Non-linear Interference Cancellation for Radio Astronomy Receivers with Strong RFI

Mahmoud E. Abdelgelil and Hlaing Minn
Information & Communications Systems Lab., University of Texas at Dallas, USA

***Abstract*—The expansion of wireless communication systems across frequency, time, and space has exacerbated the radio frequency interference (RFI) issue to radio astronomy systems (RAS). A recent solution based on a coexistence paradigm offers some RFI-free durations for RAS but further expansion of RAS observation faces with strong RFI. To enhance spectrum access opportunities for both wireless communications and RAS, we explore a new approach where both systems simultaneously access the spectrum for some durations and RAS exploits RFI cancellation. The main challenge is nonlinear distortion caused by strong RFI. We study this nonlinearity's effect on RAS observation performance and develop a nonlinear-RFI cancellation scheme and a modified RAS signal power estimation scheme. We also design a channel estimator for the nonlinear system based on the unscented Kalman filter. Simulation results in flat stationary channels and multipath Rayleigh fading channels show the proposed approach yields a great improvement in RAS signal power estimation performance even at strong RFI levels.**

I. Introduction

Radio spectrum is one of the important natural resources and its usage can be classified into passive wireless services such as radio astronomy system (RAS) and active wireless services such as cellular wireless communications (CWC). RAS is a receive-only system and hence it does not cause any radio frequency interference (RFI) to other services. On the other hand, even spectrum side lobes of active wireless services can cause harmful RFI to RAS since the signal-to-noise power ratio (SNR) of the RAS signals from outer space can be as low as -60 dB [1]. Thus, RAS receivers are typically located at remote locations with radio quiet zones around them. ITU allocated some dedicated frequency bands to RAS [2] but RAS makes astronomical observations in all available atmospheric windows ranging from 2 MHz to 1000 GHz and above. As the spectrum needs of both types of wireless services are expanding, conflicts for the rights of spectrum access arise.

Since both types of wireless services are beneficial to society, their growth and coexistence are important. In view of such needs and conflicts, recently [3] proposed a shared spectrum access paradigm between CWC and RAS. With some coordination between the two systems, the three phase time division spectrum access is developed, where both systems can geographically coexist without requiring radio quiet zones. The scheme presented in [3] focuses on disjoint spectrum access without causing RFI, and more RAS observation beyond the RFI-free duration will face with very strong RFI. In this paper, to further increase spectrum access opportunities for both systems under the above shared spectrum access scenario or other scenarios with strong RFI (e.g., from communication satellites), we explore a new additional approach where both systems simultaneously access the spectrum for some duration of time. The main challenge is how to mitigate strong RFI experienced by RAS during simultaneous access and this calls for efficient algorithms to handle RFI in the non-linear region.

RFI mitigation plays an important role for RAS [4]–[6]. In [7], a reference antenna is used to detect the interference, and a least mean square (LMS) filter is applied to remove the interference. In [8], the same technique is used on a different application with a larger filter length. In [9] a post correlation cancellation is introduced instead of a pre-correlation cancellation. All of these existing RFI mitigation methods do not consider non-linearity at the low noise amplifier (LNA). Even in the current paradigm, sometimes unintentional strong RFI could enter into RAS receiver. For our explored scenario of simultaneous spectrum access between CWC and RAS, RFI would be much stronger, thus driving the RAS's LNA into a non-linear region. This creates a much more challenging RFI mitigation problem due to the non-linearity, and the existing approaches would not be effective. In this paper, we propose an efficient solution to such problem.

Our contributions are summarized as follows:

i) We develop a new signal model which incorporates non-linearity effects of LNA and analyze how RFI in the non-linear region affects the RAS's detection/estimation performance.

ii) With the aid of a reference/auxiliary antenna, we develop an unscented Kalman filter (UKF) based approach [10]–[12] to estimate the interference channel under nonlinearity.

iii) We propose a new nonlinear-RFI cancellation scheme and a modified RAS signal power estimation scheme.

iv) We illustrate effects of non-linearity on the existing approaches and advantage of the proposed approach.

II. System Model and Non-linearity Effect

Radio astronomy receivers are used to detect the power of an astronomical signal in a desired bandwidth. The main challenge is that the signal is extremely weak, much below noise level, and is highly sensitive to RFI that comes from different radio sources around an RAS site. We consider an auxiliary receiver assisted RAS receiver similar to [7] but the difference is incorporation of nonlinearity effect in our work. Fig. 1 shows the composition of such RAS receiver where the auxiliary (reference) receiver assists the main receiver in its RFI cancellation. The reference receiver antenna can be steered toward the direction of interference in order to improve the detection performance of the interference signal which is

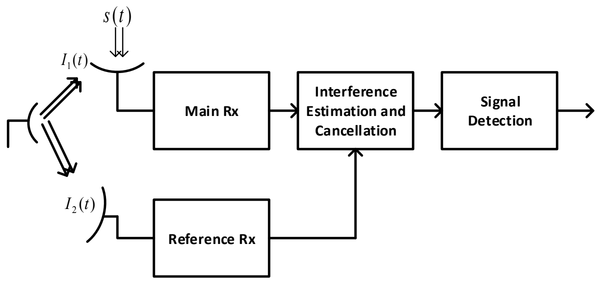

Fig. 1. RAS receiver with auxiliary antenna aided interference cancellation

to be used in the main RAS receiver's RFI cancellation. Note that the reference receiver is designed for RFI not RAS signal and hence it is operated in the linear region while the main receiver, which is designed to detect very weak RAS signals, could be in the nonlinear region when strong RFI is present.

Denote a RAS signal by $s(t)$, RFI at the main receiver by $I_1(t)$, and the received signal at the main receiver's antenna by $r(t) = s(t) + I_1(t)$. The main receiver consists of a band-pass filter to limit the bandwidth to the desired one, followed by an LNA whose functional operation is denoted by $f(\cdot)$. Then, the output of the LNA is given by

$$y(t) = f(r(t) + n(t)) \tag{1}$$

where $n(t)$ is the total noise consisting of the noise before the LNA and the input-refereed noise of LNA and devices afterwards. After the quadrature base-band conversion and sampling, the received signal becomes

$$y_{\mathrm{bb}}(k) = f_{\mathrm{bb}}(r_{bb}(k) + n_{bb}(k)) \tag{2}$$

where the subscript bb (used afterwards as well) denotes the base-band equivalent of the corresponding RF signal. The reference receiver similarly down-converts its received signal to obtain the base-band equivalent and then detect the CWC transmitted signal $u(k)$, which becomes the input to the interference estimation block at the main receiver. Then, the main receiver, under the system nonlinearity, estimates its base-band equivalent interference, denoted by $\hat{I}_{1,\mathrm{bb}}(k)$ (see Section III). After estimating the interference, a typical RFI cancellation block will cancel the estimated interference from the received signal, yielding the remaining signal $x_{\mathrm{bb}}(k)$ as

$$x_{\mathrm{bb}}(k) = y_{\mathrm{bb}}(k) - \hat{I}_{1,\mathrm{bb}}(k). \tag{3}$$

Typically, RAS performs a calibration phase where an estimate $\hat{P}_{\mathrm{n}}$ of the noise power is obtained. Then the RAS signal power is estimated by using the detector in [13] as

$$\hat{P}_{\mathrm{s}} = (\overline{|x_{\mathrm{bb}}(k)|^2} - \hat{P}_n)/a_1^2 \tag{4}$$

where a_1 is the first order gain of LNA and $\overline{()}$ is the m-sample time average.

