Interference-Controlled Transmission Schemes for Cognitive Radio in Frequency-Selective Time-Varying Fading Channels

Wenxun Qiu, *Student Member, IEEE,* Baile Xie, *Student Member, IEEE,* Hlaing Minn, *Senior Member, IEEE,* and Chia-Chin Chong, *Senior Member, IEEE*

***Abstract*—Interference control is crucial in cognitive radio networks and the existing related works focus on quasi-static channels. The expansion of wireless services and applications in everyday life has necessitated support of higher mobility, as evident from the emerging wireless standards. This results in time-varying fading channels within a transmission frame for which the interference power control schemes and power allocation strategies have not been developed yet. This paper proposes several interference control and power allocation strategies with different criteria for underlay cognitive radios in time-varying frequency-selective fading channels and extend the transmission schemes to multiple secondary systems. The transmission schemes are evaluated in terms of interference statistics at the primary receiver and the ergodic capacity of the secondary user link. Simulation results over the wide Doppler range show how the proposed schemes with different criteria shape the interference at the primary receiver and the capacity of the secondary user. The results also illustrate better interference control capability of the proposed approaches over the existing scheme.**

***Index Terms*—Underlay cognitive radio, conditional average capacity, interference control, time-varying channel.**

I. INTRODUCTION

COGNITIVE radio (CR) has emerged as a promising technology for efficient spectral utilization [1]. In CR networks, the non-cognitive users, referred to as *primary users*, usually the licensed users, have higher priority to access the frequency resource. The cognitive users, also called *secondary users*, access the frequency resource in an opportunistic manner with the knowledge of the channel information. There are three main CR network paradigms: underlay, overlay, and interweave [2]. In this paper, we focus on the underlay paradigm where the secondary users can directly occupy the licensed spectra as long as the interference caused by secondary users can be controlled under an acceptable threshold [3], which gives more freedom to secondary users.

Several works [4]–[10] in the literature address the power allocation for secondary users in underlay systems for different scenarios. Traditionally, the capacity of secondary channel is investigated under the power constraints on transmission and interference. The power allocation is addressed in [5] for AWGN channel, in [6], [7] for OFDM in frequency-selective channels, and in [8], [9] for flat fading channels using ergodic capacity. Moreover, how to control the interference to the primary users caused by secondary users is an important problem in underlay cognitive radio systems. The studies in [11] and [12] consider the effect of average interference and peak interference in CR networks, while [13]–[15] optimize the capacity under the temperature interference constraint. And in [16], the capacities of CRs under different fading channels are studied. All of the above approaches are based on the quasi-static channel assumption.

Portability and mobility have been the celebrated advantages of the wireless systems, and next generation wireless systems aim to support higher mobile speeds. Due to the congestion in current mobile bands, high frequency bands are under consideration, e.g., WiMAX uses 2-6 GHz bands and a case study in [17] considers 60 GHz at the speed of 50km/h. Higher mobile speeds and higher carrier frequency will both lead to larger Doppler spreads and consequently faster time-varying fading channels under which both controlling time-varying interference to the primary users and optimizing the CR transmission become much more challenging. To the best of our knowledge, such an important aspect has not been considered in the existing literature.

In this paper, we address the interference control and power allocation for CRs in time-varying frequency-selective fading channels. The power allocation problem is formulated to maximize the conditional average capacity of the secondary user channel under the constraints of both the secondary user transmission power and the interference power to the primary user. The main contributions of this paper are summarized as follows.

- The power allocation with the interference control for CR systems where channels vary within a transmission frame is an unexplored issue, and we propose several power control schemes to address this issue. These schemes can be classified into 3 categories: conditional mean interference power constraint per subband and per symbol, and conditional probability constraint on the intolerable interference.
- Incorporating the channel variations within the frame, we

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W. Qiu, B. Xie, and H. Minn are with the Department of Electrical Engineering, University of Texas at Dallas, Richardson, TX, 75080, USA (e-mail: {wxq081000, bxx081000, hlaing.minn}@utdallas.edu).

C. C. Chong was with DOCOMO USA Lab. Inc., Palo Alto, CA, 94304, USA (e-mail: chia-chin.chong@ieee.org).

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propose conditional average capacity as opposed to the instantaneous capacity used in quasi-static channels, as the performance metric for power control optimization. As the corresponding optimization problem is no longer convex (hence existing schemes cannot be applied), we propose two approximations to the conditional average capacity which effectively translate the problem into a convex optimization. For some of these optimization problems, we provide water-filling style solutions.
- We derive the cumulative distribution function (CDF) of the conditional channel power gain in time-varying fading channels. The lower bound of the probability that the interference is below a threshold is provided. Based on the analysis, we propose a power control scheme with the conditional probability constraint.
- We extend the power and interference control schemes to multiple secondary systems. The same interference control targets (conditional mean interference and conditional probability of intolerable interference) at the primary receivers can be achieved in multiple secondary systems as in the single secondary system.
- The performance results unveil how different control strategies affect interference on primary system and capacity of secondary systems, providing useful information for development of CR systems in time-varying channels.

The rest of the paper is organized as follows. Section II presents the system model of our considered problem. Section III provides the proposed transmission schemes developed with constraints of the mean interference power and the probability of intolerable interference. In Section IV, we extend the transmission schemes to multiple secondary systems. In Section V, performance comparisons with the conventional approaches are discussed in terms of the ergodic channel capacity and interference metrics. Finally, Section VI concludes the paper.

II. System Model

We consider the underlay CR paradigm (see Fig. 1) with a primary system and a single secondary system (studied in Section II and III) or multiple secondary systems (investigated in Section IV). In a secondary system, several pairs of secondary users may operate simultaneously. We assume that the frequency band occupied by a pair of secondary users (secondary transmitter (ST) and secondary receiver (SR)) is not overlapped with the frequency band occupied by other pairs of secondary users. Thus, the power allocation and interference control among different secondary user pairs are decoupled. Therefore, we only need to consider one pair as an example. We also assume that there are random access phase and data transmission phase for the secondary users, and the random access phase has been carried out for the considered bandwidth. During the later part of the random access phase, initial signaling between the two secondary users is accomplished, from which ST can obtain knowledge of the secondary channel. Since the primary users and secondary users can transmit simultaneously in the underlay CR systems, the ST should limit its transmit power to control the interference to

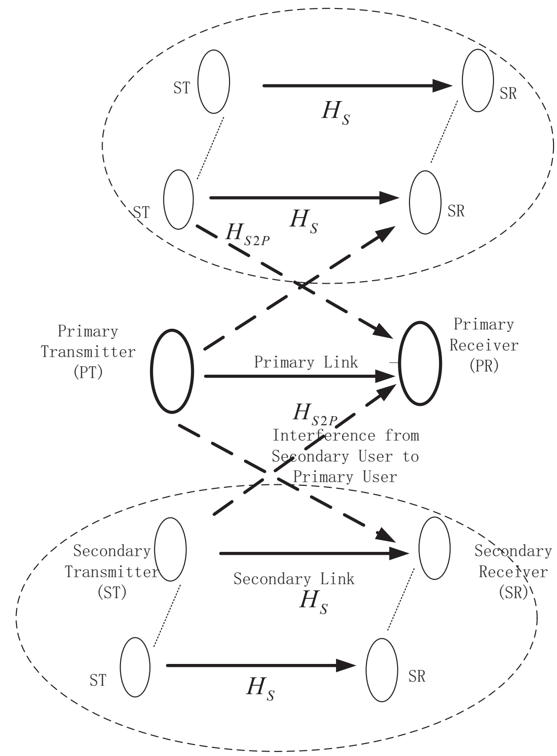

Fig. 1. An underlay CR network with a primary system and multiple secondary systems.

the PR so that the transmission of primary users would not be affected much.

Fig. 1 shows the considered system where $H_{\rm s}$ and $H_{\rm s2p}$ are respectively the low-pass equivalent secondary user channel (i.e., ST-to-SR) and interference channel (i.e., ST-to-PR). Further, we assume that only ST is mobile, while SR, PT and PR are quasi-static (e.g., the primary system can be TV broadcasting while SR can be a pico cellular base station). Thus, both $H_{\rm s}$ and $H_{\rm s2p}$ are time-varying channels. We also assume that the primary system is a two-way communication system with time division duplexing (TDD). Then the ST can obtain knowledge of the interference channel ($H_{\rm s2p}$) during the transmission phase of PR. Once PR gets into the receiving phase, SR, which is equipped with a power sensing module, can estimate the interference power from PT to SR, denoted as $P_{\rm p}(f)$. Then SR feeds back $P_{\rm p}(f)$ to ST during signaling exchange at the later part of the random access phase. After obtaining all the information, ST starts to transmit data. Note that we assume $P_{\rm p}(f)$ is quasi-static but can be frequency-selective. It is also reasonable to assume that $P_{\rm p}(f)$ is greater than the signal power at SR. A rich scattering environment with non-line-of-sight condition is assumed. Then we can model $H_{\rm s}$ and $H_{\rm s2p}$ as independent time-varying frequency-selective zero-mean complex Gaussian processes.

