A Novel OFDMA Ranging Method Exploiting Multiuser Diversity

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***Abstract*—This paper addresses initial ranging (uplink synchronization and power control) for TDD OFDMA systems. Exploiting the channel knowledge from the downlink channel together with initial power control, we develop a novel initial ranging method which provides multiuser diversity gain and significant power saving for the subscriber stations. We present new ranging signal designs which enable multiuser diversity gain and facilitate new efficient low-complexity algorithms for multiuser ranging signal detection, timing estimation and power estimation. The advantages of the proposed approach over existing methods in terms of the ranging signal detection performance, the number of ranging frames required to finish the ranging process (latency at the network entry), the timing and power estimation performance, the ranging transmission power saving at the subscriber stations, and the complexity saving at the base station are illustrated by analytical and simulation results.**

***Index Terms*—Ranging, OFDMA, signal design, multiuser diversity, power control, synchronization, multiuser detection.**

I. INTRODUCTION

ORTHOGONAL Frequency Division Multiple Access (OFDMA) has recently drawn much attention (e.g., 802.16a/e[1][2]). In OFDMA systems, uplink (UL) synchronization is accomplished by initial ranging process. In the current initial ranging scheme defined in IEEE 802.16a/e[1][2], each Ranging Subscriber Station (RSS) first attains downlink (DL) synchronization and UL transmission parameters from DL control frames, and then it randomly chooses one of the available ranging codes and transmits it twice in a phase-continuous manner over two consecutive symbols on a single ranging channel that consists of 144 subcarriers[1] specified in the UL-MAP message. After detecting ranging codes and extracting the information of timing and power from the received ranging signal, the Base Station (BS) broadcasts a ranging response message that advertises the detected ranging code and the ranging time-slot where the ranging code has been identified. The ranging response message also contains all the needed adjustment (e.g., timing and power adjustment) and a status notification (e.g., success, continue). If an RSS receives the success notification, its ranging process is complete. Otherwise the RSS continues the ranging process until it receives the success notification.

There are several OFDMA uplink synchronization approaches [3]–[8] in the literature but they are not formulated in the framework of a ranging process. [3]–[5] only deal with frequency synchronization because they consider quasi-synchronized systems where no timing synchronization is required. The methods in [6] and [7] both assume uplink signals are transmitted over disjoint subcarriers, and their receivers use a bank of bandpass filters to separate the users' signals. They cannot be applied to a scenario where multiple users share the same subchannel. In [6], after users separation, the timing offset of each user is estimated by utilizing the correlation introduced by the cyclic prefix. This timing estimator's performance degrades considerably as the number of subcarriers in one subband becomes smaller or the timing offsets become large as in systems with large cell-size. The synchronization technique in [7] requires a very complicated two-dimensional grid search over the set spanned by all possible timing and frequency offset trial values. The method in [8] assumes all other users are perfectly synchronized to the BS except one new unsynchronized user. This assumption does not fully represent a practical situation. From the perspectives of initial ranging, existing uplink synchronization methods have one or more of the following limitations/drawbacks: (i) not applicable for large timing offsets (large cell-size), (ii) idealistic assumption of system conditions, (iii) unsuitable for shared ranging subchannels, (iv) no power control involved, (v) no multiuser detection involved, (vi) large complexity, and (vii) lack of multiuser diversity exploitation.

There also exist a few papers addressing initial ranging in OFDMA systems. The ranging methods proposed in [9] and [10] both use a frequency domain correlator bank to detect the ranging codes and estimate the timing offsets. The method in [9] first pre-computes phase-compensated ranging codes to account for the effects of timing offsets. For a timing offset range of $[0, d_{\max}]$ samples, each of the N_c ranging codes gives $(d_{\max} + 1)$ phase-compensated codes. Each of these $N_c(d_{\max} + 1)$ phase-compensated codes is cross-correlated with the received frequency-domain symbols on the ranging channel in every ranging time slot to detect the codes and estimate the timing offsets. The method in [10] applies the approach in [9] only to the ranging time slots where the presence of ranging codes have been identified by an energy detector. The method in [11] uses a similar cross-correlation approach but in the time domain. A common disadvantage of these existing ranging methods is the high computational complexity (see Section VI). By means of a ranging signal design, the ranging method in [12] achieves low-complexity ranging signal detection. However, its iterative timing esti-

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[1]36 non-contiguous blocks with 4 contiguous subcarriers per block.

mator requires high complexity. In addition, these existing ranging methods have not exploited multiuser diversity.