If the LNA is linear, then $f(x) = a_1 x$ and we have

$$y_{\mathrm{bb}}(k) = a_1(r_{\mathrm{bb}}(k) + n_{\mathrm{bb}}(k)) \tag{5}$$

where $r_{\mathrm{bb}}(k)$ can be written as

$$r_{\mathrm{bb}}(k) = I_{1,\mathrm{bb}}(k) + s_{\mathrm{bb}}(k). \tag{6}$$

With $\hat{I}_{1,\mathrm{bb}}(k)$, we can cancel RFI as

$$\begin{aligned} x_{\mathrm{bb}}(k) &= y_{\mathrm{bb}}(k) - a_1\hat{I}_{1,\mathrm{bb}}(k) \\ &= a_1 s_{\mathrm{bb}}(k) + \widetilde{n}_{\mathrm{bb}}(k) + e(k) \end{aligned} \tag{7}$$

where $e(k)$ is the residual interference after RFI cancellation due to the mismatch between the actual interference and the estimated one, and $\widetilde{n}_{\mathrm{bb}}(k) = a_1 n_{\mathrm{bb}}(k)$. Then, the time-averaged RAS signal power estimate becomes

$$\overline{|x_{\mathrm{bb}}(k)|^2} = a_1^2\overline{|s_{\mathrm{bb}}(k)|^2} + \overline{|\widetilde{n}_{\mathrm{bb}}(k)|^2} + \overline{|\xi_{\mathrm{res}}(k)|} \tag{8}$$

where $\overline{|\xi_{\mathrm{res}}(k)|}$ contains the residual interference power and the sum of cross correlations between the signals in (7). Define $P_{\mathrm{n}} = \overline{|\widetilde{n}_{\mathrm{bb}}(k)|^2}$, $P_{\mathrm{s}} = \overline{|s_{\mathrm{bb}}(k)|^2}$ and $P_{\mathrm{t}} = \overline{|x_{\mathrm{bb}}(k)|^2}$. Then,

$$P_{\mathrm{t}} = a_1^2 P_{\mathrm{s}} + P_{\mathrm{n}} + \overline{|\xi_{\mathrm{res}}(k)|} \tag{9}$$

$$\hat{P}_{\mathrm{s}} = \frac{P_{\mathrm{t}} - \hat{P}_{\mathrm{n}}}{a_1^2} = P_{\mathrm{s}} + \frac{P_n - \hat{P}_{\mathrm{n}}}{a_1^2} + \frac{\overline{|\xi_{\mathrm{res}}(k)|}}{a_1^2}. \tag{10}$$

Although the second term in the above equation can be neglected if a reliable noise power estimate is obtained, the last term will not reduce to zero even with a long integration time. This shows the effect of the residual interference on the RAS signal power estimate.

Another problem arises when the interference level increases. This high received signal level could make the LNA to work in its nonlinear region, and the former estimation technique will no longer hold. From (2), we can write the LNA input RF signal as

$$x_{\mathrm{lna}}(t) = A_1(t)\cos(wt) - A_2(t)\sin(wt) \tag{11}$$

where

$$A_1(t) = \Re\{s_{\mathrm{bb}}(t) + I_{1,\mathrm{bb}}(t) + n_{\mathrm{bb}}(t)\} \tag{12}$$

$$A_2(t) = \Im\{s_{\mathrm{bb}}(t) + I_{1,\mathrm{bb}}(t) + n_{\mathrm{bb}}(t)\}. \tag{13}$$

By using a widely used third order nonlinear model of a differential output LNA as in [14], the LNA output reads as

$$y(t) = a_1 x_{\mathrm{lna}}(t) + a_3 x_{\mathrm{lna}}^3(t). \tag{14}$$

After removing out-of-band terms, the LNA output becomes

$$y(t) = y_{\mathrm{r}}(t)\cos(wt) - y_{\mathrm{i}}(t)\sin(wt) \tag{15}$$

where

$$y_{\mathrm{r}}(t) \approx a_1 A_1(t) + \frac{3}{4}a_3\left(A_1^3(t) + A_1(t)A_2^2(t)\right) \tag{16}$$

$$y_{\mathrm{i}}(t) \approx a_1 A_2(t) + \frac{3}{4}a_3\left(A_2^3(t) + A_2(t)A_1^2(t)\right). \tag{17}$$

We will simply drop the time notation for simplicity. Define $s_{\mathrm{bb}} = s_{\mathrm{r}} + js_{\mathrm{i}}$, $I_{1,\mathrm{bb}} = I_{\mathrm{r}} + jI_{\mathrm{i}}$ and $n_{\mathrm{bb}} = n_{\mathrm{r}} + jn_{\mathrm{i}}$, and by neglecting all the higher powers of s_r and s_i and their multiplications (since they are negligible), we can get

$$y_{\mathrm{r}} \approx C_1 s_{\mathrm{r}} + C_2 s_{\mathrm{i}} + C_3^+ \tag{18}$$

$$y_{\mathrm{i}} \approx D_1 s_{\mathrm{i}} + D_2 s_{\mathrm{r}} + D_3^+ \tag{19}$$

where, defining $\alpha = a_1$ and $\beta = \frac{3}{4}a_3$,

$$\begin{aligned} C_1 &= \alpha + 3\beta(n_{\mathrm{r}} + I_{\mathrm{r}})^2 + \beta(n_{\mathrm{i}} + I_{\mathrm{i}})^2 \\ C_2 &= 2\beta(n_{\mathrm{i}} + I_{\mathrm{i}})(n_{\mathrm{r}} + I_{\mathrm{r}}) \\ C_3^+ &= \alpha(n_{\mathrm{r}} + I_{\mathrm{r}}) + \beta(n_{\mathrm{r}} + I_{\mathrm{r}})^3 + \beta(n_i + I_{\mathrm{i}})^2(n_{\mathrm{r}} + I_{\mathrm{r}}) \\ &= \beta n_{\mathrm{r}}^3 + 3\beta I_{\mathrm{r}} n_{\mathrm{r}}^2 + \left(\alpha + \beta\left(3I_{\mathrm{r}}^2 + I_{\mathrm{i}}^2\right)\right)n_{\mathrm{r}} + \beta n_{\mathrm{i}}^2 n_{\mathrm{r}} \\ &\quad + 2\beta I_{\mathrm{i}} n_{\mathrm{i}} n_{\mathrm{r}} + \beta I_{\mathrm{r}} n_{\mathrm{i}}^2 + 2\beta I_{\mathrm{r}} I_{\mathrm{i}} n_{\mathrm{i}} + \alpha I_{\mathrm{r}} + \beta I_{\mathrm{r}}^3 + \beta I_{\mathrm{r}} I_{\mathrm{i}}^2 \\ D_1 &= \alpha + 3\beta(n_{\mathrm{i}} + I_{\mathrm{i}})^2 + \beta(n_{\mathrm{r}} + I_{\mathrm{r}})^2 \\ D_2 &= 2\beta(n_{\mathrm{i}} + I_{\mathrm{i}})(n_{\mathrm{r}} + I_{\mathrm{r}}) \\ D_3^+ &= \alpha(n_{\mathrm{i}} + I_{\mathrm{i}}) + \beta(n_{\mathrm{i}} + I_{\mathrm{i}})^3 + \beta(n_{\mathrm{r}} + I_{\mathrm{r}})^2(n_{\mathrm{i}} + I_{\mathrm{i}}). \end{aligned}$$

We can notice from the equations of C_3^+ and D_3^+ that noise-free terms of the interference can introduce a noticeable error.

