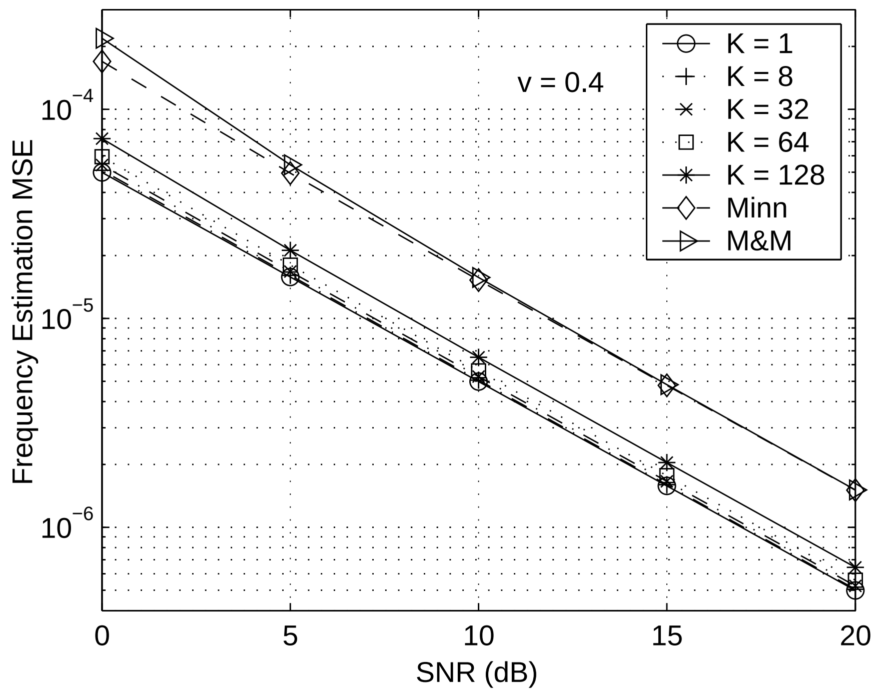

Fig. 1. The frequency estimation MSE performance comparison among the proposed method with the proposed training signal consisting of two non-zero parts and the reference methods (Minn[12] and M&M[10]) using a training signal containing 8 identical parts generated by a pseudo-noise sequence in frequency-domain

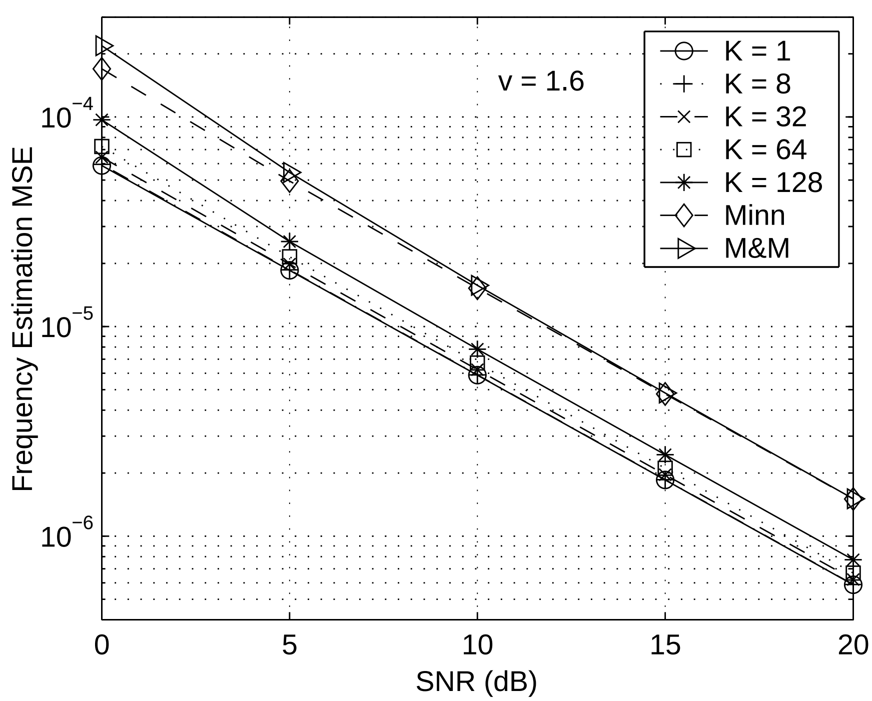

Fig. 2. The frequency estimation MSE performance comparison among the proposed method with the modified training signal consisting of 4 non-zero parts and the reference methods (Minn[12] and M&M[10]) using a training signal containing 8 identical parts generated by a pseudo-noise sequence in frequency-domain

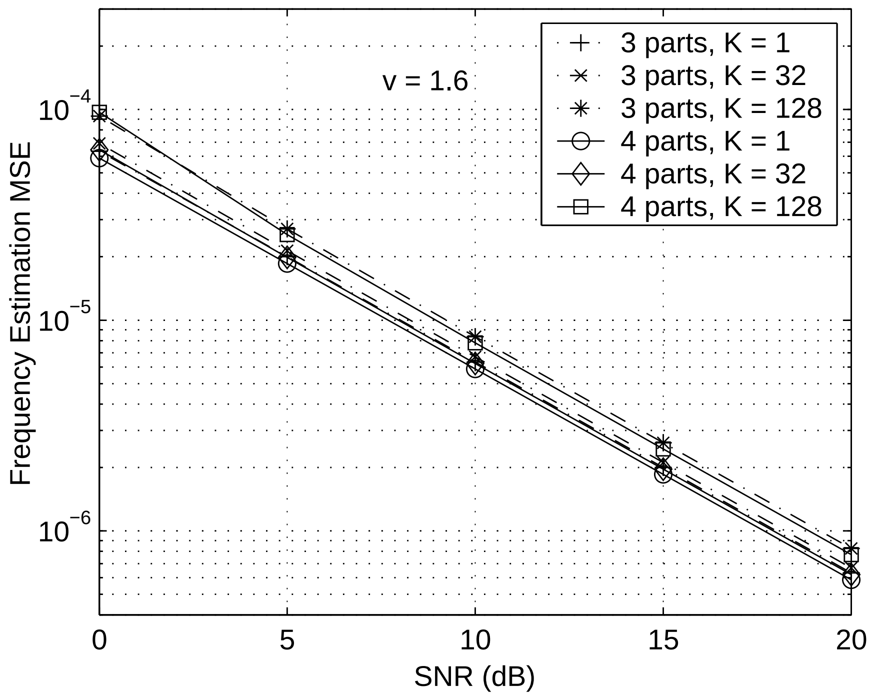

Fig. 3. The frequency estimation MSE performance comparison between the proposed method with the modified training signal consisting of 3 non-zero parts and that with 4 non-zero parts

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