Asymptotically Optimal Power Allocation for Massive MIMO Uplink

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***Abstract*—This paper considers massive MIMO uplink using maximum ratio combining (MRC) and zero forcing (ZF) receivers with perfect and imperfect channel state information (CSI). We develop pilot and data power allocation strategies among users under peak power constraint and prove their asymptotic optimality in maximizing the sum-rate lower bound. Then we investigate the optimal amount of pilot symbols. Analytical and simulation results illustrate the performance gain of the proposed power allocation, and the impact of pilot overhead on sum-rate.**

***Index Terms*—Massive MIMO, Power allocation, Sum rate**

I. Introduction

Massive MIMO is a potential technology for 5G systems [1], [2] due to its advantages in enhancing spectrum and energy efficiency [3]–[6]. To overcome complexity issue of massive MIMO, low complexity linear detectors based on MRC, ZF or minimum mean-square error (MMSE) [4], [6]–[10] have been developed. When practical conditions such as imperfect CSI are imposed, the channel orthogonality among the users is deteriorated and multiuser interferences arise. In the existing massive MIMO work, all the users transmit with the same power. However, in the presence of the above multiuser interferences, equal power allocation among the users is not necessarily the best strategy. This motivates us to explore power allocation strategies for massive MIMO uplink with ZF or MRC receiver by exploiting the knowledge of the large-scale fading coefficients of the users.

The problem of sum-rate maximization is a challenging NP-hard problem [11]–[13]. In this paper, based on the sum-rate lower bound, we propose an alternative optimization strategy for power allocation, namely maximizing the product of signal to interference plus noise power ratios (SINRs) of the users. We will show that the proposed power allocation strategy is asymptotically (as M becomes large) optimal for maximizing the sum-rate, and it provides substantial sum-rate improvement over the existing power allocation strategy.

Another important aspect we investigate is the impact of CSI overhead in massive MIMO systems. Increasing the number of pilot symbols gives two conflicting effects on sum-rate, namely, lower multiuser interferences due to improved CSI accuracy and less time slots for data transmission. We will develop guidelines on the pilot overhead to yield optimal or approximately optimal sum-rate.

Notation: Vectors (matrices) are denoted by bold face small (big) letters. The superscripts T and H stand for the transpose and conjugate transpose. $\boldsymbol{I}_K$ is the $K \times K$ identity matrix. $\preccurlyeq$ and δ_{ki} denote element-wise inequality and the Kronecker delta, respectively. $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C})$ means $\mathbf{n}$ is the zero-mean complex Gaussian vector with covariance matrix $\boldsymbol{C}$; for real-valued Gaussian, $\mathcal{CN}$ is replaced by $\mathcal{N}$.

II. System Model

We consider an uplink multi-user MIMO system where K independent single-antenna users transmit data simultaneously on the same frequency band to a single BS with M antenna elements. Channel gains are quasi-static within a frame, and channels of different users and antennas are independent. The overall $M \times K$ channel matrix is $\mathbf{G} = \mathbf{H}\mathbf{D}^{1/2}$ where $\mathbf{H}$ is an $M \times K$ matrix of i.i.d. $\mathcal{CN}(0, 1)$ elements and $\mathbf{D}$ is a diagonal matrix with the kth diagonal element β_k relating to the large-scale fading component for user k. The (m, k)th elements of $\mathbf{H}$ and $\mathbf{G}$, denoted by h_{mk} and g_{mk}, represent the small-scale fading component and the overall gain of the channel between user k and BS antenna m, respectively. Thus, we have $g_{mk} = h_{mk}\sqrt{\beta_k}$. Same as in [6], we assume $\mathbf{D}$ is known at BS; this is well justified since $\{\beta_k\}$ change slowly and hence can be estimated reliably.

