Linear Prediction Receiver for Differential Space-Time Block Codes With Spatial Correlation

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***Abstract*—The differential space-time block code is well-known to provide full spatial diversity and allows simple differential detection. Recent results have shown performance degradation when the channel is time-varying. This letter shows the effect of spatial correlation between transmit antennas on the error performance. A generalized receiver which exploits spatial correlation information, is proposed and shown to outperform an existing receiver in terms of frame error rate for slow varying channels with moderate to high spatial correlation.**

***Index Terms*—Differential space-time block code (DSTBC), maximum-likelihood (ML) receiver, spatial correlation.**

I. INTRODUCTION

TAROKH and Jafarkhani's differential space-time block codes (DSTBCs) [2] have been well-known to provide full spatial diversity and can be detected without channel state information. The transmission matrix for two transmit antennas is in the Alamouti's form [1] which allows simple differential detection (DD) with linear complexity. However, when the channel is time-varying, DD yields performance degradation and irreducible error floor appears. In [5], an approximate maximum-likelihood (ML) receiver is proposed and it shows mitigation of error floor significantly. In [7], an idea of joint channel estimation and detection with linear prediction is proposed and it further reduces the error floor. Both papers discussed the receivers which embed linear predictors which have scalar prediction coefficients and considered the channel to be time-varying and independent between each transmission paths.

When the transmission paths are correlated due to limited antenna spacing, there is some performance degradation due to loss in diversity. The idea of including spatial correlation at the receiver has been discussed in [4] for receive diversity. It is shown that exploiting spatial correlation at the receiver does not substantially improve the bit error rate (BER) compared to the receiver without it.

This letter concentrates on transmit diversity employing DSTBC when there is spatial correlation between transmit antennas. The results show performance degradation due to spatial correlation. Then, a generalized receiver, which exploits spatial correlation information, is proposed. The proposed receiver applies a linear predictor which is in a matrix form. It shows some improvement over the existing receiver which is easily seen in terms of frame error rate (FER). Exploiting transmit antennas spatial correlation at the receiver seems to have an advantage when the spatial correlation is moderate to high and fading is slow.

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II. SYSTEM MODEL

The system model and notations follow from [5] consisting of two transmit antennas and one receive antenna. An input vector $\boldsymbol{d}_n = [d_n^1 d_n^2]^t$, representing n^{th} pair of symbols, is selected from an M-ary (2^b-ary) constellation according to the $2b$ input bits. Each symbol has an amplitude of $1/\sqrt{2}$ so the total average transmit power from two transmit antennas is one. The vector $\boldsymbol{d}_n$ is multiplied by a unitary matrix to obtain a mapping vector

$$\boldsymbol{x}_n = \begin{bmatrix} A_n \\ B_n \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \boldsymbol{d}_n. \tag{1}$$

Then, differential encoding produces a transmission vector [2]

$$\boldsymbol{s}_n = \begin{bmatrix} s_{2n-1} \\ s_{2n} \end{bmatrix} = \boldsymbol{D}_{n-1}^t \boldsymbol{x}_n \tag{2}$$

where $\boldsymbol{D}_n = \begin{bmatrix} s_{2n-1} & s_{2n} \\ -s_{2n}^* & s_{2n-1}^* \end{bmatrix}$ is an Alamouti's transmission matrix. The symbol s_{2n-1} and s_{2n} are transmitted simultaneously at signaling interval $2n-1$ from the first and second antennas, respectively. The symbol $-s_{2n}^*$ and s_{2n-1}^* are transmitted simultaneously at signaling interval $2n$ from the first and second antennas, respectively.