In this paper, we develop an initial ranging method that takes advantage of multiuser diversity in OFDMA UL. We present new ranging signal designs which provide multiuser diversity gain and facilitate efficient low-complexity algorithms for multiuser ranging signal detection, timing offset estimation and power estimation. In the proposed approach, each RSS obtains channel knowledge from the DL channel in a Time-Division Duplexing (TDD) system, chooses the best ranging subchannel, and applies initial power control to yield multiuser diversity gain and power saving. Exploiting the multiuser diversity and initial power control, we also develop efficient low-complexity signal processing algorithms for ranging. The proposed method achieves significant gains in ranging performance, power saving, and complexity reduction over the existing methods while using the same amount of time and frequency resources.

The rest of the paper is organized as follows. Section II describes the system and signal model. Section III presents the proposed ranging signal design and Section IV describes the proposed ranging method. In Section V, the theoretical analysis of the variance of timing offset estimation, the variance of power estimation, and the average ranging transmission power are given. Section VI presents the computation complexity comparisons. Simulation results are presented in Section VII, and conclusions are drawn in Section VIII.

II. System Description and Signal Model

Consider a UL OFDMA system with N_r RSSs, N_d Data Subscriber Stations (DSSs) and N subcarriers. After excluding DC and null subcarriers, the remaining subcarriers are grouped into Q_r ranging subchannels and Q_d data subchannels. The numbers of left and right null subcarriers are denoted by γ_{LN} and γ_{RN}, respectively. Each ranging subchannel has γ_r subcarriers and each data subchannel has γ_d subcarriers. The indices of the subcarriers corresponding to the q-th ranging subchannel ($q \in \{0, \ldots, Q_r - 1\}$) and the q_d-th data subchannel ($q_d \in \{0, \ldots, Q_d - 1\}$) are denoted as $\mathcal{J}_q$ and $\mathcal{K}_{q_d}$, respectively. One UL frame consists of M OFDM symbols where predefined subcarriers over M_r OFDM symbols are allocated for the ranging. The ranging time slot index is denoted by t ($t \in \{0, 1, \ldots, \frac{M_r}{M_1} - 1\}$), where M_1 is the number of OFDM symbols in one ranging time slot. The indices of OFDM symbols in one ranging time slot are denoted by m_1 ($m_1 \in \{0, 1, \ldots, M_1 - 1\}$).

In the subcarrier domain at the m-th OFDM symbol interval, the length-N transmit ranging code vector for the i-th RSS and the length-N transmit data vector for the j-th DSS are denoted by $\mathbf{X}_{i,r}^{(m)}$ and $\mathbf{X}_{j,d}^{(m)}$, respectively. The corresponding k-th elements ($k \in \{0, \ldots, N - 1\}$), provided that the i-th RSS transmits during t-th ranging time slot, are given by

$$\mathrm{X}_{i,r}^{(m)}(k) = \begin{cases} A_{i,r} C_{i,r}^{(m_1)}(l), & k \in \mathcal{J}_q(l); \\ & m = M_1 \cdot t + m_1; \\ & l = 0, \ldots, \gamma_r - 1; \\ 0, & \text{otherwise}, \end{cases} \quad (1)$$

$$\mathrm{X}_{j,d}^{(m)}(k) = \begin{cases} A_{j,d} C_{j,d}^{(m)}(l), & k \in \mathcal{K}_{q_d}(l); \\ & m = 0, \ldots, M - 1; \\ & l = 0, \ldots, \gamma_d - 1; \\ 0, & \text{otherwise}, \end{cases} \quad (2)$$