If we can obtain interference estimate $\hat{I}_{\rm r}+j\hat{I}_{\rm i}$, we can cancel its effects and obtain the post-cancellation signals as

$$\begin{aligned} z_{\rm r} &= y_{\rm r} - \left(\alpha\hat{I}_{\rm r} + \beta\hat{I}_{\rm r}^3 + \beta\hat{I}_{\rm r}\hat{I}_{\rm i}^2\right) \\ &= C_1 s_{\rm r} + C_2 s_{\rm i} + C_3 + e_{\rm nl,r} \quad (20) \end{aligned}$$

$$\begin{aligned} z_{\rm i} &= y_{\rm i} - \left(\alpha\hat{I}_{\rm i} + \beta\hat{I}_{\rm i}^3 + \beta\hat{I}_{\rm i}\hat{I}_{\rm r}^2\right) \\ &= D_1 s_{\rm i} + D_2 s_{\rm r} + D_3 + e_{\rm nl,i} \quad (21) \end{aligned}$$

where

$$C_3 = C_3^+ - \alpha I_{\rm r} - \beta I_{\rm r}^3 - \beta I_{\rm r} I_{\rm i}^2 \quad (22)$$

$$D_3 = D_3^+ - \alpha I_{\rm i} - \beta I_{\rm i}^3 - \beta I_{\rm i} I_{\rm r}^2 \quad (23)$$

$$e_{\rm nl,r} = \alpha\left(I_{\rm r} - \hat{I}_{\rm r}\right) + \beta\left(I_{\rm r}^3 - \hat{I}_{\rm r}^3\right) + \beta\left(I_{\rm r}I_{\rm i}^2 - \hat{I}_r\hat{I}_i^2\right) \quad (24)$$

$$e_{\rm nl,i} = \alpha\left(I_{\rm i} - \hat{I}_{\rm i}\right) + \beta\left(I_{\rm i}^3 - \hat{I}_{\rm i}^3\right) + \beta\left(I_{\rm i}I_{\rm r}^2 - \hat{I}_{\rm i}\hat{I}_{\rm r}^2\right). \quad (25)$$

Note C_3 can be written as

$$\begin{aligned} C_3 &= \beta n_r^3 + 3\beta I_r n_r^2 + \left(\alpha + \beta\left(3I_r^2 + I_{\rm i}^2\right)\right) n_{\rm r} \\ &+ \beta n_{\rm i}^2 n_{\rm r} + 2\beta I_{\rm i} n_{\rm i} n_{\rm r} + \beta I_{\rm r} n_{\rm i}^2 + 2\beta I_{\rm r} I_{\rm i} n_{\rm i}. \quad (26) \end{aligned}$$

Next, computing the power estimate by squaring and time-averaging over m samples, we obtain

$$\begin{aligned} \overline{z_{\rm r}^2} &= \overline{C_1^2 s_{\rm r}^2} + \overline{C_2^2 s_{\rm i}^2} + \overline{C_3^2} + 2\overline{C_1 C_2 s_{\rm r} s_{\rm i}} + 2\overline{C_1 C_3 s_{\rm r}} \\ &+ 2\overline{C_2 s_{\rm i} C_3} + \overline{e_{\rm nl}^2} + \overline{e_{\rm nl}\left(C_1 s_{\rm r} + C_2 s_{\rm i} + C_3\right)} \quad (27) \\ &= \overline{C_1^2}\,\overline{s_{\rm r}^2} + \overline{C_2^2}\,\overline{s_{\rm i}^2} + \overline{C_3^2} + \overline{e_{\rm nl,r}^2} + E_{\rm res,r} \quad (28) \end{aligned}$$

$$\overline{z_{\rm i}^2} = \overline{D_1^2}\,\overline{s_{\rm i}^2} + \overline{D_2^2}\,\overline{s_{\rm r}^2} + \overline{D_3^2} + \overline{e_{\rm nl,i}^2} + E_{\rm res,i} \quad (29)$$

where $E_{\rm res,r}$ and $E_{\rm res,i}$ are the residual errors due to the finite sum in the real part and the imaginary part. Furthermore,

$$\begin{aligned} \overline{C_1^2} &= \alpha^2 + 9\beta^2\overline{I_{\rm r}^4} + \beta^2\overline{I_{\rm i}^4} + \left(54\beta^2\overline{n_{\rm r}^2} + 6\alpha\beta + 6\beta^2\overline{n_{\rm i}^2}\right)\overline{I_{\rm r}^2} \\ &+ \left(6\beta^2\overline{n_{\rm r}^2} + 2\alpha\beta + 6\beta^2\overline{n_{\rm i}^2}\right)\overline{I_{\rm i}^2} + 6\beta^2\overline{I_{\rm r}^2 I_{\rm i}^2} + 9\beta^2\overline{n_{\rm r}^4} \\ &+ \beta^2\overline{n_{\rm i}^4} + 6\alpha\beta\overline{n_{\rm r}^2} + 2\alpha\beta\overline{n_{\rm i}^2} + 6\beta^2\overline{n_{\rm r}^2 n_{\rm i}^2} \quad (30) \end{aligned}$$

$$\overline{C_2^2} = 4\beta^2\overline{n_{\rm i}^2 n_{\rm r}^2} + 4\beta^2\overline{n_{\rm i}^2}\,\overline{I_{\rm r}^2} + 4\beta^2\overline{n_{\rm r}^2}\,\overline{I_{\rm i}^2} + 4\beta^2\overline{I_{\rm r}^2 I_{\rm i}^2} \quad (31)$$

$$\begin{aligned} \overline{C_3^2} &= \left(9\beta^2\overline{n_{\rm r}^2}\right)\overline{I_{\rm r}^4} + \left(\beta^2\overline{n_{\rm r}^2}\right)\overline{I_{\rm i}^4} + \left(6\beta^2\overline{n_{\rm r}^2} + 4\beta^2\overline{n_{\rm i}^2}\right)\overline{I_{\rm r}^2 I_{\rm i}^2} \\ &+ \left(6\alpha\beta\overline{n_{\rm r}^2} + 15\beta^2\overline{n_{\rm r}^4} + 12\beta^2\overline{n_{\rm r}^2 n_{\rm i}^2} + \beta^2\overline{n_{\rm i}^4}\right)\overline{I_{\rm r}^2} \\ &+ \left(2\alpha\beta\overline{n_{\rm r}^2} + 2\beta^2\overline{n_{\rm r}^4} + 6\beta^2\overline{n_{\rm r}^2 n_{\rm i}^2}\right)\overline{I_{\rm i}^2} + \alpha\overline{n_{\rm r}^2} + 2\alpha\beta\overline{n_{\rm r}^4} \\ &+ 2\alpha\beta\overline{n_{\rm r}^2 n_{\rm i}^2} + \beta^2\overline{n_{\rm r}^6} + 2\beta^2\overline{n_{\rm r}^4 n_{\rm i}^2} + \beta^2\overline{n_{\rm r}^2 n_{\rm i}^4} \quad (32) \end{aligned}$$

