Robust and Consistent Pilot Designs for Frequency Offset Estimation in MIMO OFDM Systems

Yinghui Li, *Student Member, IEEE*, and Hlaing Minn, *Senior Member, IEEE*

***Abstract*—This paper presents pilot designs for consistent frequency offset estimation of MIMO OFDM systems in frequency-selective fading channels. We derive the sufficient consistency condition for the pilots in MIMO OFDM systems to yield unambiguous estimation, and present corresponding consistent pilot designs. We discuss robustness of the frequency offset estimation against outliers at low to moderate SNR values and present an efficient criterion to choose robust and consistent pilots. Furthermore, we develop pilot designs which satisfy both consistency over a limited frequency offset estimation range and the optimal channel estimation condition in MIMO OFDM systems. Simulation results corroborate that both the consistent pilot design condition and the robustness criterion are efficient in choosing pilot patterns yielding better frequency offset estimation performance.**

***Index Terms*—Frequency offset, MIMO OFDM, consistency, robustness, pilot design, maximum likelihood estimation.**

I. Introduction

MULTIPLE input multiple output (multiple antennas) orthogonal frequency-division multiplexing (MIMO OFDM) provides high capacity wireless links for future wireless networks [1] [2]. However, OFDM systems are very sensitive to frequency synchronization errors, requiring efficient carrier frequency offset (CFO) estimators (e.g., see [3]–[7] for single input single output (SISO) systems and [8] [9] for MIMO systems). Although crucial for emergency and other critical wireless systems, the issue of avoiding ambiguity of the CFO estimation metric trajectory within the considered estimation range has been overlooked until recently. Some training sequences, under noise-free condition, can give multiple maxima of the estimation likelihood metric function, known as the ambiguity of the CFO estimation metric, for some channel responses. This ambiguity will yield a large CFO estimation error and the link failure.

For emergency and other critical systems, the link failure may have significant consequences, while for other communication systems the CFO estimation inconsistency may affect QoS support. For example, consider a situation where the channel is not in fading but it yields inconsistent CFO estimation due to the inconsistent pilot, resulting in a packet error. If the channel is slowly varying, additional attempts using ARQ techniques would not help due to large CFO estimation errors. Some research works on this problem for SISO OFDM systems have been discussed in [3] [10] for null tone based estimators and in [11] [12] for pilot based estimators. Very recently, [13] addressed the robustness of consistent pilot designs against outliers for SISO OFDM systems and presented an efficient robustness design criterion.

All of the above existing pilot designs address only for SISO OFDM systems. Since consistency and robustness of CFO estimator are also crucial for realizing the advantages of MIMO OFDM systems, in this paper we pursue these consistent and robust pilot design issues for MIMO OFDM systems. Although CFO estimators for MIMO and SISO scenarios are similar, the pilot designs are different for MIMO and SISO systems. In MIMO systems, not only a larger set of pilot vectors needs to be designed, but also the effects of these pilot vectors on one another need to be considered, hence requiring a careful design.

In this paper, we derive the sufficient condition for the consistent CFO estimation in MIMO OFDM systems using the CFO estimators[1] from [14] (the time-domain (TD) - maximum likelihood estimator (MLE)) and [11] (the frequency-domain (FD) - MLE) (see Section III), and develop corresponding consistent pilot design patterns for MIMO OFDM systems (see Section IV). Furthermore, we propose pilot designs for both consistent CFO MLE and optimal channel estimation in MIMO systems (see Section IV), which has not been addressed in the literature. The consistency conditions in the probabilistic sense are also discussed for MIMO systems (see Section V). Then we present a new design criterion for the consistent pilots to be robust against outliers in MIMO OFDM systems (see Section VI). Simulation results corroborate the effectiveness of our pilot designs (see Section VII).

Notations: A bold small (capital) letter represents a column vector (matrix). The superscripts $*$, T, and H represent the conjugate, the transpose, and the conjugate transpose operations, respectively. $[\boldsymbol{Y}]_{k,m}$ denotes the k-th row, m-th column element of $\boldsymbol{Y}$. All indices start from 0. $\boldsymbol{\Phi}[0{:}m,\ 0{:}n]$ denotes the sub-matrix of $\boldsymbol{\Phi}$ comprising of the first $m+1$ rows and the first $n+1$ columns of $\boldsymbol{\Phi}$. The all-one (all-zero) column vector of length-k, the $k \times m$ all-zero matrix, and the $k \times k$ identity matrix are denoted by $\mathbf{1}_k$ $(\mathbf{0}_k)$, $\mathbf{0}_{k\times m}$,

Paper approved by G.-H. Im, the Editor for Equalization and Multicarrier Techniques of the IEEE Communications Society. Manuscript received September 15, 2006; revised February 24, 2007 and May 13, 2007. This paper was presented in parts at IEEE ICASSP 2007. The work of H. Minn was supported by the Erik Jonsson School Research Excellence Initiative, the University of Texas at Dallas, USA.

The authors are with the Department of Electrical Engineering, University of Texas at Dallas, P.O. Box 830688, EC 33, Richardson, TX 75080 USA (e-mail: {yxl044000, hlaing.minn}@utdallas.edu).

Digital Object Identifier 10.1109/TCOMM.2008.060522

[1]The CFO FD-MLE in [11] and the CFO TD-MLE in [14] both maximize the likelihood function for the joint estimation of the frequency offset and the channel. The former used a frequency-domain model and the channel's limited delay spread property was not exploited in its frequency-domain channel estimation. Hence, the CFO TD-MLE outperforms the CFO FD-MLE but under certain conditions both give the same performance. The advantage of FD-MLE is the lower complexity.

and $\boldsymbol{I}_k$, respectively. The i-th column of $\boldsymbol{I}_N$ is denoted by $\boldsymbol{e}_i$ and diag$\{\boldsymbol{x}\}$ represents a diagonal matrix whose diagonal elements are defined by $\boldsymbol{x}$. The l-cyclic-down-shifted version of $\boldsymbol{c}$ is denoted by $\boldsymbol{c}^{(l)}$. $\otimes$ denotes the Kronecker product. l mod N represents l modulo N. $\lfloor X \rfloor$ denotes the largest integer not greater than X while $\lceil X \rceil$ represents the smallest integer greater than or equal to X. $\boldsymbol{F}$ denotes the N-point unitary discrete Fourier transform matrix and $\boldsymbol{f}_k$ is the k-th column of $\boldsymbol{F}$.

II. Signal Model and Frequency Offset Estimation

We consider a MIMO OFDM system where training signals are transmitted from N_t transmit antennas to N_r receive antennas over one OFDM symbol which contains N subcarriers. The length-L sample-spaced channel impulse response (CIR) vector between the k-th transmit and l-th receive antenna pair is denoted by $\boldsymbol{h}_{(k,l)}$ which is quasi-static over one OFDM symbol. Let $\boldsymbol{c}_m = [c_m(0), \ldots, c_m(N-1)]^T$ be the pilot tone vector of the m-th transmit antenna. Denote the indices of non-zero pilot tones and null tones of the m-th transmit antenna by $\{t_m(k) : k = 0, \ldots, P_m - 1\}$ and $\{n_m(k) : k = 0, \ldots, N - P_m - 1\}$, where P_m denotes the number of non-zero pilots on the m-th transmit antenna. Let $\boldsymbol{s}_m = [s_m(-N_g), s_m(-N_g+1) \ldots, s_m(N-1)]$ denote the corresponding time-domain complex baseband training samples of the m-th transmit antenna, including N_g $(\geq L-1)$ cyclic prefix samples. Define $\boldsymbol{S}_m$ as the training signal matrix of size $N \times L$ for the m-th transmit antenna with elements given by $[\boldsymbol{S}_m]_{k,l} = s_m(k-l)$ for $k \in \{0, \ldots, N-1\}$ and $l \in \{0, \ldots, L-1\}$.

After removing the cyclic prefix, the received vector $\boldsymbol{r}$ from all N_r receive antennas in the presence of a normalized (by the subcarrier spacing) frequency offset v can be expressed as

$$\boldsymbol{r} = [\boldsymbol{I}_{N_r} \otimes (\boldsymbol{\Gamma}(v)\boldsymbol{S})]\,\boldsymbol{h} + \boldsymbol{w} \tag{1}$$

where
$$\boldsymbol{r} = \left[\boldsymbol{r}_0^T, \ldots, \boldsymbol{r}_{N_r-1}^T\right]^T \tag{2}$$
$$\boldsymbol{\Gamma}(v) = \mathrm{diag}\{1, e^{j2\pi v/N}, \ \ldots, \ e^{j2\pi(N-1)v/N}\} \tag{3}$$
$$\boldsymbol{S} = [\boldsymbol{S}_0, \ldots, \boldsymbol{S}_{N_t-1}] \tag{4}$$
$$\boldsymbol{h} = \left[\boldsymbol{h}_{(0,0)}^T, \boldsymbol{h}_{(1,0)}^T, \ldots, \boldsymbol{h}_{(N_t-1,N_r-1)}^T\right]^T \tag{5}$$
$$\boldsymbol{w} = [\boldsymbol{w}_0^T, \boldsymbol{w}_1^T, \ldots, \boldsymbol{w}_{N_r-1}^T]^T. \tag{6}$$

Here $\boldsymbol{r}_k$ is the $N \times 1$ received signal vector at the k-th receive antenna, and the elements of $\boldsymbol{w}$ are independent and identically distributed zero-mean circularly-symmetric complex Gaussian noise samples with variance $\sigma_n^2 = E[|w_i(k)|^2]$. $\boldsymbol{S}$ is an $N \times N_t L$ matrix.