The considered band is divided into M subbands, each with spacing W Hz (e.g., orthogonal frequency division multiplexing (OFDM) systems). Within each subband the channel is frequency non-selective. ST can obtain channel state information (CSI) of $H_{\rm s}$ and $H_{\rm s2p}$ over the whole bandwidth before each transmission, but the channel knowledge may be outdated by u and v symbols, respectively. We assume that the channel gains remain static within one symbol, but may vary from one symbol to another. Let $H_{\mathrm{s}}(t,f)$ and $H_{\mathrm{s2p}}(t,f)$ denote the low-pass equivalent channel gain of H_{s} and H_{s2p} on the fth subband at the tth symbol respectively. With the CSI outdate of u and v symbols, $H_{\mathrm{s}}(-u,f)$ and $H_{\mathrm{s2p}}(-v,f)$ can be considered as the CSI knowledge of the fth subband at ST. The temporal correlation coefficient of the fth subband channel gain between the tth symbol and the corresponding CSI knowledge is denoted by $\rho_{\mathrm{s}}(t)$ and $\rho_{\mathrm{s2p}}(t)$ for the two channels. A common model of the channel temporal correlation is given by the classical Clark-Jakes' model with a normalized autocorrelation function $J_0(2\pi f_m \tau)$, where $J_0(x)$ is the zero-order Bessel function of the first kind, τ denotes the time lag and f_m is the maximum Doppler frequency. With the CSI knowledge delay of u or v symbols, the autocorrelations at the tth symbol are $\rho_{\mathrm{s}}(t) = J_0(2\pi f_m (t+u)T_s)$ and $\rho_{\mathrm{s2p}}(t) = J_0(2\pi f_m (t+v)T_s)$ where T_s is the symbol duration. Note that $\rho_{\mathrm{s}}(t)$ and $\rho_{\mathrm{s2p}}(t)$ can assume other correlation model as well.

We have the expected value of secondary user's channel capacity at the tth symbol as

$$\begin{aligned} E[C(t)] &= E\left[\sum_{f=1}^{M} W \log_2(1+\mathrm{SINR}_f(t))\right] \\ &= E\left[\sum_{f=1}^{M} W \log_2\left(1+\frac{P_{\mathrm{s}}(t,f)|H_{\mathrm{s}}(t,f)|^2}{N_0+P_{\mathrm{p}}(f)}\right)\right], \end{aligned}$$

where $P_{\mathrm{s}}(t,f)$ is the ST's transmission power in the fth subband of the tth symbol and N_0 is the noise power in each subband. And we set $I(t,f)$ to be the interference power from ST to PR on the fth subband at the tth symbol as $I(t,f) = P_{\mathrm{s}}(t,f)|H_{\mathrm{s2p}}(t,f)|^2$.

In practice, the total transmission power is constrained by Φ as $\sum_{f=1}^{M} P_{\mathrm{s}}(t,f) \leqslant \Phi$. Our goal is to properly control the interference to PR caused by ST while optimizing the conditional average capacity for secondary users in frequency-selective time-varying channels. First, it is a constrained optimization problem with no simple closed-form solution. Hence, we develop suboptimal power allocation strategies which maximize approximates of the conditional average channel capacity. Second, due to channel variations within a frame, how to control the interference will be a challenge. We will explore several criteria to constrain the interference to PR.

III. Power Allocation for Secondary Users

A. Initial Interference Power Constraint

The existing approaches, which only address the quasi-static scenario, can be considered as the initial interference power constraint approach in time-varying channels, since actual power constraint is guaranteed only to the initial part of the transmission.

If the channel is quasi-static, we can obtain the channel capacity at the tth symbol as

$$C(t) = \sum_{f=1}^{M} W \log_2\left(1+\frac{P_{\mathrm{s}}(t,f)|H_{\mathrm{s}}(-u,f)|^2}{N_0+P_{\mathrm{p}}(f)}\right). \quad (1)$$

And the objective is to maximize $C(t)$ with constraints. For the interference control, we can have different constraint criteria according to different scenarios. If the primary systems are sensitive to the interference on each tone like uncoded systems or if multiple primary users dynamically share the considered band or we have no idea about the primary system, the interference should be constrained tone by tone. By assuming the channel power gains of the tones within each subband are the same, we can take the constraint over one subband instead. On the other hand, if the primary systems use strong coding schemes and capture frequency and time diversities and there is only one primary user in the considered band, it is more reasonable to constrain the interference per symbol or even per frame. Indeed, it relaxes the constraint on the interference, which will probably lead to better capacity performance. However, for the interference constraint over a whole frame, the computation complexity in the optimization is impractically high. Hence, in this paper, we consider the interference constraint per subband of each symbol and per symbol but it can be extended to the constraint per frame except the complexity barrier.

1) Interference Power Constraint Per Subband: In this case, as the channel power gains within the subband are essentially the same, we have the constraint as

$$I(t,f) = P_{\mathrm{s}}(t,f)|H_{\mathrm{s2p}}(-v,f)|^2 \leqslant \phi, \forall f, \quad (2)$$

where ϕ is the maximum acceptable interference power per subband. Then we can formulate the optimization problem with the initial interference power constraint as

$$\min\left\{-C(t) = -\sum_{f=1}^{M} W \log_2\left(1+\frac{P_{\mathrm{s}}(t,f)|H_{\mathrm{s}}(-u,f)|^2}{N_0+P_{\mathrm{p}}(f)}\right)\right\}$$
$$\text{s.t. } P_{\mathrm{s}}(t,f)|H_{\mathrm{s2p}}(-v,f)|^2 \leqslant \phi, -P_{\mathrm{s}}(t,f) \leqslant 0, \sum_{f=1}^{M} P_{\mathrm{s}}(t,f) \leqslant \Phi. \quad [\mathrm{P}-1]$$

It is a classical convex optimization problem, which can be well-addressed by the Karush-Kuhn-Tucker (K.K.T) condition as shown in Appendix A. The solution is as

$$P_s(t,f) = \begin{cases} P_1, & \frac{W}{\mu \ln 2} \geqslant P_1 + P_2 \\ \frac{W}{\mu \ln 2} - P_2, & P_2 < \frac{W}{\mu \ln 2} < P_1 + P_2 \\ 0, & \frac{W}{\mu \ln 2} \leqslant P_2 \end{cases} \quad (3)$$

where $P_1 = \frac{\phi}{|H_{s2p}(-v,f)|^2}$, $P_2 = \frac{N_0+P_p}{|H_s(-u,f)|^2}$ and μ is the Lagrange multiplier. It is a barrier water-filling type solution which can be solved by iterative algorithms (e.g., [18]).

2) Interference Power Constraint Per Symbol: In this case, the interference power per subband may vary across subbands and only their sum is constrained. In order to make comparison, we keep the interference power constraint per symbol to be equal to $M\phi$ as there are M subbands. Then we can formulate the optimization problem with the initial interference power constraint over one symbol as

$$\min\left\{-C(t)=-\sum_{f=1}^{M}W\log_2\left(1+\frac{P_{\rm s}(t,f)|H_{\rm s}(-u,f)|^2}{N_0+P_{\rm p}(f)}\right)\right\}$$
$$\text{s.t. } \sum_{f=1}^{M}P_{\rm s}(t,f)|H_{\rm s2p}(-v,f)|^2\leqslant M\phi, -P_s(t,f)\leqslant 0,$$
$$\sum_{f=1}^{M}P_{\rm s}(t,f)\leqslant\Phi. \qquad [\mathrm{P}-2]$$

This optimization problem takes the form of minimizing a convex function subject to a convex constraint. Numerical search algorithms such as the interior-point method can be used to find the optimal solution.

B. Conditional Mean Interference Power Constraint

In the time-varying scenario, we cannot guarantee the interference $I(t,f)$ always to be below a threshold while maintaining meaningful communication in the secondary link. We consider two options. The first one is to control the mean interference power. The other is to control the probability of the interference power exceeding a certain threshold (called the probability of intolerable interference). In this section, we investigate how to maximize the conditional average capacity of secondary link with the conditional mean interference power constraint.

When we consider the mean interference power constraint, we can have two choices: controlling the mean interference power per subband or per symbol. In the former case, the conditional mean interference power constraint can be described as

$$E\left[I(t,f)\big|_{H_{\rm s2p}(-v,f),\rho_{\rm s2p}}\right] = P_{\rm s}(t,f)E\left[|H_{\rm s2p}(t,f)|^2\big|_{H_{\rm s2p}(-v,f),\rho_{\rm s2p}}\right]\leqslant\phi. \quad (4)$$

Let $2\sigma_h^2$ denote the variance of each individual channel coefficient (σ_h^2 for each dimension). For time-varying frequency-selective channels, we can model the channel gain at the tth symbol as

$$H_{\rm s}(t,f)=\rho_{\rm s}(t)H_{\rm s}(-u,f)+W_{\rm s}(t,f), \quad (5)$$
$$H_{\rm s2p}(t,f)=\rho_{\rm s2p}(t)H_{\rm s2p}(-v,f)+W_{\rm s2p}(t,f). \quad (6)$$

Here, $W_{\rm s}(t,f)\sim\mathcal{CN}(0,(1-\rho_{\rm s}^2(t))2\sigma_h^2)$ and $W_{\rm s2p}(t,f)\sim\mathcal{CN}(0,(1-\rho_{\rm s2p}^2(t))2\sigma_h^2)$. Therefore, we have the expectation of $|H_{\rm s2p}(t,f)|^2$ conditioned on $\rho_{\rm s2p}(t)$ and $H_{\rm s2p}(-v,f)$ as

$$E\left[|H_{\rm s2p}(t,f)|^2\big|_{H_{\rm s2p}(-v,f),\rho_{\rm s2p}}\right] = \rho_{\rm s2p}^2(t)|H_{\rm s2p}(-v,f)|^2+(1-\rho_{\rm s2p}^2(t))2\sigma_h^2. \quad (7)$$

By substituting (7) into (4), we obtain the corresponding constraint on $P_{\rm s}(t,f)$ as

$$P_{\rm s}(t,f)\leqslant\frac{\phi}{\rho_{\rm s2p}^2(t)|H_{\rm s2p}(-v,f)|^2+(1-\rho_{\rm s2p}^2(t))2\sigma_h^2}. \quad (8)$$

Thus, under the conditional mean interference power constraint, we can formulate the problem straight-forwardly as

$$\min\left\{-E\left[\sum_{f=1}^{M}W\log_2(1+\frac{P_{\rm s}(t,f)|H_{\rm s}(t,f)|^2}{N_0+P_{\rm p}(f)})\Big|_{H_{\rm s}(-u,f),\rho_{\rm s}}\right]\right\},$$
$$\text{s.t. } P_{\rm s}(t,f)\leqslant\frac{\phi}{\rho_{\rm s2p}^2(t)|H_{\rm s2p}(-v,f)|^2+(1-\rho_{\rm s2p}^2(t))2\sigma_h^2},$$
$$-P_s(t,f)\leqslant 0,\quad \sum_{f=1}^{M}P_{\rm s}(t,f)\leqslant\Phi. \qquad [\mathrm{P}-3]$$

Unfortunately, $E[\cdot]$ and $\log_2(\cdot)$ cannot be exchanged. And the problem is non-convex. Hence, the conventional derivation for water-filling approach cannot be implemented. We propose two approximations to the conditional average capacity to obtain suboptimal solutions to the problem.

