Network Dimensioning and Radio Resource Management for Multi-Tier Machine-Type Communications

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***Abstract*—For a radio resource limited multi-tier Machine-type Communication (MTC) network, controlling random access congestion while satisfying the unique requirements of each tier and guaranteeing fairness among nodes is always a challenge. In this paper, we propose three approaches for network dimensioning and radio resource partitioning for the uplink of an MTC network with signal-to-interference ratio (SIR)-based clustering and relaying, where MTC gateways (MTCGs) capture and forward the packets sent from MTC devices (MTCDs) to the base station (BS). Assuming that each tier has a different outage probability constraint, our first resource allocation method achieves maximum sum network capacity, while the other two methods achieve suboptimal sum network capacity with higher degrees of fairness. The proposed methods are also applicable when the objective is the network operator's total revenue instead of sum network capacity.**

I. Introduction

Although growing rapidly, Machine-to-Machine (M2M) communication is facing various challenges due to its unique characteristics [1] and the conflict between the growth of M2M applications and limited capacity of current cellular communication systems. To be specific, the number of MTCDs is significantly larger than that of Human-to-Human (H2H) communication devices and an MTCD only sends a small amount of data in each transmission, which leads to severe congestion on the random access channel (RACH) and impairs the network's reliability [2]. Extensive research has been conducted to solve this problem [3] and some standard schemes are proposed in [4]. In general, separating the access requests from H2H and M2M is a key idea to mitigate RACH overload [3].

At the same time, the coexistence of diverse devices requiring varying Quality of Service (QoS) in terms of latency [5], transmission rate and outage, connection security [6], etc, is a main desired feature of M2M networks. How to efficiently aggregate the huge amount of transmissions while satisfying various QoS requirements and mitigating the negative effects on H2H communication is still a problem. Since MTC data can be efficiently aggregated by a hierarchical network, a number of clustering and resource allocation methods have been proposed to solve this problem. For instance, a grouping-based orthogonal radio resource management with low computation complexity was proposed in [7], and [8] investigated how to select cluster heads with channel state information. Based on a matching algorithm, a distributed resource bargaining scheme between cellular users and MTCDs was proposed in [9] and its performance was shown to be comparable to centralized optimal approaches. Considering the uncertainty of system parameters, [10] proposed a fuzzy logic based robust uplink resource allocation solution. In [11] and [12], SIR and location based clustering and relaying schemes for single-type MTCDs were proposed to maximize the transmission capacity of the MTC network under an outage probability constraint. In general, orthogonal resource allocation eradicates the interference but suffers from low spectrum efficiency while using nonorthogonal resource allocation enhances spectrum efficiency but introduces interference. We combine the orthogonal and nonorthogonal resource allocation approaches, where orthogonal resources are allocated among H2H communications, MTCGs and tiers of MTCDs; while the resources are shared among the MTCDs of the same tier. In this way, the conventional H2H communication is free of MTC caused interference and the MTC achieves a higher spectrum efficiency through spectrum reuse.

In particular, we consider the scenario with MTCGs and multiple tiers of MTCDs in the coverage of a BS. The locations of MTCGs and those of MTCDs in individual tiers form independent homogeneous spatial Poisson Point Processes (PPP) and the radio resources available for MTC are limited. The problem of interest is to maximize the sum network capacity and total revenue of the MTC network with pre-defined requirements on outage probability, MTCD density and fairness level. A cluster-based uplink radio resource allocation scheme is formulated. Instead of exclusive clustering, e.g. clustering into disjoint spatial zones, we apply SIR based clustering since it requires minimal coordination between MTCGs and MTCDs but provides the highest access opportunity for MTCDs by allowing multiple MTCGs to successfully decode the same packet [11]. Equating fairness to how equally the spectrum is partitioned among different tiers, we propose three network dimensioning methods to maximize the sum network capacity and total revenue while guaranteeing different levels of fairness. First, an optimal dimensioning scheme is presented which achieves the maximum objective value with the lowest degree of fairness. Next, a geometric mean (GM) based method is proposed where we modified the objective to geometric sum network capacity or total revenue. This scheme results in equal resource partitioning when the objective is maximized. Finally, we propose a heuristic method, namely Cauchy-Schwarz (CS) based method that offers a trade-off between fairness and sum network capacity or total revenue.