Applying the TD-MLE from [14] to the MIMO system gives the MLE of v as

$$\hat{v} = \arg\max_{\tilde{v}}\{g(\tilde{v})\} \tag{7}$$

where
$$g(\tilde{v}) = \boldsymbol{r}^H \left(\boldsymbol{I}_{N_r} \otimes \boldsymbol{\Gamma}(\tilde{v})\boldsymbol{B}\boldsymbol{\Gamma}^H(\tilde{v})\right)\boldsymbol{r} \tag{8}$$
$$\boldsymbol{B} = \boldsymbol{S}(\boldsymbol{S}^H\boldsymbol{S})^{-1}\boldsymbol{S}^H. \tag{9}$$

If we apply the FD-MLE to MIMO systems, the resulting CFO estimator is given by (7) with

$$g(\tilde{v}) = \boldsymbol{r}^H \left(\boldsymbol{I}_{N_r} \otimes \boldsymbol{\Gamma}(\tilde{v})\boldsymbol{B}_2\boldsymbol{\Gamma}^H(\tilde{v})\right)\boldsymbol{r} \tag{10}$$

where

$$\boldsymbol{B}_2 = \tilde{\boldsymbol{F}}_P^* \tilde{\boldsymbol{F}}_P^T \tag{11}$$
$$\tilde{\boldsymbol{F}}_P = [\boldsymbol{f}_{t_0(0)}, \ldots, \boldsymbol{f}_{t_0(P_0-1)}, \boldsymbol{f}_{t_1(0)}, \ldots, \boldsymbol{f}_{t_1(P_1-1)}, \ldots, \boldsymbol{f}_{t_{N_t-1}(0)}, \ldots, \boldsymbol{f}_{t_{N_t-1}(P_{N_t-1}-1)}]. \tag{12}$$

III. The CFO Estimator Consistency Condition in MIMO systems

The CFO estimation consistency condition for MIMO systems can be stated similar to the condition for SISO systems [12] as follows: "In the absence of noise, there is only one $\tilde{v}$ that maximizes the estimation metric $g(\tilde{v})$ and it is at $\tilde{v} = v$ for any $\boldsymbol{h} \neq \boldsymbol{0}$." Define

$$G(\Delta) = g(v) - g(\tilde{v}) \tag{13}$$

where $\Delta = v - \tilde{v}$ and the range of Δ is $(v - N/2, v + N/2)$. Then the consistency condition stated above can be expressed as:

$$G(\Delta) = 0 \text{ if and only if } \Delta = 0, \ \forall \boldsymbol{h} \neq \boldsymbol{0}. \tag{14}$$

A. Consistency Condition for CFO TD-MLE

For the TD-MLE, substituting (8) into (13), we obtain

$$G(\Delta) = \boldsymbol{h}^H \left[\boldsymbol{I}_{N_r} \otimes \left(\boldsymbol{S}^H \left(\boldsymbol{I}_N - \boldsymbol{\Gamma}^H(\Delta)\boldsymbol{B}\boldsymbol{\Gamma}(\Delta)\right)\boldsymbol{S}\right)\right]\boldsymbol{h}. \tag{15}$$

By singular value decomposition, we have $\boldsymbol{S} = \boldsymbol{U}\boldsymbol{\Sigma}_{\boldsymbol{S}}\boldsymbol{V}^H$ where $\boldsymbol{U}$=$[\boldsymbol{u}_0, \boldsymbol{u}_1, \ldots, \boldsymbol{u}_{N-1}]$ and $\boldsymbol{V}$ are the unitary matrices containing the eigen-vectors of $\boldsymbol{S}\boldsymbol{S}^H$ and $\boldsymbol{S}^H\boldsymbol{S}$, respectively. $\boldsymbol{\Sigma}_{\boldsymbol{S}}$ is the $N \times N_t L$ diagonal matrix with non-increasing singular values of $\boldsymbol{S}$. Here we assume $N \geq N_t L$ and $\boldsymbol{S}$ is a full-rank matrix, i.e. $\mathrm{rank}(\boldsymbol{S}) = N_t L$, which is required for identifiability of the joint TD-MLE of CFO and CIR. Then we can express (9) as

$$\boldsymbol{B} = \boldsymbol{U}\boldsymbol{\Sigma}_{\boldsymbol{S}}\left(\boldsymbol{\Sigma}_{\boldsymbol{S}}{}^H\boldsymbol{\Sigma}_{\boldsymbol{S}}\right)^{-1}\boldsymbol{\Sigma}_{\boldsymbol{S}}{}^H\boldsymbol{U}^H = \boldsymbol{U}\boldsymbol{\Sigma}_{\boldsymbol{B}}\boldsymbol{U}^H \tag{16}$$

where
$$\boldsymbol{\Sigma}_{\boldsymbol{B}} = \mathrm{diag}\{\boldsymbol{1}_{N_t L}^T, \boldsymbol{0}_{N-N_t L}^T\}. \tag{17}$$

Substituting (16) into (15), we obtain

$$G(\Delta) = \boldsymbol{z}^H(\Delta)\boldsymbol{z}(\Delta) \tag{18}$$

where
$$\boldsymbol{z}(\Delta) = \left[\boldsymbol{I}_{N_r} \otimes \left(\left(\boldsymbol{I} - \boldsymbol{\Sigma}_{\boldsymbol{B}}\right)\boldsymbol{U}^H\boldsymbol{\Gamma}(\Delta)\boldsymbol{U}\boldsymbol{\Sigma}_{\boldsymbol{S}}\boldsymbol{V}^H\right)\right]\boldsymbol{h}. \tag{19}$$

To satisfy the consistency condition in (14), we require that $\boldsymbol{z}(\Delta) = \boldsymbol{0}_{NN_r}$ *only* at $\Delta = 0$ for any $|\tilde{v}| < N/2$. In other words, we need $\boldsymbol{z} \neq \boldsymbol{0}_{NN_r}$ for any $|\hat{v}| < N/2$ except $\hat{v} = v$. By using (19), this condition can be achieved for *any* $\boldsymbol{h} \neq \boldsymbol{0}_{N_r N_t L}$ if $\boldsymbol{I}_{N_r} \otimes \left(\left(\boldsymbol{I} - \boldsymbol{\Sigma}_{\boldsymbol{B}}\right)\boldsymbol{U}^H\boldsymbol{\Gamma}(\Delta)\boldsymbol{U}\boldsymbol{\Sigma}_{\boldsymbol{S}}\boldsymbol{V}^H\right)$ is a full-rank matrix. Then, the consistency condition can be achieved for any $\boldsymbol{h} \neq \boldsymbol{0}$ if the following *necessary and sufficient* condition is satisfied:

$$\mathrm{rank}\left(\boldsymbol{I}_{N_r} \otimes \left(\left(\boldsymbol{I} - \boldsymbol{\Sigma}_{\boldsymbol{B}}\right)\boldsymbol{U}^H\boldsymbol{\Gamma}(\Delta)\boldsymbol{U}\boldsymbol{\Sigma}_{\boldsymbol{S}}\boldsymbol{V}^H\right)\right) = N_r N_t L, \quad \forall \Delta \neq 0. \tag{20}$$

By using (17), together with $\mathrm{rank}(\boldsymbol{S}) = N_tL$, we obtain from (20) a sufficient condition for consistency as

$$\mathrm{rank}(\boldsymbol{S}) = N_tL \;\; \& \;\; \mathrm{rank}\left(\boldsymbol{U}_2^H\boldsymbol{\Gamma}(\Delta)\boldsymbol{U}_1\right) = N_tL, \; \forall\Delta \neq 0 \tag{21}$$

$$\text{where} \;\; \boldsymbol{U}_1 = [\boldsymbol{u}_0, \boldsymbol{u}_1, \ldots, \boldsymbol{u}_{N_tL-1}] \tag{22}$$

$$\boldsymbol{U}_2 = [\boldsymbol{u}_{N_tL}, \boldsymbol{u}_{N_tL+1}, \ldots, \boldsymbol{u}_{N-1}]. \tag{23}$$

B. Consistency Condition for the CFO FD-MLE

Define

$$\boldsymbol{F}_L = [\boldsymbol{f}_0, \; \boldsymbol{f}_1, \; \ldots, \; \boldsymbol{f}_{L-1}] \tag{24}$$

$$\tilde{\boldsymbol{F}} = \boldsymbol{1}_{N_t} \otimes \boldsymbol{F} \tag{25}$$

$$\boldsymbol{C}_m = \mathrm{diag}\{\boldsymbol{c}_m\} \tag{26}$$

$$\boldsymbol{C} = \mathrm{diag}\{\boldsymbol{C}_0, \ldots, \boldsymbol{C}_{N_t-1}\} \tag{27}$$

$$\boldsymbol{\Sigma}_{C_m} = \boldsymbol{C}_m(\boldsymbol{C}_m^H\boldsymbol{C}_m)^{-1}\boldsymbol{C}_m^H. \tag{28}$$

$$\boldsymbol{\Sigma}_C = \boldsymbol{C}(\boldsymbol{C}^H\boldsymbol{C})^{-1}\boldsymbol{C}^H. \tag{29}$$

For the CFO FD-MLE, $G(\Delta)$ in (14) can be expressed as

$$G(\Delta) = \boldsymbol{z}_2^H(\Delta)\boldsymbol{z}_2(\Delta) \tag{30}$$

where

$$\begin{aligned}\boldsymbol{z}_2(\Delta) &= \sqrt{N}\boldsymbol{I}_{N_r} \otimes \left[(\boldsymbol{I}_{NN_t} - \boldsymbol{\Sigma}_C)\tilde{\boldsymbol{F}}\boldsymbol{\Gamma}(\Delta)\tilde{\boldsymbol{F}}^H\boldsymbol{C}(\boldsymbol{I}_{N_t} \otimes \boldsymbol{F}_L)\right]\boldsymbol{h} \\ &= \sqrt{N}\boldsymbol{I}_{N_r} \\ &\otimes \left[\left(\sum_{m=0}^{N_t-1}(\boldsymbol{I}_N - \boldsymbol{\Sigma}_{C_m})\right)\boldsymbol{F}\boldsymbol{\Gamma}(\Delta)\tilde{\boldsymbol{F}}^H\boldsymbol{C}(\boldsymbol{I}_{N_t} \otimes \boldsymbol{F}_L)\right]\boldsymbol{h}.\end{aligned} \tag{31}$$

To satisfy the consistency condition stated in (14), we requires that $\boldsymbol{z}_2 \neq \boldsymbol{0}_{NN_r}$ for any $|\hat{v}| < N/2$ except $\hat{v} = v$. By using (31), this condition can be achieved for *any* $\boldsymbol{h} \neq \boldsymbol{0}_{N_rN_tL}$ if $\boldsymbol{I}_{N_r} \otimes \left[\left(\sum_{m=0}^{N_t-1}(\boldsymbol{I}_N - \boldsymbol{\Sigma}_{C_m})\right)\boldsymbol{F}\boldsymbol{\Gamma}(\Delta)\tilde{\boldsymbol{F}}^H\boldsymbol{C}(\boldsymbol{I}_{N_t} \otimes \boldsymbol{F}_L)\right]$ is a full-rank matrix. Recall that each CIR vector between any transmit-receive antenna pair has the same length of L. Then the consistency condition for the CFO FD-MLE for MIMO systems is achieved for any $\boldsymbol{h} \neq \boldsymbol{0}$ if the following sufficient condition is satisfied:

$$\mathrm{rank}\left(\left(\sum_{m=0}^{N_t-1}(\boldsymbol{I}_N - \boldsymbol{\Sigma}_{C_m})\right)\boldsymbol{F}\boldsymbol{\Gamma}(\Delta)\tilde{\boldsymbol{F}}^H\boldsymbol{C}(\boldsymbol{I}_{N_t} \otimes \boldsymbol{F}_L)\right) = N_tL. \tag{32}$$

