PAR-Constrained Training Signal Designs for MIMO OFDM Channel Estimation in the Presence of Frequency Offsets

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***Abstract*—Training signals for OFDM channel estimation should possess low PAR to avoid nonlinear distortions at the transmit amplifier and at the same time they should be robust against frequency offsets. In this letter, we show that the above two requirements of the training signals are conflicting. We propose two new training signal designs for MIMO OFDM frequency-selective channel estimation. Our proposed training signals achieve more robust channel estimation performance against frequency offsets while satisfying the PAR constraints compared to training signals designed to achieve a fixed low PAR but without any consideration for robustness to frequency offsets.**

***Index Terms*—Channel estimation, frequency offsets, MIMO, OFDM, PAR, pilot design, training signal design.**

I. Introduction

TRAINING signal design for OFDM channel estimation has attracted significant research attention recently (e.g., for SISO OFDM systems in [1]-[4], for MIMO OFDM systems in [5]-[8]). All training signal designs mentioned above assume no carrier frequency offset (CFO). In practice, CFO is unavoidable due to local oscillator mismatches. Recently, we presented in [9] training signal designs for MIMO OFDM channel estimation in the presence of CFO.

Another important practical issue which has not been incorporated in a unified and optimized way in all existing OFDM training signal designs is peak-to-average energy ratio (PAR) of the training signal. Although occasional nonlinear distortion of OFDM data signals can be tolerable [10]-[12], nonlinear distortion of the training signal should be avoided since the distorted training signal will yield larger estimation errors and affect all data detection performance regardless of data with or without nonlinear distortion. The linearity range of the transmit power amplifier is typically designed based on the PAR characteristics of the transmitted data signals. For an OFDM system with a large number of sub-carriers, the probability P of a data sample's PAR larger than a PAR threshold PAR_c can be approximated by $e^{-\text{PAR}_\text{c}}$ [12]. A reasonable design range of P is $[10^{-2},\ 10^{-3}]$ which corresponds to the range of PAR_c of $[6.6,\ 8.4]$ dB, regardless of the number of subcarriers (which is assumed to be sufficiently large). For example, if we allow 0.1% of OFDM data samples to exceed the amplifier linearity region, the transmit power amplifier should be designed such that no nonlinear distortion occurs for signals with PAR less than $\text{PAR}_\text{c} = 8.4$ dB. Then the PAR constraint of the training signal is PAR_c. As long as the training signal satisfies the PAR constraint, it will not experience nonlinear distortion and there is no need to minimize its PAR. Hence, a more appealing training design approach is to optimize the robustness of the estimation against CFO while satisfying the PAR constraint. This approach has not been considered in the literature and is pursued here.

In this letter, we study the PAR characteristics of existing training signals and find that the requirements on the training signals to possess low PAR and be robust against CFO are conflicting. We present two new training signal designs which are robust to CFO while satisfying the imposed PAR constraints, and which provide improved estimation and BER performance.

II. Signal Model and MSE-Optimality Conditions

Consider a MIMO OFDM system with K sub-carriers where training signals from N_Tx transmit antennas are transmitted over Q OFDM symbols. Since the same channel estimation procedure is performed at each receive antenna, we only need to consider one receive antenna in designing training signals. The channel impulse response (CIR) for each transmit-receive antenna pair (including all transmit/receive filtering effects) is assumed to have L taps and is quasi-static over Q OFDM symbols. Let $[c_{n,q}[0],\ \ldots,\ c_{n,q}[K-1]]^T$ be the pilot tones vector of the n-th transmit-antenna at the q-th symbol interval and $\{s_{n,q}[k] : k = -N_g, \ldots, K-1\}$ be the corresponding time-domain complex baseband training samples, including N_g $(\geq L-1)$ cyclic prefix (CP) samples where the superscript T denotes the transpose. Define $\boldsymbol{S}_n[q]$ as the training signal matrix of size $K \times L$ for the n-th transmit antenna at the q-th symbol interval whose elements are given by $[\boldsymbol{S}_n[q]]_{m,l} = s_{n,q}[m-l]$ for $m \in \{0, \ldots, K-1\}$ and $l \in \{0, \ldots, L-1\}$. Let $\boldsymbol{h}_n$ denote the length-L CIR vector corresponding to the n-th transmit antenna.

