Optimal Periodic Training Signal for Frequency Offset Estimation in Frequency Selective Fading Channels

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Abstract— **This paper addresses optimal periodic training signal design for frequency offset estimation in frequency selective multipath Rayleigh fading channels. For a fixed transmitted training signal energy and within a fixed length block, the optimal periodic training signal structure (the optimal location of identical training sub-blocks) and the optimal training sub-block signal are presented. The optimality is based on the minimum Cramer-Rao bound (CRB) criterion. Based on the snap-shot CRB, the optimal periodic training structure is derived. The optimal training sub-block signal is obtained by utilizing the average CRB and the received training signal statistics. The optimal training structure with optimal training signals achieves substantial performance improvement over non-optimal training structure with non-optimal training signals.**

I. Introduction

Training signals are commonly used in communications systems for timing synchronization, frequency synchronization and channel estimation. Training signal design is an important issue since a proper design can significantly improve the estimation performance and can drastically reduce the estimation complexity.

In [1], periodic training sequences for channel equalization were addressed and zero autocorrelation (ZAC) sequences were shown to be optimal for the cyclic convolution type channel equalization. Periodic and aperiodic training sequences for least-square channel estimation were considered in [2] where best sequences obtained by computer search were presented for several channel lengths and training sequence lengths. Several design criteria and corresponding methods for efficient computer search of optimal sequences for channel estimation were presented in [3]-[4]. Constructions of optimal complex sequences were discussed in [5]-[6] and references therein. Optimal training sequences and pilot tones for OFDM channel estimation were addressed in [7]. Recent treatments on the optimal training structure and training signal design for channel estimation can be found in [8], [9] and references therein. Training sequence design for timing synchronization was discussed in [10] in the context of MSK signal. The best pattern (+ or - signs) of repetitive training sub-blocks for timing synchronization were presented in [11].

Regarding training design for frequency synchronization, [12] addressed optimal training signals in AWGN channel by using the CRB. Optimal training signal design for frequency offset estimation in frequency selective channels is not an easy task. Recently, [13] nicely addressed this problem by applying a minmax approach based on asymptotic CRB. The channel gains remain constant within the training block and the asymptotic CRB is obtained by setting the training block length (in samples) $N \rightarrow \infty$. For a fixed channel energy, the minmax approach minimizes the asymptotic CRB for the worst-case channel response. The optimality of the training signal from [13] is limited by the minmax approach and the asymptotic CRB but [13] does provide a neat solution to a long standing problem. It is also noted that due to the fixed channel energy constraint, the channel fading effect is not included in the training signal design of [13]. The optimal training signal design for frequency offset estimation in frequency selective fading channel is still an open problem which will be addressed in this paper.

For the ease of estimation complexity, we consider periodic training signal consisting of several identical sub-blocks. Periodic training signals have been extensively used in practice (e.g., GSM, IEEE 802.11a, 802.15, 802.16a). In this paper, we will answer the following question: "For a fixed transmitted training signal energy, what is the optimal periodic training structure (the location of the training sub-blocks) within a fixed-length block and what is the optimal training signal (of a sub-block) that gives the minimum CRB of the frequency offset estimation in frequency selective multipath Rayleigh fading channels?". It will be addressed by two steps. In the first step, we will find the optimal training structure for a fixed block length and a fixed received training energy. In the second step, we will investigate the optimal training signal (of a sub-block) for a fixed transmitted training energy in a frequency selective multipath Rayleigh fading channel. The combination of the results from the two steps will give the solution to the above question.

The rest of the paper is organized as follows. Section II describes the signal model and the CRB. Section III presents the optimal periodic training structure. In Section IV, the optimal training signal (of a sub-block) is addressed. Simulation results and discussions are provided in Section V and conclusions are given in Section VI.