where $C_{i,r}^{(m_1)}(l)$ and $C_{j,d}^{(m)}(l)$ are ranging and data symbols, respectively, with $|C_{i,r}^{(m_1)}(l)| = E[|C_{j,d}^{(m)}(l)|^2] = 1$, and $\{A_{i,r}, A_{j,d} > 0\}$ are amplitude scaling factors at the RSS and DSS transmitters. Denote the unitary N-point inverse discrete Fourier transform (IDFT$_N$) of $\mathbf{X}_{i,r}^{(m)}$ and $\mathbf{X}_{j,d}^{(m)}$ by $[x_{i,r}^{(m)}(0), \ldots, x_{i,r}^{(m)}(N-1)]^T$ and $[x_{i,d}^{(m)}(0), \ldots, x_{i,d}^{(m)}(N-1)]^T$. After cyclic prefix (CP) insertion, the time domain signal samples of the i-th RSS are given by

$$x_{i,r}(n) = \begin{cases} x_{i,r}^{(m)}(l - N_g), & n = m(N + N_g) + l; \\ & m = 0, \ldots, M_r - 1; \\ & l = 0, \ldots, N + N_g - 1; \\ 0, & \text{otherwise}, \end{cases} \quad (3)$$

where $\{x_{i,r}^{(m)}(-l) : l = 1, \ldots, N_g\}$ represent CP samples and hence, $x_{i,r}^{(m)}(-l) = x_{i,r}^{(m)}(N-l)$. Note that $\mathrm{X}_{i,r}^{(m)}(k)$ is nonzero only for the corresponding ranging subcarriers and ranging time slot, and $\{x_{i,r}(n)\}$ are all zeros outside the corresponding ranging time slot. Similarly, the time domain signal samples of the j-th DSS are given by

$$x_{j,d}(n) = \begin{cases} x_{j,d}^{(m)}(l - N_g), & n = m(N + N_g) + l; \\ & m = 0, \ldots, M - 1; \\ & l = 0, \ldots, N + N_g - 1; \\ 0, & \text{otherwise}. \end{cases} \quad (4)$$

Recall that RSSs transmit ranging signals only during the M_r OFDM symbols allocated for ranging out of M symbols, while DSSs can transmit data during the whole UL frame.

We consider independent multi-path Rayleigh fading channels with slow fading (In developing the proposed ranging method, we assume the channel is quasi-static from the last OFDM symbol of the DL frame to the last OFDM symbol of the ranging allocation in the UL frame. But in our simulation we also consider time-varying channels during the above mentioned interval.). The sample-spaced channel impulse response taps (including the transmit and receive filters) for the i-th RSS/DSS are denoted by $\{h_{i,\star}(l), l = 0, \ldots, L-1\}$, where L is the channel length.[2] The channel output samples for the i-th RSS/DSS are given by

$$y_{i,\star}(n) = \sum_{l=0}^{L-1} h_{i,\star}(l)\; x_{i,\star}(n - l - d_{i,\star}) \quad (5)$$

where $d_{i,r}$ ($d_{i,d}$) is the transmission delay for the i-th RSS (DSS). The time domain received signal at the BS is

$$y(n) = \sum_{i=0}^{N_r-1} y_{i,r}(n) + \sum_{j=0}^{N_d-1} y_{j,d}(n) + \omega(n) \quad (6)$$

where $\{\omega(n)\}$ are independent and identically-distributed (i.i.d.), circularly-symmetric complex Gaussian noise samples with zero mean and variance $\sigma_\omega^2 = E[|\omega(n)|^2]$.

[2]In the rest of the paper, the subscript $\star$ denotes r for RSS or d for DSS.