1) Proposed Approximation 1: As assumed, $P_{\rm p}(f)$ is greater than the signal power at SR. Therefore, Taylor expansion can be used to make an approximation as

$$\log_2(1+X)=\frac{1}{\ln 2}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{X^n}{n} \approx\frac{1}{\ln 2}\sum_{n=1}^{L}(-1)^{n+1}\frac{X^n}{n}, -1<X<1. \quad (9)$$

By (9), the conditional average channel capacity is given as

$$E\left[C(t)\big|_{H_{\rm s}(-u,f),\rho_{\rm s}}\right] \approx W\sum_{f=1}^{M}\sum_{n=1}^{L}\frac{(-1)^{n+1}}{n\ln 2}\frac{P_{\rm s}^n(t,f)E\left[|H_{\rm s}(t,f)|^{2n}\big|_{H_{\rm s}(-u,f),\rho_{\rm s}}\right]}{[N_0+P_{\rm p}(f)]^n}.$$

By $R_n(t,f)\triangleq E\left[|H_{\rm s}(t,f)|^{2n}\big|_{H_{\rm s}(-u,f),\rho_{\rm s}}\right]$, $Q_n(f)\triangleq[N_0+P_{\rm p}(f)]^n$, we can simplify (10) as

$$E\left[C(t)\big|_{H_{\rm s}(-u,f),\rho_{\rm s}}\right]=W\sum_{f=1}^{M}\sum_{n=1}^{L}\frac{(-1)^{n+1}}{n\ln 2}\frac{P_{\rm s}^n(t,f)R_n(t,f)}{Q_n(f)}. \qquad [A-1]$$

Consequently, we can reformulate the optimization problem as

$$\min\left\{-W\sum_{f=1}^{M}\sum_{n=1}^{L}\frac{(-1)^{n+1}}{n\ln 2}\frac{P_{\rm s}^n(t,f)R_n(t,f)}{Q_n(f)}\right\},$$
$$\text{s.t. } P_{\rm s}(t,f)\leqslant\frac{\phi}{\rho_{\rm s2p}^2(t)|H_{\rm s2p}(-v,f)|^2+(1-\rho_{\rm s2p}^2(t))2\sigma_h^2},$$
$$-P_s(t,f)\leqslant 0,\quad \sum_{f=1}^{M}P_{\rm s}(t,f)\leqslant\Phi. \qquad [\mathrm{P}-4]$$

This convex optimization problem can be solved by the interior-point method. Especially, for the truncation of $L=2$, the solution similar to water-filling type can be obtained as a suboptimal solution, which is more computationally efficient.

As a standard convex optimization problem, by the K.K.T condition, we obtain

$$\frac{W}{\ln 2}\sum_{n=1}^{L}(-1)^{n+1}\frac{P_{\rm s}^{n-1}(t,f)R_n(t,f)}{Q_n(f)}-\mu\geqslant 0, \quad (10)$$

where μ is the Lagrange multiplier. We truncate the series by using $L=2$ to make a further approximation and obtain

$$\mu - \frac{W}{\ln 2}\left[\frac{R_1(t,f)}{Q_1(f)} - \frac{P_{\rm s}(t,f)R_2(t,f)}{Q_2(f)}\right] = 0. \quad (11)$$

By (5), we have

$$\begin{aligned} R_1(t,f) &= E\left[|H_{\rm s}(t,f)|^2\big|_{H_{\rm s}(-u,f),\rho_{\rm s}}\right] \\ &= |\rho_{\rm s}(t)H_{\rm s}(-u,f)|^2 + E\left[|W_{\rm s}(t,f)|^2\right], \\ R_2(t,f) &= |\rho_s(t)H_{\rm s}(-u,f)|^4 + \\ & 2|\rho_s(t)H_{\rm s}(-u,f)|^2 E[|W_{\rm s}(t,f)|^2] + E[|W_{\rm s}(t,f)|^4]. \end{aligned}$$

Considering that $W_{\rm s}(t,f)$ is a complex Gaussian, we have

$$\begin{aligned} E[|W_{\rm s}(t,f)|^2] &= (1-\rho_{\rm s}^2(t))2\sigma_h^2, \\ E[|W_{\rm s}(t,f)|^4] &= 8(1-\rho_{\rm s}^2(t))^2\sigma_h^4. \end{aligned}$$

Then from transmission power constraints in [P-4] and (11), the solution can be obtained as

$$P_{\rm s}(t,f) = \begin{cases} P_3, & \frac{\mu \ln 2}{W} \leqslant I_1 - P_3 I_2 \\ \left[I_1 - \frac{\mu\ln 2}{W}\right]/I_2, & I_1 - P_3 I_2 < \frac{\mu \ln 2}{W} < I_1 \\ 0, & I_1 \leqslant \frac{\mu \ln 2}{W} \end{cases} \quad (12)$$

where $P_3 = \frac{\phi}{\rho_{\rm s2p}^2(t)|H_{\rm s2p}(-v,f)|^2 + (1-\rho_{\rm s2p}^2(t))2\sigma_h^2}$, $I_1 = \frac{R_1(t,f)}{Q_1(f)}$, $I_2 = \frac{R_2(t,f)}{Q_2(f)}$. Due to the similar format as in the barrier water-filling approach, the iterative method (e.g., [18]) for water-filling can be applied to this approach as well. Here, we can see that P_3 is the upper limit of the transmit power which assures the mean interference power is under control. And if $0 \le P_{\rm s}(t,f) < P_3$, then the power follows a solution similar to water-filling type to optimize the capacity of the secondary link under the constraint of $\sum_{f=1}^{M} P_s(t,f) \leqslant \Phi$.

2) Proposed Approximation 2: By Jensen's inequality [19], we can have the approximation

$$E\left[\log_2(1+X)\right] \approx \log_2(1+E[X]). \quad (13)$$

Therefore, we can approximate conditional average channel capacity as

$$\begin{aligned} & E\left[C(t)\big|_{H_{\rm s}(-u,f),\rho_{\rm s}}\right] \quad [A-2] \\ & \approx W\sum_{f=1}^{M}\log_2\left(1 + \frac{P_{\rm s}(t,f)E\left[|H_{\rm s}(t,f)|^2\big|_{H_{\rm s}(-u,f),\rho_{\rm s}}\right]}{N_0 + P_{\rm p}(f)}\right). \end{aligned}$$

Then, the optimization problem can be reformulated as

$$\begin{aligned} & \min\left\{-W\sum_{f=1}^{M}\log_2\left(1 + \frac{P_{\rm s}(t,f)E\left[|H_{\rm s}(t,f)|^2\big|_{H_{\rm s}(-u,f),\rho_{\rm s}}\right]}{N_0 + P_{\rm p}(f)}\right)\right\}, \\ & \text{s.t. } P_{\rm s}(t,f) \leqslant \frac{\phi}{\rho_{\rm s2p}^2(t)|H_{\rm s2p}(-v,f)|^2 + (1-\rho_{\rm s2p}^2(t))2\sigma_h^2}, \\ & -P_{\rm s}(t,f) \leqslant 0, \quad \sum_{f=1}^{M} P_{\rm s}(t,f) \leqslant \Phi. \quad [\text{P}-5] \end{aligned}$$

According to Appendix A, we can have the barrier water-filling type solution as

$$P_{\rm s}(t,f) = \begin{cases} P_3, & \frac{W}{\mu\ln 2} \geqslant P_3 + P_4 \\ \frac{W}{\mu \ln 2} - P_4, & P_4 < \frac{W}{\mu\ln 2} < P_3 + P_4 \\ 0, & \frac{W}{\mu \ln 2} \leqslant P_4 \end{cases}, \quad (14)$$

where $P_4 = \frac{N_0 + P_{\rm p}(f)}{|\rho_{\rm s}(t)H_{\rm s}(-u,f)|^2 + (1-\rho_{\rm s}^2(t))2\sigma_h^2}$. The same as [P-4], iterative water-filling methods (e.g., [18]) can be applied to this approach as well. Since the interference control is the same as [P-4], (14) has the same upper limit as (12). And this upper limit is not approximated but accurate. Due to different approximations to the average capacity, the power allocation schemes in $0 < P_{\rm s}(t,f) < P_3$ are different between [P-3] and [P-4]. Note that all the schemes with constraint per subband can be directly applied to the scenario where multiple primary users operate in the considered band but occupy different subbands from each other. The only difference is that the variables related to the primary user (i.e., $\rho_{\rm s2p}(t)$, $H_{\rm s2p}(-v,f)$, $P_{\rm p}(f)$) should be replaced by the variables for the corresponding primary user who occupies the specific subband.