II. System Model

We consider a single-hop MTC uplink relay scenario as shown in Fig. 1. A group of packets sent from MTCDs are gathered by nearby MTCGs and then relayed to the BS so that random access requests to the BS are moderated. The MTCDs are classified into different tiers according to their required transmission outage rate ζ_j. We assume that there are N tiers of devices and the locations of the jth tier MTCDs form a homogeneous spatial PPP, $\Phi_{D_j} = \{X_{j,k}\}$ with density λ_j, $\forall j = 1, \dots, N$. In compact form, we use vector $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_N]^{\mathrm{T}}$ to represent the densities. Similarly the MTCG locations form an independent homogeneous spatial PPP, $\Phi_G = \{Y_i\}$ with density λ_G.

The total amount of radio spectrum resources for MTC is limited and it should be partitioned for MTCDs and MTCGs. Assuming that there are Q_M resource blocks (RBs) reserved for M2M communication and $Q_1 = \gamma Q_M$ RBs are allocated for MTCD-to-MTCG link (link 1), then $Q_2 = (1-\gamma)Q_M$ RBs are allocated for MTCG-to-BS link (link 2), where $\gamma \in [0, 1]$. Next, $Q_1^j = \beta_j Q_1$ RBs are allocated for the jth tier of MTCDs, where $0 \leq \beta_j$ and $\sum_{j=1}^N \beta_j = 1$. We denote the spectrum partitioning factors in compact form with vector $\boldsymbol{\beta} = [\beta_1, \dots, \beta_N]^{\mathrm{T}}$. Since [11] stated equal resource partitioning among MTCGs is optimal by showing that all MTCGs capture a packet with the same probability, Q_2/G RBs are allocated for each MTCG, where G is the number of MTCGs in the coverage of a BS. Furthermore, it is assumed that, each MTCD sends a fixed-length packet and needs δ_1 RBs for transmitting one packet, and each MTCG needs δ_2 RBs in order to relay a MTCD's packet to the BS. Therefore, the jth tier of MTCDs have $U_1^j = \lfloor \beta_j Q_1/\delta_1 \rfloor$ orthogonal data channels, while each MTCG can relay at most $U_2 = \lfloor Q_2/(G\delta_2) \rfloor$ packets.

We assume i.i.d. Rayleigh fading with unit average power gain on each channel, and exponential path loss with exponent α over all channels. In the transmission process, each active MTCD randomly selects a channel among those that are allocated for its tier. Since two or more MTCDs can potentially transmit on the same channel, they may interfere with each other. MTCGs can successfully decode a packet only when their received SIR is higher than a threshold η. MTCGs treat packets from all MTCDs equally in the sense that if no greater than U_2 packets are decoded, all of them will be relayed; otherwise, the MTCG will randomly drop some packets and only relays the remaining U_2 packets.

We consider that each one of N MTCD tiers has a transmission outage probability constraint ζ_j and may also have a minimum transmission density requirement $\tilde{\lambda}_j$. According to [13], the transmission capacity or area spectral efficiency for a single type of device is the density of the simultaneously

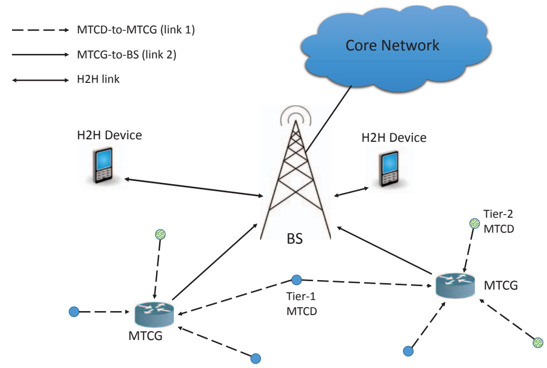

Fig. 1. Network Structure

transmitting MTCDs multiplied with their end-to-end transmission success probability. Thus, for a multi-tier M2M, the sum network capacity can be expressed by $\sum_{j=1}^N \lambda_j(1-\epsilon_j)$, where ϵ_j is the end-to-end outage probability of the jth tier of MTCDs. We may also consider the economic revenue of this M2M network as $\sum_{j=1}^N \lambda_j \pi_j (1-\epsilon_j)$, where π_j denotes the price or revenue per unit capacity of tier j.