II. Signal Model and CRB

We consider a wide-sense stationary uncorrelated scattering frequency selective Rayleigh fading channel characterized by L taps with uncorrelated complex baseband tap gains $h(0)$, $h(1)$, …, $h(L-1)$ and tap-spacing of symbol duration T_s. The channel is assumed to be quasi-static where the tap gains remain essentially constant over the block length NT_s. The complex baseband received signal sampled at the symbol rate can be expressed by

$$r(n) = x(n)e^{j2\pi nv} + w(n),\ n = 0, 1,\ \ldots,\ N-1 \qquad (1)$$

where v is the carrier frequency offset normalized by the symbol rate $1/T_s$, $\{w(n)\}$ are the uncorrelated samples of a zero-mean complex Gaussian noise process each having a variance of σ_n^2, and $\{x(n)\}$ are the channel output signal samples corresponding to the transmitted signal samples $\{s_n : n = -L+1,\ -L+2,\ \ldots,\ N-1\}$ and can be given by

$$x(n) = \sum_{k=0}^{L-1} h(k)\ s_{n-k},\quad n = 0,\ 1,\ \ldots,\ N-1. \qquad (2)$$

In matrix form, the received signal vector is expressed as

$$\boldsymbol{r} = \Lambda(v)\boldsymbol{Sh} + \boldsymbol{w} \qquad (3)$$

where $\boldsymbol{r}$ = $[r(0),\ r(1),\ \ldots,\ r(N-1)]^T$, $\boldsymbol{h}$= $[h(0),\ h(1),\ \ldots,\ h(L-1)]^T$, $\boldsymbol{w}$= $[w(0),\ w(1),\ \ldots,\ w(N-1)]^T$, $\Lambda(v)$ = $diag\{1, e^{j2\pi v}, \ldots, e^{j2\pi(N-1)v}\}$ is a diagonal matrix, $\boldsymbol{S}$ is an $N \times L$ matrix with entries $\boldsymbol{S}_{ij} = s_{i-j}$, $0 \le i \le N-1$, $0 \le j \le L-1$, and the superscript T denotes the transpose operation. The covariance matrices for $\boldsymbol{w}$ and $\boldsymbol{h}$ are given by $\boldsymbol{C_w}$ = $E[\boldsymbol{ww}^H]$ = $\sigma_n^2\boldsymbol{I}_N$ and $\boldsymbol{C_h}$ = $E[\boldsymbol{hh}^H]$ = $diag\{\sigma_{h_0}^2, \sigma_{h_1}^2, \ldots, \sigma_{h_{L-1}}^2\}$ where $trace(\boldsymbol{C_h})$ is assumed to be unity and $\boldsymbol{I}_N$ is a $N \times N$ identity matrix. The superscript H represents the Hermitian transpose.

If $\{s_k\}$ are known training signal samples, then for a given channel realization $\boldsymbol{h}$, the conditional CRB (or the snap-shot CRB) for the estimation of v based on the received vector $\boldsymbol{r}$ is given by [14]

$$\mathrm{CRB}_{|\boldsymbol{h}} = \frac{\sigma_n^2}{8\pi^2 \boldsymbol{h^H S^H M (I_N - B) M S h}} \qquad (4)$$

where $\boldsymbol{M}$ = $diag\{0,\ 1, \ldots,\ N-1\}$ and $\boldsymbol{B = S(S^H S)^{-1} S^H}$ is a projection matrix. If the whole training block of length N+L samples is constructed by repeating (P+1) times the same training signal sub-block of length L samples, the snap-shot CRB for the obtained periodic training signal can be simplified as [14]

$$\mathrm{CRB}_{|\boldsymbol{h}} = \frac{3\ \mathrm{SNR}_i^{-1}}{2\pi^2(PL^3)(P^2-1)} = \mathrm{CRB}_{|\mathrm{SNR}_i}. \qquad (5)$$

where

$$\mathrm{SNR}_i = \frac{\mathcal{E}_1/L}{\sigma_n^2} \qquad (6)$$

and $\mathcal{E}_1$ denotes the energy of a received training sub-block. The snap-shot CRB depends only on SNR_i for fixed values of P and L. Hence, $\mathrm{CRB}_{|\boldsymbol{h}}$ of (5) can be considered as $\mathrm{CRB}_{|\mathrm{SNR}_i}$, the CRB conditioned on the snap-shot SNR of the received training signal.