The data signal vector received at BS is

$$\mathbf{y} = \mathbf{G}\mathbf{P}^{1/2}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ is the noise vector, $\mathbf{P} \triangleq \mathrm{diag}\{p_1, \cdots, p_K\}$ with p_k denoting the transmit data power of user k, and $\mathbf{x} \triangleq [x_1, \cdots, x_K]^T$ with $\{x_k\}$ being i.i.d. data with $\mathbb{E}\left\{|x_k|^2\right\} = 1$. Thus, p_k equals data Tx SNR of user k. The ZF or MRC receiver computes

$$\mathbf{r} = \tilde{\mathbf{A}}^H\mathbf{y} \tag{2}$$

where $\tilde{\mathbf{A}}$ for the perfect CSI case is denoted by $\mathbf{A}$ and for the imperfect CSI case by $\hat{\mathbf{A}}$. $\mathbf{A}$ is given by $\mathbf{G}$ for MRC and $\mathbf{G}\left(\mathbf{G}^H\mathbf{G}\right)^{-1}$ for ZF. $\hat{\mathbf{A}}$ is obtained by substituting $\mathbf{G}$ with its estimate $\hat{\mathbf{G}}$ in $\mathbf{A}$. For channel estimation, each user transmits τ pilot symbols where $\tau \geq K$. Let $\mathbf{S}$ denote the pilot matrix whose kth row is the pilot sequence of user k with pilot power q_k. We use orthogonal pilot sequences (i.e., $\mathbf{S}\mathbf{S}^H = \tau\mathbf{Q}$), where $\mathbf{Q} \triangleq \mathrm{diag}\{q_1, \cdots, q_K\}$. With the MMSE channel estimation, the error variance of each element of kth column of $\hat{\mathbf{G}}$ is $\frac{\beta_k}{\tau q_k \beta_k + 1}$. The kth element of $\mathbf{r}$, r_k, is the decision variable for detecting data of user k.

With the same approach of [6], we can find the following lower bounds $\{\underline{R}\}$ for the achievable rate of MRC and ZF receiver with arbitrary power allocation:

$$\underline{R}^{\mathrm{mrc}}_{\mathrm{P},k} = \log_2\left(1 + \mathrm{SINR}^{\mathrm{mrc}}_{\mathrm{P},k}\right), \tag{3}$$

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$$\mathrm{SINR}^{\mathrm{mrc}}_{\mathrm{P},k} \triangleq \frac{p_k\beta_k(M-1)}{\sum_{i=1,i\neq k}^{K} p_i\beta_i + 1} \tag{4}$$

$$\underline{R}^{\mathrm{mrc}}_{\mathrm{IP},k} = (1-\lambda)\log_2\left(1+\mathrm{SINR}^{\mathrm{mrc}}_{\mathrm{IP},k}\right), \tag{5}$$

$$\mathrm{SINR}^{\mathrm{mrc}}_{\mathrm{IP},k} \triangleq \frac{\tau p_k q_k \beta_k^2(M-1)}{(\tau q_k\beta_k+1)\left(1+\sum_{i=1,i\neq k}^{K} p_i\beta_i\right)+p_k\beta_k} \tag{6}$$

$$\underline{R}^{\mathrm{zf}}_{\mathrm{P},k} = \log_2\left(1+\mathrm{SINR}^{\mathrm{zf}}_{\mathrm{P},k}\right) \tag{7}$$

$$\mathrm{SINR}^{\mathrm{zf}}_{\mathrm{P},k} \triangleq p_k\beta_k(M-K) \tag{8}$$

$$\underline{R}^{\mathrm{zf}}_{\mathrm{IP},k} = (1-\lambda)\log_2\left(1+\mathrm{SINR}^{\mathrm{zf}}_{\mathrm{IP},k}\right) \tag{9}$$

$$\mathrm{SINR}^{\mathrm{zf}}_{\mathrm{IP},k} \triangleq \frac{\tau p_k q_k \beta_k^2(M-K)}{(\tau q_k\beta_k+1)\left(1+\sum_{i=1}^{K}\frac{p_i\beta_i}{\tau q_i\beta_i+1}\right)} \tag{10}$$

where $\lambda \triangleq \frac{\tau}{T}$ with T being the total number of symbols in a transmission frame, and the subscripts P and IP refer to perfect CSI and imperfect CSI, respectively.