Instead of the mean interference power constraint on subband, we can also constrain the mean interference power per symbol as in Section III-A2 as follows:

$$\sum_{f=1}^{M} P_{\rm s}(t,f)E\left[|H_{\rm s2p}(t,f)|^2\big|_{H_{\rm s2p}(-v,f) \quad \rho_{\rm s2p}}\right] \leqslant M\phi. \quad (15)$$

Then [P-4] can be reformulated as

$$\begin{aligned} & \min\left\{-W\sum_{f=1}^{M}\sum_{n=1}^{L}\frac{(-1)^{n+1}}{n\ln 2}\frac{P_{\rm s}^n(t,f)R_n(t,f)}{Q_n(f)}\right\}, \\ & \text{s.t. } \sum_{f=1}^{M} P_{\rm s}(t,f)E\left[|H_{\rm s2p}(t,f)|^2\big|_{H_{\rm s2p}(-v,f), \quad \rho_{\rm s2p}}\right] \leqslant M\phi, \\ & -P_{\rm s}(t,f) \leqslant 0, \quad \sum_{f=1}^{M} P_{\rm s}(t,f) \leqslant \Phi. \quad [\text{P}-6] \end{aligned}$$

Similarly, [P-5] can be reformulated as

$$\begin{aligned} & \min\left\{-W\sum_{f=1}^{M}\log_2\left(1 + \frac{P_{\rm s}(t,f)E\left[|H_{\rm s}(t,f)|^2\big|_{H_{\rm s}(-u,f),\rho_{\rm s}}\right]}{N_0 + P_{\rm p}(f)}\right)\right\}, \\ & \text{s.t. } \sum_{f=1}^{M} P_{\rm s}(t,f)E\left[|H_{\rm s2p}(t,f)|^2\big|_{H_{\rm s2p}(-v,f),\rho_{\rm s2p}}\right] \leqslant M\phi, \\ & -P_{\rm s}(t,f) \leqslant 0, \quad \sum_{f=1}^{M} P_{\rm s}(t,f) \leqslant \Phi, \quad [\text{P}-7] \end{aligned}$$

where $E\left[|H_{\rm s2p}(t,f)|^2\big|_{H_{\rm s2p}(-v,f),\rho_{\rm s2p}}\right]$ is obtained in (7). Both [P-6] and [P-7] can be solved by the interior-point method.

Here, we optimize the approximated conditional average capacity symbol by symbol for each channel realization. Effectively, the optimization for each realization also approximately leads to the optimization of ergodic capacity. The approaches in the following section also have the same property.

C. Conditional Probability Constraint on Intolerable Interference

1) Probability Analysis on Conditional Interference Control: From (6), if given $\rho_{\rm s2p}(t)$ and $H_{\rm s2p}(-v,f)$, we have

$$H_{\rm s2p}(t,f) \sim \mathcal{CN}\left(\rho_{\rm s2p}(t)H_{\rm s2p}(-v,f), (1-\rho_{\rm s2p}^2(t))2\sigma_h^2\right).$$

The real and imaginary parts (given $\rho_{\rm s2p}(t)$ and $H_{\rm s2p}(-v,f)$) follow Gaussian distribution respectively as

$$\mathcal{R}\left[H_{\rm s2p}(t,f)\right] \sim \mathcal{N}\left(\rho_{\rm s2p}(t)\mathcal{R}\{H_{\rm s2p}(-v,f)\}, (1-\rho_{\rm s2p}^2(t))\sigma_h^2\right),$$
$$\mathcal{I}\left[H_{\rm s2p}(t,f)\right] \sim \mathcal{N}\left(\rho_{\rm s2p}(t)\mathcal{I}\{H_{\rm s2p}(-v,f)\}, (1-\rho_{\rm s2p}^2(t))\sigma_h^2\right).$$

We introduce a new variable

$$V(t,f) \triangleq \frac{1}{(1-\rho_{\rm s2p}^2(t))\sigma_h^2}|H_{\rm s2p}(t,f)|^2.$$

Conditioned on $\rho_{\rm s2p}(t)$ and $H_{\rm s2p}(-v,f)$, we have $V(t,f)$ following a Non-Central Chi Square distribution $\chi_k'^2(\beta)$ with the degrees of freedom $k=2$ and the non-centrality parameter $\beta=\lambda(t,f)$, where $\lambda(t,f)=\frac{\rho_{\rm s2p}^2(t)}{(1-\rho_{\rm s2p}^2(t))\sigma_h^2}|H_{\rm s2p}(-v,f)|^2$. The conditional cumulative distribution function (CDF) of $V(t,f)$ is

$$F_{V|\rho,H}\left(v(t,f)\big|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)}\right) = \sum_{j=0}^{\infty} e^{-\lambda(t,f)/2}\frac{(\lambda(t,f)/2)^j}{j!}\frac{\gamma(j+k/2, v(t,f)/2)}{\Gamma(j+k/2)} \tag{16}$$

where $\gamma(\cdot)$ is the lower incomplete Gamma function and $\Gamma(\cdot)$ is the Gamma function.

Consequently, we can obtain the conditional CDF of the secondary-to-primary channel power gain $G_{\rm s2p}(t,f)=|H_{\rm s2p}(t,f)|^2$ as

$$F_{G|\rho,H}\left(G_{\rm s2p}(t,f)\big|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)}\right) = F_{V|\rho,H}\left[\frac{1}{(1-\rho_{\rm s2p}^2(t))\sigma_h^2}G_{\rm s2p}(t,f)\right]. \tag{17}$$

We rewrite the interference power to PR caused by ST as

$$I(t,f)=P_{\rm s}(t,f)|H_{\rm s2p}(t,f)|^2=P_{\rm s}(t,f)G_{\rm s2p}(t,f). \tag{18}$$

Obviously, by controling the variance of $I(t,f)$, we can control the probability of the interference exceeding the tolerant threshold. As the conditional expectation and variance of $G_{\rm s2p}(t,f)$ are functions of correlation coefficient $\rho_{\rm s2p}(t)$, in the optimization the transmission power $P_{\rm s}(t,f)$ should be adjusted symbol by symbol.

In this paper, we consider the probability constraint of intolerable interference per subband only. In the case of interference power constraint per symbol, the CDF of $I(t,f)$ is nonlinearly related to $P_s(t,f)$ and hence obtaining a closed-form PDF or CDF is intractable. In the case of interference power constraint per subband, as described before, we assume the tolerant threshold of each subband is ϕ. The interference constraint (2) used in [P-1] only constrains the interference power at $t=-v$. The interference constraints (8) used in [P-3]-[P-5] control the mean interference power to the primary users for all t. However, since the channel power gain $G_{\rm s2p}$ is a random variable with the range $[0,\infty)$, the interference power would definitely exceeds the tolerant threshold by a certain probability. Suppose the transmission power is constrained as $P_{\rm s}(t,f)\leqslant\phi_0$. The probability of the interference power not larger than the threshold conditioned on $\rho_{\rm s2p}(t)$ and $H_{\rm s2p}(t,f)$ can be shown as

$$\Psi_{\rm in}(t,f)|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)} \triangleq {\rm Pro}\left\{P_{\rm s}(t,f)|H_{\rm s2p}(t,f)|^2\le\phi\big|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)}\right\} \tag{19}$$
$$\ge{\rm Pro}\left\{|H_{\rm s2p}(t,f)|^2\le\frac{\phi}{\phi_0}\big|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)}\right\} = F_{{\rm G}|\rho,H}\left\{\frac{\phi}{\phi_0}\right\}. \tag{20}$$

Till here, we have obtained the lower bound of $\Psi_{\rm in}|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)}$ in the case of $P_{\rm s}(t,f)\le\phi_0$. We can see that the conditional probability and the lower bound are both functions of ϕ and ϕ_0. For [P-1], we rewrite the interference constraint (2) as $P_{\rm s}(t,f)\leqslant\frac{\phi}{|H_{\rm s2p}(-v,f)|^2}$. By substituting $\phi_0=\frac{\phi}{|H_{\rm s2p}(-v,f)|^2}$ into the lower bound (20), we can have

$$\Psi_{\rm in}(t,f)\Big|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)} \ge F_{{\rm G}|\rho,H}\left\{|H_{\rm s2p}(-v,f)|^2\right\}$$

which means that the inference power constraint used in [P-1] can only guarantee $I(t,f)\leqslant\phi$ with the probability of $F_{{\rm G}|\rho,H}\left\{|H_{\rm s2p}(-v,f)|^2\right\}$.

For the interference constraint (8) used in [P-3]-[P-5], we have $\phi_0=\frac{\phi}{\rho_{\rm s2p}^2(t)|H_{\rm s2p}(-v,f)|^2+2(1-\rho_{\rm s2p}^2(t))\sigma_h^2}$. The conditional probability of the interference power below the threshold can be shown as

$$\Psi_{\rm in}(t,f)\Big|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)} \ge F_{{\rm G}|\rho,H}\left\{\left[\rho_{\rm s2p}^2(t)|H_{\rm s2p}(-v,f)|^2+2(1-\rho_{\rm s2p}^2(t))\sigma_h^2\right]\right\}. \tag{21}$$

From (21), we can see that the interference constraint used in [P-3]-[P-5] can only guarantee the lower bound of (21). In practice, the primary users may not be able to tolerate this. In order to avoid too much intolerable interference at primary users, some techniques to control the variance of interference should be introduced.