Each transmit-receive antenna path is a time-varying, frequency-flat Rayleigh fading channel with Doppler power spectrum according to Jakes model. Let $a_i[n]$ denote fading gains corresponding to the i^{th} transmit antenna where $i = 1$, 2. Then, $a_1[n]$ and $a_2[n]$ are zero mean complex Gaussian random variables, each with unit variance and autocorrelation $R_{a_i}[m] = R_a[m] = J_0(2\pi f_d T m)$ where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, $f_d T$ is the normalized maximum Doppler frequency. Spatial correlation between the two channels can be represented by a covariance matrix [3]

$$E \begin{bmatrix} a_1^*[n] & a_2^*[n] \\ a_1[n] & a_2[n] \end{bmatrix} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}. \tag{3}$$

As in [5], the receiver is derived based on the assumption of constant channel gains within a transmission matrix. This assumption is approximately valid for slow varying channel. The received signal vector $\boldsymbol{r}_n = [r_{2n-1}\ r_{2n}]^t$ can be written as [5]

$$\boldsymbol{r}_n = \boldsymbol{D}_n \boldsymbol{a}_n + \boldsymbol{w}_n \tag{4}$$

where $\boldsymbol{a}_n = [a_1[2n-1]a_2[2n-1]]^t$ is the channel gain vector and $\boldsymbol{w}_n = [w_{2n-1}\ w_{2n}]^t$ is the noise vector. The elements w_{2n-1} and w_{2n} are zero mean complex Gaussian random variables, each with variance $1/(2\mathrm{SNR}) = \sigma_w^2/2$ per real dimension. The knowledge of maximum Doppler frequency and spatial correlation are assumed to be available at the receiver.

III. An Approximate ML Receiver With a Scalar Linear Predictor

The receiver in [5] is an approximate ML receiver searching for the sequence which minimizes Euclidean distances between channel gains computed from the present received signal and those from the linear predictor. Viterbi algorithm applies the following branch metric

$$\Lambda(\Gamma_n, \boldsymbol{d}_{n+1}) = \left\| \boldsymbol{D}_{n+1}^H \boldsymbol{r}_{n+1} - \sum_{k=1}^{Q} b_k \boldsymbol{D}_{n+1-k}^H \boldsymbol{r}_{n+1-k} \right\|^2 \quad (5)$$

where b_k is the k^{th} linear prediction coefficient of the *two-step* linear predictor of order Q. We can determine coefficients $[b_1 b_2 \ldots b_Q]$ from Cholesky decomposition of the matrix $\tilde{\boldsymbol{M}}^{-1} = (\boldsymbol{C}_{\dot{a}} + \sigma_w^2 \boldsymbol{I})^{-1}$, where $\boldsymbol{C}_{\dot{a}} = E[\dot{\boldsymbol{a}}\dot{\boldsymbol{a}}^H]$ and $\dot{\boldsymbol{a}} = [a_1[-1]a_1[1]\ldots a_1[2N-3]]^t$ [5] or from solving *Yule–Walker* equation [8] for two-step linear predictor

$$\begin{bmatrix} R_a[0]+\sigma_w^2 & R_a[-2] & \ldots & R_a[-2Q] \\ R_a[2] & R_a[0]+\sigma_w^2 & \ldots & R_a[-2(Q-1)] \\ \vdots & \vdots & \vdots & \vdots \\ R_a[2Q] & R_a[2(Q-1)] & \ldots & R_a[0]+\sigma_w^2 \end{bmatrix} \cdot \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_Q \end{bmatrix} = \begin{bmatrix} \sigma_e^2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (6)$$

and $[b_1 b_2 \ldots b_Q] = -[c_1 c_2 \ldots c_Q]/c_0$ where σ_e^2 is the square prediction error which can be assumed as a constant. The coefficients of the linear predictor are scalar due to the assumption of independent channel paths. We refer this receiver as a *scalar predictor receiver (SPR)*.

IV. An Approximate ML Receiver With a Matrix Linear Predictor

When two transmission channels are correlated in a spatial domain, the behavior of one channel effects the other. It is natural to exploit spatial correlation information at the receiver. The prediction coefficient in this case will be in a matrix form. Let $\boldsymbol{R}_{a_1a_2}[m]$ denote cross correlation of channel gains $a_1[n]$ and $a_2[n+m]$, which is expressed as

$$\boldsymbol{R}_{a_1a_2}[m] = E\left[\begin{bmatrix} a_1^*[n] \\ a_2^*[n] \end{bmatrix} [a_1[n+m]a_2[n+m]]\right] = \begin{bmatrix} R_a[m] & \rho R_a[m] \\ \rho R_a[m] & R_a[m] \end{bmatrix}. \quad (7)$$