III. Power Allocation

The lower bounds in (3), (5), (7), and (9) serve as our metrics for power allocation among users. Similar lower bounds are also used in other existing works (e.g., [6]).

For the prefect CSI, we maximize sum-rate of all users as

$$\begin{array}{ll}\max & \sum_{k=1}^{K}\log_2(1+\mathrm{SINR}_{\mathrm{P},k})\\ \text{s.t.} & \mathbf{0}\preccurlyeq\mathbf{p}\preccurlyeq\mathbf{p}_{\max}\end{array} \tag{11}$$

where $\mathrm{SINR}_{\mathrm{P},k}$ is given in (4) for MRC and (8) for ZF. Here, $\mathbf{p}\triangleq[p_1,\cdots,p_K]^T$ and $\mathbf{p}_{\max}\triangleq[p_{1,\max},\cdots,p_{K,\max}]^T$ while p_k and $p_{k,\max}$ represent actual transmit data power for user k and its maximum limit. This peak power constraint is relevant for practical systems because transmit powers are constrained by peak power of transmit amplifiers.

Theorem 1. For ZF receiver with perfect CSI, all users transmit full power, i.e. $p_k = p_{k,\max}$, is optimum in (11).

Proof: By considering (7)-(8), the proof is obvious. □

For the imperfect CSI case, pilot powers become part of the optimization parameters. With the same reasoning, we consider peak power constraint for both data and pilot. With $\mathrm{SINR}_{\mathrm{IP},k}$ given in (6) for MRC and (10) for ZF receiver, the optimization problem becomes

$$\begin{array}{ll}\max & \sum_{k=1}^{K}\log_2(1+\mathrm{SINR}_{\mathrm{IP},k})\\ \text{s.t.} & 0\preccurlyeq\mathbf{p}\preccurlyeq\mathbf{p}_{\max},\ 0\preccurlyeq\mathbf{q}\preccurlyeq\mathbf{q}_{\max}\end{array} \tag{12}$$

where $\mathbf{q}\triangleq[q_1,\cdots,q_K]^T$ and $\mathbf{q}_{\max}\triangleq[q_{1,\max},\cdots,q_{K,\max}]^T$ while q_k and $q_{k,\max}$ denote actual transmit pilot power of user k and its maximum limit.

Theorem 2. Maximum power for pilot is optimum in (12):

$$\mathbf{q}_{\mathrm{opt}} = \mathbf{q}_{\max}. \tag{13}$$

Proof: From (6) and (10) by dividing nominator and denominator by$\{q_k\}$, we can see when $\{q_k\}$ increase, SINRs of all users increase. Thus, using maximum allowable pilot power is optimal to maximize the sum-rate in (12). □

By exploiting the result of Theorem 2, the two optimization problems of (11) and (12) can be combined into one as:

$$\begin{array}{ll}\max & \sum_{k=1}^{K}\log_2(\frac{1}{\alpha}+\eta_k(\mathbf{p}))\\ \text{s.t.} & 0\preccurlyeq\mathbf{p}\preccurlyeq\mathbf{p}_{\max}\end{array} \tag{14}$$

where

$$\alpha \triangleq \begin{cases} M-1 & \text{MRC with prefect CSI}\\ M-1 & \text{MRC with imperfect CSI}\\ M-K & \text{ZF with imperfect CSI}\end{cases} \tag{15}$$

and $\eta_k(\mathbf{p})$ for these three interested scenarios are defined as

$$\eta^{\mathrm{mrc}}_{\mathrm{P},k} \triangleq \frac{p_k\beta_k}{\sum_{i=1,i\neq k}^{K}p_i\beta_i+1}, \tag{16}$$