III. Problem Formulation

In [11], a single-tier SIR based clustering and relaying M2M communication scheme was proposed and optimized to achieve maximum network capacity. It was clarified that successful transmission is equivalent to the joint occurrence of three events, namely a) the typical MTCD is the nearest MTCD on a randomly selected channel u to an MTCG at Y_i, $\mathcal{N}^u_{X_0,Y_i}$, b) the typical MTCD's packet on channel u is successfully captured by an MTCG at Y_i, $\mathcal{C}^u_{X_0,Y_i}$, c) the typical MTCD's packet received on u is successfully relayed by an MTCG at Y_i, $\mathcal{R}^u_{X_0,Y_i}$, where X_0 represents the nearest MTCD to Y_i transmitting on u. Provided that the MTCD density is λ_D, the (conditional) probabilities of these three events are

$$\Pr(\mathcal{N}^u_{X_0,Y_i}) = \exp\left(-\pi \frac{\lambda_D}{U_1}\|X_0 - Y_i\|^2\right) \tag{1}$$

$$\Pr(\mathcal{C}^u_{X_0,Y_i}|\mathcal{N}^u_{X_0,Y_i}) = \exp\left(-\pi \frac{\lambda_D}{U_1}\eta^{\frac{2}{\alpha}}\|X_0 - Y_i\|^2 K_{\alpha,\eta}\right) \tag{2}$$

$$\begin{aligned}\Pr(\mathcal{R}^u_{X_0,Y_i}|\mathcal{C}^u_{X_0,Y_i}) &= 1-\varepsilon \\ &= 1 - \sum_{u=U_2}^{U_1-1}\begin{pmatrix} U_1-1 \\ u \end{pmatrix} p^u_{c,\mathrm{SIR}} \\ &\quad \times (1-p_{c,\mathrm{SIR}})^{U_1-1-u}\left[1-\frac{U_2}{1+u}\right]\end{aligned} \tag{3}$$

where $K_{\alpha,\eta} = \int_{\eta^{-\frac{2}{\alpha}}}^{\infty} \frac{dt}{1+t^{\alpha/2}}$, $p_{c,\mathrm{SIR}} = \left(1+\eta^{\frac{2}{\alpha}}K_{\alpha,\eta}\right)^{-1}$ is the probability of an MTCG capturing a packet on any link 1 channel and ε represents the link 2 outage probability. We note that if $U_2 < U_1 - 1$, then $\varepsilon = 0$.

A typical MTCD can transmit successfully on channel u if and only if, first, it is the nearest MTCD on u to an MTCG, second, there is at least one MTCG that can capture and relay the MTCD's packet successfully. Therefore, applying the chain rule on (conditional) probabilities in (1) - (3), the end-to-end outage probability of a typical MTCD transmission is expressed as

$$\begin{aligned}\epsilon(\lambda_D,\gamma) =& \mathbb{E}\prod_{Y_i\in\Phi_G}\Big[1-\Pr(\mathcal{R}^u_{X_0,Y_i}|\mathcal{C}^u_{X_0,Y_i})\\ &\times\Pr(\mathcal{C}^u_{X_0,Y_i}|\mathcal{N}^u_{X_0,Y_i})\Pr(\mathcal{N}^u_{X_0,Y_i})\Big] \\ =& \exp\left[-\frac{\lambda_G U_1}{\lambda_D\left(1+\eta^{2/\alpha}K_{\alpha,\eta}\right)}(1-\varepsilon)\right].\end{aligned} \tag{4}$$

The details of the derivation for (1)-(4) can be found in [11].

We now consider the scenario of N tiers of MTCDs of which the required outage probabilities and minimum densities are $\zeta_1,\ldots,\zeta_N$ and $\tilde{\lambda}_1,\ldots,\tilde{\lambda}_N$ respectively. In particular, assuming the MTCGs treat all the packets equally, for the jth tier, the MTCD density is lower bounded by $\tilde{\lambda}_j$ and the outage probability $\epsilon(\lambda_j,\gamma)\equiv\epsilon_j$ is upper bounded by ζ_j. Accordingly, the sum network capacity maximization (SNCM) problem and total revenue maximization (TRM) problem can be jointly formulated as

$$\begin{aligned}&\max_{\boldsymbol{\lambda},\boldsymbol{\beta},\gamma}\ \sum_{j=1}^{N}\lambda_j\pi_j\left(1-\exp\left[-\frac{\beta_j\phi(\gamma)}{\lambda_j}\right]\right)\\ &\text{s.t.}\ \ \tilde{\lambda}_j\le\lambda_j\le\frac{\beta_j\phi(\gamma)}{\ln(1/\zeta_j)},\ 0\le\gamma\le1,\\ &\qquad\sum_{j=1}^{N}\beta_j=1,\ 0\le\beta_j\end{aligned} \tag{5}$$

where we define

$$\phi(\gamma)\triangleq\frac{\lambda_G U_1}{1+\eta^{2/\alpha}K_{\alpha,\eta}}(1-\varepsilon(\gamma)) \tag{6}$$

as a function[1] of γ and the right-hand-side inequality in the first constraint of (5) represents $\epsilon(\lambda_j,\gamma)\le\zeta_j$. Note that when π_j's denote the revenue per unit capacity of tier j, the objective can be regarded as total revenue of the M2M network, and that for $\pi_j=1, j\in\{1,...,N\}$, the objective is the sum network capacity of the M2M network.