The matrix prediction coefficients $\boldsymbol{B}_k$ are computed from *multichannel Yule-Walker* equation [8] for two-step linear predictor

$$\begin{bmatrix} \boldsymbol{R}_{a_1a_2}[0]+\sigma_w^2\boldsymbol{I} & \boldsymbol{R}_{a_1a_2}[-2] & \ldots & \boldsymbol{R}_{a_1a_2}[-2(Q-1)] \\ \boldsymbol{R}_{a_1a_2}[2] & \boldsymbol{R}_{a_1a_2}[0]+\sigma_w^2\boldsymbol{I} & \ldots & \boldsymbol{R}_{a_1a_2}[-2(Q-2)] \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{R}_{a_1a_2}[2(Q-1)] & \boldsymbol{R}_{a_1a_2}[2(Q-2)] & \ldots & \boldsymbol{R}_{a_1a_2}[0]+\sigma_w^2\boldsymbol{I} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{A}_1^t \\ \boldsymbol{A}_2^t \\ \vdots \\ \boldsymbol{A}_Q^t \end{bmatrix} = -\begin{bmatrix} \boldsymbol{R}_{a_1a_2}[2] \\ \boldsymbol{R}_{a_1a_2}[4] \\ \vdots \\ \boldsymbol{R}_{a_1a_2}[2(Q-1)] \end{bmatrix} \quad (8)$$

and

$$[\boldsymbol{B}_1\ \boldsymbol{B}_2\ \cdots\ \boldsymbol{B}_Q] = -[\boldsymbol{A}_1\ \boldsymbol{A}_2\ \cdots\ \boldsymbol{A}_Q]. \quad (9)$$

The multichannel Yule-Walker equation can be solved efficiently by Levinson-Wiggins-Robinson algorithm [8]. The branch metric for Viterbi algorithm is modified to be

$$\Lambda(\Gamma_n, \boldsymbol{d}_{n+1}) = \left\| \boldsymbol{D}_{n+1}^H \boldsymbol{r}_{n+1} - \sum_{k=1}^{Q} \boldsymbol{B}_k \boldsymbol{D}_{n+1-k}^H \boldsymbol{r}_{n+1-k} \right\|^2 \quad (10)$$

With this metric, the receiver takes spatial correlation into account. The receiver will be called a *matrix predictor receiver (MPR)*. MPR can be considered as a generalized version of SPR. When channels become spatially uncorrelated ($\rho = 0$), each matrix coefficient $[\boldsymbol{B}_1\boldsymbol{B}_2\ldots\boldsymbol{B}_Q]$ becomes a diagonal matrix and the coefficients reduce to the scalar version.

V. Performance Analysis

From the metric in (5) or (10), we can derive a pairwise error probability (PEP) given a transmitted sequence $\boldsymbol{D}$ and a corresponding error sequence $\tilde{\boldsymbol{D}}$. Following the approach in [4], the PEP is given as

$$P(\boldsymbol{D} \to \tilde{\boldsymbol{D}}) = \frac{1}{2} + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\Phi(\xi)}{j\xi} d\xi \quad (11)$$

where $\Phi(\xi)$ is a characteristic function of a Gaussian quadratic form $\vec{\boldsymbol{r}}^H \boldsymbol{Y} \vec{\boldsymbol{r}}$ and $\vec{\boldsymbol{r}} = [\boldsymbol{r}_{-Q}^t, \boldsymbol{r}_{-Q+1}^t, \ldots, \boldsymbol{r}_0^t, \ldots, \boldsymbol{r}_{L-1}^t]^t$ is a $2(L+Q)\times 1$ complex vector and $\boldsymbol{Y}$ is a $2(L+Q)\times 2(L+Q)$ Hermitian symmetric matrix. The matrix $\boldsymbol{Y}$ consists of an array of 2×2 block matrix $\boldsymbol{y}_{ij}, i,j = -Q, -Q+1, \ldots, L-1$ where

$$\boldsymbol{y}_{ij} = \begin{cases} \boldsymbol{0}_{2\times 2}, & |i-j| > Q \\ \sum_{m=u}^{v} \left(\boldsymbol{D}_i \tilde{\boldsymbol{B}}_{m-i}^H \tilde{\boldsymbol{B}}_{m-j} \boldsymbol{D}_j^H - \tilde{\boldsymbol{D}}_i \tilde{\boldsymbol{B}}_{m-i}^H \tilde{\boldsymbol{B}}_{m-j} \tilde{\boldsymbol{D}}_j^H \right), & \text{otherwise.} \end{cases} \quad (12)$$