IV. Proposed Consistent Pilot Designs

A. TD-MLE Consistent Pilot Design Condition

Denote the union of the non-zero pilot tones indices over N_t transmit antennas by $\{t(k) : k = 0, \ldots, P-1\} = \bigcup_{m=0}^{N_t-1}\{t_m(k) : k = 0, \ldots, P_m - 1\}$, and the intersection of the null pilot tones indices over N_t transmit antennas by $\{n(k) : k = 0, \ldots, N-P-1\} = \bigcap_{m=0}^{N_t-1}\{n_m(k) : k = 0, \ldots, N - P_m - 1\}$, respectively. Note that $\{t(k) : k = 0, \ldots, P-1\} \bigcup \{n(k) : k = 0, \ldots, N-P-1\} = \{0, \ldots, N-1\}$. Define

$$t^{(l)}(k) = (t(k) + l) \mod N, \quad l \in \{0, 1, 2, \ldots, N-1\} \tag{33}$$

$$X = \left\{t^{(l)}(k) : k = 0, \ldots, P-1\right\} \bigcap \{n(k) : k = 0, \ldots, N-P-1\} \tag{34}$$

$$\boldsymbol{Z} = \left[\boldsymbol{I}_P, \boldsymbol{0}_{P\times(N-P)}\right] \tag{35}$$

$$\boldsymbol{\Theta} = [\boldsymbol{e}_{t(0)}, \boldsymbol{e}_{t(1)}, \ldots, \boldsymbol{e}_{t(P-1)}, \boldsymbol{e}_{n(0)}, \boldsymbol{e}_{n(1)} \ldots, \boldsymbol{e}_{n(N-P-1)}] \tag{36}$$

$$\boldsymbol{D} = \boldsymbol{Z}\boldsymbol{\Theta}^T[\boldsymbol{C}_0\boldsymbol{F}_L, \boldsymbol{C}_1\boldsymbol{F}_L, \ldots, \boldsymbol{C}_{N_t-1}\boldsymbol{F}_L] \tag{37}$$

where $\boldsymbol{e}_{t(i)}$ and $\boldsymbol{e}_{n(l)}$ are the $t(i)$-th and $n(l)$-th columns of $\boldsymbol{I}_N$, respectively, with $0 \leq i \leq P-1$, $0 \leq l \leq N-P-1$. Then, for a MIMO OFDM system with N_t transmit antennas and length-L CIR vectors of all transmit-receive antenna pairs, a sufficient pilot design condition for the consistency of the TD-MLE is comprised of the following two parts (see Appendix-A for the proof):

Part 1. The design condition to choose the locations of the consistent pilots over all transmit antennas: "$(N-P) \geq P$, $P > N_tL$, $\mathrm{rank}(\boldsymbol{S}) = N_tL$, *and for any cyclic-shifting distance* $l \in \{1, \; 2, \; \ldots, N-1\}$, *the cardinality of the set* X *defined in (34) is always larger than or equal to* N_tL."

Part 2. The design condition to arrange the locations and values of the pilots on each transmit antenna: "$\boldsymbol{D}_K$ *is always a full-rank matrix, i.e.* $\mathrm{rank}(\boldsymbol{D}_K) = N_tL$, where $\boldsymbol{D}_K$ denotes the matrix formed by any K $(N_tL \leq K \leq P)$ rows of $\boldsymbol{D}$ in (37)."

The above consistent pilot design condition is derived for the full range of CFO estimation, i.e. $v \in (-\frac{N}{2}, \frac{N}{2}]$. For an arbitrary limited CFO estimation range such as $v \in (-\frac{\Omega}{2}, \frac{\Omega}{2}]$, where Ω is an integer and $\Omega \leq N$, the two parts of the sufficient consistent pilot design condition remain the same except that $l \in \{1, 2, \ldots, N-1\}$ is replaced by $l \in \{-\lfloor\Omega/2\rfloor, \ldots, -1, 1, \ldots, \lfloor\Omega/2\rfloor - 1\}$.

Let $\boldsymbol{c}$ denote the length-N (virtual pilot) vector whose indices of non-zero pilot tones and null tones are the same as $\{t(k) : k = 0, \ldots, P-1\}$ and $\{n(k) : k = 0, \ldots, N-P-1\}$, respectively, where P is the number of non-zero pilot tones of $\boldsymbol{c}$. Then the sufficient consistent pilot design condition can be rewritten as:

Part 1. "$(N-P) \geq P$, $P > N_tL$, $\mathrm{rank}(\boldsymbol{S}) = N_tL$, *and for any cyclic-shifting distance* $l \in \{1, \; 2, \; \ldots, N-1\}$ *for the full range CFO estimation, or* $l \in \{-\lfloor\Omega/2\rfloor, \ldots, -1, 1, \ldots, \lfloor\Omega/2\rfloor - 1\}$ *for the limited range CFO estimation,* $\boldsymbol{c}^{(l)}$ *always has at least* N_tL *non-zero pilot tones located at the null-tone indices of the original unshifted virtual pilot vector* $\boldsymbol{c}$."

Part 2. "$\mathrm{rank}(\boldsymbol{D}_K) = N_tL$ *for any* K *where* $N_tL \leq K \leq P$."

B. TD-MLE Consistent Pilot Designs

In the following, we will only discuss for the full-range consistent pilot design since this process can be easily extended for an arbitrary limited estimation range. According to the two parts of the consistent pilot design condition proposed above, we separate the consistency pilot design procedure for MIMO OFDM systems into two steps accordingly:

Step 1) The design of $\boldsymbol{c}$, the virtual pilot vector with P $(> N_tL)$ non-zero pilot tones, for the consistency in MIMO OFDM systems:

This task is very similar to the pilot design we proposed in [15] for the SISO systems. Several efficient consistent pilot design patterns for the full or limited range CFO estimation were introduced in [15] based on different code structures, such as BCH codes, cyclic codes, m-sequence, distinctive-spacing, etc. We can choose any of them as our $\boldsymbol{c}$ according to the CFO estimation range of interest.

Step 2) The design of the pilot vector $\boldsymbol{c}_m$ on each m-th transmit antenna, satisfying $\bigcup_{m=0}^{N_t-1}\{t_m(k): k=0,\ldots,P_m-1\} = \{t(k): k=0,\ldots,P-1\}$, $\text{rank}(\boldsymbol{S}) = N_tL$ and $\text{rank}(\boldsymbol{D}_K) = N_tL$ for any K where $N_tL \le K \le P$:

The design patterns to assign pilot tones over N_t transmit antennas are not unique. In the following, we present two patterns while the proof for their validity is provided in Appendix-B.

Pattern (a):

Construct a Vandermonde matrix $\boldsymbol{\Psi}$ with the (k,m)-th element defined by

$$[\boldsymbol{\Psi}]_{k,m} = e^{jm\theta_k} \tag{38}$$

where $\theta_k \neq \theta_n$, $\forall k \neq n$. Note that we already have the virtual pilot vector $\boldsymbol{c}$ with P non-zero pilot tones from Step 1. Choosing an arbitrary integer J satisfying $\lceil P/L \rceil \le J \le P$ and defining $Q = \lfloor P/J \rfloor$, we construct a $P \times N_t$ matrix $\boldsymbol{\Xi}_1$ as

$$\begin{aligned}\boldsymbol{\Xi}_1 =& [\boldsymbol{e}_{\lfloor 0/Q \rfloor}, \boldsymbol{e}_{\lfloor 1/Q \rfloor}, \ldots, \boldsymbol{e}_{\lfloor (P-1)/Q \rfloor}]^T \\ &\times \boldsymbol{\Psi}[0:(J-1),\ 0:(N_t-1)].\end{aligned} \tag{39}$$

Then, the pilot tones for the m-th ($0 \le m \le N_t-1$) transmit antenna are given by

$$\boldsymbol{c}_m[t(k)] = \boldsymbol{c}[t(k)][\boldsymbol{\Xi}_1]_{k,m} \quad \& \quad \boldsymbol{c}_m[n(k)] = 0. \tag{40}$$

An alternative form of Pattern (a) is given by

$$\boldsymbol{c}_m[t(k)] = \boldsymbol{c}[t(k)]e^{jm\theta_{\lfloor k/J \rfloor}} \quad \& \quad \boldsymbol{c}_m[n(k)] = 0. \tag{41}$$

Pattern (b):

We construct a $P \times N_t$ matrix $\boldsymbol{\Xi}_2$ as

$$\begin{aligned}\boldsymbol{\Xi}_2 =& [\boldsymbol{e}_{\lfloor 0/Q \rfloor}, \boldsymbol{e}_{\lfloor 1/Q \rfloor}, \ldots, \boldsymbol{e}_{\lfloor (P-1)/Q \rfloor}]^T \\ &\times [\boldsymbol{I}_{N_t}, \boldsymbol{\Psi}[0:(J-N_t-1), 0:(N_t-1)]^T]^T.\end{aligned} \tag{42}$$

Then the pilot tones for the m-th transmit antenna are given by

$$\boldsymbol{c}_m[t(k)] = \boldsymbol{c}_m[t(k)][\boldsymbol{\Xi}_2]_{k,m} \quad \& \quad \boldsymbol{c}_m[n(k)] = 0. \tag{43}$$

An alternative expression for Pattern (b) is given by $\boldsymbol{c}_m[n(k)] = 0$ and

$$\boldsymbol{c}_m[t(k)] = \begin{cases} \boldsymbol{c}[t(k)], & mQ \le k < (m+1)Q \\ \boldsymbol{c}[t(k)]e^{jm\theta_{\lfloor (k-N_tQ)/J \rfloor}}, & k \ge N_tQ \\ 0, & \text{else} \end{cases} \tag{44}$$

C. Pilot Designs For Both Consistent CFO Estimation and Optimal Channel Estimation

The designed training sequences for CFO estimation may also be used to estimate the channels. The optimal training signals for the estimation of frequency-selective channels in MIMO OFDM systems [16] satisfy the following condition:

$$\boldsymbol{S}^H\boldsymbol{S} = E_{\text{av}}\boldsymbol{I} \tag{45}$$

$$\text{where } \ E_{\text{av}} = \frac{1}{N_t}\sum_{m=0}^{N_t-1} E_m \tag{46}$$

$$\text{and } \ E_m = \sum_{k=0}^{N-1} |s_m(k)|^2. \tag{47}$$

In [16], the pilot tone allocations among transmit antennas are classified as frequency division multiplexing (FDM), time division multiplexing (TDM), code division multiplexing in time-domain (CDM-T), code division multiplexing in frequency-domain (CDM-F), and combinations thereof. In this paper we only consider one OFDM symbol, and hence the TDM and CDM-T allocations cannot be used here. The FDM type training vector $\boldsymbol{c}_m$ for the m-th transmit antenna in frequency domain can be defined as

$$c_m[k] = \sum_{v=0}^{V_m-1}\sum_{l=1}^{L_0-1} b_m^{(l,v)}\delta[k - \frac{lN}{L_0} - i_{m,v}]; m=0,\ldots,N_t-1; \tag{48}$$

$$\begin{aligned}&i_{m,v} \in [0, \frac{N}{L_0}-1]; \\ &i_{m_1,v_1} = i_{m_2,v_2} \text{ only if } (m_1 = m_2 \ \&\ v_1 = v_2); \\ &\sum_{m=0}^{N_t-1} V_mL_0 \le N; \ \sum_{v=0}^{V_m-1}\sum_{l=1}^{L_0-1} |b_m^{(l,v)}|^2 = E_{\text{av}}\end{aligned} \tag{49}$$

where V_m is an integer greater than zero, $\{b_m^{(l,v)}\}$ are constant-modulus symbols and L_0 is *any* integer such that N/L_0 is an integer while $L \le L_0 \le N/N_t$. The training vector $\boldsymbol{c}_m$ for the m-th transmit antenna of CDM-F type can be defined as

$$c_m[k] = \sum_{v=0}^{V-1}\sum_{l=1}^{L_0-1} b_m^{(l,v)}\delta[k - \frac{lN}{L_0} - i_v]; \ m=0,\ldots,N_t-1; \tag{50}$$

$$\begin{aligned}&i_m \in [0, \frac{N}{L_0}-1]; i_{v_1} = i_{v_2} \text{ only if } v_1 = v_2; \\ &\sum_{v=0}^{V} b_m^{(l,v)*}b_n^{(l,v)} = 0, \ \forall m \le n; \ m,n \in \{0,\ldots,N_t-1\}\end{aligned}$$

where V is *any* integer satisfying $N_t \le V \le N/L_0$. The details of the FDM and CDM-F type pilots designs are referred to [16].