$$\begin{aligned} \overline{D_1^2} &= \alpha^2 + 9\beta^2\overline{I_{\rm i}^4} + \beta^2\overline{I_{\rm r}^4} + \left(54\beta^2\overline{n_{\rm i}^2} + 6\alpha\beta + 6\beta^2\overline{n_{\rm r}^2}\right)\overline{I_{\rm i}^2} \\ &+ \left(6\beta^2\overline{n_{\rm i}^2} + 2\alpha\beta + 6\beta^2\overline{n_{\rm r}^2}\right)\overline{I_{\rm r}^2} + 6\beta^2\overline{I_{\rm r}^2 I_{\rm i}^2} + 9\beta^2\overline{n_{\rm i}^4} \\ &+ \beta^2\overline{n_{\rm r}^4} + 6\alpha\beta\overline{n_{\rm i}^2} + 2\alpha\beta\overline{n_{\rm r}^2} + 6\beta^2\overline{n_{\rm r}^2 n_{\rm i}^2} \quad (33) \end{aligned}$$

$$\overline{D_2^2} = 4\beta^2\overline{n_{\rm i}^2 n_{\rm r}^2} + 4\beta^2\overline{n_{\rm i}^2}\,\overline{I_{\rm r}^2} + 4\beta^2\overline{n_{\rm r}^2}\,\overline{I_{\rm i}^2} + 4\beta^2\overline{I_{\rm r}^2 I_{\rm i}^2} \quad (34)$$

$$\begin{aligned} \overline{D_3^2} &= \left(9\beta^2\overline{n_{\rm i}^2}\right)\overline{I_{\rm i}^4} + \left(\beta^2\overline{n_{\rm i}^2}\right)\overline{I_{\rm r}^4} + \left(6\beta^2\overline{n_{\rm i}^2} + 4\beta^2\overline{n_{\rm r}^2}\right)\overline{I_{\rm r}^2 I_{\rm i}^2} \\ &+ \left(6\alpha\beta\overline{n_{\rm i}^2} + 15\beta^2\overline{n_{\rm i}^4} + 12\beta^2\overline{n_{\rm r}^2 n_{\rm i}^2} + \beta^2\overline{n_{\rm r}^4}\right)\overline{I_{\rm i}^2} \\ &+ \left(2\alpha\beta\overline{n_{\rm i}^2} + 2\beta^2\overline{n_{\rm i}^4} + 6\beta^2\overline{n_{\rm r}^2 n_{\rm i}^2}\right)\overline{I_{\rm r}^2} + \alpha\overline{n_{\rm i}^2} + 2\alpha\beta\overline{n_{\rm i}^4} \\ &+ 2\alpha\beta\overline{n_{\rm r}^2 n_{\rm i}^2} + \beta^2\overline{n_{\rm i}^6} + 2\beta^2\overline{n_{\rm i}^4 n_{\rm r}^2} + \beta^2\overline{n_{\rm i}^2 n_{\rm r}^4}. \quad (35) \end{aligned}$$

So, by using (10), we get

$$\hat{P}_{\rm s} = (\overline{z_{\rm r}^2} + \overline{z_{\rm i}^2} - \overline{n_{\rm r}^2} - \overline{n_{\rm i}^2})/\alpha^2. \quad (36)$$

Now, to observe the effect of non-linearity only, in the rest of this section, we assume $\overline{n_{\rm r}^2} = \overline{n_{\rm i}^2} = \sigma_{\rm n}^2 = \frac{P_{\rm n}}{2}$, $\overline{n_{\rm r}^4} = \overline{n_{\rm i}^4} = 3\sigma_{\rm n}^4$ and $\overline{n_{\rm r}^6} = \overline{n_{\rm i}^6} = 15\sigma_{\rm n}^6$. Then, we have

$$\begin{aligned} \overline{C_1^2} &= 9\beta^2\overline{I_{\rm r}^4} + \beta^2\overline{I_{\rm i}^4} + \left(60\beta^2\sigma^2 + 6\alpha\beta\right)\overline{I_{\rm r}^2} + 6\beta^2\overline{I_{\rm r}^2 I_{\rm i}^2} \\ &+ \left(12\beta^2\sigma^2 + 2\alpha\beta\right)\overline{I_{\rm i}^2} + \alpha^2 + 36\beta^2\sigma^4 + 8\alpha\beta\sigma^2 \quad (37) \end{aligned}$$

$$\overline{C_2^2} = 4\beta^2\sigma^4 + 4\beta^2\sigma^2\overline{I_{\rm r}^2} + 4\beta^2\sigma^2\overline{I_{\rm i}^2} + 4\beta^2\overline{I_{\rm r}^2 I_{\rm i}^2} \quad (38)$$

$$\begin{aligned} \overline{C_3^2} &= \sigma^2\Big(\left(9\beta^2\right)\overline{I_{\rm r}^4} + \left(\beta^2\right)\overline{I_{\rm i}^4} + \left(10\beta^2\right)\overline{I_{\rm r}^2 I_{\rm i}^2} \\ &+ \left(6\alpha\beta + 60\beta^2\sigma^2\right)\overline{I_{\rm r}^2} + \left(2\alpha\beta + 12\beta^2\sigma^2\right)\overline{I_{\rm i}^2} \\ &+ \left(\alpha^2 + 8\alpha\beta\sigma^2 + 24\beta^2\sigma^4\right)\Big) \quad (39) \end{aligned}$$

$$\begin{aligned} \overline{D_1^2} &= 9\beta^2\overline{I_{\rm i}^4} + \beta^2\overline{I_{\rm r}^4} + \left(60\beta^2\sigma^2 + 6\alpha\beta\right)\overline{I_{\rm i}^2} + 6\beta^2\overline{I_{\rm r}^2 I_{\rm i}^2} \\ &+ \left(12\beta^2\sigma^2 + 2\alpha\beta\right)\overline{I_{\rm r}^2} + \alpha^2 + 36\beta^2\sigma^4 + 8\alpha\beta\sigma^2 \quad (40) \end{aligned}$$

$$\overline{D_2^2} = 4\beta^2\sigma^4 + 4\beta^2\sigma^2\overline{I_{\rm r}^2} + 4\beta^2\sigma^2\overline{I_{\rm i}^2} + 4\beta^2\overline{I_{\rm r}^2 I_{\rm i}^2} \quad (41)$$