2) Probability Constraint: The variance of the interference can be controlled by limiting $P_{\rm s}(t,f)$. Consequently, the conditional probability of the interference power exceeding the interference threshold denoted by $\Psi_{\rm out}|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)}=1-\Psi_{\rm in}|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)}$ can be controlled. By (17), we can easily obtain the CDF of $I(t,f)$ conditioned on $P_{\rm s}(t,f)$ as

$$F_{I(t,f)|\rho,H,P_{\rm s}}\left(I(t,f)\big|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f),P_{\rm s}(t,f)}\right) = F_{V|\rho,H}\left[\frac{1}{(1-\rho_{\rm s2p}^2(t))\sigma_h^2P_{\rm s}(t,f)}\cdot I(t,f)\right]. \tag{22}$$

Suppose that the system requires $\Psi_{\rm in}(t,f)|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)}\ge A$ where A is a predefined constant, e.g., $A=0.9$. Here, $(1-A)$ is the desired probability of intolerable interference. Then we have

$$\Psi_{\rm in}(t,f)|_{\rho_{\rm s2p}(t),H_{\rm s2p}(-v,f)} \ge {\rm Pro}\left\{P_{\rm s}(t,f)|H_{\rm s2p}(t,f)|^2<\phi\big|P_{\rm s}(t,f)\right\}\ge A, \tag{23}$$

$$F_{I(t,f)|\rho,H,P_s}\left(I(t,f)\big|_{\rho_{s2p}(t),H_{s2p}(-v,f),P_s(t,f)}\right) = F_{V|\rho,H}\left(\frac{\phi}{(1-\rho_{s2p}^2(t))\sigma_h^2 P_s(t,f)}\right) \geq A. \quad (24)$$

By solving this inequality and incorporating the case of $\rho_{s2p}(t) = 1$, we obtain

$$P_s(t,f) \leq P_C(t,f,\phi,A), \quad (25)$$

where

$$P_C(t,f,\phi,A) = \begin{cases} \frac{\phi}{|H_{s2p}(-v,f)|^2}, & \rho_{s2p}(t) = 1 \\ \frac{\phi}{(1-\rho_{s2p}^2(t))\sigma_h^2 F_{V|\rho,H}^{-1}(A)}, & \text{otherwise.} \end{cases} \quad (26)$$

We have the new constraint on the variance of $I(t,f)$ as

$$\mathrm{Var}[I(t,f)] \leq \frac{\{4[(1-\rho_{s2p}^2(t))\sigma_h^2] + 4\rho_{s2p}^2(t)|H_{s2p}(-v,f)|^2\}\phi^2}{[(1-\rho_{s2p}^2(t))\sigma_h^2][F_{V|\rho,H}^{-1}(A)]^2}, \quad \rho_{s2p}(t) \neq 1.$$

By constraining the $P_s(t,f)$ with parameters ϕ and A, we can control the variance of $I(t,f)$. Moreover, when A is close to 1, e.g., $A = 0.9$, $F_{V|\rho,H}^{-1}(A)$ will increase substantially. Consequently, the mean and variance of $I(t,f)$ will face a stricter constraint in this case.

By using (25) instead of the interference constraint used in [P-4]-[P-5], we can have new approaches, which control the $\Psi_{in}(t,f)|_{\rho_{s2p}(t),H_{s2p}(-v,f)}$. We reformulate [P-4] to obtain [P-8] as

$$\min\left\{-W\sum_{f=1}^{M}\sum_{n=1}^{L}\frac{(-1)^{n+1}}{n\ln 2}\frac{P_s^n(t,f)R_n(t,f)}{Q_n(f)}\right\}$$
$$\text{s.t. } P_s(t,f) \leqslant P_C(t,f,\phi,A), -P_s(t,f) \leqslant 0, \sum_{f=1}^{M} P_s(t,f) \leqslant \Phi. \quad [\mathrm{P}-8]$$

We can use the interior-point method to solve it. Especially, when $L = 2$, the iterative water-filling approach can be used. By the K.K.T. condition, the solution can be obtained as

$$P_s(t,f) = \begin{cases} P_5, & \mu\ln 2 \leqslant I_1 - P_5 I_2 \\ \left[I_1 - \frac{\mu\ln 2}{W}\right]/I_2, & I_1 - P_3 I_2 < \frac{\mu\ln 2}{W} < I_1 \\ 0, & I_1 \leqslant \frac{\mu\ln 2}{W} \end{cases}, \quad (27)$$

where $P_5 = P_C(t,f,\phi,A)$. Then the iterative water-filling approach (e.g., [18]) can be applied.

Similarly, we can reformulate [P-5] to obtain [P-9] as

$$\min\left\{-W\sum_{f=1}^{M}\log_2\left(1+\frac{P_S(t,f)E\left[|H_S(t,f)|^2\big|_{H_s(-u,f),\rho_s}\right]}{N_0+P_p(f)}\right)\right\}$$
$$\text{s.t. } P_s(t,f) \leqslant P_C(t,f,\phi,A), \quad -P_s(t,f) \leqslant 0, \quad \sum_{f=1}^{M} P_s(t,f) \leqslant \Phi. \quad [\mathrm{P}-9]$$

According to Appendix A, the barrier water-filling form solution is

$$P_s(t,f) = \begin{cases} P_5, & \frac{W}{\mu\ln 2} \geqslant P_5 + P_4 \\ \frac{W}{\mu\ln 2} - P_4, & P_4 < \frac{W}{\mu\ln 2} < P_5 + P_4 \\ 0, & \frac{W}{\mu\ln 2} \leqslant P_4 \end{cases}. \quad (28)$$

In (27) and (28), the upper limits are the same (P_5) due to the same conditional probability constraint. And when $0 < P_s(t,f) < P_5$, the power allocations follow different rules due to the different adopted approximations. Furthermore, since we only adopt the approximations for conditional average capacity, the interference control is not affected by the approximation. Thus, even in the high SINR case in which the two proposed approximations are quite loose, the interference control can still achieve the interference control target. The only difference is that the average capacity of the secondary link would be affected.

IV. Multiple Secondary Systems

In this section, we investigate the case of one primary system with several secondary systems in the considered geographical area. We suppose each secondary system adopts the same strategy to control the interference and optimize the average throughput. Additionally, we assume different secondary systems cannot hear from each other which could be established via contention process. Thus, the different secondary systems do not interfere each other. Then for this multiple secondary systems scenario, the average capacity formula for each secondary system is the same as the single secondary system case. Therefore, the proposed approximations in Section III can still be used. The differences between single and multiple secondary systems are in interference control. Additionally, as we assumed in Section II, the frequency bands occupied by different pairs of secondary users within one secondary system are not overlapped. Therefore, in the multiple secondary system scenario, for a specific frequency band, we only need to consider one pair of secondary users from each secondary system.

A. Conditional Mean Interference Power Constraint

Since the different secondary systems operate independently, the tolerable interference threshold can be equally shared by the secondary systems in mean interference control. Thus, for a ST in ith secondary system, we have $\phi_i = \frac{\phi}{N_s}$, where N_s is the number of secondary systems. Using ϕ_i to substitute ϕ in (8) and (15), we can obtain the new conditional mean interference constraint per subband and per symbol for ST in the ith secondary system, respectively as

$$P_{s,i}(t,f) \leqslant \frac{\phi_i}{\rho_{s2p,i}^2(t)|H_{s2p,i}(-v_i,f)|^2 + (1-\rho_{s2p,i}^2(t))2\sigma_h^2}, \quad (29)$$

$$\sum_{f=1}^{M} P_{s,i}(t,f)E\left[|H_{s2p,i}(t,f)|^2\big|_{H_{s2p,i}(-v_i,f)\quad \rho_{s2p,i}}\right] \leqslant M\phi_i. \quad (30)$$

Here, $P_{s,i}(t,f), \rho_{s2p,i}(t), H_{s2p,i}, v_i$ respectively correspond to $P_s(t,f), \rho_{s2p}(t), H_{s2p}, v$ in ith secondary system. By substituting (29) into [P-4] and [P-5], we can obtain the multiple secondary system versions of [P-4] and [P-5], denoted by [P-4m] and [P-5m]. Similarly, by substituting (30) into [P-6] and [P-7], we obtain the new versions [P-6m] and [P-7m].

B. Conditional Probability Constraint on Intolerable Interference

In the scenario with multiple secondary systems, each secondary system have no information about other systems except the number of secondary systems. And we assume that every secondary system adopts the same principle to control the interference to PR. If each ST uses the control principle as below

$$\Upsilon_i \triangleq \text{Pro}\left\{ P_{\mathrm{s,i}}(t,f)|H_{\mathrm{s2p,i}}(t,f)|^2 < \frac{\phi}{N_{\mathrm{s}}}\Big|_{\rho_{\mathrm{s2p,i}},H_{\mathrm{s2p,i}}(-v_i,f)}\right\} \geq \eta, \quad (31)$$

since all the STs in different systems control the interference independently, we can easily guarantee that with only local knowledge of $\rho_{\mathrm{s2p,i}}(t)$ and $H_{\mathrm{s2p,i}}(-v_i,f)$ at each ST,

$$\text{Pro}\left\{ \sum_{i=1}^{N_{\mathrm{s}}} P_{\mathrm{s,i}}(t,f)|H_{\mathrm{s2p,i}}(t,f)|^2 < \phi \right\} > \eta^{N_{\mathrm{s}}}. \quad (32)$$

As in Section III-C, the interference control target is to control the probability of tolerable interference to PR to be larger than A as

$$\text{Pro}\left\{ \sum_{i=1}^{N_{\mathrm{s}}} P_{\mathrm{s,i}}(t,f)|H_{\mathrm{s2p,i}}(t,f)|^2 < \phi \right\} > A. \quad (33)$$

Thus, by setting $\eta = A^{\frac{1}{N_{\mathrm{s}}}}$, we can have the interference constraint for ST from ith secondary system as

$$\Upsilon_i = \text{Pro}\left\{ P_{\mathrm{s,i}}(t,f)|H_{\mathrm{s2p,i}}(t,f)|^2 < \frac{\phi}{N_{\mathrm{s}}}\Big|_{\rho_{\mathrm{s2p,i}},H_{\mathrm{s2p,i}}(-v_i,f)}\right\} \geq (A)^{\frac{1}{N_{\mathrm{s}}}}. \quad (34)$$

Following the same derivation in Section III-C, we can have the constraint as

$$P_{\mathrm{s,i}}(t,f) \leq P_{\mathrm{C}}(t,f,\frac{\phi}{N_{\mathrm{s}}},(A)^{\frac{1}{N_{\mathrm{s}}}}). \quad (35)$$

Obviously, (35) is stricter than our control target, which would sacrifice the capacity. In practice, we can use $(\alpha A)^{\frac{1}{N_{\mathrm{s}}}}$ in place of $(A)^{\frac{1}{N_{\mathrm{s}}}}$ in (35) to counter the rather loose inequality in (32), where $0 < \alpha \leq 1$ and it can be set by simulation study. By substituting (35) into [P-8] and [P-9], we can obtain the multiple secondary system version of [P-8m] and [P-9m].