$$\eta^{\mathrm{mrc}}_{\mathrm{IP},k} \triangleq \frac{\tau p_k q_{k,\max}\beta_k^2}{(\tau q_{k,\max}\beta_k+1)\left(1+\sum_{i=1,i\neq k}^{K}p_i\beta_i\right)+p_k\beta_k}, \tag{17}$$

$$\eta^{\mathrm{zf}}_{\mathrm{IP},k} \triangleq \frac{\tau p_k q_{k,\max}\beta_k^2}{(\tau q_{k,\max}\beta_k+1)\left(1+\sum_{i=1}^{K}\frac{p_i\beta_i}{\tau q_{i,\max}\beta_i+1}\right)}. \tag{18}$$

The optimization problem of (14) is equivalent to

$$\max\prod_{k=1}^{K}\left(\frac{1}{\alpha}+\eta_k(\mathbf{p})\right)\quad \text{s.t.}0\preccurlyeq\mathbf{p}\preccurlyeq\mathbf{p}_{\max} \tag{19}$$

The problem (19) is still challenging. Thus, we consider

$$\begin{array}{ll}\max & \prod_{k=1}^{K}\eta_k(\mathbf{p})\ \text{ or }\ \min\prod_{k=1}^{K}\eta_k^{-1}(\mathbf{p})\\ \text{s.t.} & 0\prec\mathbf{p}\preccurlyeq\mathbf{p}_{\max}\end{array} \tag{20}$$

and we show its solution with growth of M is asymptotically optimal for the optimization problem of (19). The optimization in (20) can be solved by geometric programming [14] which can be converted to a convex optimization problem by change of variable. The following theorem builds a relationship between the optimization problems of (20) and (19).

Theorem 3. The optimum solution of the optimization problem in (20) is asymptotically (with respect to increasing the number of BS antennas) the optimal solution of (19).

Proof: We denote the maximum of (20) and (19) as $l^\star$ and l_{opt}, respectively and define $u_k\triangleq\max_{\mathbf{p}}\{\eta_k(\mathbf{p})\}$ with $0\preccurlyeq\mathbf{p}\preccurlyeq\mathbf{p}_{\max}$. Then we have

$$u^{\mathrm{mrc}}_{\mathrm{P},k} = p_{k,\max}\beta_k, \tag{21}$$

$$u^{\mathrm{mrc}}_{\mathrm{IP},k} = u^{\mathrm{zf}}_{\mathrm{IP},k} = \frac{\tau p_{k,\max}q_{k,\max}\beta_k^2}{\tau q_{k,\max}\beta_k+p_{k,\max}\beta_k+1}. \tag{22}$$

Without loss of generality, we assume $u_1\geq u_2\geq\cdots\geq u_K$. Then, for any i different indexes $j_1,j_2,\cdots,j_i\in[1,\cdots,K]$,

$$\eta_{j_1}(\mathbf{p})...\eta_{j_i}(\mathbf{p})\leq u_1u_2\cdots u_i. \tag{23}$$

Obviously from (19) and (20), $l^\star$ is a lower bound for l_{opt}. By expanding the objective function of (19) and exploiting (23), we can bound l_{opt} from upper as

$$l^\star\leq l_{\mathrm{opt}}\leq l^\star+f(\alpha) \tag{24}$$

where α is given in (15) and $f(\alpha)$ is defined as

$$f(\alpha)\triangleq\frac{1}{\alpha^K}+\sum_{i=1}^{K-1}\frac{1}{\alpha^{K-i}}\binom{K}{i}\prod_{j=1}^{i}u_j. \tag{25}$$

As M increases, so does α, and $f(\alpha)$ approaches zero. Hence, asymptotically $l_{\rm opt} = l^\star$. □

Theorem 4. For ZF receiver with imperfect CSI, if $q_{i,\max} \geq p_{i,\max}$ $\forall i$, then the solution to the optimization problem of (20) is that all users transmit full power, i.e., $\mathbf{p}_{\rm opt} = \mathbf{p}_{\max}$.