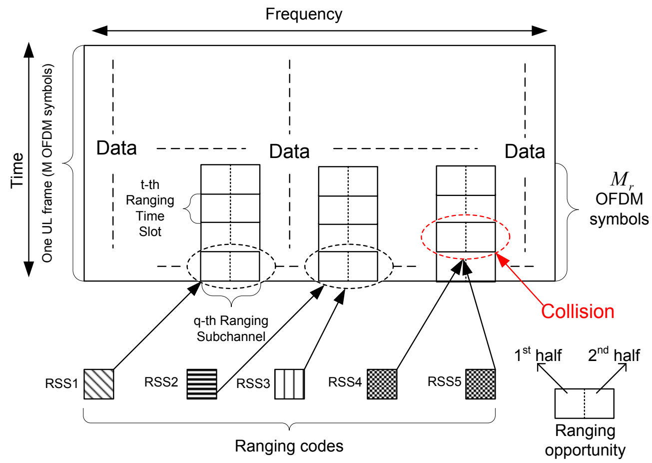

Fig. 1. Illustration of ranging resource structuring.

III. Proposed Ranging Signal Designs

The underlying rationale/principles of our ranging signal designs are multiuser diversity exploitability, multiuser separability/detectability in frequency-selective channels with frequency offsets and possibly large timing offsets, and low complexity implementation. We consider an OFDMA system with Q_r ranging subchannels over M_r OFDM symbols (i.e., M_r/M_1 ranging time-slots of M_1 OFDM symbols each). A ranging signal is constructed from one of the N_c orthogonal codes $\{C_c(l) : c, l = 0, 1, \ldots, N_c - 1\}$ (e.g., BPSK modulated Walsh codes) of length-N_c each, where c is the ranging code index and l is the orthogonal code element index. This allows N_c RSSs to share the same ranging subchannel and ranging time-slot. Hence, the total number of ranging opportunities is $N_{\text{total}} = Q_r \cdot N_c \cdot \frac{M_r}{M_1}$ for the proposed designs. Fig. 1 illustrates the proposed ranging resource structuring. Detailed characteristics will be discussed in the following.

For multiuser diversity exploitability, each ranging subchannel is composed of γ_r adjacent subcarriers over M_1 OFDM symbols (one ranging time slot) and the ranging subchannels are spread out over the whole frequency band with approximately equal spacing. The subcarrier assignment for the q-th ranging subchannel is defined as[3]

$$\mathcal{J}_q = \{\gamma_{LN} + q \cdot \Delta + l : l = 0, \ldots, \gamma_r - 1\} \qquad (7)$$

where $\Delta = \lfloor \frac{N_{\text{used}} - \gamma_r}{Q_r - 1} \rfloor$, $N_{\text{used}} = \gamma_r Q_r + \gamma_d Q_d$, and $\lfloor x \rfloor$ denotes the largest integer not greater than x. Note that $\{\mathcal{J}_q\}$ are disjoint for different values of q. Since the channels of different RSSs are independent, designing each RSS to pick its best ranging subchannel instead of a randomly chosen one out of Q_r subchannels does not increase the collision probability while providing an SNR gain for all RSSs. This is known as a multiuser diversity in a distributed access manner, which we incorporate into our ranging process.

For low complexity implementation, the proposed designs introduce a frequency domain repetition of the chosen orthogonal code within the chosen ranging subchannel and time-slot. In other words, the first half and the second half of the ranging subchannel contain the same length-N_c ranging code (see Fig. 1). A timing offset introduces linear phase shifts across the subcarriers and the above frequency domain repetition facilitates an easy capture of this phase shift and hence a very low complexity timing offset estimation.