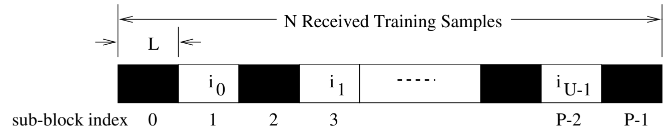

Fig. 1. An arbitrary periodic training structure illustrating the sub-blocks with indexes $[i_0,\ i_1,\ \ldots,\ i_{U-1}]$ used in the estimation. (The shaded sub-blocks serve as CPs and hence are not included in the observation vector of the estimation)

III. Optimal Periodic Training Structure

Periodic training signals are commonly used in practice due to their advantage of complexity reduction in estimation. For example, using a periodic training block, which consists of $(P+1)$ sub-blocks of length L samples each, rather than a non-periodic training block of the same length $N + L = (P+1)L$ would reduce the estimation complexity approximately by a factor of L [14]. For the same reason, we consider periodic training signals in this paper. Our interest is to find optimal periodic training structure that gives the minimum CRB. The CRB is derived for a joint estimation of the frequency offset and the channel impulse response as in [14].

The problem is formulated as follows. For a block with a fixed length of $(N+L)$ samples, which may contain training signals only or both training and data signals, and for a fixed received training signal energy $\mathcal{E}$, what is the best periodic training structure? This problem is divided into two sub-problems. In the first sub-problem, we will investigate the best structure consisting of V identical training sub-blocks, i.e., we have to find the best locations of V identical training sub-blocks within the block of length $N+L$ samples. Note that $\mathcal{E} = V\mathcal{E}_1$ and $V \le (P+1)$ if $N+L = (P+1)L$. In the second sub-problem, we will find the best value of V.

Note that if $\mathcal{E}_1 = \mathcal{E}/(P+1)$, then $V = P+1$. The first received sub-block which serves as a cyclic prefix part to absorb the channel dispersion, is discarded in the estimation. Let U be the number of training sub-blocks used in the estimation. Then $U = P$ in this case. In general, if all V sub-blocks are consecutively located as one group, $U = V-1$. If V sub-blocks form two groups of consecutive sub-blocks, where data signals are located between the two groups, then $U = V-2$. If there are G groups of consecutive training sub-blocks, we have $U = V - G$. Hence, forming more groups of consecutive training sub-blocks (a group may contain one or more consecutive sub-blocks) will result in more loss of training energy used in the estimation. We will denote this training energy loss as the energy loss in the cyclic prefixes.

A general training structure can be defined by the location index vector $\boldsymbol{J} = [i_0,\ i_1,\ \ldots,\ i_{U-1}]$ of the U sub-blocks used in the estimation (see Fig. 1). The indexes within the block are from -1 to $P-1$ and hence, we have $i_0 \ge 0$, $i_{U-1} \le P-1$ and $i_k < i_m$ for $k < m$. The corresponding snap-shot CRB can be given by (details are skipped due to space limitation)

$$\mathrm{CRB}_{|\mathrm{SNR}_i}(\boldsymbol{J}) = \frac{\mathrm{SNR}_i^{-1}}{8\pi^2 L^3 (\sum_{k=0}^{U-1} i_k^2 - \frac{(\sum_{n=0}^{U-1} i_n)^2}{U})}. \quad (7)$$

For a general training structure consisting of $V = 2(K+1)$ identical training sub-blocks, each having a received sub-block energy $\mathcal{E}_1 = \mathcal{E}/(2K+2)$, the best training structure can be obtained by

$$\boldsymbol{J}_{2K+2}^{\dagger} = arg\ min_{\boldsymbol{J}_{2K+2}} \mathrm{CRB}_{|\mathrm{SNR}_i}(\boldsymbol{J}_{2K+2}) \quad (8)$$

where the vector $\boldsymbol{J}_{2K+2}$ is of variable length U, $0 \le U \le (2K+1)$, corresponding to the location indexes of U sub-blocks used in the estimation out of $2K+2$ training sub-blocks. For each fixed value of U, there are several different $\boldsymbol{J}_{2K+2}$'s corresponding to different location index vectors.