After cyclic prefix removal at the receiver, denote the received vector of length K at the q-th symbol interval by $\boldsymbol{r}_q$. Then, the received vector over the Q symbol intervals in the presence of a normalized (by the sub-carrier spacing) CFO v is

$$\boldsymbol{r} = \boldsymbol{W}(v)\ \boldsymbol{S}\boldsymbol{h} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{r} = [\boldsymbol{r}_0^T\ \boldsymbol{r}_1^T\ \ldots,\ \boldsymbol{r}_{Q-1}^T]^T$, $\boldsymbol{h} = [\boldsymbol{h}_0^T\ \boldsymbol{h}_1^T \ldots \boldsymbol{h}_{N_\text{Tx}-1}^T]^T$, the (q,n)-th sub-block of $\boldsymbol{S}$ is $\boldsymbol{S}_n[q]$, $\boldsymbol{W}(v) = \boldsymbol{W}_1(v) \otimes \boldsymbol{W}_0(v)$, $\boldsymbol{W}_1(v) = \text{diag}\{1, e^{\frac{j2\pi v(K+N_g)}{K}}, \ldots, e^{\frac{j2\pi v(Q-1)(K+N_g)}{K}}\}$, $\boldsymbol{W}_0(v) = \text{diag}\{1, e^{\frac{j2\pi v}{K}}, \ldots, e^{\frac{j2\pi(K-1)v}{K}}\}$,

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$$\mathrm{NMSE} = \frac{\mathrm{E}[\|\boldsymbol{h}-\hat{\boldsymbol{h}}\|^2]}{LN_{\mathrm{Tx}}} = \underbrace{\frac{\sigma_n^2 \ \mathrm{Tr}\left[(\boldsymbol{S}^H\boldsymbol{S})^{-1}\right]}{LN_{\mathrm{Tx}}}}_{\mathrm{NMSE}_0} + \underbrace{\frac{\mathrm{Tr}\left[(\boldsymbol{S}^H\boldsymbol{S})^{-1}\boldsymbol{S}^H\left(\boldsymbol{I}-\boldsymbol{W}(v)\right)\boldsymbol{S}\boldsymbol{C}_{\boldsymbol{h}}\boldsymbol{S}^H\left(\boldsymbol{I}-\boldsymbol{W}(v)\right)^H\boldsymbol{S}(\boldsymbol{S}^H\boldsymbol{S})^{-1}\right]}{LN_{\mathrm{Tx}}}}_{\Delta_{\mathrm{NMSE}}} \tag{2}$$

$\otimes$ is the Kronecker product, and $\boldsymbol{n}$ is a length-KQ vector of independent and identically-distributed circularly-symmetric complex Gaussian noise samples with zero-mean and variance of σ_n^2.

The normalized mean square error (NMSE) of the channel estimate $\hat{\boldsymbol{h}} = (\boldsymbol{S}^H\boldsymbol{S})^{-1}\boldsymbol{S}^H\boldsymbol{r}$ is given in (2), where the first term is the NMSE obtained without any CFO and the second term is the extra NMSE caused by the normalized CFO v.

Since using one OFDM training symbol is more robust to frequency offsets than using multiple symbols [9], we will consider one OFDM training symbol. Our pilot design will be based on frequency-division multiplexing (FDM) and code-division multiplexing in frequency-domain (CDM-F) pilot structures from [8] which are optimal in the absence of frequency offset. Let L_0 be the smallest integer satisfying $L_0 = K/M$ (M is a positive integer) and $L_0 \geq L$. The pilot tones at the k-th sub-carrier for the m-th antenna in the FDM and CDM structures are respectively given by

$$\begin{aligned}
\mathrm{FDM:}\quad & c_m[k] = \sum_{p=0}^{U-1}\sum_{l=0}^{L_0-1} b_m^{(l,p)}\ \delta[k - \frac{lK}{L_0} - i_{m,p}]; \qquad (3)\\
& \sum_{p=0}^{U-1}\sum_{l=0}^{L_0-1} |b_m^{(l,p)}|^2 = KE_{\mathrm{av}};\\
& i_{m,p} \in [0, \frac{K}{L_0}-1];\\
& i_{m_1,p_1} = i_{m_2,p_2} \text{ only if } (m_1 = m_2 \ \&\ p_1 = p_2)\\
\mathrm{CDM:}\quad & c_m[k] = \sum_{p=0}^{V-1}\sum_{l=0}^{L_0-1} b_m^{(l,p)}\ \delta[k - \frac{lK}{L_0} - i_p]; \qquad (4)\\
& b_m^{(l,p)} = b_0^{(l,p)}\ e^{-j2\pi pm/V};\\
& i_p \in [0, \frac{K}{L_0}-1];\ i_{p_1} = i_{p_2} \text{ only if } p_1 = p_2
\end{aligned}$$

where $\{b_0^{(l,p)}\}$ are constant modulus symbols, $1 \leq U \leq K/(N_{\mathrm{Tx}}L_0)$, and $N_{\mathrm{Tx}} \leq V \leq K/L_0$. These pilot structures are constructed based on pilot-combs where each pilot-comb consists of L_0 cyclically[1] equi-spaced, equal energy disjoint pilot tones (see Fig. 1). In the FDM structure, pilots on each antenna are composed of U pilot-combs which are disjoint from those on other antennas. In the CDM structure, all antennas share V pilot-combs and the pilots are designed to be code-orthogonal among all antennas. The total pilot energy per antenna, KE_{av}, is the same for all antennas of both structures.