Regarding multiuser separability/detectability, effects of timing offsets, frequency offsets, and channel frequency selectivity need to be considered. RSSs using different ranging subchannels or ranging time-slots can easily be separated due to the use of disjoint resources. RSSs over the same ranging subchannel and time-slot rely on the code orthogonality for user separation. However, the received ranging codes are affected by the timing-offset-induced phase shifts and frequency-selective channel gains. The use of adjacent subcarriers provides better protection against frequency-offset-induced intercarrier interferences from DSSs as well as code orthogonality degradation due to frequency selective channel gains. However, in systems with large cell-size, timing offsets of RSSs can be longer than CP. Large timing offsets introduce large linear phase shifts across subcarriers, degrade the orthogonality of the frequency-domain ranging codes, and may cause inter-symbol interferences as well. This degradation can be alleviated by transmitting the orthogonal ranging codes across time instead of across frequency since the timing-offset-induced phase shifts on the same subcarrier of adjacent OFDM symbols remain the same. But, for small timing offsets, frequency-domain ranging codes may be preferred over time-domain code due to smaller DSS interferences. Taking the above discussion into consideration, we propose three ranging signal designs for small, medium, and large cell-sizes. In the following signal designs, ν_1 and ν_2 are defined as $\nu_1 = 0, \ldots, \frac{\gamma_r}{2} - 1$ and $\nu_2 = \frac{\gamma_r}{2}, \ldots, \gamma_r - 1$, respectively.

Ranging Signal Design A: Under the condition of $N_g \geq d_{\max} + L - 1$ (i.e., small cell-size), we can use only one OFDM symbol per ranging time slot, i.e. $M_1 = 1$. Then, with $\gamma_r = 2N_c$, the ranging signal for the i-th RSS picking the c-th ranging code, the t-th ranging time slot and the q-th ranging subchannel can be expressed as

$$\mathrm{X}_{i,r}^{(m)}(k) = \begin{cases} A_{i,r} C_c(\nu_1), & k \in \mathcal{J}_q(\nu_1),\ m = t, \\ A_{i,r} C_c(\nu_2 - \frac{\gamma_r}{2}), & k \in \mathcal{J}_q(\nu_2),\ m = t, \\ 0, & \text{otherwise}. \end{cases} \qquad (8)$$

Ranging Signal Design B: For a medium or large cell-size, the timing offset of an unsynchronized RSS plus the channel length can be larger than the CP length, i.e. $d_{\max} + L - 1 > N_g$. In this case, two phase-continuous OFDM ranging symbols (i.e. $M_1 = 2$) are used to absorb large timing offsets as applied in IEEE 802.16a/e standard, and the ranging code detection is performed based on the second OFDM ranging symbol. The ranging signal B can be expressed for $m = 2t$ as

$$\mathrm{X}_{i,r}^{(m)}(k) = \begin{cases} A_{i,r} C_c(\nu_1) e^{-\frac{j2\pi \mathcal{J}_q(\nu_1) N_g}{N}}, & k \in \mathcal{J}_q(\nu_1) \\ A_{i,r} C_c(\nu_2 - \frac{\gamma_r}{2}) e^{-\frac{j2\pi \mathcal{J}_q(\nu_2) N_g}{N}}, & k \in \mathcal{J}_q(\nu_2) \\ 0, & \text{otherwise}, \end{cases} \qquad (9)$$

[3]All $\mathcal{J}_q$ can be shifted by a fixed amount as long as they are within the allowed subcarrier indexes.

and for $m = 2t+1$ as

$$\mathrm{X}_{i,r}^{(m)}(k) = \begin{cases} A_{i,r}C_c(\nu_1), & k \in \mathcal{J}_q(\nu_1) \\ A_{i,r}C_c(\nu_2 - \frac{\gamma_r}{2}), & k \in \mathcal{J}_q(\nu_2) \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

where $\gamma_r = 2N_c$. The phase rotations in the $(2t)$th symbol interval are to maintain the phase continuity of the time domain ranging signal between $(2t)$th and $(2t+1)$th symbols.

Ranging Signal Design C: To provide greater robustness against the channel delay spread and the timing offset (for a large cell-size), we propose a modified ranging signal design by only using 2 adjacent subcarriers in a ranging subchannel, i.e. $\gamma_r = 2$, and by moving the domain of our orthogonal ranging code design from the frequency domain to the time domain. To absorb large timing offsets, each ranging code is transmitted over two adjacent OFDM symbols in a phase-continuous manner. Hence, the number of ranging OFDM symbols per ranging time slot is $M_1 = 2N_c$. The ranging signal C can be expressed as

$$\mathrm{X}_{i,r}^{(m)}(k) = \begin{cases} A_{i,r}C_c(l)e^{-\frac{j2\pi\mathcal{J}_q(\gamma)N_g}{N}}, & l = 0,\ldots,N_c-1, \\ & k \in \mathcal{J}_q,\ m = 2l, \\ & \gamma = \{0,1\} \\ A_{i,r}C_c(l), & l = 0,\ldots,N_c-1, \\ & k \in \mathcal{J}_q,\ m = 2l+1 \\ 0, & \text{otherwise.} \end{cases} \quad (11)$$