$$\begin{aligned} \overline{D_3^2} &= \sigma^2\Big(\left(9\beta^2\right)\overline{I_i^4} + \left(\beta^2\right)\overline{I_r^4} + \left(10\beta^2\right)\overline{I_{\rm r}^2 I_{\rm i}^2} \\ &+ \left(6\alpha\beta + 60\beta^2\sigma^2\right)\overline{I_{\rm i}^2} + \left(2\alpha\beta + 12\beta^2\sigma^2\right)\overline{I_{\rm r}^2} \\ &+ \left(\alpha^2 + 8\alpha\beta\sigma^2 + 24\beta^2\sigma^4\right)\Big) \quad (42) \end{aligned}$$

and we can do the same for the imaginary part. Assuming circularly symmetric signals and interference, we have $\overline{s_{\rm r}^2} = \overline{s_{\rm i}^2} = \sigma_s^2 = \frac{P_{\rm s}}{2}$ and $\overline{I_r^2} = \overline{I_i^2}$. Define $C = \overline{C_1^2} + \overline{C_2^2}$ and $D = \overline{D_1^2} + \overline{D_2^2}$. Then, we have

$$\overline{z_{\rm r}^2} = \sigma_s^2 C + \overline{C_3^2} + \overline{e_{\rm nl,r}^2} + E_{\rm res,r}, \quad (43)$$

$$\overline{z_{\rm i}^2} = \sigma_s^2 D + \overline{D_3^2} + \overline{e_{\rm nl,i}^2} + E_{\rm res,i}. \quad (44)$$

By using the same estimator in (36), we get

$$\hat{P}_{\rm s} = 2\hat{\sigma}_s^2 = (\overline{z_{\rm r}^2} + \overline{z_{\rm i}^2} - 2\sigma_{\rm n}^2)/\alpha^2. \quad (45)$$

For simplification, it can be noticed that $\overline{C_3^2} \simeq \sigma_{\rm n}^2 C$. By neglecting all the residual errors and putting $\alpha = 1$, we get

$$\hat{P}_s = (C + D)\,\sigma_s^2 + \sigma_{\rm n}^2\,(C + D) - 2\sigma_{\rm n}^2 \quad (46)$$

and $C = D$ due to circular symmetry of the signals. As our performance metric, we define the absolute normalized error of the estimated RAS signal power as

$$e_{\rm norm} = \left|(P_s - \hat{P}_s)/P_s\right|. \quad (47)$$

With (46) and $P_n = 2\sigma_{\rm n}^2$, we have

$$e_{\rm norm} = |C - 1|\left(\sigma_{\rm s}^2 + \sigma_{\rm n}^2\right)/\sigma_{\rm s}^2. \quad (48)$$

Fig. 2 plots (48) to show the effect of non-linearity using different interference to noise power ratios (INRs) and SNRs. The normalized error of the RAS power estimate increases with INR (i.e., when entering more into nonlinearity region). Furthermore, decreasing SNR makes the situation more severe.

III. Nonlinearity Compensation

Due to nonlinearity, the equivalent gain on the desired RAS signal becomes a function of the interference level. We propose a new approach consisting of a nonlinear based interference estimator and a nonlinear based RA signal power estimator. By putting (28) and (29) in a matrix form, we get

$$\underbrace{\begin{bmatrix} \overline{z_{\rm r}^2} \\ \overline{z_{\rm i}^2} \end{bmatrix}}_{\mathbf{z}} = \underbrace{\begin{bmatrix} \overline{C_1^2} & \overline{C_2^2} \\ \overline{D_2^2} & \overline{D_1^2} \end{bmatrix}}_{\mathbf{G}} \underbrace{\begin{bmatrix} \overline{s_{\rm r}^2} \\ \overline{s_{\rm i}^2} \end{bmatrix}}_{\mathbf{s}} + \underbrace{\begin{bmatrix} \overline{C_3^2} \\ \overline{D_3^2} \end{bmatrix}}_{\mathbf{n}_{\rm new}} + \underbrace{\begin{bmatrix} e_{\rm tot,r} \\ e_{\rm tot,i} \end{bmatrix}}_{\mathbf{e}_{\rm tot}}$$

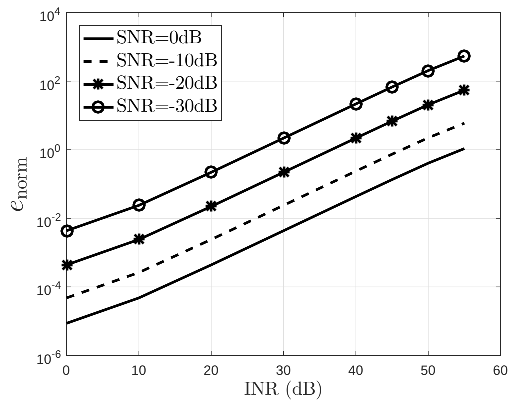

Fig. 2. Average absolute normalized error (48) of the RAS signal power estimation at different INRs and SNRs

where $e_{\text{tot,r}}$ is the sum of all the errors in (43) for the real part and $e_{\text{tot,i}}$ is for the imaginary part. With estimates of $\mathbf{G}$ and $\mathbf{n}_{\text{new}}$ denoted by $\widehat{\mathbf{G}}$ and $\hat{\mathbf{n}}_{\text{new}}$, we estimate $\mathbf{s}$ as

$$\widehat{\mathbf{s}} = \widehat{\mathbf{G}}^{-1}\left(\mathbf{z} - \hat{\mathbf{n}}_{\text{new}}\right). \tag{49}$$

To get $\hat{\mathbf{G}}$, we have to estimate the received interference that depends on both the channel and the CWC-transmitted interference. We assume the reference receiver has detected the transmitted interference perfectly. The task is to estimate the channel for a nonlinear system and we will use UKF. Define the CWC-transmitted data vector $\mathbf{u}(k) = \left[u\left(k\right), \cdots, u\left(k - \mathrm{L} + 1\right)\right]^{\mathrm{T}} \in \mathbb{C}^{L}$ and the channel vector to the main RAS receiver $\mathbf{h} = [h_0, \cdots, h_{L-1}]^{\mathrm{T}} \in \mathbb{C}^{L}$. The received interference at time k is

$$I_{1,\text{bb}}(k) = \mathbf{u}^T(k)\mathbf{h} = \underbrace{\mathbf{v}^T\left(k\right)\mathbf{h}_{\text{new}}}_{I_{\text{r}}(k)} + j\underbrace{\mathbf{w}^T\left(k\right)\mathbf{h}_{\text{new}}}_{I_{\text{i}}(k)} \tag{50}$$

where $\mathbf{v}(k) \in \mathbb{R}^{2L}$, $\mathbf{w}(k) \in \mathbb{R}^{2L}$, $\mathbf{h}_{\text{new}} \in \mathbb{R}^{2L}$ are defined as

$$\mathbf{v}(k) = [u_{\text{r}}(k), -u_{\text{i}}(k), \cdots, u_{\text{r}}(k-L+1), -u_{\text{i}}(k-L+1)]^{\mathrm{T}},$$
$$\mathbf{w}(k) = [u_{\text{i}}(k), u_{\text{r}}(k), \cdots, u_{\text{i}}(k-1), u_{\text{r}}(k-L+1)]^{\mathrm{T}},$$
$$\mathbf{h}_{\text{new}} = [h_{\text{r},0}, h_{\text{i},0}, \cdots, h_{\text{r},L-1}, h_{\text{i},L-1}]^{\mathrm{T}},$$