V. Performance Evaluation

A. Simulation Parameters

Here, we consider the OFDM system as an example with $N = 512$ subcarriers (the same as the IDFT/DFT size). We take each 32 consecutive subcarriers as a subband. Then there are $M = 16$ subbands. The simulation parameters are adopted from the LTE system: 2 GHz carrier frequency, 15 kHz subcarrier spacing, and the OFDM symbol duration is 71.355μs. The frame length (subframe length in LTE terminology) of $L_f = 14$ OFDM symbols is considered. We take the delay $u = 0$ and $v = 0$ as an example.

The channel model is a frequency-selective time-varying channel with 16 uncorrelated Rayleigh fading taps which have an exponential power delay profile with a 3 dB per tap decay factor. The channel gains are assumed to be static within one OFDM symbol duration but vary from symbol to symbol according to the Clark-Jakes' model. In order to get the performance in various mobile environments, we consider eight values of Doppler spread ranging from 5 Hz to 650 Hz (if normalized by the frame rate, the range is from 0.005 to 0.65).

Since PR can tolerate interference power up to ϕ, we introduce two metrics, *excess interference power* and *probability of intolerable interference*, to reflect the interference tolerance aspect. The excess interference power within one OFDM symbol I_e and within one frame I_E are defined as

$$I_e(t) = \sum_{f=1}^{M}(I(t,f)-\phi)(\text{sgn}(I(t,f)-\phi)+1)/2, \quad (36)$$

$$I_E = \sum_{t=1}^{L_f}\sum_{f=1}^{M}(I(t,f)-\phi)(\text{sgn}(I(t,f)-\phi)+1)/2. \quad (37)$$

We set the noise power N_0 as 1. For simplicity, we set $P_{\mathrm{p}}(f)$ at 10 dB above N_0 in the first eight subbands and 12 dB above N_0 in the second eight subbands. Also, we set $\sigma_h^2 = 0.5$. The maximum transmission power of ST is set as $\Phi = 64N_0$, and the constraint on the interference is set as $\phi = 2N_0$. For [P-8] and [P-9], we set $A = 0.9$. And we use $L = 2$ for [A-1].

B. Simulation Results

First, consider the single secondary system scenario and recall the classification of related criteria in this paper. The approaches [P-1], [P-4] and [P-5] apply interference power constraint per subband, while [P-2], [P-6] and [P-7] adopt interference power constraint per symbol. [P-8] and [P-9] use the probability of intolerable interference constraint. Additionally, we should emphasize that all the approaches with constraint per subband can be directly applied to the problem of constraint per tone if desired.

The mean interference power is shown in Figs. 2–4 at the frame level, symbol level and subband level, respectively. Here, all the data are normalized by its time and frequency scale. For example, the mean interference power at the frame level is normalized by ML_f. From the figures, we find the conventional approaches [P-1] and [P-2] have the mean interference power increase substantially when the Doppler spread goes high. On the other hand, Figs. 2– 4 illustrate that [P-4] and [P-5] can control the mean interference power per subband within the constraint of $\phi = 2$. At the symbol level, it is also validated by Figs. 2 and 3 that the proposed approaches [P-6] and [P-7] can exactly keep the mean interference power per symbol under control.

The mean transmit power per subband is shown in Fig. 5. For the approaches with interference power constraint per symbol, more power is allocated to the subbands with better channel (both channel power gain and interference from PT are considered) in order to optimize the ergodic channel capacity. For cases with interference power constraint per subband, the

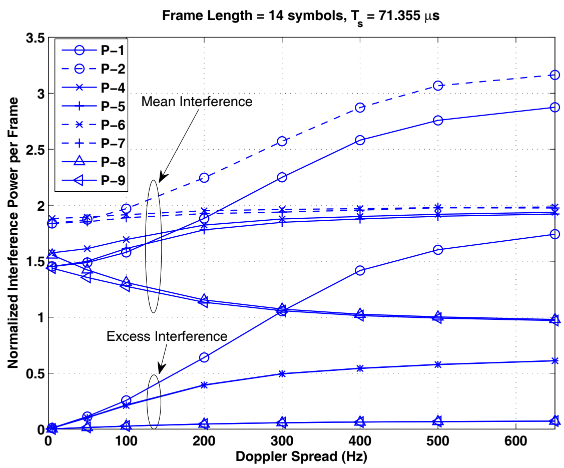

Fig. 2. Mean Interference and excess interference power per frame at the primary receiver.

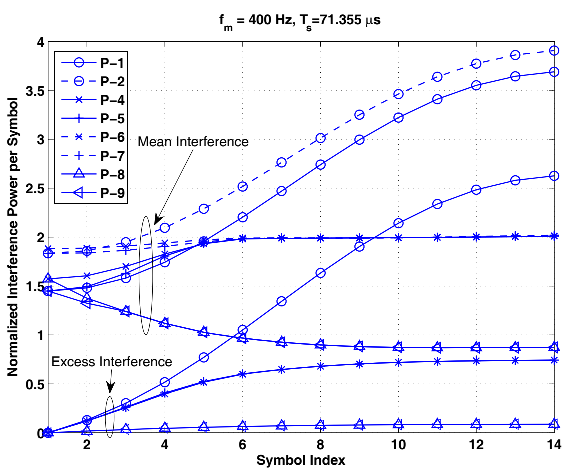

Fig. 3. Mean interference and excess interference power per symbol at the primary receiver.

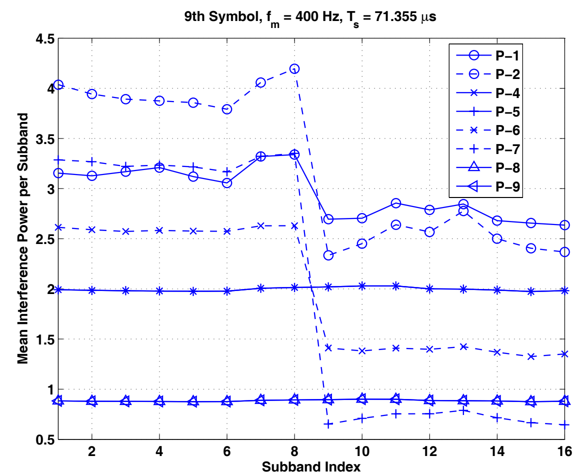

Fig. 4. Mean interference power per subband at the primary receiver.

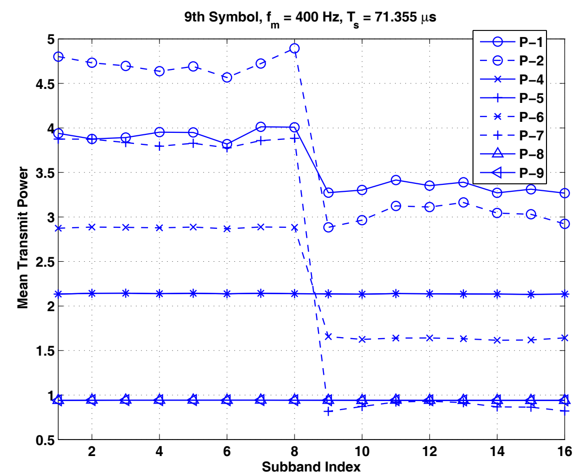

Fig. 5. Mean transmit power per subband of the secondary transmitter.

proposed approaches ([P-4] and [P-5]) have the same power allocation as [P-1] at the beginning of each transmission, which allocates more power on the subbands with better channel. When the CSI becomes outdated, the interference constraint becomes stricter than the transmission power constraint. Thus, [P-4] and [P-5] move towards allocating the power equally across subbands to meet the interference constraint, which leads to equal interference to PR on each subband. Comparing the constraint per subband and per symbol, the approaches with constraint per symbol ([P-6] and [P-7]) can allocate more power on the subbands with better channel than those with constraint per subband ([P-4] and [P-5]), which leads to better ergodic capacity performance in secondary link. Comprehensively, compared with constraint per subband, the interference power constraint per symbol relaxes the constraint, which leads to better capacity performance.

For the approaches with interference constraint per subband, we compare their excess interference power also in Figs. 2 and 3 at the frame level and symbol level, respectively. The conventional constraint [P-1] generates huge excess interference power. [P-4] and [P-5] have almost the same excess interference power control performance, which is much better than [P-1] but still cannot be ignored. On the other hand, the excess interference generated by [P-8] and [P-9] is much smaller due to the strict interference power constraint.

The probabilities of intolerable interference are shown in Figs. 6 and 7 at the frame level and symbol level, respectively. They illustrate that the proposed probability control scheme [P-8] and [P-9] can accurately control the probability of intolerable interference, while other approaches have large probability of intolerable interference. In fact, [P-8] and [P-9] yield the best interference control performance as can be seen in Figs. 2-7. If the primary system has strict requirement on the reliability, the [P-8] and [P-9] should be applied.