Proof: For ZF receiver with imperfect CSI, $\eta_k(\mathbf{p})$ is given by $\eta^{\rm zf}_{{\rm IP},k}$ in (18), so (20) becomes:

$$\begin{array}{ll} \min & \prod_{k=1}^{K} \frac{1+\sum_{i=1}^{K} a_i p_i}{\kappa_k p_k} = \frac{\left(1+\sum_{i=1}^{K} a_i p_i\right)^K}{\kappa_1 \cdots \kappa_K p_1 \cdots p_K} \\ \text{s.t.} & 0 < p_k \leq p_{k,\max};\ k = 1, \cdots, K \end{array} \quad (26)$$

where $a_i \triangleq \frac{\beta_i}{\tau q_{i,\max}\beta_i + 1}$ and $\kappa_i \triangleq \frac{q_{i,\max}\beta_i^2}{\tau q_{i,\max}\beta_i+1}$. Next, we consider log of the objective function and constraints, and by change of variable $x_i \triangleq \ln(p_i)$ and ignoring the constant terms $\kappa_1, \cdots, \kappa_K$, (26) becomes

$$\min\ f_o(\mathbf{x}) \ \text{ s.t. } x_k - b_k \leq 0, \quad k = 1, \cdots, K \quad (27)$$

where $\mathbf{x} \triangleq [x_1, \cdots, x_K]^T$, $f_o(\mathbf{x}) \triangleq K \ln\left(1 + \sum_{i=1}^{K} a_i e^{x_i}\right) - \sum_{i=1}^{K} x_i$, and $b_k \triangleq \ln(p_{k,\max})$. f_o is convex [15] and hence (27) is a convex optimization problem. The Lagrangian of (27) is

$$L(\mathbf{x}, \nu) = f_o(\mathbf{x}) + \sum_{i=1}^{K} \nu_i (x_i - b_i) \quad (28)$$

where $\{\nu_i\}_{i=1}^{K}$ are Lagrange multipliers, and $\nu \triangleq [\nu_1, \cdots, \nu_K]^T$. If $(\mathbf{x}^\star, \nu^\star)$ satisfy KKT conditions, then they are optimal [15]. The KKT conditions for (27) are given by $\nu_i^\star \geq 0$, $x_i^\star - b_i \leq 0$, $\nu_i^\star(x_i^\star - b_i) = 0, \forall i$, and $\nabla L(\mathbf{x}^\star, \nu^\star) = \mathbf{0}$, where ∇ denotes the gradient operator, $\mathbf{x}^\star \triangleq [x_1^\star, \cdots, x_K^\star]^T$ and $\nu^\star \triangleq [\nu_1^\star, \cdots, \nu_K^\star]^T$. The kth element of $\nabla L(\mathbf{x}, \nu)$ is

$$[\nabla L(\mathbf{x}, \nu)]_k = \frac{\partial L(\mathbf{x}, \nu)}{\partial x_k} = \frac{K a_k e^{x_k}}{1 + \sum_{i=1}^{K} a_i e^{x_i}} - 1 + \nu_k. \quad (29)$$

If we choose $x_k^\star \triangleq b_k$, $\forall k$ and $\nu_k^\star \triangleq 1 - \frac{K a_k e^{x_k^\star}}{1+\sum_{i=1}^{K} a_i e^{x_i^\star}}$, then obviously $x_k^\star$ and $\nu_k^\star$ satisfy the last three KKT conditions. Next, we will show $\nu_k^\star > 0$. We have $K a_k e^{x_k^\star} = \frac{K \beta_k p_{k,\max}}{\tau q_{k,\max}\beta_k + 1}$. With $\tau \geq K$ for orthogonal pilots and $p_{k,\max} \leq q_{k,\max}$, we arrive at $K a_k e^{x_k^\star} < 1$. Then $\frac{K a_k e^{x_k^\star}}{1+\sum_{i=1}^{K} a_i e^{x_i^\star}} < 1$ and $\nu_k^\star > 0$. In summary, $(\mathbf{x}^\star, \nu^\star)$ satisfy all of the KKT conditions and hence they are optimal. □