and the subscript r and i denote the real and imaginary parts, respectively. We can define a system with a state space representation of system states vector $\mathbf{x}(k) \in \mathbb{R}^{N}$, where $N = 2L$, and an observation vector $\boldsymbol{\theta}(k) \in \mathbb{R}^2$. As we need to estimate the channel, we will assign $\mathbf{h}_{\text{new}}$ as the system states. Since we assume a constant channel over an observation block of size M, the system states' dynamics will be

$$\mathbf{x}(k) = \mathbf{x}(k-1) \tag{51}$$

where the states are independent. For the observation dynamic equation, we will use the LNA output as the system observation. From (18) and (19), after neglecting insignificant terms, the LNA output can be approximated as

$$y_{\text{r}}(k) \simeq \alpha I_{\text{r}}(k) + \beta\left(I_{\text{r}}^3(k) + I_{\text{r}}(k)I_{\text{i}}^2(k)\right) + n_{\text{r}}(k) \tag{52}$$
$$y_{\text{i}}(k) \simeq \alpha I_{\text{i}}(k) + \beta\left(I_{\text{i}}^3(k) + I_{\text{i}}(k)I_{\text{r}}^2(k)\right) + n_{\text{i}}(k) \tag{53}$$

and the observation vector can be defined as

$$\boldsymbol{\theta}(k) = \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \end{bmatrix} = \begin{bmatrix} y_{\text{r}}(k) \\ y_{\text{i}}(k) \end{bmatrix} = \mathbf{g}\left(\mathbf{x}(k)\right) + \textbf{noise}. \tag{54}$$

So, using (50), we can write the observations as

$$\theta_1(k) = \alpha\mathbf{v}^T(k)\mathbf{x}(k) + \beta\big(\mathbf{v}^T(k)\mathbf{x}(k)\big)^3 + \beta\Big(\big(\mathbf{v}^T(k)\mathbf{x}(k)\big)\big(\mathbf{w}^T(k)\mathbf{x}(k)\big)^2\Big) + n_{\text{r}}(k) \tag{55}$$
$$\theta_2(k) = \alpha\mathbf{w}^T(k)\mathbf{x}(k) + \beta\left(\mathbf{w}^T(k)\mathbf{x}(k)\right)^3 + \beta\Big(\big(\mathbf{w}^T(k)\mathbf{x}(k)\big)\big(\mathbf{v}^T(k)\mathbf{x}(k)\big)^2\Big) + n_{\text{i}}(k). \tag{56}$$

UKF uses $(2N+1)$ probability distribution samples which are transformed using the nonlinear function instead. Then a filtering part is done in the same way as the Kalman filter. In the following, for compactness, we use $(k|k_1)$ to denote the estimation at time k given k_1 samples. First, by sampling the distribution, we get (57) shown at the bottom of the page where $(\,)_i$ represents the ith column vector of the matrix inside the bracket and $\boldsymbol{\chi}_i\left(k-1|k-1\right) \in \mathbb{R}^N$ has a mean of $\hat{\mathbf{x}}\left(k-1|k-1\right) \in \mathbb{R}^N$ and covariance $\mathbf{P}_{\mathbf{xx}}\left(k-1|k-1\right) \in \mathbb{R}^{N\times N}$. From (51), the estimated states' samples are

$$\boldsymbol{\chi}_i(k|k-1) = \boldsymbol{\chi}_i(k-1|k-1) \tag{58}$$

with the estimated mean

$$\hat{\mathbf{x}}(k|k-1) = \sum_{i=0}^{2N} w_{\text{m}}(i)\boldsymbol{\chi}_i(k|k-1) \tag{59}$$

and the covariance

$$\mathbf{P}_{\mathbf{xx}}(k|k-1) = \sum_{i=0}^{2N}\Big(w_{\text{c}}(i)\big[\boldsymbol{\chi}_i(k|k-1) - \hat{\mathbf{x}}(k|k-1)\big]\big[\boldsymbol{\chi}_i(k|k-1) - \hat{\mathbf{x}}(k|k-1)\big]^{\mathrm{T}}\Big) \tag{60}$$

where $w_{\text{m}}(i) = w_{\text{c}}(i) = \frac{1-w_{\text{m}}(0)}{2N}$ and $w_{\text{c}}(0) = w_{\text{m}}(0) + (1 - c^2 + \epsilon)$. Note that c^2, $w_{\text{m}}(0)$ and ϵ are constants and they are adjusted as in [12]. By using $\mathbf{g}(.)$ in (54), the estimated samples of observations become

$$\boldsymbol{\psi}_i(k|k-1) = \mathbf{g}\big(\boldsymbol{\chi}_i(k|k-1)\big) \tag{61}$$

with the estimated mean

$$\hat{\boldsymbol{\theta}}(k|k-1) = \sum_{i=0}^{2N} w_{\text{m}}(i)\boldsymbol{\psi}_i(k|k-1) \tag{62}$$

where $\hat{\boldsymbol{\theta}}\left(k|k-1\right) \in \mathbb{R}^2$. The estimated cross covariance between states and observations is

$$\mathbf{P}_{\mathbf{x}\boldsymbol{\theta}}(k|k-1) = \sum_{i=0}^{2N}\Big(w_{\text{c}}(i)\big[\boldsymbol{\chi}_i(k|k-1) - \hat{\mathbf{x}}(k|k-1)\big]\big[\boldsymbol{\psi}_i(k|k-1) - \hat{\boldsymbol{\theta}}(k|k-1)\big]^{\mathrm{T}}\Big) \tag{63}$$

where $\mathbf{P}_{\mathbf{x}\boldsymbol{\theta}}(k|k-1) \in \mathbb{R}^{N\times 2}$. Define $\boldsymbol{\nu}(k) = \boldsymbol{\theta}(k) - \hat{\boldsymbol{\theta}}(k|k-1)$ as the error between the actual observation and the estimated one. Since the noise is Gaussian with zero mean and covariance $\mathbf{R}_n \in \mathbb{R}^{2\times 2}$, $\boldsymbol{\nu}(k)$ has zero mean and covariance

$$\mathbf{P}_{\boldsymbol{\nu\nu}}(k|k-1) = \sum_{i=0}^{2N}\Big(w_{\text{c}}(i)\big[\boldsymbol{\psi}_i(k|k-1) - \hat{\boldsymbol{\theta}}(k|k-1)\big]\big[\boldsymbol{\psi}_i(k|k-1) - \hat{\boldsymbol{\theta}}(k|k-1)\big]^{\mathrm{T}}\Big) + \mathbf{R}_{\text{n}} \tag{64}$$

where $\mathbf{P}_{\boldsymbol{\nu\nu}}(k|k-1) \in \mathbb{R}^{2\times 2}$. To filter the estimated states' vector, we calculate the Kalman filter gain

$$\mathbf{K}(k) = \mathbf{P}_{\mathbf{x}\boldsymbol{\theta}}(k|k-1)\mathbf{P}_{\boldsymbol{\nu\nu}}^{-1}(k|k-1) \tag{65}$$

where $\mathbf{K}(k) \in \mathbb{R}^{N\times 2}$. The filtered system states' mean is