The FDM type violates Part 2 of the consistent pilot design condition we proposed in Section IV.A. We can use the necessary and sufficient design condition in (20) to evaluate whether a specific FDM pilot design is consistent or not. However, an exhausted search of consistent FDM pilot may be impractical. Hence, we will consider the consistent pilot designs based on the CDM-F type defined in [16].

It is very difficult (if not impossible) to obtain pilot patterns which give both the CFO estimation consistency over the *full* range CFO estimation and the optimal channel estimation performance. The optimal pilot tone allocation patterns from

[16] follow certain periodic structures (e.g., cyclically equi-spacing), which conflict with the consistent pilot design patterns over the full CFO estimation range [12] (e.g., distinctive spacings). To illustrate this, consider a cyclically equi-spaced pilot with a spacing of M tones. After cyclically shifting a distance of M, the cyclically equi-spaced pilots will have no non-zero pilot tones located at the null-tone indices of the original unshifted pilot vector, which obviously violates the design condition for the full CFO estimation range.

For a limited CFO estimation range (smaller than the full range), it is possible to design pilots satisfying both the consistency condition in the limited CFO estimation range such as $(-\frac{N}{2L}, \frac{N}{2L}]$ and the optimal channel estimation condition. In our approach, we use periodic pilot allocation patterns to satisfy the optimal channel estimation condition and design a consistent pilot tones vector for one period of the whole training sequence on one antenna, and develop the pilot tones on different antennas according to the optimal pilot allocation for channel estimation while maintaining CFO estimation consistency over the considered range. In the following, we present the design patterns assuming that N is a multiple of the CIR vector length L, i.e. $N = ML$.

CDM-F based design:

This design consists of the following two steps.

i) Design the length-M consistent pilot tone subvector $\boldsymbol{d}$ ($\boldsymbol{d}$ is consistent over the full-range of $(-M/2, M/2]$) according to the design described in Step 1 of Section IV.B, i.e. following the same consistent pilot design process in [15] to design a virtual consistent pilot sub-vector $\boldsymbol{d}$.

Denote the number of non-zero pilot tones of $\boldsymbol{d}$ by $P^{(d)}$ and the corresponding tone indices by $\{t(k)_d\}$. Then $\boldsymbol{d}$ satisfies

$$|d[t(k)_d]| = d_0,\ \forall k, d_0 > 0 \ \ \&\ \ P^{(d)} > N_t L. \tag{51}$$

Next, construct the virtual pilot vector $\boldsymbol{c}$ by repeating $\boldsymbol{d}$ L times, i.e. $\boldsymbol{c} = \mathbf{1}_L \otimes \boldsymbol{d}$, and its $P^{(d)}L$ nonzero pilot tones' indices are denoted by $\{t(k)\}$ and the remaining $(N - P^{(d)}L)$ null tones' indices are denoted by $\{n(k)\}$. The virtual pilot vector $\boldsymbol{c}$ inherits the consistency over the limited estimation range $v \in (-M/2, M/2]$ from $\boldsymbol{d}$.

ii) The pilot tones of the m-th ($0 \le m \le N_t - 1$) transmit antenna are given by one of the CDM-F pilot allocation patterns defined in [16]. An example design is as follows:

$$\boldsymbol{c}_m[t(k)] = \boldsymbol{c}[t(k)]e^{\frac{-j2\pi mk}{P^{(d)}}} \quad \& \quad \boldsymbol{c}_m[n(k)] = 0. \tag{52}$$

Note that the CDM-F type pilots satisfy $\boldsymbol{c}_k^H \boldsymbol{c}_m = 0$, $\forall k \neq m$, which is a subset of the consistent pilot patterns proposed as Pattern (a) in Section IV.B. For example, setting $Q = L$, $J = P^{(d)}$, and $\theta_{\lfloor k/J \rfloor} = -j2\pi k/P^{(d)}$ in (40), we immediately obtain the same form as in (52). Hence, the proposed CDM-F based pilot design satisfies the two parts of the consistency condition presented in Section IV.A. The above design has flexibility in the phases of $\{d[t(k)_d]\}$ which can be optimized to yield lower peak-to-average ratio of the time domain training (pilot) signal.

We can also have other design patterns besides the above CDM-F based designs. For example, we can follow the same procedure except that we may apply other pilot allocations (e.g. some antennas may have disjoint pilot locations in addition to common locations) as long as they satisfy both the consistent pilot design conditions and the optimal channel estimation condition. The consistent CFO estimation range is proportional to $1/L$, which indicates that the longer the CIR vector length is, the narrower the consistent CFO estimation range is. In practice, if the exact CIR vector length is unknown, we can use an upper bound for the value of L.

D. Consistent Pilot Design for the CFO FD-MLE

The consistent pilot design condition for the TD-MLE also satisfies that for the CFO FD-MLE in [11]. The proof is provided in Appendix-A. Consequently, all of the consistent pilot patterns proposed for the TD-MLE in Section IV.B also hold for the FD-MLE.

V. Consistency in the Probabilistic Sense for MIMO Systems

As discussed in the previous section, the consistency holding for any $\boldsymbol{h}(\neq \mathbf{0}_{N_t L})$ (in absolute sense) is desirable for the emergency and critical wireless systems. On the other hand, the absolute consistency condition may not always be required for other wireless systems. In this case, we can relax the consistent pilot design condition by introducing the consistency in the probabilistic sense where there can be some $\boldsymbol{h}$ which yield inconsistency but the probability of their occurrence is zero. In the following, we address pilot designs satisfying the consistency in the probabilistic sense for MIMO OFDM systems.

For the TD-MLE, define $\boldsymbol{R}(\Delta) = \left(\boldsymbol{I}_{N_r} \otimes (\boldsymbol{I} - \boldsymbol{\Sigma_B})\, \boldsymbol{U}^H \boldsymbol{\Gamma}(\Delta) \boldsymbol{U} \boldsymbol{\Sigma_S} \boldsymbol{V}^H\right)$. Since $\boldsymbol{h}$ is a continuous random vector, if $\boldsymbol{R}(\Delta \neq 0) \neq \mathbf{0}_{NN_r \times N_t L}$, the probability of $(\boldsymbol{z}(\Delta) = \boldsymbol{R}(\Delta)\boldsymbol{h} = \mathbf{0}_{NN_r})$ is equal to zero, i.e., the probability of $(G(\Delta \neq 0) = 0)$ is zero. Hence, as long as $\boldsymbol{R}(\Delta \neq 0) \neq \mathbf{0}_{NN_r \times N_t L}$, the consistency in the probabilistic sense is achieved. If we assume that each CIR vector between transmit-receive antenna pair has the same length of L, then a sufficient consistent pilot design condition in the probabilistic sense is given by

$$\mathrm{rank}\,(\boldsymbol{S}) = N_t L \ \ \&\ \ \boldsymbol{U}_2^H \boldsymbol{\Gamma}(\Delta) \boldsymbol{U}_1 \neq \mathbf{0}_{(N - N_t L) \times N_t L},\ \forall \Delta \neq 0 \tag{53}$$

$$\text{where}\ \ \boldsymbol{U}_1 = [\boldsymbol{u}_0, \boldsymbol{u}_1, \ldots, \boldsymbol{u}_{N_t L - 1}] \tag{54}$$

$$\boldsymbol{U}_2 = [\boldsymbol{u}_{N_t L}, \boldsymbol{u}_{N_t L + 1}, \ldots, \boldsymbol{u}_{N-1}]\,. \tag{55}$$

For the length-N vector $\boldsymbol{c}$, which has been defined in Section IV, whose indices of non-zero pilot tones and null tones satisfy $\{t(k) : k = 0, \ldots, P-1\} = \bigcup_{m=0}^{N_t - 1}\{t_m(k) : k = 0, \ldots, P_m - 1\}$ and $\{n(k) : k = 0, \ldots, N - P - 1\} = \bigcap_{m=0}^{N_t - 1}\{n_m(k) : k = 0, \ldots, N - P_m - 1\}$, respectively, we propose the sufficient pilot design condition for the consistency in the probabilistic sense over the estimation range $v \in (-\frac{\Omega}{2}, \frac{\Omega}{2}]$ with $\Omega \le N$ as

$$\begin{aligned} &\mathrm{rank}\,(\boldsymbol{S}) = N_t L \\ &\&\ \{t_k^{(l)}\} \neq \{t_k\},\quad l = -\lfloor \Omega/2 \rfloor, \ldots, -1, 1, \ldots, \lfloor \Omega/2 \rfloor - 1. \end{aligned} \tag{56}$$

For the FD-MLE, a sufficient condition for the consistency in the probabilistic sense is given by

$$\boldsymbol{F}_Z^T \boldsymbol{\Gamma}(\Delta) \boldsymbol{F}_P^* \neq \boldsymbol{0}_{(N-P)\times P} \tag{57}$$

from which we obtain the pilot design satisfying consistency in the probabilistic sense over the estimation range $v \in (-\frac{\Omega}{2}, \frac{\Omega}{2}]$ for the FD-MLE as

$$\{t_k^{(l)}\} \neq \{t_k\}, \quad l = -\lfloor \Omega/2 \rfloor, \ldots, -1, 1, \ldots, \lfloor \Omega/2 \rfloor - 1. \tag{58}$$

The proofs for (56) and (58) are provided in Appendix-C.

VI. Robust Consistent Pilot Designs for MIMO OFDM Systems

Outliers cause significant degradation of CFO estimation performance at moderate to low SNR range. For SISO systems, [13] reports that different consistent pilot designs can have different outlier statistics, resulting in different CFO estimation performance at moderate to low SNR values. In this section, we address this robustness issue of the consistent pilots against outliers for MIMO OFDM systems, and propose a new robustness design criterion. For MIMO systems, the cost function in (15) can be written as

$$\begin{aligned} G(\Delta) &= \boldsymbol{h}^H \left(\boldsymbol{I}_{N_r} \otimes \boldsymbol{S}^H \left(\boldsymbol{I}_N - \boldsymbol{\Gamma}^H(\Delta)\boldsymbol{B}\boldsymbol{\Gamma}(\Delta) \right) \boldsymbol{S} \right) \boldsymbol{h} \\ &= \sum_{i=0}^{N_r-1} \boldsymbol{h}_i^H \left(\boldsymbol{S}^H \left(\boldsymbol{I}_N - \boldsymbol{\Gamma}^H(\Delta)\boldsymbol{B}\boldsymbol{\Gamma}(\Delta) \right) \boldsymbol{S} \right) \boldsymbol{h}_i \end{aligned} \tag{59}$$

where $\boldsymbol{h}_i = \left[\boldsymbol{h}_{(0,i)}^T, \ldots, \boldsymbol{h}_{(N_t-1,i)}^T\right]^T$. The above equation shows that for multiple receive antennas systems, the cost function is equivalent to the summation of N_r cost functions corresponding to N_r different receive antennas. In other words, multiple receive antennas provide the cost function an averaging effect across different receive antennas which decreases the possibility of drastic fluctuation of the cost function over $\boldsymbol{h}$. Hence, more receive antennas imply greater robustness of CFO estimation against outliers.