Based on the snap-shot CRB in (7) and the result of the optimal training signal design in the AWGN channel from [12], the optimal training structure can be given by its location index vector as follows:

$$\boldsymbol{J}_{2K+2}^{\dagger} = [0, 1, \ldots, K-1, P-K, P-K+1, \ldots, P-1]. \quad (9)$$

The corresponding snap-shot CRB can be expressed as

$$\mathrm{CRB}_{|\mathrm{SNR}_i}(\boldsymbol{J}_{2K+2}^{\dagger}) = \frac{3\ \mathrm{SNR}_i^{-1}}{4K\pi^2 L^3 (4K^2 + 3P^2 - 6PK - 1)}. \quad (10)$$

If the block contains an odd number of training sub-blocks, say $2K+3$ sub-blocks with the received sub-block energy of $\mathcal{E}_1 = \mathcal{E}/(2K+3)$, the optimal structure will take either of the following two forms:

$$\boldsymbol{J}_{2K+3}^{\dagger} = [0, 1, \ldots, K, P-K, P-K+1, \ldots, P-1] \quad (11)$$

$$\boldsymbol{J}_{2K+3}^{\dagger} = [0, 1, \ldots, K-1, , P-K-1, P-K, \ldots, P-1]. \quad (12)$$

Both structures will give the same performance. It can be easily checked from (7) that $\boldsymbol{J}_{2K+2}^{\dagger}$ gives a smaller snap-shot CRB than $\boldsymbol{J}_{2K+3}^{\dagger}$. Hence, it is sufficient to consider an even number of training sub-blocks, i.e., $(2K+2)$.

Next, we investigate what value of K is the best for a block with a fixed length of $(P+1)L$ samples, a fixed total received training energy $\mathcal{E}$ and a fixed noise variance σ_n^2. Eq.(10) can be expressed as

$$\begin{aligned} \mathrm{CRB}_{|\mathrm{SNR}_i}(\boldsymbol{J}_{2K+2}^{\dagger}) &= \frac{3\sigma_n^2/\mathcal{E}}{2L^2\pi^2} \frac{K+1}{4K^3 - 6PK^2 + (3P^2-1)K} \\ &= \frac{3\sigma_n^2/\mathcal{E}}{2L^2\pi^2} f(K) \quad (13) \end{aligned}$$

where

$$f(K) = \frac{K+1}{4K^3 - 6PK^2 + (3P^2-1)K}. \quad (14)$$

Hence, the best value of K is determined by

$$K^{\dagger} = arg\ min_{1 \le K \le \lceil (P-1)/2 \rceil} f(K) \quad (15)$$

which can be easily evaluated numerically. A close form solution can be obtained but due to space limitation, it is not included. Note that since $f(K)$ is independent of SNR_i, the best value $K^{\dagger}$ holds for any SNR_i and hence, for fading channels as well.

IV. Optimal Training Sub-block Signal

In the previous section, we design the optimal periodic training structure for a fixed block length and a fixed received training energy. Although the channel dispersion effect is included in the design, the channel fading effect is excluded from the design due to the condition of the fixed *received* training energy. In this section, we will investigate the effect of the channel fading on the average CRB and we will find optimal training signal within a training sub-block so that the average CRB is minimized. We consider a frequency selective multipath Rayleigh fading channel. The problem can be formulated as follows: "For a fixed *transmitted* energy of a periodic training signal composed of several identical training sub-blocks, what is the best training sub-block signal that minimizes the average CRB in a frequency-selective multipath Rayleigh fading channel ?"