First, we notice that the objective monotonically increases with both $\phi(\gamma)$ and any λ_j. Since λ_j is upper bounded by the term $\frac{\beta_j\phi(\gamma)}{\ln(1/\zeta_j)}$, maximizing $\phi(\gamma)$ not only maximizes the objective but also maximizes the upper bound of λ_j, which in turn relaxes the constraint on MTCD densities. However, representing $\max\phi(\gamma)$ in closed form may not be easy because U_1 and U_2 are floor functions of γ. Thus, the optimal value, $\gamma^*=\arg\max\phi(\gamma)$, is obtained by numerically computing $\phi(\gamma)$ over $\gamma\in[0,1]$. Therefore, the simplified problem is formulated as

$$\begin{aligned}\text{P1}:\quad&\max_{\boldsymbol{\lambda},\boldsymbol{\beta}}\ \sum_{j=1}^{N}\lambda_j\pi_j\left(1-\exp\left[-\frac{\beta_j\phi(\gamma^*)}{\lambda_j}\right]\right)\\ &\text{s.t.}\ \ \tilde{\lambda}_j\le\lambda_j\le\frac{\beta_j\phi(\gamma^*)}{\ln(1/\zeta_j)},\\ &\qquad\sum_{j=1}^{N}\beta_j=1,\ 0\le\beta_j.\end{aligned} \tag{7}$$

In the rest of this paper, we will discuss network dimensioning and resource partitioning methods that solve 1) SNCM and TRM subject to minimum MTCD density requirements (i.e. $\tilde{\lambda}_j>0$), 2) SNCM and TRM without minimum MTCD density requirements (i.e. $\tilde{\lambda}_j=0$). Meanwhile, the fairness issue is considered throughout this paper.

IV. Optimal Network Dimensioning

A. Network Dimensioning with Minimum Transmission Density Requirements

To solve P1, where $\tilde{\lambda}_j>0$, two steps are needed, 1) allocating minimum resource to each tier to meet the minimum transmission density requirements and 2) maximizing the sum network capacity or total revenue with the residual resources. To be specific, we first set $\lambda_j=\tilde{\lambda}_j$ so that the minimum transmission density requirements are satisfied and we have

$$\frac{\tilde{\lambda}_j\ln(1/\zeta_j)}{\phi(\gamma^*)}=\frac{\lambda_j\ln(1/\zeta_j)}{\phi(\gamma^*)}\le\beta_j \tag{8}$$

according to the first constraint in (7). The left-hand-side of (8) specifies the minimum resource ratio, $\tilde{\beta}_j=\frac{\tilde{\lambda}_j\ln(1/\zeta_j)}{\phi(\gamma^*)}$, for tier j to satisfy the outage probability constraint and minimum density requirement.

With (8) and $\sum_{j=1}^{N}\beta_j=1$, we have

$$\frac{\sum_{j=1}^{N}\tilde{\lambda}_j\ln(1/\zeta_j)}{\phi(\gamma^*)}\le1. \tag{9}$$

Thus, after allocating $\tilde{\beta}_j$ resources to tier j for all $j\in\{1,\ldots,N\}$, the normalized amount of unused resource is

$$\Delta\beta=1-\sum_{j=1}^{N}\tilde{\beta}_j=1-\frac{\sum\tilde{\lambda}_j\ln(1/\zeta_j)}{\phi(\gamma^*)}\ge0. \tag{10}$$

Next, we want to allocate the residual resources to maximize the objective function.

If $\Delta\beta$ is allocated to tier j, we obtain at most $\Delta\lambda_j=\frac{\Delta\beta\phi(\gamma^*)}{\ln(1/\zeta_j)}$ additional density according to the first constraint in (7). Therefore, the revenue or capacity for tier j increases by

$$\Delta R_j=\Delta\lambda_j\pi_j(1-\zeta_j)=\Delta\beta\phi(\gamma^*)\frac{(1-\zeta_j)\pi_j}{\ln(1/\zeta_j)}. \tag{11}$$

To maximize the objective, all the residual resources should be allocated to one tier that gives the largest ΔR_{j^*}, i.e., $j^*=\arg\max_j\frac{\pi_j(1-\zeta_j)}{\ln(1/\zeta_j)}$. Therefore, the maximum total revenue or