Though outlier phenomenon in MIMO systems may not be as prevalent as in SISO systems since the spatial diversity may mitigate the cost function fluctuation due to random $\boldsymbol{h}$, improper pilot designs can still cause outliers and hence performance degradation at low SNR. In the following we will present a new robustness criterion for the consistent pilots in MIMO OFDM systems. The new criterion evaluates pilots by means of two factors which affect robustness of CFO estimation in MIMO OFDM systems. The two factors are the fluctuations of the cost functions due to (i) random channel $\boldsymbol{h}$ and (ii) different trial values within the estimation range.

A. The Criterion Based on the Cost Function Fluctuation over Random $\boldsymbol{h}$

Very recently, [13] introduces two pilot design criteria for alleviating the outliers caused by random $\boldsymbol{h}$ of the CFO TD-MLE in SISO OFDM systems, which can also be used to evaluate for MIMO OFDM systems. Let $\delta_0, \ldots, \delta_{N_tL-1}$ (recall that $\mathrm{rank}(\boldsymbol{S}) = N_tL$) be the ordered eigenvalues of $\bar{\boldsymbol{Q}} = \boldsymbol{S}^H\boldsymbol{S}$ and $\delta_i \geq \delta_{i+1}$. For MIMO OFDM systems, $\boldsymbol{S}$ is defined in (4). The two criteria in [13] are minimizing δ_0/δ_{N_tL-1} and maximizing $\Pi_{i=0}^{N_tL-1}\delta_i$. A revised form of the criterion we adopt in this paper is given by

$$\min \left(C_1 \triangleq \frac{\sum_{i=0}^{N_tL-1} \delta_i}{(N_tL)\left(\prod_{i=0}^{N_tL-1} \delta_i\right)^{1/(N_tL)}} \right). \tag{60}$$

The above criterion compares the arithmetic and geometric means of the eigenvalues of $\boldsymbol{S}^H\boldsymbol{S}$, and it reflects the degree of fluctuation of the cost function in (8) over random $\boldsymbol{h}$. Due to the inequality between the arithmetic and geometric means, we conclude that the minimum value of C_1 is 1, which is achieved if and only if $\delta_i = \delta_j, \forall i \neq j$. Hence the smaller the value of C_1 is, the less the fluctuations of the eigenvalues of $\boldsymbol{S}$ are. In this sense, the best pilot design patterns against outliers caused by the fluctuation of the cost function over random $\boldsymbol{h}$ are those that satisfy $\boldsymbol{S}^H\boldsymbol{S} = E_{\mathrm{av}}\boldsymbol{I}$, which happens to be the same as the optimal pilot design condition for channel estimation in [16]. Note that our pilot designs in Section IV.C satisfy $\boldsymbol{S}^H\boldsymbol{S} = E_{\mathrm{av}}\boldsymbol{I}$.

B. The Criterion Based on the Cost Function Fluctuation over the Whole Estimation Range

The robustness of the CFO TD-MLE not only depends on the fluctuation of the cost function over random $\boldsymbol{h}$, which is evaluated by the criterion proposed above, but also relates to the fluctuation over the whole estimation range of the cost function. The former fluctuation dominates the occurrence of the outliers in SISO systems. For MIMO OFDM systems, the diversity gain mitigates the former fluctuation, and the effect of the latter becomes more prominent, which leads us to evaluate the fluctuation of the cost function over the whole estimation range as

$$\min \left(C_2 \triangleq \frac{\sum_{n=0}^{N_s-1}\sum_{i=0}^{N_tL-1} \lambda_i(\Delta_n)}{N_tL\left(\prod_{i=0}^{N_tL-1} \lambda_i(\Delta_n = 0)\right)^{1/(N_tL)}} \right). \tag{61}$$

Here we have evaluated for the whole CFO estimation range using $N_s = KN$ (K is the up-sampling factor) equi-spaced trial points $\{\Delta_n\}$. And $\{\lambda_0(\Delta_n), \lambda_1(\Delta_n), \cdots, \lambda_{N_tL-1}(\Delta_n)\}$ are the ordered eigenvalues of $\bar{\boldsymbol{G}}(\Delta_n)$, which is given by

$$\begin{aligned} \bar{\boldsymbol{G}}(\Delta_n) &= \boldsymbol{S}^H\boldsymbol{\Gamma}(\Delta_n)\boldsymbol{B}\boldsymbol{\Gamma}^H(\Delta_n)\boldsymbol{S} \\ &= \boldsymbol{U}_q \mathrm{diag}\{\lambda_0(\Delta_n), \lambda_1(\Delta_n), \cdots, \lambda_{N_tL-1}(\Delta_n)\}\boldsymbol{U}_q^H \end{aligned} \tag{62}$$

where $\lambda_i(\Delta_n) \geq \lambda_{i+1}(\Delta_n)$, and $\boldsymbol{U}_q$ is a unitary matrix with the ith column $\boldsymbol{u}_{qi}$ representing the eigen-vector of $\bar{\boldsymbol{G}}(\Delta_n)$ corresponding to $\lambda_i(\Delta_n)$.

C. The Combined Criterion for Robustness of the Consistent Pilot Designs

Based on the two criteria in (60) and (61), we obtain a combined criterion for robustness against outliers as

$$\min\left(C_3 = C_1 \cdot C_2\right). \tag{63}$$

The criterion in (63) includes both of the main factors that affect robustness of the CFO TD-MLE in MIMO OFDM systems. Note that in addition to the above criterion based on

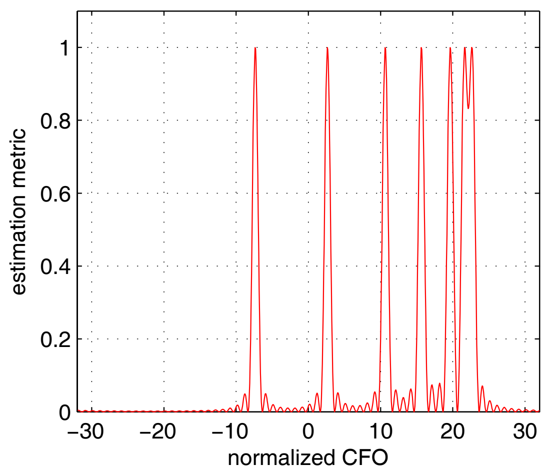

Fig. 1. The CFO estimation metric (normalized) associated with an inconsistent pilot for a MIMO OFDM system with $N = 64$, $L = 4$, $N_t = 2$ and $N_r = 2$. (A consistent pilot will give only one maximum.)

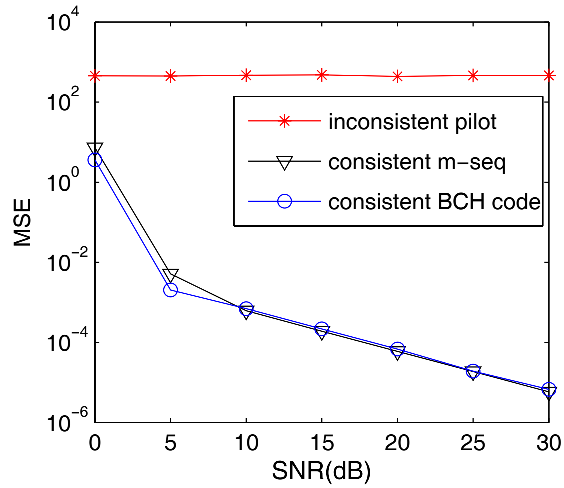

Fig. 2. The CFO estimation MSE performance associated with different preambles for a particular channel realization which yields inconsistency for a MIMO OFDM system with $N_t = 2$ and $N_r = 1$.

the cost function fluctuation, the consistency condition also contributes to the robustness since inconsistent pilots will more likely have large sidelobe peaks of the estimation likelihood metric which are prone to noise yielding outliers.

VII. Simulation Results

We consider a MIMO OFDM system with $N = 64$ subcarriers in multipath Rayleigh fading channels with $L = 4$ (unless otherwise stated) sample-spaced taps having an exponential power delay profile with a 3dB per tap decaying factor (unless otherwise stated). Simulation results are obtained from 10^5 independent runs.

Fig. 1 and Fig. 2 illustrate the effects of inconsistent pilot design. The inconsistent pilot vector used as the benchmark is D108080200000000 (in hexadecimal). In Fig. 1, we present the estimation metric of an inconsistent pilot pattern which shows multiple maxima, the sources of CFO estimation failure. Fig. 2 presents the CFO estimation mean-

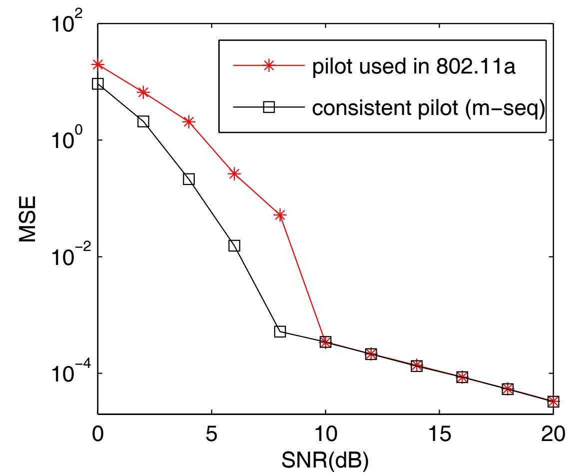

Fig. 3. The CFO estimation MSE performance for the inconsistent pilot used in 802.11a and a proposed consistent pilot for a MIMO OFDM system with $N_t = 2$ and $N_r = 1$.

square error (MSE) comparison between the inconsistent pilot and other two consistent pilots (the m-sequence type pilot (FC10C8E4725B766A) and the BCH code type pilot (9EF8153225B1D0D6)) for a fixed channel which yields inconsistency for the inconsistent pilot in Fig. 1. Due to this inconsistency, a complete estimation failure is observed for the inconsistent pilot.

We also present the CFO estimation MSE performance for the inconsistent pilot used in the standard 802.11a in Fig. 3, compared with the MSE performance for one of our proposed consistent pilots (9248244911021120). At SNR values less than 10 dB, we observe that the proposed consistent pilot provides a significant MSE performance gain (up to 20 dB) over the inconsistent pilot. The reason is that the inconsistent pilot is more likely to yield relatively-high sidelobe peaks in the estimation metric which result in a higher probability of the outlier occurrences. This result also highlights that our design enhances the performance of regular communication systems in addition to its impact on the link resilience of time-sensitive emergency and other critical communication systems.