In the presence of CFO, [9] derived the best training signals among those from [8] by minimizing the extra NMSE Δ_{NMSE}. For $K > N_{\mathrm{Tx}}L$, the best design (most robust to CFO) is of CDM-F type over all sub-carriers and is given by [9]

$$\begin{aligned}
&\{c_k[n]:\ k = 0,\ \ldots,\ N_{\mathrm{Tx}}-1\}\\
&= \{\sqrt{E_{\mathrm{av}}}\ e^{j\phi_m}\ e^{\frac{-j2\pi mnL}{K}}\ :\ m = 0,\ \ldots,\ N_{\mathrm{Tx}}-1\} \quad (5)
\end{aligned}$$

where $\{\phi_m\}$ are arbitrary phases. For $K = N_{\mathrm{Tx}}L_0$, the best training signals in the presence of CFO require an additional condition that for each transmit antenna k, the optimal pilot tone symbols $c_k[n - lN_{\mathrm{Tx}} - m]$ for different l are the same [9].

[1] modulo K

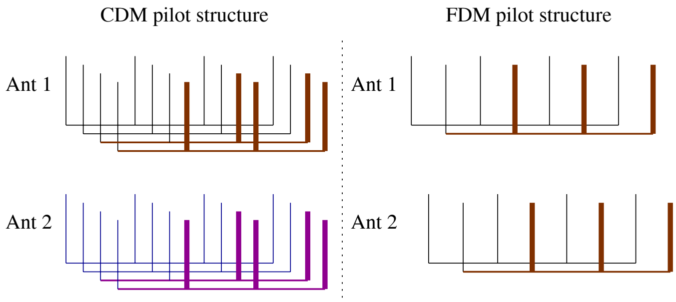

Fig. 1. An illustration of the proposed FD training design for a 16 subcarriers OFDM system with 2 transmit antennas in 4-tap channels. In the CDM structure, $V = 4$ and $\lambda = 5$. In the FDM structure, $V = 2$ and $\lambda = 3$. The pilots in thicker lines denote the modified pilots based on Newman's sequences.

III. PAR-Constrained Training Signal Designs

The instantaneous training signal energy can be related to the aperiodic autocorrelation $R(m)$ of the corresponding pilot tones $c[n]$ with the sub-carrier spacing of Δ_f as

$$\begin{aligned}
|s(t)|^2 &= \sum_{m=-K+1}^{K-1} R(m)e^{j2\pi m\Delta_f t}\\
&= R(0) + 2\Re\{\sum_{m=1}^{K-1} R(m)e^{j2\pi m\Delta_f t}\} \qquad (6)
\end{aligned}$$

where $R(m) = \sum_{n=0}^{K-1-m} c[n]c^*[n+m]$. This indicates that pilot tones with small aperiodic autocorrelation (i.e., small $R(m)$ for $m \neq 0$) give small PAR. However, the robustness of the pilots against CFO requires a high correlation property of the pilot tones (c.f. (5)). Due to these conflicting requirements, it is impossible to obtain a training signal possessing both the minimum PAR and the highest robustness against CFO.

In the absence of CFO, the use of low aperiodic autocorrelation sequences for the pilots of CDM and FDM structures will give low PAR training signals and hence there is no PAR difference between CDM and FDM structures. However, in the presence of CFO, the minimum PAR of the most robust optimal training signals from [9] obtained with FDM structure is smaller than that obtained with CDM(F) or FDM+CDM(F) structure. Due to space limitation, the detailed

PAR characteristics of the existing pilot designs in [8] and [9] are referred to [15]. In the following we propose two pilot designs which satisfy the PAR constraint (PAR $\leq$ PAR$_c$) while striving to maintain the robustness against CFO.

A. The Frequency-Domain Design

In the CDM structure, the time-domain signals of different antennas are just cyclic-shifted versions of one another and hence they have the same PAR. In the FDM structure, by using the same set of pilot symbols on the assigned sub-carriers of each antenna, we can maintain the same PAR for all antennas. Hence, we only need to design the pilots for the first transmit antenna. The pilots for the other antennas are constructed according to the structure (CDM or FDM) used. The algorithm starts with a training signal most robust to CFO as presented in [9], and then gradually replaces some pilot tones with a low correlation sequence in an attempt to lower the PAR. The number of pilot tones replaced is gradually increased until the PAR constraint is satisfied. This approach will be termed frequency-domain (FD) design. The FD design procedure is summarized below.