[1] $\phi(\gamma)$ is also a function of λ_G and $\max\phi(\gamma)$ determines whether the MTCG deployment is able to support the required MTCD densities with the outage probability constraints as we can see later.

sum network capacity is

$$\begin{aligned}R_{\mathrm{w}}^{*} =& \Delta R_{j^*} + \sum_{j=1}^{N} \tilde{\lambda}_j \pi_j (1-\zeta_j) \\ =& \left[\phi(\gamma^*) - \sum_{j=1}^{N} \tilde{\lambda}_j \pi_j \ln(1/\zeta_j)\right] \frac{(1-\zeta_{j^*})\pi_{j^*}}{\ln(1/\zeta_{j^*})} \\ &+ \sum_{j=1}^{N} \tilde{\lambda}_j \pi_j (1-\zeta_j)\end{aligned} \tag{12}$$

where the subscript w implies that the minimum transmission density requirements are considered. Defining vector $\tilde{\boldsymbol{\beta}} \triangleq [\tilde{\beta}_1, \ldots, \tilde{\beta}_N]^{\mathrm{T}}$, $\tilde{\boldsymbol{\lambda}} \triangleq [\tilde{\lambda}_1, \ldots, \tilde{\lambda}_N]^{\mathrm{T}}$ and $\boldsymbol{e}_i \triangleq [0, \ldots, 0, 1, 0, \ldots, 0]^{\mathrm{T}}$ where only the ith element is 1, the optimal resource partitioning and corresponding network densities can be expressed in compact form as

$$\boldsymbol{\beta}^* = [\beta_1^*, \ldots, \beta_N^*]^{\mathrm{T}} = \tilde{\boldsymbol{\beta}} + \Delta\beta \boldsymbol{e}_{j^*} \tag{13}$$

and

$$\boldsymbol{\lambda}^* = [\lambda_1^*, \ldots, \lambda_N^*]^{\mathrm{T}} = \tilde{\boldsymbol{\lambda}} + \Delta\lambda \boldsymbol{e}_{j^*} \tag{14}$$

respectively.

Noticing that function $f(\zeta) = \frac{1-\zeta}{\ln(1/\zeta)}$ monotonically increases with ζ, when $\pi_j = 1$, we obtain the maximum sum network capacity by allocating all the $\Delta\beta$ resource to the tier with the maximum outage probability constraint, i.e., to the j^*th tier where $j^* = \underset{j}{\arg\max}\ \zeta_j$.

B. Network Dimensioning without Minimum Transmission Density Requirements

This section considers SNCM and TRM without minimum transmission density requirements, i.e., $\tilde{\lambda}_j = 0$. Since the objective of P1 in (7) increases monotonically with λ_j, optimal MTCD density of the jth tier relates to its bandwidth proportion,

$$\lambda_j^* = \frac{\phi(\gamma^*)\beta_j}{\ln(1/\zeta_j)} \tag{15}$$

Substituting λ_j^* in (15) into P1, we simplify the original problem (P1) to

$$\begin{aligned}\mathrm{P2}: \quad & \max_{\boldsymbol{\beta}} \sum_{j=1}^{N} \frac{\beta_j \pi_j (1-\zeta_j)}{\ln(1/\zeta_j)} \phi(\gamma^*) \\ & \text{s.t.} \quad \sum_{j=1}^{N} \beta_j = 1,\ 0 \le \beta_j.\end{aligned} \tag{16}$$

As we can always find a tier k such that $k = \arg\max_j \frac{\pi_j(1-\zeta_j)}{\ln(1/\zeta_j)}$. Allocating all resources to this tier, i.e., $\boldsymbol{\beta}^* = \boldsymbol{e}_k$ and thereby $\boldsymbol{\lambda}^* = \frac{\phi(\gamma^*)}{\ln(1/\zeta_k)} \boldsymbol{e}_k$, will achieve the maximum sum network capacity or total revenue,

$$R_{\mathrm{w/o}}^* = \frac{\pi_k(1-\zeta_k)}{\ln(1/\zeta_k)} \phi(\gamma^*) \tag{17}$$

where the subscript w/o implies that the minimum transmission density requirements are not considered. For the convenience of analysis, let $\mathbf{z} = [z_1, z_2, ..., z_N]^T$ where $z_j = \frac{\pi_j(1-\zeta_j)}{\ln(1/\zeta_j)} \ge 0$, and we rewrite (17) as

$$R_{\mathrm{w/o}}^* = \|\mathbf{z}\|_\infty \phi(\gamma^*) \tag{18}$$

where $\|\cdot\|_\infty$ represents the infinity norm. We note this resource allocation method strongly favors the tier with less stringent outage constraint, or the one paying the highest price. In the following sections, we will provide two other resource allocation methods focusing on the fairness issue.