The ergodic channel capacities are shown in Figs. 8 and 9 at the frame level and the symbol level, respectively. The conventional approaches [P-1] and [P-2] have better capacity than the corresponding proposed approaches, but their interference is unacceptable to the primary system as shown in the previous figures. It is also observed that the approaches with constraint per symbol have better ergodic capacity performance than those with constraint per subband. Thus, if a strong coding scheme is applied in the primary system, the interference con-

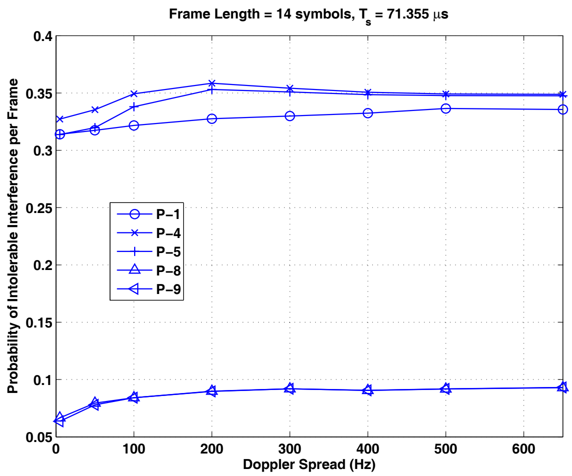

Fig. 6. The probability of intolerable interference per frame at the primary receiver.

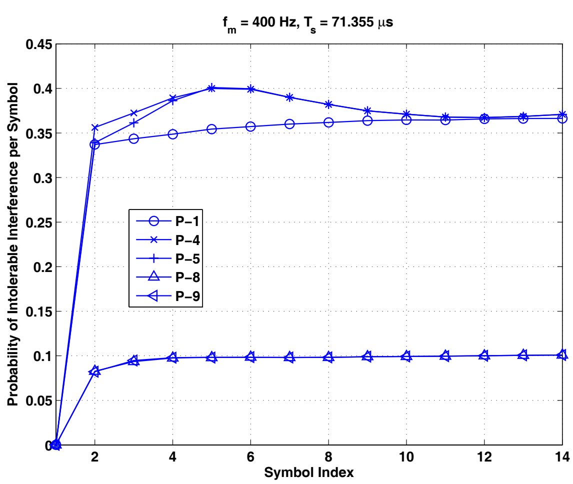

Fig. 7. The probability of intolerable interference per symbol at the primary receiver.

trol per symbol can substantially increase the performance of secondary link compared to the constraint per subband. For [P-8] and [P-9], due to their strict constraint on interference, the capacity performance is compromised as predicted. Moreover, the approaches using [A-2] still outperform those using [A-1] when $L = 2$. Additionally, we also test the sensitivity of the schemes to the error of $P_{\mathrm{p}}(f)$ (error variance is 10% of the actual value of $P_{\mathrm{p}}(f)$). The results (omitted due to the figure limitation) show that our proposed schemes are quite robust to the error of $P_{\mathrm{p}}(f)$.

For multiple secondary systems scenario, we consider $N_s = 2$, and the other parameters are the same as those in single secondary system. The simulation results are shown in Figs. 10-12. Since each secondary system follows the same transmission scheme as the single secondary system scenario (except some changes in parameters' values), the simulation results show the same trends as those in the single secondary system. As shown in Figs. 10-11, the control targets of the mean interference constraint and probability of intolerable interference constraint are achieved. Therefore, the proposed

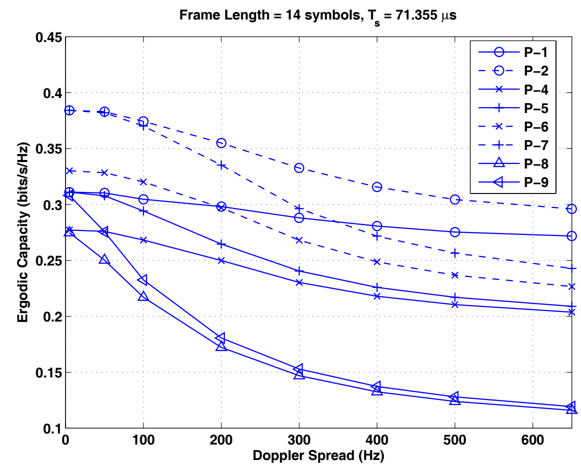

Fig. 8. Ergodic channel capacity (averaged over all symbols within a frame) of the secondary user link.

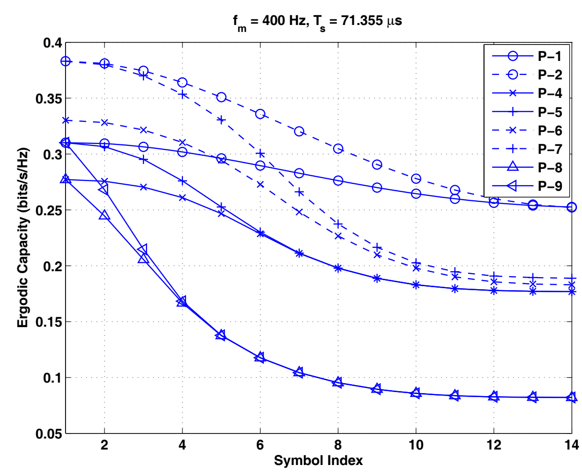

Fig. 9. Symbol by symbol ergodic channel capacity within a frame of the secondary user link.

schemes in multiple secondary systems scenario are effective as well. Especially, as we mentioned before, the proposed control scheme with conditional probability constraint for multiple secondary systems scenario is much stricter than the targets, which yields the simulation results for the probability of intolerable interference much smaller than 0.1. This strict constraint limits the ergodic capacity performance of the secondary systems. We can adjust α to loose the constraint so that the probability of intolerable interference is closer to the design target. As shown in Figs. 11 and 12, in our simulation system, when we set $\alpha A = 0.75$, the probability of intolerable interference is close to 0.1. Consequently, the ergodic capacity under this new constraint is better than the original schemes.

VI. Conclusions

In the underlay CR systems, constraining the interference power to the primary user to be below a certain level is essential for the coexistence of primary and secondary user systems, and the power allocation of the secondary user plays an important role. The existing interference constraint and power allocation strategies are based on the quasi-static

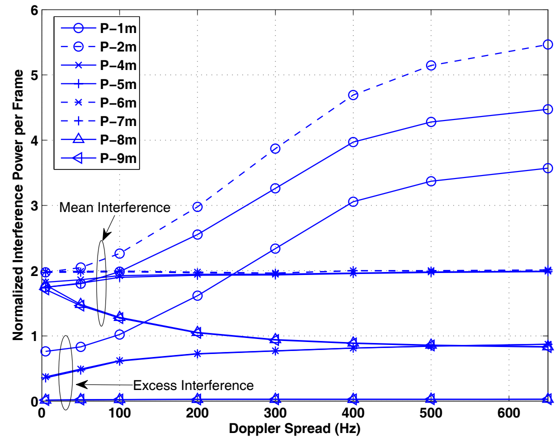

Fig. 10. Mean interference power per frame at the primary receiver in multiple secondary systems scenario.

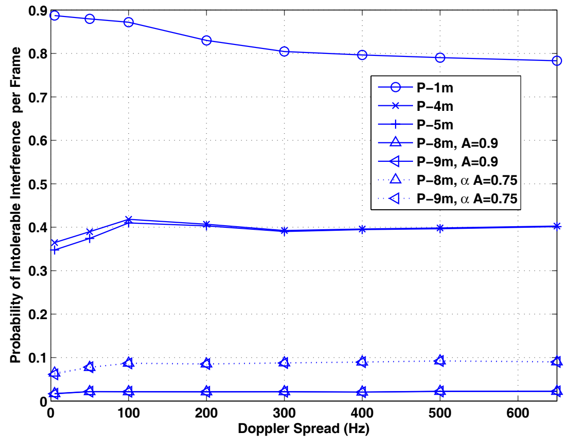

Fig. 11. The probability of intolerable interference per frame at the primary receiver in multiple secondary systems scenario.

channels and hence, their application to mobile environments can cause interference much well above the acceptable level at the primary user due to the channel variations within the transmission frame. In this paper, we have developed time-varying interference control schemes and power allocation strategies for CR in time-varying frequency selective channels and extend them to a multiple-secondary-systems scenario. For secondary users operating in the low SINR regime, the proposed approaches can achieve the interference control target and provide substantial advantages over the existing strategy for a wide range of Doppler spread. For the primary system with a strong coding scheme, the transmission scheme with interference constraint per symbol can be applied to maximize the conditional average channel capacity and approximately maximize the ergodic capacity. Otherwise, if uncoded scheme is used in primary user or no such knowledge is available, transmission scheme with interference constraint per subband or per tone should be adopted. Especially, when the primary system is very sensitive to the intolerable interference or has strict requirement on reliability, then we should adopt the transmission scheme with probability control of the intolerable interference.

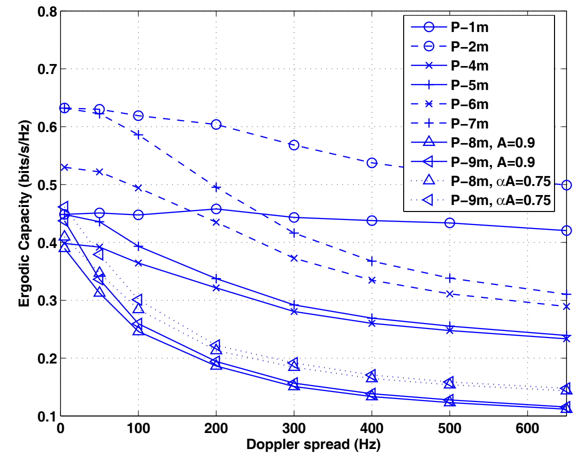

Fig. 12. Ergodic channel capacity (averaged over all symbols within a frame) for all considered secondary user links in multiple secondary sytems scenario.