The considered estimators are joint MLE of CFO and channel, and a larger N_r does not improve the channel estimation performance. Hence, the effect of N_r on the CFO estimation MSE is of interest to investigate, which we present in Fig. 4 using the same consistent preambles used in Fig. 2). About 3dB SNR advantage is observed for two receive antennas compared with only one receive antenna system. We can also notice that the cost function averaging effect of multiple receive antennas yields smaller outlier occurrence than single receive antenna case.

We cannot draw a general conclusion that a particular consistent pilot design performs better than others. For example, in Figs. 2 and 4, we observe that the specific m-sequence performs slightly better (worse) than the BCH code based sequence at high (low to moderate) SNR. We checked the CFO estimation performance of other BCH code based sequences (not listed in Figs. 2 and 4), and some of them perform slightly better than the m-sequence based sequences at high SNR. Recall that zero autocorrelation (ZAC) signals or noise-like

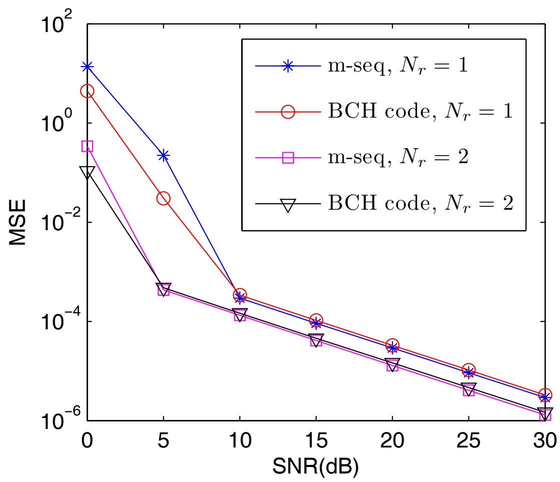

Fig. 4. The CFO estimation MSE performance associated with different numbers of receive antennas N_r for a MIMO OFDM system with $N_t = 2$.

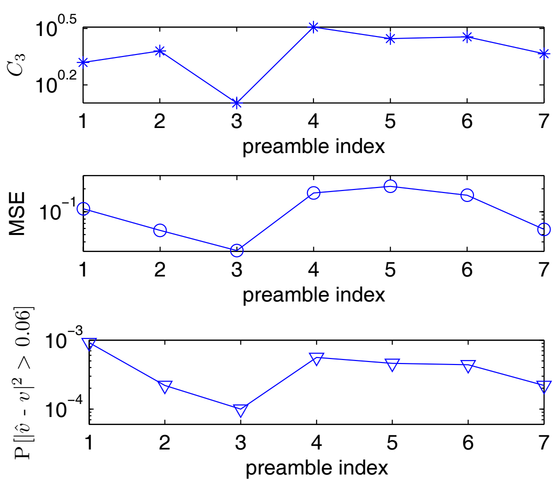

Fig. 5. The proposed robustness design metrics (top), the simulation CFO estimation MSE performance (middle), and the probability of outlier (defined by $P[|\hat{v} - v|^2 > 0.06]$) (bottom) for the proposed consistent pilot designs in a MIMO OFDM system with $N = 64$, $L = 4$, $N_t = 2$, $N_r = 2$ and SNR $= 1$ dB.

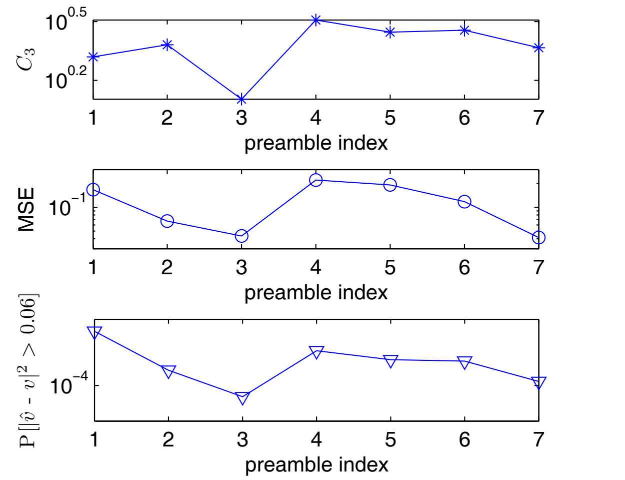

Fig. 6. The proposed robustness design metrics (top), the simulation CFO estimation MSE performance (middle), and the probability of outlier (defined by $P[|\hat{v} - v|^2 > 0.06]$) (bottom) for the proposed consistent pilot designs in a MIMO OFDM system with $N = 64$, $L = 4$, $N_t = 2$, $N_r = 1$ and SNR $= 6$ dB.

sequences provide very good CFO estimation performance [17] [18]. Both the m-sequence and BCH code based pilot designs yield approximately noise-like time domain sequences and hence their MSE performances are similar. In fact, the MSE performance differences of consistent pilot designs are very minor.

In Figs. 5-6, we present the effectiveness of our robustness criterion. Note that, although the robust pilot designs always yield less outliers than the unrobust ones at all SNRs, obvious CFO estimation MSE differences between robust and unrobust pilots can be best observed only at certain different SNRs for different MIMO OFDM systems (e.g., 1 dB in Fig. 5 and 6 dB in Fig. 6). Well below these certain SNR values, the outliers occur too often so that the differences in MSE due to the different outlier statistics are indistinguishable, while well above these SNR values, the outliers are too rare so that the estimation MSEs are hardly affected. For MIMO OFDM systems, the SNR values where we can best observe the MSE differences between robust and unrobust pilot design patterns also depend on the number of the receive antennas and the CFO estimation range.

We compare the robustness criterion C_3 defined in (63) with the MSE performance for a $N_t = 2$ system with $N_r = 2$ in Fig. 5 and $N_r = 1$ in Fig. 6. The preamble indices {1-3} and {4-7} correspond to the m-sequence and BCH code based pilot designs, respectively. In the bottom of the Figs. 5 and 6, we also present the probability of the outlier occurrence, which is evaluated by the probability of estimation error (or MSE) larger than a certain threshold (we use $P[|\hat{v} - v|^2 > 0.06]$ in this simulation). Our robustness criterion C_3 concurs with the MSE performance and the outlier probability. Among several consistent pilot patterns, the preamble #3 (9248244911021120) gives the most robust performance.

Fig. 7 compares the uncoded BER performance in a BPSK MIMO OFDM (V-BLAST) system between two of the proposed pilot designs: i) the design satisfying both the CFO estimation consistency and the optimal channel estimation condition (we use the pilot D1080000D1080000); ii) the design satisfying the CFO estimation consistency condition only (we use the pilot F8DC00000 000A12C). Both CFO and channel estimation use the same pilot signal and maximum likelihood detection is performed. Fig. 7 shows that our design with both CFO estimation consistency and optimal channel estimation gives about 1 dB SNR advantage over our design with CFO estimation consistency only.

Next, we compare the pilot design with absolute consistency and that with consistency in the probabilistic sense. Since the latter does not yield consistency for some channel realizations, it can result in large CFO estimation errors and link failures for those channel realizations (not necessarily in fading) similar to an inconsistent design as shown in Fig. 2. If the CFO estimation error is above a certain value at a certain SNR, the packet would not be detected correctly. In this case, a larger CFO estimation error would not make it worse, and

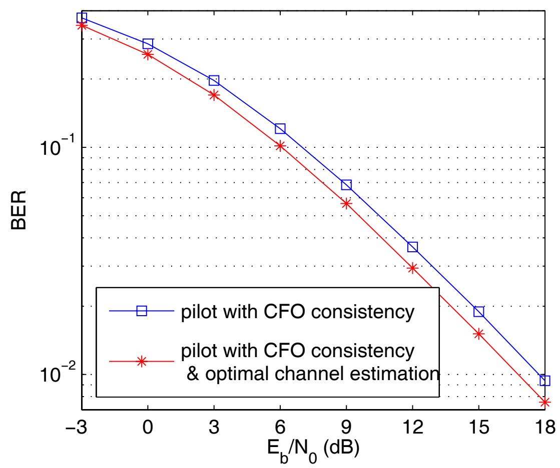

Fig. 7. The uncoded BER comparison between the proposed pilot design satisfying CFO estimation consistency and optimal channel estimation and the proposed pilot design satisfying CFO estimation consistency only, in a BPSK MIMO OFDM system (V-BLAST) with $N = 64$, $L = 2$, $N_t = 2$ and $N_r = 1$.

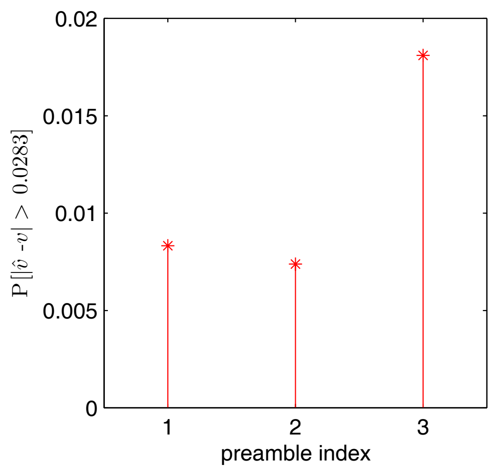

Fig. 8. The probability of large CFO estimation error occurrence (defined by $P[|\hat{v}-v| > 0.0283]$) for the proposed pilot designs with absolute consistency (preamble 1 for m-sequence: FC10C53C8E4B766A, preamble 2 for BCH code: 787CD94FC637278A) and the consistent pilot design in probabilistic sense (preamble 3: D108080200000000) in a MIMO OFDM system with $N = 64$, $L = 4$, $N_t = 2$, $N_r = 1$ and SNR = 15 dB.

$P[|\hat{v}-v|^2 > \varepsilon]$, where ε is a certain threshold value, would be a better performance measure than the MSE performance. In Fig. 8, we compare pilot designs with absolute consistency and probabilistic consistency in terms of $P[|\hat{v}-v|^2 > 8\times 10^{-4}]$ at SNR= 15dB. The design with probabilistic consistency yields more frequent occurrence of large CFO estimation errors or packet errors than the designs with absolute consistency.

VIII. Conclusions

Robust and consistent CFO estimation is important in maintaining a reliable wireless link for any system especially emergency or other critical communication systems. For MIMO OFDM systems, we have developed sufficient conditions for both the absolute consistency and the probabilistic consistency of the pilot-based CFO estimation, and presented the corresponding pilot designs. We have also developed pilot designs which satisfy the CFO estimation consistency over a limited range as well as the optimal channel estimation condition. We have proposed a new criterion for robustness against outliers in the CFO estimation. Our proposed pilot design criteria for the consistency and the robustness constitute an efficient pilot design tool as corroborated by the simulation results.

Appendix-A

In this appendix, we will prove that the pilot design sufficient condition proposed in Section IV satisfies the consistency condition in (20) for the CFO TD-MLE in [14] and in (32) for the CFO FD-MLE in [11].