Remark 1. If the interference level is negligible compared to the noise level, i.e. $\sum_{i=1, i\neq k}^{K} p_i \beta_i \ll 1$ for (16) and (17), and $\sum_{i=1, i\neq k}^{K} \frac{p_i \beta_i}{\tau q_{i,\max}\beta_i} \ll 1$ for (18), then the maximum power allocation for each user is approximately optimal.

IV. Effect of Number of Pilot Symbols

Here, we explore the effect of the number of pilot symbols τ on the sum-rate of MRC and ZF receiver with imperfect CSI. Choosing the optimum value of τ is an optimization problem which may also depend on the power allocation strategy. For MRC receiver with fixed power allocation for users and $\underline{R}^{\rm mrc}_{{\rm IP},k}$ given in (5), we have the following optimization problem

$$\max \sum_{k=1}^{K} \underline{R}^{\rm mrc}_{{\rm IP},k} \ \text{ s.t. } \tau \in \{K, K+1, \cdots, T\} \quad (30)$$

For finding the optimum τ in (30), first we relax τ to be in continuous range $[K, T]$ and find an optimal value $\tilde{\tau}$. Then we obtain the optimal integer τ^* from two adjacent integer values of $\tilde{\tau}$. We can see $\frac{\partial^2}{\partial \tau^2} \underline{R}^{\rm mrc}_{{\rm IP},k} < 0$, and hence the objective function in (30) is concave and τ^* can be found numerically.

For the proposed power allocation strategy in (20), we find the sum-rate for each integer value τ and then we pick the τ that maximizes the sum-rate.

Remark 2. At low SINR, we can approximate (6) and (10) as ${\rm SINR}^{\rm mrc}_{{\rm IP},k} \approx c_k^{\rm mrc}\tau$ and ${\rm SINR}^{\rm zf}_{{\rm IP},k} \approx c_k^{\rm zf}\tau$ where $c_k^{\rm mrc} = p_k q_k \beta_k^2 (M-1)$ and $c_k^{\rm zf} = p_k q_k \beta_k^2 (M-K)$. Then using $\log_2(1+{\rm SINR}) \approx {\rm SINR}/\ln(2)$, we can approximate sum-rate as $\sum_{k=1}^{K} \underline{R}_{{\rm IP},k} \approx (1 - \frac{\tau}{T}) \sum_{k=1}^{K} c_k \tau / \ln(2)$ which yields the optimum τ at half of the frame length, i.e., $\tau^* = T/2$, for both MRC and ZF receivers at low SINR.

Remark 3. If we fix data power but increase pilot power to a high value, then (6) and (10) imply that τ has little effect on ${\rm SINR}_{{\rm IP},k}$ for MRC and ZF receivers. Hence, the optimum value for τ that maximizes the rate in (5) and (9) is its minimum, i.e., $\tau^* = K$. Suppose both pilot and data powers of users are high. Then for MRC receiver the above result still holds. For ZF receiver, (10) reduces to ${\rm SINR}^{\rm zf}_{{\rm IP},k} \approx \frac{\zeta_k \tau}{\tau + K}$ where $\zeta_k \triangleq p_k \beta_k (M - K)$, and (9) can be approximated as $\tilde{\underline{R}}^{\rm zf}_{{\rm IP},k} \triangleq \left(1 - \frac{\tau}{T}\right) \log_2\left(\frac{\zeta_k \tau}{\tau + K}\right)$. Since ζ_k is large, $\frac{\partial}{\partial \tau} \tilde{\underline{R}}^{\rm zf}_{{\rm IP},k} = \frac{K(T-\tau)}{T\tau(K+\tau)} - \frac{1}{T} \ln\left(\frac{\zeta_k \tau}{\tau+K}\right)$ is negative for $\tau \in [K, T]$. So $\tilde{\underline{R}}^{\rm zf}_{{\rm IP},k}$ is a decreasing function of τ in that range, and $\tau^* = K$.