1) Calculate the PAR of the training signal most robust to frequency offsets [9].
2) If PAR $\leq$ PAR$_c$, the algorithm stops. Otherwise, set $\lambda = 2$. Replace the last λ pilot tones of the last pilot-comb with a length-λ low-correlation sequence and calculate the corresponding PAR.
3) If PAR $\leq$ PAR$_c$, the algorithm stops. Otherwise, increase λ by one. Replace the last λ pilot tones of the last $\lceil \lambda / L_0 \rceil$ pilot-combs with a length-λ low-correlation sequence and calculate the corresponding PAR where $\lceil x \rceil$ denotes the smallest integer larger than or equal to x. Repeat Step 3.

Note that a pilot tone sequence with low correlation has a very low PAR and hence the above algorithm is guaranteed to satisfy the PAR constraint. The algorithm can be repeated with the training signal of a different pilot allocation type (e.g., FDM over all or some sub-carriers, CDM over some sub-carriers) with the same pilot tone (for robustness against frequency offsets) as an initial training signal. Among the obtained training signals satisfying the PAR constraint, the one with the smallest NMSE can be chosen. Note that occasionally a training signal with a lower PAR may give a smaller NMSE. Hence, the above procedure can continue for some PARs lower than the PAR constraint and we can choose the training signal with the smallest NMSE among those obtained with PAR $\leq$ PAR$_c$.

There are several works on sequences with low periodic or aperiodic correlation properties. In our algorithm, a sequence with low aperiodic autocorrelation property is required. Examples of such sequences are Newman's sequence [13] and Schroeder's sequence [14]. The length-P Newman's sequence is given by

$$c[m] = e^{j\pi m^2/P}, \; m = 0, \; 1, \; \ldots, P-1. \qquad (7)$$

The length-P Schroeder's sequence is defined by $c[m] = c[0] \; e^{-j\pi(m+1)m/P}$, $m = 1, 2, \ldots, P-1$, where $c[0] = e^{j\phi}$ is arbitrary. Note that both sequences when used as pilot tones give the same PAR. In this paper, we adopt the Newman sequence, for simplicity.

In Fig. 1, an illustration of the proposed FD design is presented for a 16 sub-carriers OFDM system with 2 tansmit antennas in 4-tap channels. The pilot structures are composed of several pilot-combs, each having 4 cyclically equi-spaced equal energy pilot tones, all of which for the first antenna are originally set to 1 (except a scaling factor). In this example, the CDM structure uses $V = 4$ pilot-combs while the FDM structure has $U = 2$ pilot-combs. The pilots modified to yield lower PAR are depicted in thicker lines. In the CDM (FDM) example, $\lambda = 5$ ($\lambda = 3$) pilot tones of the first antenna are modified to be a Newman's sequence. The CDM (FDM) pilots for the second antenna are constructed according to the CDM (FDM) designs.

B. The Time-Domain Design

In the following, we will discuss an alternative approach termed the time-domain (TD) training design which uses single-carrier-type training signals with CP. Based on the training signal design from [9], we just need to consider the following training signals (in the time-domain):

$$s_{k,q}[n] = \sum_{i=0}^{d_{k,q}-1} A_{k,q,i}\delta[n - l_{k,q,i}], \; k = 0, \ldots, N_{\mathrm{Tx}} - 1 \qquad (8)$$

$$d_q \equiv \sum_{k=0}^{N_{\mathrm{Tx}}-1} d_{k,q} \leq \frac{K}{L}; \quad \sum_{q=0}^{Q-1} \sum_{i=0}^{d_{k,q}-1} |A_{k,q,i}|^2 = E_{\mathrm{av}}, \; \forall k \qquad (9)$$

where $d_{k,q}$ is the number of non-zero samples of the q-th signaling interval (excluding CP) for the k-th transmit antenna, and for each q, $\{l_{k,q,i} : \; \forall k, i\}$ are any permutation of $\{m_p\}$ with $m_{p+1} - m_p \geq L$, $K + m_0 - m_{d_q - 1} \geq L$, and $0 \leq m_p \leq K - 1$. Let V_l denote the l-th diagonal element of $\boldsymbol{V} \equiv (\boldsymbol{I} - \boldsymbol{W}(v))$ from (2). Then the training signal design from [9] that minimizes the extra NMSE also minimizes the following function:

$$\mathrm{Tr}[\boldsymbol{X_d}] = \sum_{k=0}^{N_{\mathrm{Tx}}-1} \sum_{m=0}^{L-1} \sigma_m^2 \left| \sum_{q=0}^{Q-1} \sum_{i=0}^{d_{k,q}-1} |A_{k,q,i}|^2 V_{m+l_{k,q,i}+Kq} \right|^2 . \qquad (10)$$