1. Proof of Consistent Pilot Design Condition for the TD-MLE in [14]

Recall that the proposed pilot design requires $P > N_tL$. We first express the orthogonal vectors $\{\boldsymbol{u}_1, \ldots, \boldsymbol{u}_{N_tL}\}$ in (22) as linear transforms of the Fourier orthogonal vectors as

$$\boldsymbol{U}_1 = \boldsymbol{F}_P^* \boldsymbol{\Phi}. \tag{64}$$

Define

$$\boldsymbol{\Phi} = \boldsymbol{F}_P^T \boldsymbol{U}_1 \tag{65}$$

$$\boldsymbol{E} = \boldsymbol{V}\boldsymbol{\Sigma_S}^H (\boldsymbol{\Sigma_S}\boldsymbol{\Sigma_S}^H)^{-1} \left[\boldsymbol{I}_{N_tL}, \boldsymbol{0}_{N_tL\times(N-N_tL)}\right]^T. \tag{66}$$

Then, $\boldsymbol{U}_1$ in (22) can be expressed as

$$\boldsymbol{U}_1 = \boldsymbol{U}\boldsymbol{\Sigma_S}\boldsymbol{V}^H\boldsymbol{E} = \sqrt{N}\tilde{\boldsymbol{F}}^H \boldsymbol{C}\left(\boldsymbol{I}_{N_t} \otimes \boldsymbol{F}_L\right)\boldsymbol{E}. \tag{67}$$

Substituting (67) into (65), and then applying (37) give

$$\begin{aligned}\boldsymbol{\Phi} &= \sqrt{N}\boldsymbol{F}_P^T\tilde{\boldsymbol{F}}^H \boldsymbol{C}\left(\boldsymbol{I}_{N_t} \otimes \boldsymbol{F}_L\right)\boldsymbol{E}\\ &= \sqrt{N}\left(\boldsymbol{1}_{N_t}^T \otimes (\boldsymbol{Z}\boldsymbol{\Theta}^T)\right)\boldsymbol{C}\left(\boldsymbol{I}_{N_t} \otimes \boldsymbol{F}_L\right)\boldsymbol{E}\\ &= \sqrt{N}\boldsymbol{D}\boldsymbol{E}.\end{aligned} \tag{68}$$

Note that $\boldsymbol{E}$ is an $N_tL \times N_tL$ full-rank matrix. Next, $\boldsymbol{U}_2^H$ can be decomposed as

$$\boldsymbol{U}_2^H = \boldsymbol{A}\left[\boldsymbol{F}_Z,\ \boldsymbol{Y}\right]^T \tag{69}$$

where $\boldsymbol{Y}$ is an $N \times (P - N_tL)$ matrix whose columns form the basis of the null space of the columns of $\boldsymbol{U}_1$ within the sub-space of the columns of $\boldsymbol{F}_P$, and $\boldsymbol{A}$ is an $(N - N_tL) \times (N - N_tL)$ full-rank matrix. Then, from (37) and (69), we have

$$\boldsymbol{U}_2^H\boldsymbol{\Gamma}(\Delta)\boldsymbol{U}_1 = \boldsymbol{A}\begin{bmatrix}\boldsymbol{T}_1\boldsymbol{\Phi}\\ \boldsymbol{T}_2\boldsymbol{\Phi}\end{bmatrix} \tag{70}$$

where

$$\boldsymbol{T}_1 = \boldsymbol{F}_Z^T\boldsymbol{\Gamma}(\Delta)\boldsymbol{F}_P^* \tag{71}$$

$$\boldsymbol{T}_2 = \boldsymbol{Y}^T\boldsymbol{\Gamma}(\Delta)\boldsymbol{F}_P^*. \tag{72}$$

Consider the following two cases:

(a) When Δ is any non-zero integer, from (72) we have

$$\boldsymbol{T}_1 = [\boldsymbol{e}_{l_1}, \boldsymbol{e}_{l_2}, \ldots, \boldsymbol{e}_{l_K}, \boldsymbol{0}_{(N-P)\times(P-K)}]\,\boldsymbol{\mathcal{I}}_{P\times P} \tag{73}$$

$$\boldsymbol{T}_1\boldsymbol{\Phi} = \sqrt{N}\boldsymbol{D}_K\boldsymbol{E} \tag{74}$$

where the value of K depends on $\{t_k\}$ and Δ, $\boldsymbol{e}_{l_i}$ is the l_i-th column of the $(N-P)\times(N-P)$ identity matrix and $\boldsymbol{\mathcal{I}}_{P\times P}$ denotes a $P \times P$ permutation matrix determined by Δ.

Part 1 of our design condition in Section IV.A requires that after cyclic-shifting, at least N_tL pilots fall into the initial null tone locations, so we must have $N_tL \le K \le P$. Part 2 of the consistent condition requires $\mathrm{rank}(\boldsymbol{D}_K) = N_tL$. Since $\boldsymbol{A}$ and $\boldsymbol{E}$ are full-rank square matrix, applying our design condition to (74) and (70), we obtain that $\mathrm{rank}(\boldsymbol{U}_2^H \boldsymbol{\Gamma}(\Delta)\boldsymbol{U}_1) = N_tL$.

(b) When Δ is not an integer, from (72) we have

$$[\boldsymbol{T}_1]_{m,n} = \frac{(1-e^{j2\pi\Delta})\cdot e^{j2\pi n_{k_m}/N}}{e^{j2\pi n_{k_m}/N} - e^{j2\pi(t_{k_n}+\Delta)/N}} \triangleq \frac{c\cdot a_m}{a_m - b_n} \quad (75)$$

where $(1-e^{j2\pi\Delta}) \neq 0$, $\{a_m \neq b_n, \forall m, n\}$, $\{a_m \neq a_n, \forall m \neq n\}$, and $\{b_m \neq b_n, \forall m \neq n\}$ for $m = 0, 1, \ldots, N-P-1$; $n = 0, 1, \ldots, P-1$. Let $\overline{\boldsymbol{T}}_1$ denote the square matrix formed by any P rows of $\boldsymbol{T}_1$ (recall that $(N-P) \ge P$ in our design condition). Then

$$\det(\overline{\boldsymbol{T}}_1) = c^P \prod_{m=2}^{P} \prod_{n=1}^{(m-1)} \frac{(b_m - b_n)(a_n - a_m)}{(a_m - b_n)(a_n - b_m)} \cdot \prod_{l=1}^{P} \frac{1}{a_l - b_l} \neq 0 \quad (76)$$

which shows that $\boldsymbol{T}_1$ is a full-rank (tall) matrix, i.e., $\mathrm{rank}(\boldsymbol{T}_1) = P$. Define an $(N-P) \times (N-P)$ full-rank matrix $\widetilde{\boldsymbol{T}} = [\boldsymbol{T}_1, \boldsymbol{T}_1^{\perp}]$ and an $(N-P) \times N_tL$ matrix $\widetilde{\boldsymbol{\Phi}} = [\boldsymbol{\Phi}^T, \boldsymbol{0}_{N_tL\times(N-2P)}]^T$. Then, we have $\boldsymbol{T}_1\boldsymbol{\Phi} = \widetilde{\boldsymbol{T}}_1\widetilde{\boldsymbol{\Phi}}$ and hence $\mathrm{rank}(\boldsymbol{T}_1\boldsymbol{\Phi}) = \mathrm{rank}(\widetilde{\boldsymbol{\Phi}}) = N_tL$ [19]. Multiplying with the square full-rank matrix $\boldsymbol{A}$ does not change the rank. Hence, $\mathrm{rank}(\boldsymbol{U}_2^H \boldsymbol{\Gamma}(\Delta)\boldsymbol{U}_1) = N_tL$ for any non-integer Δ.

From (a) and (b), we have established that the proposed pilot design condition proposed in IV.A is sufficient for the consistency condition in (21).

2. Proof of Consistent Pilot Design Condition for the CFO FD-MLE

We can express the first part of the argument from the left side of (32) as

$$\left(\sum_{m=0}^{N_t-1} (\boldsymbol{I}_N - \boldsymbol{\Sigma}_{C_m})\right)\boldsymbol{F} = \left(\boldsymbol{A}_Z[\boldsymbol{F}_Z\ \boldsymbol{0}_{N\times P}] + \boldsymbol{A}_P[\boldsymbol{0}_{N\times(N-P)}\ \boldsymbol{F}_P]\right) \quad (77)$$

where $\boldsymbol{A}_Z$ is a full-rank matrix. The sub-spaces spanned by $[\boldsymbol{F}_Z\ \boldsymbol{0}_{N\times P}]$ and $[\boldsymbol{0}_{N\times(N-P)}\ \boldsymbol{F}_P]$ are orthogonal. Then applying (77) to the left side of (32), we obtain

$$\begin{aligned}&\mathrm{rank}\left(\left(\sum_{m=0}^{N_t-1}(\boldsymbol{I}_N - \boldsymbol{\Sigma}_{C_m})\right)\boldsymbol{F}\boldsymbol{\Gamma}(\Delta)\tilde{\boldsymbol{F}}^H\boldsymbol{C}(\boldsymbol{I}_{N_t}\otimes\boldsymbol{F}_L)\right)\\ &\ge \mathrm{rank}\left(\boldsymbol{F}_Z^T\boldsymbol{\Gamma}(\Delta)\tilde{\boldsymbol{F}}^H\boldsymbol{C}(\boldsymbol{I}_{N_t}\otimes\boldsymbol{F}_L)\right)\\ &= \mathrm{rank}\left(\boldsymbol{F}_Z^T\boldsymbol{\Gamma}(\Delta)\boldsymbol{F}_P^*\boldsymbol{D}\right) = \mathrm{rank}\left(\boldsymbol{T}_1\boldsymbol{D}\right). \end{aligned} \quad (78)$$

We already proved in the first part of the Appendix that the proposed consistent pilot design condition for the TD-MLE satisfies $\mathrm{rank}(\boldsymbol{T}_1\boldsymbol{D}) \ge N_tL$. Applying this result to (78), we conclude that the proposed sufficient consistent pilot design condition for the TD-MLE also satisfies (32), which is the consistency condition for CFO FD-MLE.

APPENDIX-B

In this appendix, we will prove that the proposed pilot design patterns in Section IV.B satisfy the consistent pilot design condition described in Section IV.A.

Step 1 of the consistent pilot design patterns presented in Section IV.B obviously fulfills the part 1 of the consistent pilot design conditions proposed in Section IV.A. Hence, we only need to prove that the two design patterns in Step 2, which assign the values of the pilot tones on each antenna, satisfy Part 2 of the consistent pilot design condition described in Section IV.A.

Pattern (a): Substituting the values of $\boldsymbol{c}_m$ defined by (40) into (37), we obtain (79), where the $1\times L$ vector $\boldsymbol{f}_{L,k}$ is the $t(k)$-th row of the Vandermonde matrix $\boldsymbol{F}_L$. Since we have designed $\theta_k \neq \theta_m$ for any $k \neq m$, and due to the Vandermonde type group-wise phase shifts, we can see that any N_tL rows of $\boldsymbol{D}$ are independent. In other words, taking any $K(N_tL \le K \le P)$ rows of $\boldsymbol{D}$ to form the matrix $\boldsymbol{D}_K$, we have $\mathrm{rank}(\boldsymbol{D}_K) = N_tL$. Then, Part 2 of the consistent condition described in Section IV.A is satisfied.