V. Simulation Results and Discussions

In the simulation, $K = 10$ users are uniformly located in the disk around BS and the pdf of d_k, the distance of user k to BS with $M = 100$ antenna, is given by $f_{d_k}(r) = \frac{2r}{d_m^2 - d_0^2}$ with $d_0 = 100 \leq r \leq d_m = 1000$. Small-scale channel fadings of users are i.i.d. Rayleigh distributed. $\{\beta_k\}$ are independently generated by $\beta_k = z_k / (\frac{d_k}{d_0})^\nu$ where the path loss exponent is $\nu = 3.8$ and z_k represents lognormal shadowing with $10\log_{10}(z_k) \sim \mathcal{N}\left(0, \sigma^2_{\rm shadow,dB}\right)$ and $\sigma_{\rm shadow,dB}$ =8. The results are obtained by averaging over 1000 realizations of $\{\beta_k\}$. For simulations with fixed $\{\beta_k\}$, we set $\beta_k = \beta_1 e^{-a(k-1)}$ with $\beta_1 = 0.1$ and $a = \ln(10)/3$. We use a frame length of $T = 200$ symbols and equal peak power constraints $p_{k,\max} = q_{k,\max} = E$, $\forall k$. As references, we include maximum power allocation scheme (where each user transmits with full maximum power) and binary power allocation scheme [13] (where a user either transmits with full power or stays silent) for which we search over all combinations of transmitting users to maximize the sum-rate. To solve (20), we use CVX package [16].

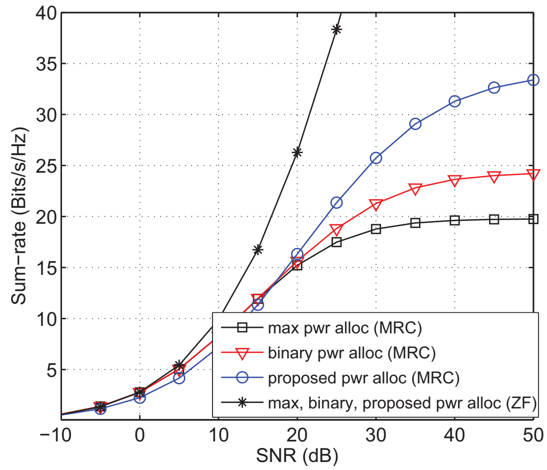

Fig. 1. Power allocation performance (sum-rate versus maximum Tx SNR) for MRC and ZF receiver with imperfect CSI.

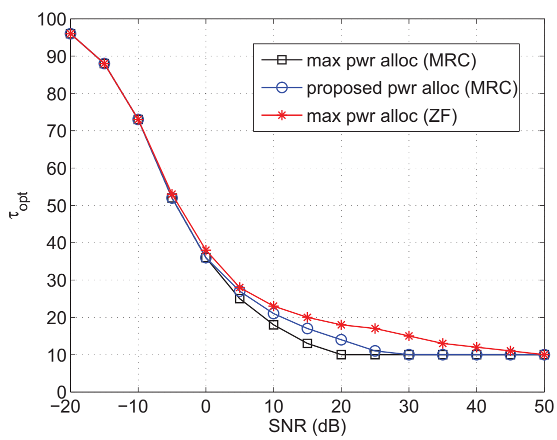

Fig. 2. The optimal number of pilot symbols versus maximum Tx SNR for MRC and ZF receivers with power allocation.