For small values of v, we have $V_l = 1 - e^{j2\pi k_l v/K} \simeq -j2\pi k_l v/K$ where $k_l = \lfloor l/K \rfloor N_g + l$ with $\lfloor x \rfloor$ denoting the largest integer not greater than x. Since V_l increases approximately linearly with l, we obtained in [9] the most robust pilot tones (against frequency offsets) as given in (5) whose time-domain signal is given by

$$\begin{aligned} &\{|s_k[n]| : \; k = 0, \; 1, \; \ldots, N_{\mathrm{Tx}} - 1\} \\ &= \{\sqrt{E_{\mathrm{av}}}\delta[n - mL] : m = 0, \; 1, \; \ldots, N_{\mathrm{Tx}} - 1\}. \end{aligned} \qquad (11)$$

Intuitively, while satisfying the optimality conditions in [8], equation (5) (hence (11)) simply allocates training signal energies to the leading samples (in time-domain) so that they are weighted by the smallest V_l values. In our TD design, we apply the same approach while satisfying PAR constraints. We start with the most robust training signal from (5) whose corresponding PAR is K. If the PAR constraint is less than

K, we modify the training signal from (5) in the time-domain as follows:

$$\{s_k[n]: \ k=0, \ \ldots, N_{\mathrm{Tx}}-1\} = \left\{\sum_{i=0}^{d-1} A_{m,i}\ \delta[n-mL^\dagger-i] : m=0, \ \ldots, N_{\mathrm{Tx}}-1\right\} \quad (12)$$

$$L^\dagger = \begin{cases} d+L, & (d+L)N_{\mathrm{Tx}} \le K \\ \lfloor K/N_{\mathrm{Tx}} \rfloor, & \text{else} \end{cases} \quad (13)$$

where d is the number of non-zero time-domain training samples for each antenna. The above definition[2] of $L^\dagger$ attempts to satisfy the conditions in [8] first, while the positions of d non-zero samples follow the robustness condition (allocating energies to leading samples).

Next, we consider the design of $\{A_{m,i}\}$. In the frequency-offset-robust design from [9], $\boldsymbol{S}^H\boldsymbol{S} = E_{\mathrm{av}}\boldsymbol{I}$ and $\boldsymbol{Y} = \boldsymbol{S}^H\boldsymbol{V}\boldsymbol{S}$ is a diagonal matrix. The TD design attempts to closely follow these diagonal conditions by suppressing off-diagonal elements of $\boldsymbol{S}^H\boldsymbol{S}$ and $\boldsymbol{Y}$ as follows. With the structure in (12), the off-diagonal elements of $\boldsymbol{S}^H\boldsymbol{S}$ are just the aperiodic autocorrelation and cross-correlation samples of $\{A_{m,i}\}$. Similarly, the off-diagonal elements of $\boldsymbol{Y}$ are the weighted (by V_l) aperiodic autocorrelation and crosscorrelation of $\{A_{m,i}\}$. By simply neglecting the weighting, the TD design finds $\{A_{m,i}\}$ with low aperiodic autocorrelation and crosscorrelation. If we set $s_{m+l}[n] = s_m[n-lL^\dagger]$, the aperiodic crosscorrelation becomes the same as aperiodic autocorrelation and we just need to find a low aperiodic autocorrelation sequence of length d for which we adopted the Newman's sequence.

Define $d_{\min} = \lceil E_{\mathrm{av}}/E_{\mathrm{peak}} \rceil$ where E_{peak} is the allowable peak sample energy defined by the PAR constraint [3]. Then, the TD design evaluates all training signals defined in (12) for $d = d_{\min}, d_{\min}+1,, \ \ldots \ , K$ and chooses the one with the smallest NMSE.

IV. Performance Results and Discussions

We consider an OFDM system with $K = 64$ sub-carriers in an uncorrelated 8-tap multipath Rayleigh fading channel with a 3 dB per tap decay factor. In this case, $L_0 = L = 8$ and the maximum number of transmit antennas that can be supported while guaranteeing channel identifiability is 8. In calculating PAR, $N_{\mathrm{up}} = 16$ is used. When $V = UN_{\mathrm{Tx}}$, the total number of pilot tones for all antennas is the same for both CDM and FDM structures. For $N_{\mathrm{Tx}} = 1$, CDM and FDM structures are the same.