The matrices $\tilde{\boldsymbol{B}}_i = -\boldsymbol{B}_i$ for the case of MPR and can be replaced by $\tilde{b}_i = -b_i$ for the case of SPR. The indices of the summation u, v are expressed as

$$u = \begin{cases} 0, & i \le 0;\ j \le 0 \\ \max(i,j); & \text{otherwise} \end{cases}$$
$$v = \begin{cases} L-1, & i \ge L-Q-1;\ j \ge L-Q-1 \\ \min(i+Q, j+Q), & \text{otherwise.} \end{cases} \quad (13)$$

The integral in (11) can be evaluated using Residue Theorem. Finally, the upper bound on the BER or FER is determined from

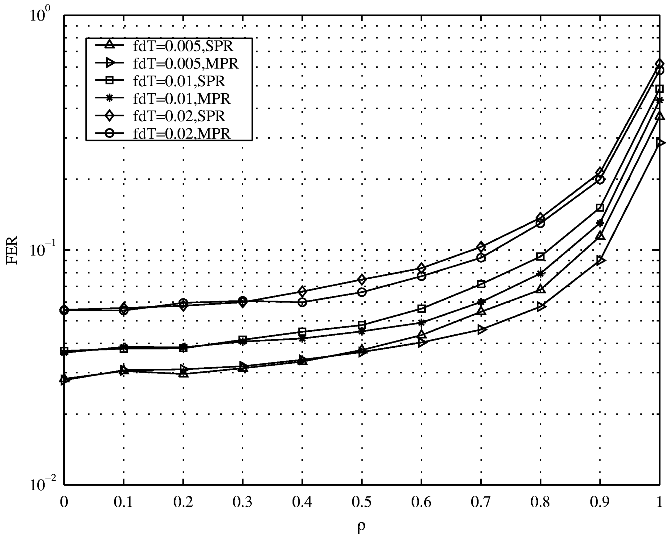

Fig. 1. FER simulation performance of DSTBC with BPSK for different spatial correlation values, at $\mathrm{SNR} = 20$ dB, with SPR and MPR receivers.

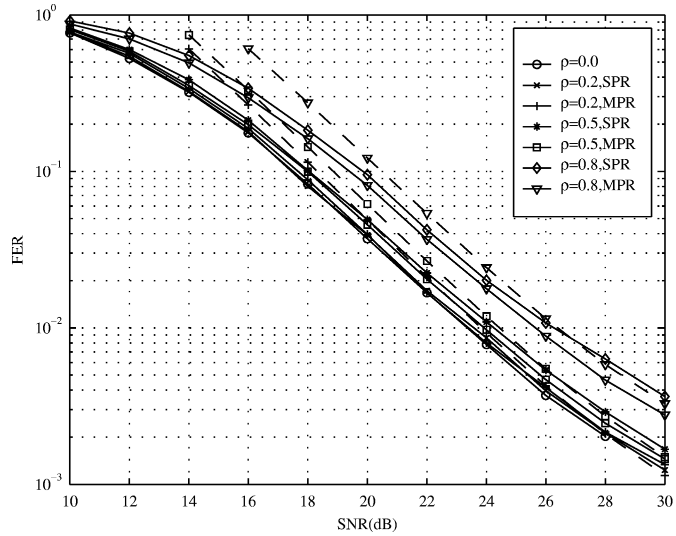

Fig. 2. FER simulation performance (solid line) of DSTBC with BPSK, at fading rate 0.01, $\text{cross correlation} = 0.0,\ 0.2,\ 0.5,\ 0.8$, with SPR and MPR receivers, and corresponding upper bounds of MPR (dash lines).

a standard union bound. In this letter, we evaluate the upper bound on FER which can be approximated as [6]

$$P_f \approx \frac{N}{\log_2 M \times \text{no. of states}} \sum_{\boldsymbol{D},\tilde{\boldsymbol{D}}} P(\boldsymbol{D} \rightarrow \tilde{\boldsymbol{D}}) \tag{14}$$

where N is the frame length (total symbol interval). It is complicated to include all possible sequence pairs. Nevertheless, the bound is fairly accurate by including only short length sequence pairs which are dominant especially at high SNR.