V. Suboptimal Network Dimensioning without Minimum Transmission Density Requirements

In this section, we propose two resource allocation methods to achieve different levels of fairness when minimum transmission density requirement $\tilde{\lambda}_j = 0$. These methods can also be used to allocate the residual resource that was mentioned in (10) in section IV-A, i.e., instead of being allocated to only one tier, the residual $\Delta\beta$ resource can also be partitioned to all tiers by either suboptimal method.

A. Geometric Mean (GM) based Resource Allocation

Instead of maximizing sum network capacity or total revenue, we replace the revenue-sum objective function of P1 in (7) with the geometric mean of the revenues of all tiers as

$$\log R_{\mathrm{total}} = \log\left(\prod_{j=1}^{N} R_j\right) = \sum_{j=1}^{N} \log R_j \tag{19}$$

where $R_j = \lambda_j \pi_j \left(1 - \exp\left[-\frac{\beta_j \phi(\gamma^*)}{\lambda_j}\right]\right)$ is the revenue or transmission capacity of the jth tier. The motivation of using this logarithmic product function comes from a branch of cooperative game theory, namely bargaining theory [14] and the notion of proportional fairness [15]. Further justification of this bargaining model is beyond the scope of this paper, and interested readers can refer to section 7.1.3 of [14] and [15], [16] for more details. Noting that the objective increases monotonically with λ_j, to satisfy the first constraint of P1 by equality, optimal MTCD density of the jth tier should also be the λ_j^* specified in (15). With this knowledge, we obtain a simplified problem,

$$\begin{aligned}\mathrm{P3}: \quad & \max_{\boldsymbol{\beta}} \sum_{j=1}^{N} \log\left[\frac{\beta_j \pi_j (1-\zeta_j)}{\ln(1/\zeta_j)} \phi(\gamma^*)\right] \\ & \text{s.t.} \quad \sum_{j=1}^{N} \beta_j = 1,\ 0 \le \beta_j.\end{aligned} \tag{20}$$

The objective function in (20) can be rewritten as

$$\max_{\boldsymbol{\beta}} \sum_{j=1}^{N} \log \beta_j \ + \sum_{j=1}^{N} \log\left[\frac{\pi_j(1-\zeta_j)}{\ln(1/\zeta_j)} \phi(\gamma^*)\right] \tag{21}$$

which allows us to maximize the objective function over β_j. Due to the concavity of log function, equal partition is the optimal GM based resource allocation among MTCDs, i.e., $\beta_j^{\mathrm{GM}} = \frac{1}{N}$. Thus, the corresponding MTCD density is $\lambda_j^{\mathrm{GM}} = \frac{\phi(\gamma^*)}{N \ln(1/\zeta_j)}$ according to (15). Substituting λ_j and β_j in the objective function of P1 with λ_j^{GM} and β_j^{GM} respectively, the sum network capacity or total revenue achieved by GM method is

$$R^{\mathrm{GM}} = \frac{1}{N} \sum_{j=1}^{N} \frac{\pi_j(1-\zeta_j)}{\ln(1/\zeta_j)} \phi(\gamma^*) = \frac{1}{N} \|\mathbf{z}\|_1 \phi(\gamma^*) \tag{22}$$

where $\|\cdot\|_1$ represents the l_1-norm. Different from the method introduced in section IV-B, this GM method achieves absolute fairness over all tiers of MTCDs, regardless of the outage probability constraints.

Comparing the sum network capacity or total revenue achieved by the GM method in (22) with the maximum value in (18), we have $R^{\mathrm{GM}} \leq R^*_{\mathrm{w/o}}$, where equality holds when all the elements in $\mathbf{z}$ are equal. Clearly, the fairness achieved by the GM method is at the cost of sum network capacity or total revenue. On the other hand, if the price value π_j are adjusted to make the value of elements in $\mathbf{z}$ more uniform, the difference between R^{GM} and $R^*_{\mathrm{w/o}}$ will be reduced.

B. Cauchy-Schwarz (CS) based Resource Allocation

While the GM method results in equal partitioning, and optimal allocation yields maximum sum network capacity or total revenue, we are also interested in a trade-off between fairness and capacity, which motivated us to propose CS based resource allocation.