The multipath channel fading causes fluctuation of the received training energy (although the long-term average or the expected value of the received training energy is fixed) which in turn affects the average CRB. To minimize the average CRB, the training signal should be designed such that the received training energy fluctuation is minimized. This fact will be proved later. In the context of a periodic training signal, only one training sub-block is needed to be considered in the training signal design.

Define the following:

$$Z = \sum_{k=0}^{L-1} |x(k)|^2 = \mathcal{E}_1. \quad (16)$$

Then for a given periodic training structure, we know from (7) that

$$\mathrm{CRB}_{|\mathrm{SNR}_i} = \frac{\alpha}{Z} = \mathrm{CRB}_{|Z} \quad (17)$$

where

$$\alpha = \frac{\sigma_n^2}{8\pi^2 L^2 (\sum_{k=0}^{U-1} i_k^2 - \frac{(\sum_{n=0}^{U-1} i_n)^2}{U})}. \quad (18)$$

In a multipath Rayleigh fading channel, Z can be well approximated by a Gamma random variable with the parameter n. Note that $|x(k)|^2$ is a chi-square random variable. The sum of chi-square random variables is often represented by another chi-square random variable (see [15] and references therein). The Gamma distribution is a generalization of the chi-square distribution in that n is an integer in the latter but can be any positive real number in the former. In our case, $n \ge 2$. The mean and variance of Z are given by $E[Z] = n\sigma^2$ and $\sigma_Z^2 = 2n\sigma^4$ where the values of n and σ^2 depend on the training signal but $E[Z] = n\sigma^2 = \sum_{k=0}^{L-1} |s_k|^2$ is a constant equal to the transmitted energy of one training sub-block (assuming the total channel power transfer gain is unity). Let $p_Z(z)$ denote the probability density function of Z. The average CRB in the multipath Rayleigh fading channel can be given by

$$\begin{aligned} \mathrm{CRB} &= \int_0^{\infty} \mathrm{CRB}_{|Z}\ p_Z(z)\ dz \quad (19) \\ &\simeq \int_0^{\infty} \frac{\alpha}{z} \frac{1}{\sigma^2\ 2^{n/2}\ \Gamma(n/2)} z^{n/2}\ e^{-\frac{z}{2\sigma^2}}\ dz \quad (20) \\ &= \frac{\alpha}{E[Z] - \sigma_Z^2/E[Z]}. \quad (21) \end{aligned}$$

Eq.(21) indicates that the larger received training energy fluctuation (larger σ_Z^2) causes the larger CRB in the multipath Rayleigh fading channel. In other words, the training signal which gives the minimum fluctuation of the received training energy is the optimal training signal.

After some calculation, the variance of Z is given by

$$\begin{aligned}\sigma_Z^2 &= \sum_{k=0}^{L-1}(C_S(k,k))^2 E[|h(k)|^4] \\ &+ \sum_{m=0}^{L-1}\sum_{n=0,n\neq m}^{L-1} C_S(m,n)C_S(n,m)\sigma_{\boldsymbol{h}_m}^2\ \sigma_{\boldsymbol{h}_n}^2 \\ &+ \sum_{i=0}^{L-1}\sum_{l=0,l\neq i}^{L-1} C_S(i,i)C_S(l,l)\sigma_{\boldsymbol{h}_i}^2\ \sigma_{\boldsymbol{h}_l}^2 - (E[Z])^2 \quad (22)\end{aligned}$$

where $C_S(m,n)$ is the (m,n)-th element of $\boldsymbol{C_S} = \boldsymbol{S^H S}$ which represents the periodic autocorrelation of the training signal $\{s_k : k = 0,\ 1,\ \ldots, L-1\}$. Note that $C_S(n,n) = \sum_{k=0}^{L-1}|s_k|^2$ for all n and $C_S(m,n) = C_S^*(n,m)$. From (22), it is clear that σ_Z^2 is minimized when $C_S(l,k) = 0$ for $l \neq k$. In other words, the training signal which possesses zero periodic autocorrelation for any non-zero correlation lag (usually referred to as ZAC signal) minimizes the received training energy fluctuation in the multipath Rayleigh fading channel. Combining this result with that of (21) indicates that the ZAC training signals are optimal for frequency offset estimation in multipath Rayleigh fading channels.