In the training designs, the calculation of NMSE requires the knowledge of $\boldsymbol{C_h}$ and v. Since $\boldsymbol{C_h}$ can easily be obtained from channel measurement, we use the exact $\boldsymbol{C_h}$. On the other hand, v which represents the residual random CFO after CFO estimation and compensation will be typically quite small and can be set to an upper bound, say $v = 0.1$. In sub-section IV-A, which compares analytical channel estimation NMSE, we use the exact value of v in the designs. In sub-section IV-B, which presents simulation results, we use a design value of 0.1 for v in the designs. For a small v, we have $V_l \simeq -j2\pi k_l v/K$ in (10) and hence the use of a mismatched value of v will not affect the training designs since it just introduces a different scaling of the extra NMSE term and that scaling is the same for all training signals.

In the design process, the FD design would require a larger complexity than the TD design since several pilot structures need to be investigated in the FD design. However, both designs can be finished momentarily (in MATLAB). Furthermore, the training designs are performed offline and the obtained training signals are stored in the memory of the transceiver. Hence, there is no complexity burden for practical applications.

A. Channel Estimation NMSE Performance in the Presence of a Fixed CFO

In this section, we compare the analytical channel estimation NMSE in the presence of a fixed normalized CFO[4] among the training signals obtained from the FD design (based on CDM with $V = 8$), TD design and those with a fixed low PAR. The low PAR training signals are of CDM type where one of the antennas uses a Newman's sequence over all sub-carriers. Note that all of the considered training signals give the same optimal channel estimation performance in the absence of CFO. Due to the space limitation, the analytical NMSE performance plots are referred to [15]. Our proposed training signals achieve better performance than the training signals with a fixed low PAR. For any N_{Tx}, the NMSE differences between our proposed training signals and those with a fixed low PAR become larger at larger SNR and CFO.

Next, we evaluate the effect of different pilot structures (CDM and FDM pilot structures) on the FD design. The corresponding performance plots and detailed discussions are referred to [15]. The following observations are in order:

1) For $N_{\mathrm{Tx}} < K/L_0$, training structures with larger V or U give better NMSE with mild PAR constraints while those with smaller V or U are better with stringent PAR constraints.
2) For $N_{\mathrm{Tx}} < K/L_0$, when the same number of total pilot tones is used in the CDM and FDM structures, the CDM structure yields a slightly better NMSE.
3) For $N_{\mathrm{Tx}} = K/L_0$, the FDM structure gives a marginal NMSE advantage at mild PAR constraints.

B. Performance of Different Training Signals When Used for Both CFO and Channel Estimation

In this section, we investigate the performance of different training signals when used for joint CFO and channel estimation. In the simulation, we set the normalized CFO to 0.3. The receiver performs CFO estimation and compensation first, and then channel estimation and maximum likelihood detection. Hence, the channel estimation is affected by the random residual CFO. We use a spatial multiplexing scheme where data streams from different antennas are independent BPSK symbols. For simplicity, we assume that a packet contains a preamble of one OFDM symbol duration followed by one

[2]Note that we used $L^\dagger = L$ in [15] but we modified it here to achieve a slight performance improvement.

[3]In our design, the maximum PAR allowed may be set smaller than the PAR constraint to allow PAR increase due to filtering.

[4]It corresponds to the residual normalized CFO after CFO estimation and compensation.

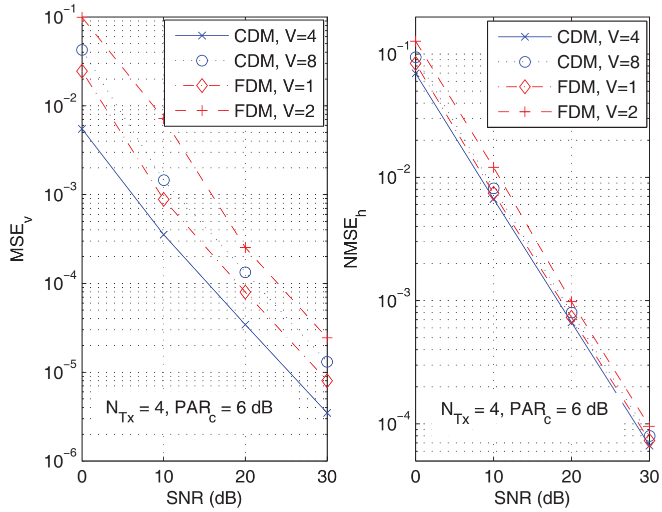

Fig. 2. Estimation performance comparison among different training structures of the FD design with $\text{PAR}_c = 6$ dB and $N_{\text{Tx}} = 4$.