We consider the optimization problem P2 since we assume $\tilde{\lambda}_j = 0$. Denoting $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_N]^{\mathrm{T}}$, with Cauchy-Schwarz inequality, we have

$$(\boldsymbol{\beta}^T \mathbf{z})^2 \leq \|\boldsymbol{\beta}\|_2^2 \|\mathbf{z}\|_2^2 \tag{23}$$

where $\|\cdot\|_2$ represents l_2-norm and the equality holds when $\beta_1/z_1 = \ldots = \beta_N/z_N$. Specifically, with this equality and the fact that the summation of all the nonnegative β_j is 1, we have

$$\beta_j^{\mathrm{CS}} = \frac{z_j}{\|\mathbf{z}\|_1} = \frac{\frac{\pi_j(1-\zeta_j)}{\ln(1/\zeta_j)}}{\sum_{l=1}^{N} \frac{\pi_l(1-\zeta_l)}{\ln(1/\zeta_l)}} \tag{24}$$

We can see that this solution relates the resource partitioning parameter to the outage probability constraints and prices. For each tier, the stricter the outage constraint is, less resource is allocated; and the higher the price is, more resources are allocated. Therefore, we set β_j^{CS} as the resource partitioning parameter and name this method as CS based resource allocation.

By substituting β_j with β_j^{CS} in (15), the maximum transmission density for tier j is

$$\lambda_j^{\mathrm{CS}} = \frac{\frac{\pi_j(1-\zeta_j)}{[\ln(1/\zeta_j)]^2}}{\sum_{l=1}^{N} \frac{\pi_l(1-\zeta_l)}{\ln(1/\zeta_l)}} \phi(\gamma^*) \tag{25}$$

which results in the sum network capacity or total revenue,

$$R^{\mathrm{CS}} = \frac{\sum_{j=1}^{N} \left(\frac{\pi_j(1-\zeta_j)}{\ln(1/\zeta_j)}\right)^2}{\sum_{j=1}^{N} \frac{\pi_j(1-\zeta_j)}{\ln(1/\zeta_j)}} \phi(\gamma^*) = \frac{\|\mathbf{z}\|_2^2}{\|\mathbf{z}\|_1} \phi(\gamma^*). \tag{26}$$

Compared with the GM method that provides equal resource partitioning for all tiers, the CS method provides less uniform resource allocation for different tiers. Furthermore, since $\|\mathbf{z}\|_1^2 \leq N\|\mathbf{z}\|_2^2$ for $\mathbf{z} \succeq 0$, we can conclude $R^{\mathrm{GM}} \leq R^{\mathrm{CS}}$. The equality holds when all the elements in $\mathbf{z}$ are equal. In other words, the equality condition is

$$\pi_j = \frac{\ln(1/\zeta_j)}{1-\zeta_j} a, \quad j = 1, ..., N \tag{27}$$

where a is a nonnegative constant. Equation (27) indicates that the GM based resource allocation can be regarded as a special case of the CS based method and that, this pricing strategy serves as a reference to evaluate fairness level in the CS based resource allocation process. In general, the closer to this pricing strategy, the fairer resource allocation will be. We notice that for all $\tilde{\lambda}_j = 0$, since $R^*_{\mathrm{w/o}}$ is the maximum objective value, the relationship between the three resource allocation methods is

$$R^{\mathrm{GM}} \leq R^{\mathrm{CS}} \leq R^*_{\mathrm{w/o}} \tag{28}$$

where both equalities hold when (27) holds. Besides, the GM based method and optimal allocation method achieve absolute fairness and absolute unfairness respectively, while the CS based method offers a trade-off between the two extremes.

VI. Numerical Results

Comparisons among three methods are shown in Fig. 2-4 respectively. To be specific, we consider 10 tiers of MTCDs with outage probability constraints, $\zeta_1, \ldots, \zeta_{10}$, ranging from 0.05 to 0.5 (the tier with larger number has a larger outage probability), set the minimum MTCD density requirements to be 95 percent of the MTCD density achieved by CS based method, i.e., $\tilde{\lambda}_j = 0.95\lambda_j^{\mathrm{CS}}$, and assume that $\phi(\gamma^*) = 1, \pi_j = 1, \forall j$. As can be seen from Fig. 2, while GM based method allocates the resources equally, the CS based method allocates less resources to the tiers with stricter outage constraints. The tallest yellow bar in Fig. 2 indicates that while satisfying minimum MTCD density requirements, the optimal method allocates all the residual resources to the tier with the least stringent outage constraint. Under our definition of fairness, the GM-based method provides the fairest allocation, the optimal resource allocation method is the most unfair one, and the CS based method achieves a degree of fairness between the two extremes. The differences in the focus on fairness lead to the differences in MTCD density and transmission capacity which are shown in Fig. 3 and Fig. 4 respectively. Compared with the optimal resource allocation and CS based method, the equal partitioning of GM based method results in larger (less) densities and per-tier capacities of the tiers with stricter (looser) outage constraints, i.e., tier 1 to tier 5 (tier 6 to tier 10), than those of the other two methods. On the other hand, given the same settings, the corresponding sum network capacities are $R^{\mathrm{GM}} = 0.5460$, $R^{\mathrm{CS}} = 0.5749$, $R^*_{\mathrm{w}} = 0.5822$ and $R^*_{\mathrm{w/o}} = 0.7213$ for $\tilde{\lambda}_j = 0, \ \forall j$, which is in line with the inequality (28).