V. Simulation Results and Discussions

We assume that the channel gains remain unchanged over a block of length $40L$ samples (i.e., P = 39) where the channel length L is 16 samples. The channel is assumed to have a power delay profile with a 3 dB per tap decaying factor. We first evaluate various training structures which have the same total transmitted training energy E_{Tx} and are composed of identical training sub-blocks of length L samples each. Structure#1 denotes a structure where the first 20 sub-blocks are training signals. Structure#2 represents a structure where all 40 sub-blocks are training signals (sub-block energy in this case will be smaller than that in Structure#1). The Optimal structure is the one where the first and the last $K+1$ sub-blocks are training signals. The training sub-block signal used is the one from IEEE 802.11a (OFDM). Note that Structure#1 corresponds to the training signal proposed in [16]. The frequency offset estimation method of [17] (Method-B) is used in the evaluation of the training structures. For very low SNR, the initial frequency offset compensation of [17] is done by the method of [16] with 64-point FFT. The results are presented for different values of $\gamma = E_{\text{Tx}}/(160\ \sigma_n^2)$. For the optimal structure with the optimal K value (K = 4), γ equals to SNR.

Fig. 2 shows the frequency offset estimation performance in terms of the snap-shot CRBs (7), the average CRBs ((19) evaluated by simulation) and MSE (simulation results) for different training structures in the multipath Rayleigh fading channel. Both CRBs and MSEs indicate the same information on the training structures. Structure#2 performs better than Structure#1 but at the cost of more system resource (time). The optimal structures with $K = 4$ and $K = 9$ perform significantly better than Structure#1 and Structure#2. The optimal structure with $K = 9$ utilizes the same system resource (time) as Structure#1. The optimal structure with $K = 4$ uses less system resource (time). Although smaller K needs less system resource (time), it may not give a better estimation performance. The best value of K, denoted by $K^\dagger$, in terms of estimation performance is given by (15). For the considered parameters, $K^\dagger = 4$. The simulation results and the snap-shot CRBs are presented in Fig. 3 for the optimal structure with different values of K. They agree with the theoretical result of (15) indicating $K^\dagger = 4$.

Next, using the optimal training structure with the best value of K, we investigate several training sub-block signals. The considered signals are the OFDM training sub-block signal from IEEE 802.11a, several constant amplitude sequences including a m-sequence (1-bit augmented to have an even length) and a constant amplitude ZAC sequence (CAZAC) and two arbitrary non-ZAC sequences. The snap-shot CRB (which is independent of the training sub-block signal), the average CRBs and the MSEs for different training sub-block signals are presented in Fig. 4. Among them, the ZAC sequence is the best. Since the IEEE 802.11a training sub-block signal and the m-sequence have correlation properties very close to that of ZAC, their performance are almost the same as ZAC sequence (not distinguishable in the figure). The performances of two arbitrary non-ZAC sequences are worse than the ZAC sequence, as expected.

The optimal training sub-block (ZAC sequence) improves about 1.5 to 2 dB in CRB or MSE performance over the two arbitrary non-ZAC sequences. For the considered parameters, the proposed optimal training structure has approximately 9 dB improvement in CRB or MSE performance over the conventional consecutive periodic training structure such as Structure#1.

VI. Conclusions

We have presented an optimal periodic training signal design for frequency offset estimation in frequency selective multipath Rayleigh fading channels. The optimality is based on the minimum average CRB in the frequency selective fading channel within the framework of a fixed total transmitted training signal energy and a fixed block length over which the channel gains remain unchanged. The training design is addressed in terms of the optimal periodic training structure (the optimal location of identical training sub-blocks within the block) and the optimal training sub-block signal. Signals having zero autocorrelation (ZAC) property experience the minimum received energy fluctuation in frequency selective Rayleigh fading channels which translates into the minimum average CRB. Hence, the optimal training sub-block is a ZAC signal. This result also provides the missing proof of the optimality or near-optimality of several training signals adopted in many standards. The optimal periodic training structure consists of $(2K^\dagger + 2)$ identical training sub-blocks, each having length L samples (the channel length) within the block of length $(P+1)L$ samples. The first $K^{\dagger}+1$ training sub-blocks are located at the beginning and the rest resides at the end of the block. The value of $K^{\dagger}$ is a function of P and can easily be calculated. The optimal periodic training structure achieves a significant estimation performance improvement over the conventional consecutive periodic training structure.