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)\boldsymbol{\nu}(k) \tag{66}$$

and the updated states' covariance is

$$\mathbf{P}_{\mathbf{xx}}(k|k)=\mathbf{P}_{\mathbf{xx}}(k|k-1)-\mathbf{K}(k)\mathbf{P}_{\mathbf{x}\boldsymbol{\theta}}^{T}(k|k-1). \quad (67)$$

Since the system is nonlinear, it depends intensely on the initial states. For our considered scenario in [3], there is a coordination between RAS and CWC. So, CWC system can send a small packet (say 100 samples) with lower power (to mitigate nonlinearity effect) at the beginning of each long transmission packet so that the RAS main receiver can obtain a good initial channel estimate. After M iterations, we can get $\hat{\mathbf{h}}_{\mathrm{new}}=\hat{\mathbf{x}}\left(M|M\right)$, and from (50) the estimated interference at the main receiver is obtained as

$$\hat{I}_{1,\mathrm{bb}}(k)=\mathbf{v}^{T}(k)\hat{\mathbf{h}}_{\mathrm{new}}+j\mathbf{w}^{T}(k)\hat{\mathbf{h}}_{\mathrm{new}}. \quad (68)$$

Next, substituting (68) in (20) and (21), we obtain $\mathbf{z}$, and using (30) to (35), we compute $\hat{\mathbf{G}}$ and $\hat{\mathbf{n}}_{\mathrm{new}}$. Then, we obtain $\hat{\mathbf{s}}$ from (49) and compute the astronomical signal power estimate as

$$\hat{P}_{\mathrm{s}}=\left[\begin{array}{cc}1 & 1\end{array}\right]\hat{\mathbf{s}}. \quad (69)$$

IV. Performance Evaluation

Our simulation setup is as follows. For the LNA parameters, we set $a_1=1$ and $a_3=-72.5$ to give 1 dB input point at 0 dBm for a single tone input signal. CWC transmits QPSK symbols, i.e., $u_{\mathrm{r}}\left(k\right)=\pm\sqrt{\frac{P_{\mathrm{I}}}{2}}$ and $u_{\mathrm{i}}\left(k\right)=\pm\sqrt{\frac{P_{\mathrm{I}}}{2}}$, and the INR at the main receiver is $\frac{P_{\mathrm{I}}}{P_{\mathrm{n}}}$ where P_{I} and P_{n} are the baseband interference power and noise power. We set P_{n} at -50 dBm, so at low INR the system will tend to be linear, and at high INR the system turns to be in the nonlinear range. We will assume that till the last INR used, the LNA does not settle and follows the third order nonlinear relation in (14). Of course a real LNA has a lower noise level and a wider range of INR, but we intentionally shrink that range to save computational time. We assume the RAS signal and the CWC interference are in the same band with a bandwidth of 10 MHz. To show the impact of our method, we consider six cases as shown in Table I where PCE stands for perfect channel estimation and LE represents a linear estimator which assumes the system is linear. In case 3, we use the Wiener filter interference canceler as an optimum solution for the LMS used in [7], and we derive it for a nonlinear desired signal in a single tap channel as

$$w_{\mathrm{opt}}=\left(h_{\mathrm{r}}+2\beta\frac{P_{\mathrm{I}}}{2}\left(h_{\mathrm{r}}^{3}+h_{\mathrm{r}}h_{\mathrm{i}}^{2}\right)\right)+j\left(h_{\mathrm{i}}+2\beta\frac{P_{\mathrm{I}}}{2}\left(h_{\mathrm{i}}^{3}+h_{\mathrm{r}}^{2}h_{\mathrm{i}}\right)\right) \quad (70)$$

and in a multi-path frequency-selective channel as in (71) (shown at the bottom of the page). In all cases, we assume the noise power is estimated perfectly and the time averaging for the RAS power estimate is over one second (10^7 samples).

Fig. 3 presents the average absolute normalized error (47) of the RAS signal power estimate for the considered six cases under various INRs. The channel here has a single tap with coefficient $0.8+j0.6$ (having a unity power). By comparing case 5 with the first three cases, we can observe that under the perfect channel estimation scenario, our RAS power estimator outperforms the other schemes at all considered INRs. This illustrates importance of incorporating nonlinearity effect in the RFI cancellation and RAS power estimation. The performance difference between case 5 and case 6 reflects the effect of channel estimtion error on the proposed power estimator. To observe the nonlinearity effect on channel estimation, we compare case 6 and case 4 (both use the proposed power estimator but differs in the channel estimators). At low INR, the performances are the same since nonlinearity is very mild and the noise-induced channel estimation error is more dominant. But as INR increases beyond 30 dB, the proposed channel estimator (case 6) shows substantial performance gains. This verifies importance of incorporating nonlinearity in the channel estimation. For both perfect and practical (imperfect) channel estimation cases, the performance gain of the proposed RAS power estimator is larger at higher INR where nonlinearity is more severe.

TABLE I
Cases used in simulation

Case	Ch.Est.	Interference Cancellation	RAS Estimation
1	PCE	(3)	basic (4)
2	PCE	(20) (21)	basic (4)
3	PCE	(70)/ (71)	basic (4)
4	LE	(20) (21)	Proposed
5	PCE	(20) (21)	Proposed
6	UKF	(20) (21)	Proposed

Fig. 4 shows more results on how the channel estimation accuracy in terms of the channel estimation block length affects the RAS power estimation performance. The normalized error decreases as the block length (thus the channel estimation accuracy) increases, as expected. Up to a relatively strong INR of 40 dB, the rate factor of this performance improvement is the same as that of the block length increase. However, beyond 40 dB INR, the same relationship between performance improvement and the block length increase does not hold especially for the 10^5 block length and the reason is that the contribution of the cross correlation term to the RAS signal power estimation based on a window of 10^7samples (1 sec.) becomes more dominant than the contribution of the channel estimation errors obtained with the 10^5 block length. This behavior indicates that at a very strong INR, there may be a practical trade-off between the channel estimation complexity and RAS power estimation performance improvement when the channel estimation block length is designed.

In Fig. 5, we compare performance of different channel estimation methods (the last four cases) in a $L=6$ taps Rayleigh fading channel with an exponential power delay profile. The channel is quasi-static over a block of 10^4 samples and SNR is set at -20 dB. The results verify for a frequency-

$$\boldsymbol{\chi}_i(k-1|k-1)=\begin{cases}\hat{\mathbf{x}}(k-1|k-1) & i=0\\ \hat{\mathbf{x}}(k-1|k-1)+\left(\sqrt{N\mathbf{P}_{\mathbf{xx}}(k-1|k-1)}\right)_i & 1\le i\le N\\ \hat{\mathbf{x}}(k-1|k-1)-\left(\sqrt{N\mathbf{P}_{\mathbf{xx}}(k-1|k-1)}\right)_{i-N} & N+1\le i\le 2N\end{cases} \quad (57)$$

selective interference channel the effectiveness of our method (case 6 using the UKF for the channel estimation and the nonlinear RAS signal power estimator) over case 3 and case 4, especially at a strong INR region.