VI. Results and Discussion

We perform simulation to compare the performance of DSTBC with SPR and MPR at different cross-correlation ρ values. The frame length is 260 symbol intervals. The number of states is four corresponding to a combination of two BPSK symbols. The optimal order of the linear predictor is not clear to us but it should be chosen such that the output of the predictor properly represents the channels which are modeled as two correlated autoregressive (AR) processes. The order of five is chosen in this letter. Fig. 1 shows the effect of spatial correlation and the improvement of using MPR over SPR at $\mathrm{SNR} = 20$ dB when $f_dT = 0.005,\ 0.01,\ 0.02$. We can see that the degradation from spatial correlation is not a linear function of ρ. The degradation becomes more severe at higher spatial correlation $(\rho > 0.5)$. At $\rho < 0.3$, the performance is very close to the $\rho = 0.0$ case and MPR has no visible gain over SPR. However, as the spatial correlation increases, MPR seems to have better performance over SPR. At $f_dT = 0.005,\ 0.01$, MPR achieves higher gain over SPR than at $f_dT = 0.02$. Fig. 2 shows FER performance when $\rho = 0.0, 0.2, 0.5, 0.8$ and corresponding upper bound of MPR when $f_dT = 0.01$. The upper bounds include only length two error sequences while including longer sequences does not effect the bounds at high SNR. They seem to be fairly tight and follow the simulation performance. MPR has about 0.4 and 0.6 dB gain over SPR when $\rho = 0.5$ and 0.8, respectively. Note that this gain is not visible in terms of BER which is essentially the same as in the case of receive diversity [4]. At higher fading rate, it is possible for SPR to perform better than MPR. This is because the assumption of fixed channel gains within a block is less valid at high fading rate. The receiver becomes less optimal and it is found that MPR is more sensitive to this effect.

VII. Conclusions

The receiver for DSTBC with matrix linear predictor is proposed and compared to the receiver with scalar linear predictor. When there is spatial correlation between antennas, it is found that a small gain can be achieved using matrix linear predictor, especially at slow fading. Although this letter shows an example of DSTBC with BPSK and two transmit antennas, the receiver can be easily extended to the case of higher modulation schemes and more than two transmit antennas DSTBC (e.g., [9]).

References

[1] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.

[2] V. Tarokh and H. Jafarkhani, "A differential detection scheme for transmit diversity," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 1169–1174, July 2000.

[3] J. Ventura-Traveset, G. Caire, E. Biglieri, and G. Taricco, "Impact of diversity reception on fading channels with coded modulation—Part I: Coherent detection," *IEEE Trans. Commun.*, vol. 45, no. 5, pp. 563–572, May 1997.

[4] B. D. Hart and D. P. Taylor, "Maximum-likelihood synchronization, equalization, and sequence estimation for unknown time-varying frequency-selective Rician channels," *IEEE Trans. Commun.*, vol. 46, pp. 211–221, Feb. 1998.

[5] E. Chiavaccini and G. M. Vitetta, "Further results on Tarokh's space-time differential technique," in *Proc. IEEE ICC 2002*, vol. 3, 28 Apr.–2 May 2002.

[6] G. Caire and E. Viterbo, "Upper bound on the frame error probability of terminated trellis Codes," *IEEE Commun. Lett.*, vol. 2, pp. 2–4, Jan. 1998.

[7] P. Tarasak, H. Minn, and V. K. Bhargava, "Multistage receiver for differential space-time block codes," in *Proc. IEEE ICC 2003*, vol. 5, May 11–15, 2003, pp. 3155–3159.

[8] S. M. Kay, *Modern Spectral Estimation*. Englewood Cliffs, NJ: Prentice Hall, 1987.

[9] H. Jafarkhani and V. Tarokh, "Multiple transmit antenna differential detection from generalized orthogonal design," *IEEE Trans. Inform. Theory.*, vol. 47, pp. 2626–2631, Sept. 2001.