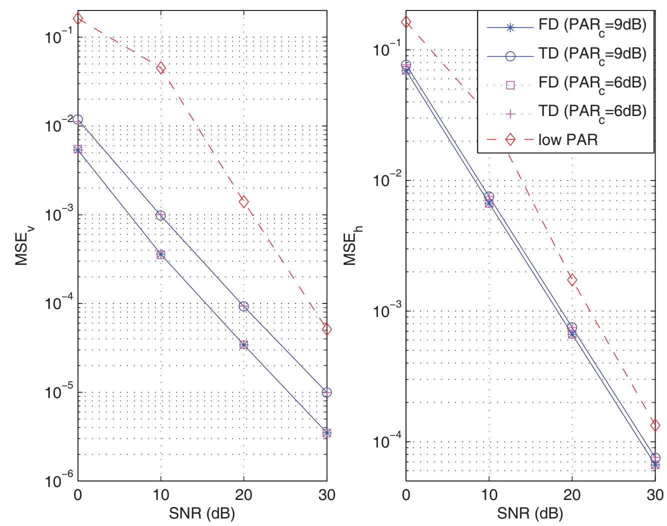

Fig. 4. Estimation performance comparison between the low PAR design and the proposed designs in an OFDM system with $N_{\text{Tx}} = 4$ transmit antennas (FD design is based on the CDM structure with $V = 4$).

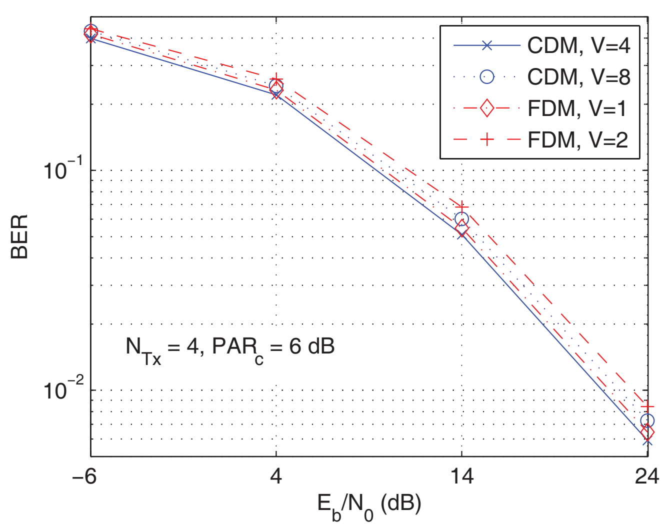

Fig. 3. BER performance comparison among different training structures of the FD design with $\text{PAR}_c = 6$ dB and $N_{\text{Tx}} = 4$.

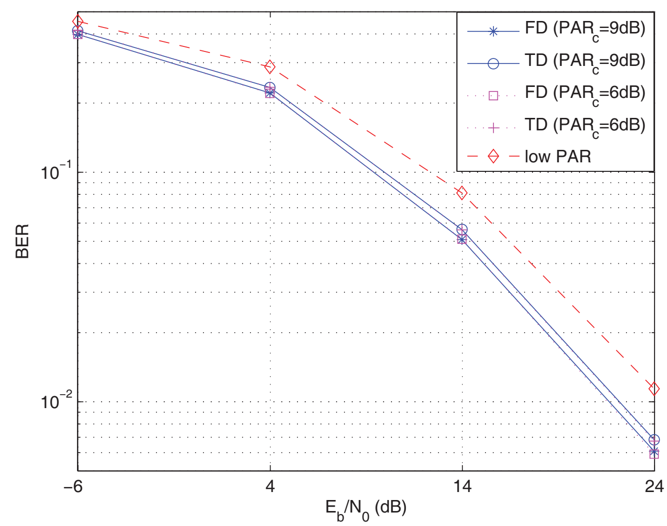

Fig. 5. BER performance comparison between the low PAR design and the proposed designs in an OFDM system with $N_{\text{Tx}} = 4$ transmit antennas (FD design is based on the CDM structure with $V = 4$).

OFDM data symbol simultaneously from each antenna. The SNRs of the preamble and data symbols are assumed the same, and hence $E_b/N_0 = \text{SNR}/N_{\text{Tx}}$.

In Figs. 2 and 3, we show the simulation results for the effects of different training structures on the FD design in terms of CFO and channel estimation, and BER performance, respectively, for $N_{\text{Tx}} = 4$ and $\text{PAR}_c = 6$ dB. The results follow the same trend as the channel estimation NMSE results in the presence of a fixed CFO, and hence the same remarks apply here. For $N_{\text{Tx}} = 1$ (the corresponding plots are omitted due to the space limitation), BER differences due to the different training structures of the FD design are negligible. For a larger N_{Tx}, the FD design based on the CDM structure with smaller V gives a slight performance advantage.