Pattern (b): Substituting the values of $\boldsymbol{c}_m$ defined by (43) into (37), we have (80), where $\boldsymbol{F}_{L,k}$ is a $Q\times L$ sub matrix corresponding to the kQ-th to $((k+1)Q-1)$-th rows of $\boldsymbol{F}_L$. From the diagonal structure of the upper sub-matrix of $\boldsymbol{D}$ and the Vandermonde type group-wise phase shifts in the lower sub-matrix of $\boldsymbol{D}$, we can see that any N_tL rows of $\boldsymbol{D}$ are independent. In other words, the matrix $\boldsymbol{D}_K$ formed by taking any $K(N_tL \le K \le P)$ rows of $\boldsymbol{D}$ has full column rank N_tL. Hence, Pattern (b) satisfies the consistent condition described in Section IV.A.

APPENDIX-C

In this appendix, we will prove that the proposed pilot design conditions in Section V satisfy the consistency condition in the probabilistic sense, i.e., (56) satisfies (53) for the TD-MLE and (58) satisfies (57) for the FD-MLE. Define $\bar{\boldsymbol{R}}(\Delta) = \boldsymbol{U}_2^H\boldsymbol{\Gamma}(\Delta)\boldsymbol{U}_1$. First, we will prove it for the TD-MLE.

From (70) we have

$$\bar{\boldsymbol{R}}(\Delta) = \boldsymbol{A}\begin{bmatrix}\boldsymbol{T}_1\boldsymbol{\Phi}\\ \boldsymbol{T}_2\boldsymbol{\Phi}\end{bmatrix}. \quad (81)$$

For a non-zero integer Δ, the condition in (56) yields $\boldsymbol{T}_1 = [\boldsymbol{e}_{l_1}, \boldsymbol{e}_{l_2}, \ldots, \boldsymbol{e}_{l_K}, \boldsymbol{0}_{(N-P)\times(P-K)}]\ \boldsymbol{\mathcal{I}}_{P\times P}$. Also $\boldsymbol{A}$ and $\boldsymbol{\Phi}$ are full-rank matrices, and hence, $\bar{\boldsymbol{R}}(\Delta) \neq \boldsymbol{0}$.

For a non-integer Δ, we have proved in Appendix-A that $\boldsymbol{T}_1$ is a full-rank matrix if $(N-P) \ge P$. Then, it follows that $\bar{\boldsymbol{R}}(\Delta) \neq \boldsymbol{0}$ for any non-integer Δ. Hence, we conclude that (56) is the sufficient condition for (53).

Next, we will prove it for the FD-MLE.

If Δ is a non-zero integer, the condition in (58) yields $\{n_k^{(l)}\} \cap \{t_k\} \neq \oslash$ which straightforwardly guarantees (57).

If Δ is not an integer, $\boldsymbol{F}_Z^T\boldsymbol{\Gamma}^H(\Delta)\boldsymbol{F}_P^*$ is always a full-rank matrix (see Appendix-A), and hence (57) is guaranteed. Hence, we complete the proof for the FD-MLE.

REFERENCES

[1] IEEE standards for local and metropolitan area networks, part 16: air interface for fixed broadband wireless access systems, IEEE LAN/MAN Standards Committee, 2004.

[2] G. Stuber, J. Barry, S. McLaughlin, Y. Li, M. Ingram, and T. Pratt, “Broadband MIMO-OFDM wireless communications,” *Proc. IEEE*, vol. 92, no. 2, pp. 271–294, Feb. 2004.

$$D = \begin{bmatrix} f_{L,0}, & e^{j\theta_0} f_{L,0}, & e^{j2\theta_0} f_{L,0}, & \dots, & e^{j(N_t-1)\theta_0} f_{L,0} \\ f_{L,1}, & e^{j\theta_{\lfloor 1/Q \rfloor}} f_{L,1}, & e^{j2\theta_{\lfloor 1/Q \rfloor}} f_{L,1}, & \dots, & e^{j(N_t-1)\theta_{\lfloor 1/Q \rfloor}} f_{L,1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{L,P-1}, & e^{j\theta_{\lfloor (P-1)/Q \rfloor}} f_{L,P-1}, & e^{j2\theta_{\lfloor (P-1)/Q \rfloor}} f_{L,P-1}, & \dots, & e^{j(N_t-1)\theta_{\lfloor (P-1)/Q \rfloor}} f_{L,P-1} \end{bmatrix} \quad (79)$$

$$D = \begin{bmatrix} F_{L,0}, & 0_{Q\times L}, & 0_{Q\times L}, & \dots, & 0_{Q\times L} \\ 0_{Q\times L}, & F_{L,1}, & 0_{Q\times L}, & \dots, & 0_{Q\times L} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{Q\times L}, & 0_{Q\times L}, & 0_{Q\times L}, & \dots, & F_{L,N_t-1} \\ f_{L,QN_t}, & e^{j\theta_0} f_{L,QN_t}, & e^{j2\theta_0} f_{L,QN_t}, & \dots, & e^{j(N_t-1)\theta_0} f_{L,QN_t} \\ f_{L,QN_t+1}, & e^{j\theta_{\lfloor 1/Q \rfloor}} f_{L,QN_t+1}, & e^{j2\theta_{\lfloor 1/Q \rfloor}} f_{L,QN_t+1}, & \dots, & e^{j(N_t-1)\theta_{\lfloor 1/Q \rfloor}} f_{L,QN_t+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{L,P-1}, & e^{j\theta_{\lfloor (P-1)/Q \rfloor - N_t}} f_{L,P-1}, & e^{j2\theta_{\lfloor (P-1)/Q \rfloor - N_t}} f_{L,P-1}, & \dots, & e^{j(N_t-1)\theta_{\lfloor (P-1)/Q \rfloor - N_t}} f_{L,P-1} \end{bmatrix} \quad (80)$$

[3] X. Ma, C. Tepedelenlioglu, G. Giannakis, and S. Barbarossa, "Non-data-aided carrier offset estimators for OFDM with null subcarriers: identifiability, algorithms, and performance," *IEEE J. Select. Areas Commun.*, vol. 19, no. 12, pp. 2504–2515, Dec. 2001.

[4] H. Minn, P. Tarasak, and V. Bhargava, "Some issues of complexity and training symbol design for OFDM frequency offset estimation methods based on BLUE principle," in *Proc. IEEE VTC*, vol. 2, 22-25 Apr. 2003, pp. 1288–1292.

[5] M. Morelli and U. Mengali, "An improved frequency offset estimator for OFDM applications," *IEEE Commun. Lett.*, vol. 3, no. 3, pp. 75–77, Mar. 1999.

[6] H. Minn, V. Bhargava, and K. Letaief, "A combined timing and frequency synchronization and channel estimation for OFDM," in *Proc. IEEE ICC*, vol. 2, 20-24 June 2004, pp. 872–876.

[7] D. Huang and K. Letaief, "Enhanced carrier frequency offset estimation for OFDM using channel side information," *IEEE Trans. Wireless Commun.*, vol. 5, no. 10, pp. 2784–2793, Oct. 2006.

[8] Y. Yao and G. Giannakis, "Blind carrier frequency offset estimation in SISO, MIMO, and multiuser OFDM systems," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 173–183, Jan. 2005.

[9] H. Minn and N. Al-Dhahir, "Estimation of carrier-frequency offset and frequency-selective channels in MIMO OFDM systems using a common training signal," in *Proc. WNCG Wireless Networking Symposium*, 2004.

[10] H. Liu and U. Tureli, "A high-efficiency carrier estimator for OFDM communications," *IEEE Commun. Lett.*, vol. 2, no. 4, pp. 104–106, Apr. 1998.

[11] J. Lei and T.-S. Ng, "A consistent OFDM carrier frequency offset estimator based on distinctively spaced pilot tones," *IEEE Trans. Wireless Commun.*, vol. 3, no. 2, pp. 588–599, Mar. 2004.

[12] H. Minn, Y. Li, N. Al-Dhahir, and R. Calderbank, "Pilot designs for consistent frequency offset estimation in OFDM systems," in *Proc. IEEE ICC*, vol. 10, June 2006, pp. 4566–4571.

[13] Y. Li, H. Minn, N. Al-Dhahir, and R. Calderbank, "Robust pilot design for consistent carrier frequency offset estimation," in *Proc. IEEE MILCOM*, Oct. 2006, pp. 1–7.

[14] M. Morelli and U. Mengali, "Carrier-frequency estimation for transmissions over selective channels," *IEEE Trans. Commun.*, vol. 48, no. 9, pp. 1580–1589, Sept. 2000.

[15] Y. Li, H. Minn, N. Al-Dhahir, and R. Calderbank, "Pilot designs for consistent frequency offset estimation in OFDM systems," *IEEE Trans. Commun.*, vol. 55, no. 5, pp. 864–877, May 2007.

[16] H. Minn and N. Al-Dhahir, "Optimal training signals for MIMO OFDM channel estimation," *IEEE Trans. Wireless Commun.*, vol. 5, no. 5, pp. 1158–1168, May 2006.

[17] H. Minn, X. Fu, and V. Bhargava, "Optimal periodic training signal for frequency offset estimation in frequency-selective fading channels," *IEEE Trans. Commun.*, vol. 54, no. 6, pp. 1081–1096, June 2006.

[18] D. Huang and K. Letaief, "Carrier frequency offset estimation for OFDM systems using subcarriers," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 813–823, May 2006.

[19] R. A. Horn and C. R. Johnson, *Matrix analysis,* Cambridge University Press, 1985.

Yinghui Li (S'05) received the B.E. and M.S. degrees in Electrical Engineering from the Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2000 and 2003, respectively, and the Ph.D. degree in Electrical Engineering from the University of Texas at Dallas in 2007. Since 2008, she has been with LitePoint Inc, CA, USA. Her research interests include signal processing algorithms and training signal designs for broadband wireless communications.

Hlaing Minn (S'99-M'01-SM'07) received his B.E. degree in Electronics from Yangon Institute of Technology, Yangon, Myanmar, in 1995, M.Eng. degree in Telecommunications from Asian Institute of Technology (AIT), Thailand, in 1997 and Ph.D. degree in Electrical Engineering from the University of Victoria, Victoria, BC, Canada, in 2001.

He was with the Telecommunications Program in AIT as a laboratory supervisor during 1998. He was a research assistant from 1999 to 2001 and a post-doctoral research fellow during 2002 in the Department of Electrical and Computer Engineering at the University of Victoria. He has been with the University of Texas at Dallas, USA, since 2002, and currently he is an Associate Professor. His research interests include wireless communications, statistical signal processing, error control, detection, estimation, synchronization, signal design, cross-layer design, and cognitive radios. He is an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS and the JOURNAL OF COMMUNICATIONS AND NETWORKS.