V. Conclusions

We have explored a scenario where both CWC and RAS systems simultaneously access the spectrum for some duration of time, and RAS is equipped with RFI cancellation. Our study shows that strong RFI causes nonlinear distortions and severely degrades the performance of the conventional RAS signal power estimator. We have developed an auxiliary receiver assisted nonlinear-RFI cancellation scheme and a modified RAS signal power estimator for the scenario with strong RFI. Our simulation results clearly demonstrate substantial performance advantage of the proposed approach over the existing approaches, and the need for incorporating nonlinear system model into the channel estimation, interference cancellation, and the RAS signal power estimation.

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References

[1] ITU-R, "Protection creteria used for radio astronomical measurements," ITU, Recommendation RA.769-2, Mar. 2003.

[2] ITU, "Radio regulations articles," ITU, Tech. Rep., 2016.

[3] H. Minn, Y. R. Ramadan, and Y. Dai, "A new shared spectrum access paradigm between cellular wireless communications and radio astronomy," in *IEEE GLOBECOM*, 2016.

[4] W. A. Baan, "RFI mitigation in radio astronomy," in *General Assembly and Scientific Symposium, 2011 XXXth URSI*. IEEE, Oct. 2011.

[5] J. M. Ford and K. D. Buch, "RFI mitigation techniques in radio astronomy," in *IEEE IGARSS*, 2014.

[6] ITU-R, "Techniques for mitigation of radio frequency interference in radio astronomy," ITU, Tech. Rep. RA.2126-1, Nov. 2013.

[7] C. Barnbaum and R. Bradley, "A new approach to interference excision in radio astronomy: Real-time adaptive cancellation," *The Astronomical Journal*, pp. 2598–2614, Nov. 1998.

[8] A. J. Poulsen, et al., "Programmable real-time cancellation of GLONASS interference with the green bank telescope," *The Astronomical Journal*, pp. 2916–2927, Dec. 2005.

[9] F. H. Briggs, et al., "Removing radio interference from contaminated astronomical spectra using an independent reference signal and closure relations," *The Astronomical Journal*, pp. 3351–3361, Dec. 2000.

[10] S. Julier, et al., "A new approach for filtering nonlinear systems," in *IEEE American Control Conf.*, June 1995.

[11] S. J. Julier and J. K. Uhlmann, "New extension of the Kalman filter to nonlinear systems," in *AeroSense*, 1997.

[12] S. Julier and J. Uhlmann, "Unscented filtering and nonlinear estimation," *Proc. of the IEEE*, vol. 92, 2004.

[13] M. E. Tiuri, "Radio astronomy receivers," *IEEE Trans. Military Electronics*, vol. 8, July 1964.

[14] F. J. Casas, et al., "Simple nonlinearity evaluation and modeling of low-noise amplifiers with application to radio astronomy receivers," *Review of Scientific Instruments*, vol. 81, 2010.

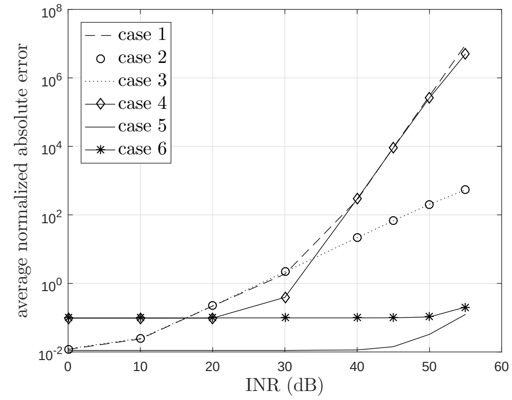

Fig. 3. Performance comparison of different RFI cancellation and RAS signal power estimation methods (SNR=−30 dB)

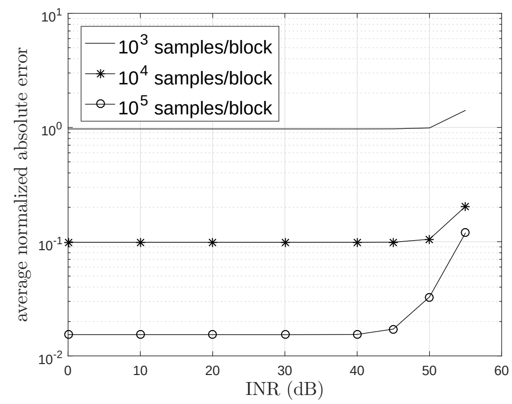

Fig. 4. Effect of channel estimation block length of the proposed estimator (case 6) on the RAS signal power estimation performance (SNR=−30 dB)

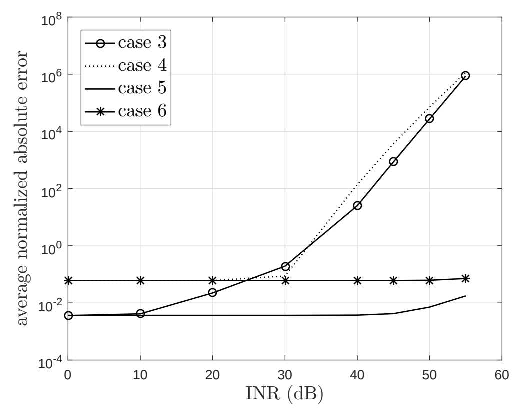

Fig. 5. Effects of different channel estimation methods in the multipath fading channel on the RAS power estimation performance (SNR=−20 dB)

$$\begin{aligned} w_{\mathrm{opt}}(k) =& \alpha h_{r,k} + \beta\frac{P_{\mathrm{I}}}{2}\Big(h_{r,k}^3 + h_{r,k}h_{i,k}^2 + 3h_{r,k}\sum_{n=0,\ n\neq k}^{L-1} h_{r,n}^2 + 3h_{r,k}\sum_{l=0}^{L-1} h_{i,l}^2 + 2h_{i,k}\sum_{n=0,\ n\neq k}^{L-1} h_{r,n}h_{i,n} + h_{r,k}\sum_{n=0,\ n\neq k}^{L-1} h_{i,n}^2\Big) \\ &+ \beta\frac{P_{\mathrm{I}}}{2}\Big(+h_{r,k}\sum_{l=0}^{L-1} h_{r,l}^2 - 2h_{i,k}\sum_{l=0}^{L-1} h_{r,l}h_{i,l}\Big) + j\Big(\alpha h_{i,k} + \beta\frac{P_{\mathrm{I}}}{2}\Big(h_{i,k}^3 + h_{i,k}h_{r,k}^2 + 3h_{i,k}\sum_{n=0,\ n\neq k}^{L-1} h_{i,n}^2\Big)\Big) \\ &+ j\beta\frac{P_{\mathrm{I}}}{2}\Big(3h_{i,k}\sum_{l=0}^{L-1} h_{r,l}^2 + 2h_{r,k}\sum_{n=0,\ n\neq k}^{L-1} h_{i,n}h_{r,n} + h_{i,k}\sum_{n=0,\ n\neq k}^{L-1} h_{r,n}^2 + h_{i,k}\sum_{l=0}^{L-1} h_{i,l}^2 - 2h_{r,k}\sum_{l=0}^{L-1} h_{i,l}h_{r,l}\Big) \end{aligned} \tag{71}$$