Next, we present the simulation results of the proposed designs with $\text{PAR}_c = 6$ and 9 dB and the low PAR design in terms of CFO and channel estimation in Fig. 4 and BER performance in Fig. 5. There are no noticeable performance differences for the PAR constraints of 6 and 9 dB. Both proposed FD and TD designs outperform the low PAR design in all estimation MSEs and BER performance. Our proposed designs yield over 2 dB SNR advantage at the (uncoded) BER of 10^{-2} and $N_{\text{Tx}} = 4$ over the low PAR design.

V. Conclusions

The requirements on the MIMO OFDM training signals to possess low PAR and be robust against frequency offsets are conflicting. We presented two training signal designs (frequency-domain (FD) and time-domain (TD)) which are robust to frequency offsets while satisfying the PAR constraints. Both designs give better channel estimation and BER performance than using training signals with a fixed low PAR. The FD and TD designs have almost the same performance. The FD design has better flexibility (applicable with different pilot structures, compatible with OFDM systems, applicable to pilot-data multiplexed schemes, etc.) and hence is a better choice for OFDM systems while the TD design would be suitable for single-carrier MIMO systems.

REFERENCES

[1] S. Adireddy, L. Tong, and H. Viswanathan, "Optimal placement of training for frequency-selective block-fading channels," *IEEE Trans. Inform. Theory,* vol. 48, no. 8, pp. 2338–2353, Aug. 2002.

[2] S. Ohno and G. B. Giannakis, "Optimal training and redundant precoding for block transmissions with application to wireless OFDM," *IEEE Trans. Commun.,* vol. 50, no. 12, pp. 2113–2123, Dec. 2002.

[3] R. Negi and J. Cioffi, "Pilot tone selection for channel estimation in a mobile OFDM system," *IEEE Trans. Consumer Electron.*, vol. 44, no. 3, pp. 1122–1128, Aug. 1998.

[4] J. H. Manton, "Optimal training sequences and pilot tones for OFDM systems," *IEEE Commun. Lett.*, pp. 151–153, Apr. 2001.

[5] I. Barhumi, G. Leus, and M. Moonen, "Optimal training design for MIMO OFDM systems in mobile wireless channels," *IEEE Trans. Signal Processing,* pp. 1615–1624, June 2003.

[6] Y. Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," *IEEE Trans. Wireless Commun.*, pp. 67–75, Jan. 2002.

[7] X. Ma, L. Yang, and G. B. Giannakis, "Optimal training for MIMO frequency-selective fading channels," *IEEE Trans. Wireless Commun.,* vol. 4, no. 2, pp. 453–466, Mar. 2005.

[8] H. Minn and N. Al-Dhahir, "Optimal training signals for MIMO OFDM channel estimation," *IEEE Trans. Wireless Commun.,* vol. 5, no. 5, pp. 1158–1168, May 2006.

[9] H. Minn, N. Al-Dhahir, and Y. Li, "Optimal training signals for MIMO OFDM channel estimation in the presence of frequency offsets and phase noise," *IEEE Trans. Commun.,* vol. 54, no. 10, pp. 1754–1759, Oct. 2006.

[10] X. Li and L. J. Cimini, "Effects of clipping and filtering on the performance of OFDM," *IEEE Commun. Lett.,* vol. 2, no. 5, pp. 131–133, May 1998.

[11] J. Armstrong, "Peak-to-average power reduction for OFDM by repeated clipping and frequency domain filtering," *Electron. Lett.,* vol. 28, no. 5, pp. 246–247, 28 Feb. 2002.

[12] R. Dinis and A. Gusmao, "A class of nonlinear signal-processing schemes for bandwidth-efficient OFDM transmission with low envelope fluctuation," *IEEE Trans. Commun.,* vol. 52, no. 11, pp. 2009–2018, Nov. 2004.

[13] D. J. Newman, "An L1 extremal problem for polynomials," *Proc. American Math. Soc.*, vol. 16, pp. 1287–1290, Dec. 1965.

[14] M. R. Schroeder, "Synthesis of low-peak-factor signals and binary sequences with low autocorrelation," *IEEE Trans. Inform. Theory,* pp. 85–89, Jan. 1970.

[15] H. Minn and N. Al-Dhahir, "PAR-constrained training signal designs for MIMO OFDM channel estimation in the presence of frequency offsets," in *Proc. IEEE VTC05* (Fall), vol. 3, pp. 1465–1470, Sept. 2005.