APPENDIX A

The considered convex problem can be given as

$$\min_{x_i} F(x_i) = -\sum_{i=1}^{M} W \log_2(1 + \frac{x_i a_i}{b_i}) \tag{38}$$

$$\text{s.t.} \sum_{i=1}^{M} x_i \leq \Phi, x_i \leq c_i, -x_i \leq 0, (a_i, b_i, c_i \geq 0). \tag{39}$$

The Lagrangian formulation is given by

$$L(x_i) = -\sum_{i=1}^{M} W \log_2(1 + \frac{x_i a_i}{b_i}) + \sum_{i=1}^{M} \nu_i(-x_i) + \sum_{i=1}^{M} \lambda_i(x_i - c_i) + \mu(\sum_{i=1}^{M} x_i - \Phi) \tag{40}$$

where ν_i, λ_i and μ are Lagrange multipliers. Then applying K.K.T. condition, we have

$$\frac{\partial L(x_i)}{\partial x_i} = -\frac{W}{\ln 2}\frac{a_i}{b_i + a_i x_i} - \nu_i + \lambda_i + \mu = 0 \tag{41}$$

$$\nu_i \geq 0, \nu_i x_i = 0, \lambda_i \geq 0, \lambda_i(x_i - c_i) = 0. \tag{42}$$

We have two cases for this optimization problem as below:

- Case I: $\sum_{i=1}^{M} x_i < \Phi$. This is equivalent to $x_i = c_i, \forall i$.
- Case II: $\sum_{i=1}^{M} x_i = \Phi$. If $x_i = c_i$, then the solution is $x_i = c_i$.
  Otherwise, $x_i < c_i$. It becomes a typical waterfilling problem. The solution is

$$x_i = \left[\frac{W}{\mu \ln 2} - \frac{b_i}{a_i}\right]^+, \tag{43}$$

where $[y]^+ \triangleq \max(0, y)$. Thus, overall, we have the solution as

$$x_i = \begin{cases} c_i, & \frac{W}{\mu \ln 2} \geqslant c_i + \frac{b_i}{a_i} \\ \frac{W}{\mu \ln 2} - \frac{b_i}{a_i}, & \frac{b_i}{a_i} < \frac{W}{\mu \ln 2} < c_i + \frac{b_i}{a_i} \\ 0, & \frac{W}{\mu \ln 2} \leqslant \frac{b_i}{a_i} \end{cases}. \tag{44}$$

REFERENCES

[1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.,* vol. 23, no. 2, pp. 201–220, Feb. 2005.

[2] A. Goldsmith, S. A. Jafar, I. Marić, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proc. IEEE,* vol. 97, no. 5, pp. 894–914, May 2009.

[3] T. C. Clancy, "Achievable capacity under the interference temperature model," in *Proc. IEEE INFOCOM 2007,* pp. 794–802.

[4] H. Yao, Z. Zhou, H. Liu, and L. Zhang, "Optimal power allocation in joint spectrum underlay and overlay cognitive radio networks," in *Proc. IEEE CROWNCOM 2009,* pp. 1–5.

[5] M. Gastpar, "On capacity under receive and spatial spectrum-sharing constraint," *IEEE Trans. Inf. Theory,* vol. 53, no. 2, pp. 471–487, Feb. 2007.

[6] G. Bansal, M. J. Hossain, and V. K. Bhargava, "Optimal and suboptimal power allocation schemes for OFDM-based cognitive radio systems," *IEEE Trans. Wireless Commun.,* vol. 7, no. 11, pp. 4710–4718, Nov. 2008.

[7] Y. Zhang and C. Leung, "An efficient power loading scheme for OFDM-based cognitive radio systems," *IEEE Trans. Veh. Technol.,* vol. 59, no. 4, pp. 1858–1864, May, 2010.

[8] X. Kang, Y. Liang, A. Nallanathan, H. K. Garg, and R. Zhang, "Optimal power allocation for fading channels in cognitive radio networks: ergodic capacity and outage capacity," *IEEE Trans. Wireless Commun.,* vol. 8, no. 2, pp. 940–950, Feb. 2009.

[9] L. Musavian and S. Aïssa, "Capacity and power allocation for spectrum-sharing communications in fading channels," *IEEE Trans. Wireless Commun.,* vol. 8, no. 1, pp. 148–156, Jan. 2009.

[10] L. Le and E. Hossain, "Resource allocation for spectrum underlay in cognitive radio networks," *IEEE Trans. Wireless Commun.,* vol. 7, no. 12, pp. 5306–5315, Dec. 2008.

[11] R. Zhang, "On peak versus average interference power constraints for protecting primary users in cognitive radio networks," *IEEE Trans. Wireless Commun.,* vol. 8, no. 4, pp. 2112–2120, Apr. 2009.

[12] X. Hong, C. Wang, H. Chen, and J. Thompson, "Performance analysis of cognitive radio networks with average interference power constraints," in *IEEE ICC 2008,* pp. 3578–3582.

[13] C. Wang, X. Hong, H. Chen, and J. Thompson, "On capacity of cognitive radio networks with average interference power constraints," *IEEE Trans. Wireless Commun.,* vol. 8, no. 4, pp. 1620–1625, Apr. 2009.

[14] J. Pang, G. Scutari, D. P. Palomar, and F. Facchinel, "Design of cognitive radio systems under temperature-interference constraints: a variational inequality approach," *IEEE Trans. Signal Process.,* vol. 58, no. 6, pp. 3251–3271, June 2010.

[15] X. Kang, Y. Liang, and A. Nallanathan, "Optimal power allocation for fading channels in cogniteve radio networks under transmit and interference power constraints," in *Proc. IEEE ICC 2008,* pp. 3568–3572.

[16] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading enviorments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[17] A. Plattner, "Technology and demonstrator of the RACE project Mobile Broadband System," in *Proc. 1994 IEEE MTT-S Intl. Microwave Symp.,* vol. 2, pp. 639–642.

[18] Y. Pan, A. Nix, and M. Beach, "Distributed resource allocation for OFDMA-based relay networks," *IEEE Trans. Veh. Technol.*, vol. 60, no. 3, pp. 919–931, Mar. 2011.

[19] S. Pandula and B.-P. Paris, "Capacity optimization for Ricean correlated MIMO channels with decorrelator receiver," in *Proc. 2008 Conference of Information Sciences and Systems,* pp. 374–378.

Wenxun Qiu (S'09) received the B.E. and M.S. degree in Electrical Engineering from Harbin Institute of Technology in 2005 and 2008. He is currently working towards his Ph.D. degree in Electrical Engineering at the University of Texas at Dallas, TX, USA. His research interests include resource allocation in multicarrier systems, cognitive radio networks and advanced transmission schemes.

Baile Xie (S'10) received the B.S. degree in Communication Engineering from University of Electronic Science and Technology of China, Chengdu, China in 2008. Now she is working towards the Ph.D. degree in Electrical Engineering at the University of Texas at Dallas, TX, USA. Her research interests include wireless communications and wireless health-care technologies.

Hlaing Minn (S'99, M'01, SM'07) received the B.E. degree in Electronics from the Yangon Institute of Technology, Yangon, Myanmar, in 1995, the M.Eng. degree in Telecommunications from the Asian Institute of Technology (AIT), Pathumthani, Thailand, in 1997, and the Ph.D. degree in Electrical Engineering from the University of Victoria, Victoria, BC, Canada, in 2001. He was a laboratory supervisor in the Telecommunications Program at AIT during Feb.-Dec. 1998, and a post-doctoral fellow at the University of Victoria during Jan.-Aug. 2002. He has been with the Erik Jonsson School of Engineering and Computer Science, the University of Texas at Dallas, Richardson, TX, USA since Sept. 2002, and currently is an Associate Professor. His research interests include wireless communications, statistical signal processing, error control, detection, estimation, synchronization, signal design, cross-layer design, cognitive radios, and wireless health-care applications. Prof. Minn serves as an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS and for the *International Journal of Communications and Networks*. He served as a Co-Chair of the Wireless Access Track in the IEEE VTC 2009 (Fall).

Chia-Chin Chong (S'00, M'03, SM'08) is a Senior Manager at Orange France Telecom in San Francisco, CA. Prior to joining Orange, she spent over 5 years at NTT DOCOMO USA Labs and 2 years at Samsung Electronics R&D. Dr. Chong has done research in cellular systems, short range communications, wireless home energy management and wireless healthcare. She has been an invited speaker and panelist in many international conferences. Dr. Chong has authored a book chapter and published more than 90 international journals, conference papers, and international standard contributions. She also has more than 30 granted and pending patents. Dr. Chong has received numerous awards including the IEE Prize and the IEE Vodafone Research Award in 1999 and 2001, respectively, the IEEE International Conference on Ultra-Wideband (ICUWB) Best Paper Award and the DOCOMO USA Labs President Award in 2006, the URSI Young Scientist Award and the DOCOMO USA Labs President Award in 2008. Dr. Chong was honored with The Outstanding Young Malaysian Award 2006 (the most prestigious achievement award for people under age 40 in Malaysia) by Junior Chamber International and Ministry of Youth and Sports, Malaysia under the category "Scientific and Technological Development". Currently, Dr. Chong serves as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and the Associate Editor for IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY and *EURASIP Journal on Wireless Communications and Networking (JWCN)*. She has served on the committee members and organizers of various international conferences including the TPC Co-Chair for the Broadband Wireless Access Symposium of the IEEE ICCCN 2007, the TPC Co-Chair for the Wireless Communications Symposium of the IEEE ICC 2008, the Publicity Co-Chair for the IEEE PIMRC 2008, the Sponsorship Chair for the CrownCom 2008, and the Tutorial Chair for the IEEE ICCCN 2009. She was also the Former Chair for the DG-EVAL Channel Model standardization group within the ITU-R WP5D for IMT systems.