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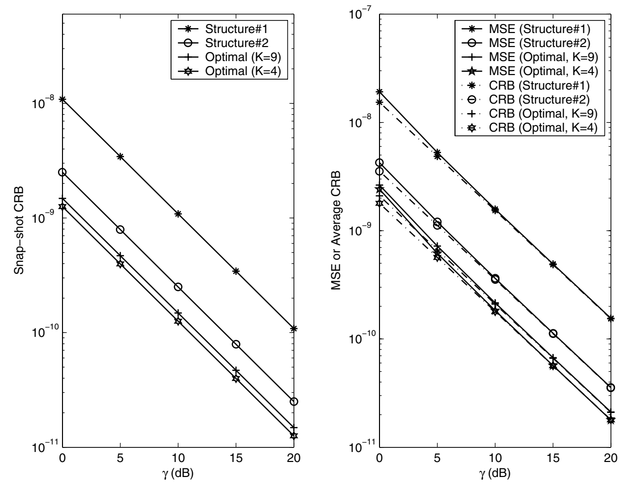

Fig. 2. Performance of different periodic training structures in the multipath Rayleigh fading channel

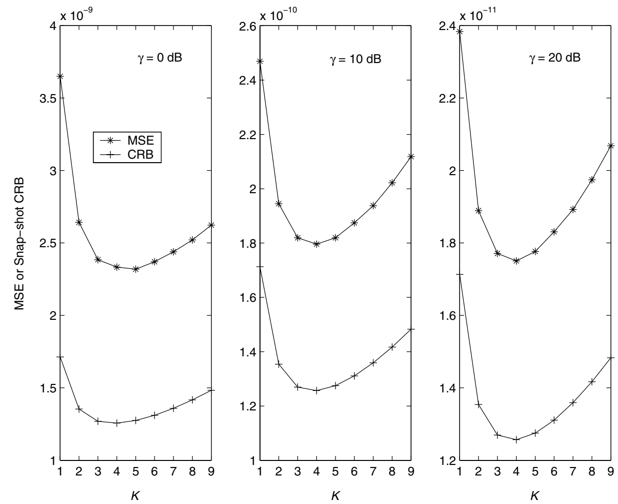

Fig. 3. Performance at different values of K for the optimal training structure containing $(2K+2)$ sub-blocks

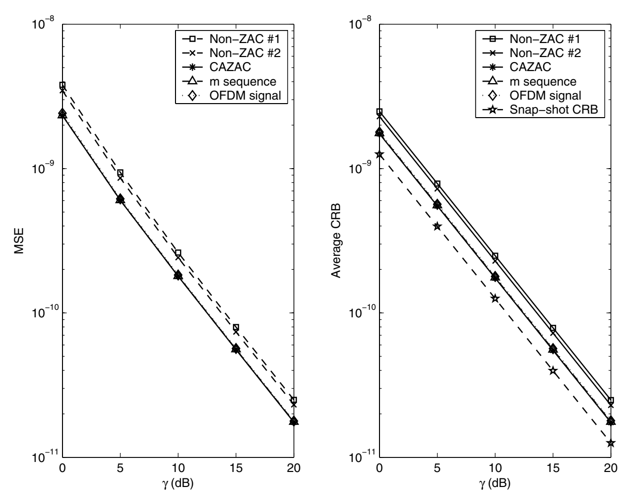

Fig. 4. Performance of different training signals (of a sub-block) with the optimal training structure (K=4) in the multipath Rayleigh fading channel