VII. Conclusion

We proposed three network dimensioning and resource allocation schemes for the relay structure in MTC. First, SNCM and TRM with and without minimum transmission density requirements were considered and optimal solutions were obtained. However, both solutions involve unfair resource partitioning. Then, we proposed GM based resource allocation which achieves absolute fairness and CS based resource allocation that trades off some fairness for sum network capacity or total revenue. While the GM based resource allocation is not sensitive to the outage probability constraints or pricing

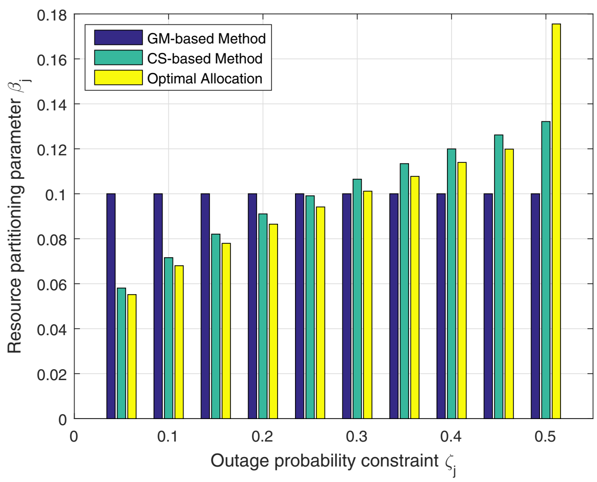

Fig. 2. Comparison of resource partitioning parameter.

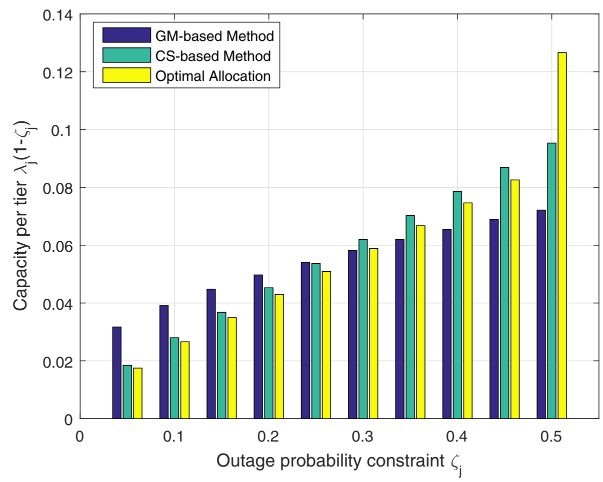

Fig. 4. Comparison of capacity per tier.

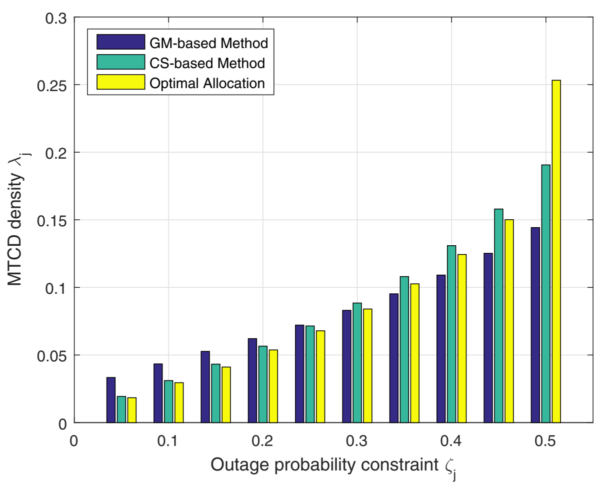

Fig. 3. Comparison of MTCD density.

strategy, the CS based method uses both parameters, ζ_j and π_j, in the resource allocation strategy. Even though both GM and CS based methods only achieve suboptimal sum network capacity or total revenue of the M2M network, they provide each tier with a certain level of fairness in sharing the radio resources. Network operators can choose among all methods discussed above to serve the needs of their M2M networks and maximize their income.

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