Now, we present the sum-rate versus Tx SNR plots of the proposed power allocation scheme and two reference schemes in Fig. 1 for imperfect CSI case. The plots for the perfect CSI case show the same trend as Fig. 1 except slightly higher sum-rates, and hence they are omitted. We observe the following. (i) For MRC, the proposed scheme offers significant sum-rate gains over the reference schemes, e.g., about 75% rate improvement over the maximum power allocation scheme at Tx SNR of 50 dB. (ii) For low Tx SNR, sum-rates of all three schemes converge to the same approximately optimal rate (c.f. Remark 1). (iii) For ZF, all the three schemes yield the same power allocation of maximum power to each user and hence the same sum-rate. In fact for ZF, we know from Theorem 1 and 4, the optimal power allocation of (20) is the maximum power allocation scheme. Thus, for ZF, optimization is not needed in practice. (iv) MRC and ZF receivers have similar sum-rates at low Tx SNR but as Tx SNR increases, ZF offers substantially higher rate than MRC receiver at the cost of higher complexity.

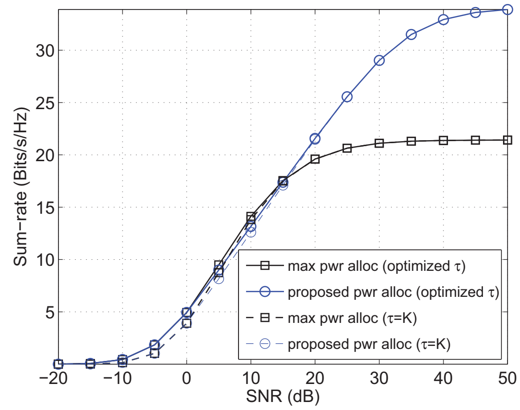

Fig. 3. Comparison between optimized τ and $\tau = K$ in terms of sum-rate versus maximum Tx SNR for MRC receiver.

Next, we investigate the effect of τ (the number of pilots) on the sum-rate under the setting of fixed $\{\beta_k\}$ and $M = 100$. Fig. 2 shows the optimum values of τ for the maximum power allocation and the proposed power allocation schemes. The corresponding sum-rate performances obtained with the optimized τ and the minimum τ $(= K)$ are presented in Fig. 3 for MRC receiver; the sum-rate for ZF receiver with optimized τ is almost the same as that with $\tau = K$ shown in Fig. 1 and hence the plot is omitted. From the above results, we observe the following: (i) The optimum τ decreases to its minimum value $K = 10$ as transmit pilot power (here Tx SNR) increases from medium to high values (consistent with Remark 3) and approaches $T/2 = 100$ (half of the frame length) when Tx SNR becomes very low (concurred with Remark 2). (ii) Only at medium Tx SNR range, different receivers or power allocation schemes yield noticeably different optimum τ values. At very low or very high Tx SNR, the optimum τ is the same for all receivers or power allocation schemes. (iii) Although optimum τ changes substantially across Tx SNR, it has little effect on the sum-rate (marginal gain only at medium Tx SNR if compared with minimum τ). Thus, for practical systems, the use of $\tau = K$ is essentially optimum.

VI. Conclusion

For massive MIMO uplink using MRC and ZF receivers with perfect and imperfect CSI, we proposed a power allocation strategy, and investigated impact of the number of pilot symbols. The proposed power allocation strategy, which maximizes the product of SINRs and can be solved by geometric programming, is asymptotically optimal in maximizing the lower bound sum-rate, and offers substantial rate improvement for MRC receiver over the existing schemes. When the pilot peak power is at least the same as the data peak power, we prove that the maximum power allocation strategy for all users is optimal for ZF receiver. We also show that the number of pilot symbols has insignificant effect on the sum-rate and the use of minimum number of pilot symbols (equal to the number of simultaneous users) for maintaining pilot orthogonality is practically optimum.

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