The phase term for the $(2l)$th symbol is to maintain the phase continuity of the time domain ranging signal.

The channel responses within a ranging subchannel of the proposed scheme (e.g., $\gamma_r = 8$ adjacent subcarriers in our Design A/B or $\gamma_r = 2$ in our Design C for an OFDMA system with $N = 1024$ subcarriers) are almost the same. Hence, the proposed ranging signals still maintain orthogonality at the BS (under negligible frequency offsets), while the ranging signals from 802.16a/e suffer from loss of orthogonality due to the channel frequency selectivity.

IV. The Proposed Ranging Method

We consider a TDD OFDMA system, and we assume all RSSs first perform frequency synchronization based on the downlink frame and using an appropriate method (e.g., [13], [14]). Hence, there are only small residual frequency offsets in the uplink. Our proposed method first performs multi-user ranging signal detection, and then estimates timing offset for each detected RSS by incorporating the ranging signal detection results. Next, by combining the ranging signal detection results and the timing offsets estimation results, the received power of each detected RSS is estimated. The proposed ranging process is summarized in the following:

1) Each RSS estimates its channel power gains across the band by utilizing the DL preamble and the pilots embedded in the subsequent OFDM symbols (Note that this channel estimation provides channel information across the entire band because the pilots are spread out over the entire band). According to the estimated channel power gains, the RSS picks the ranging subchannel with the largest channel power gain, and adjusts the transmission power (i.e., adjusts $A_{i,r}$) so that the received ranging signal power per subcarrier approximately equals to the target received ranging signal power per subcarrier P_r. If the adjusted transmission power per subcarrier is greater than the maximum allowable transmission power per subcarrier $P_{\max}$, then the RSS does not initiate the ranging process. As user channels are independent, ranging subchannel selection rules will not affect collision probability, while choosing the best ranging subchannel provides each RSS an additional SNR advantage known as multiuser diversity.

2) Each RSS randomly chooses one of the N_c orthogonal ranging codes and one of the $\frac{M_r}{M_1}$ ranging time slots, and then it transmits the ranging signal over the selected ranging subchannel in the selected ranging time slot.

3) At the BS, the receiver identifies whether or not a ranging code is transmitted without any collision, and extracts the timing and power information based on the detection results. For each detected active RSS, the BS compares estimated values of the timing offset and power with the ranging requirements. If they satisfy the requirements, then the BS informs the RSS that its ranging process is successful. If not, the BS sends timing and power adjustment parameters and a "continue" status notification to the RSS. The ranging response message from the BS also contains the code, the ranging subchannel, and the time slot where the corresponding RSS is detected. The BS need not send the collision status notification to the collided users.

4) When the RSSs receive the ranging response message, the RSSs with "success" status complete the ranging process, while each RSS with "continue" status adjusts the timing accordingly and then repeats the step 1) and step 2). The RSSs which do not find their ranging information in the ranging response message (i.e., the RSSs with collision) repeat the step 1) and step 2) to re-initiate the ranging process in the next frame or according to the truncated exponential back-off.

When an RSS cannot detect the DL control channel, or the ranging transmit power needed is larger than the maximum transmit power limit, the situation can be reflected in the received signal strength indicator at the RSS. This will alert the user to move around to get a good channel to be able to start its ranging process.

Due to the space limitation, we only present the derivations of the ranging signal detection, timing offset estimation and power estimation based on Design B. Those for Design C can be straightly obtained.

A. Multi-User Ranging Signal Detection

At the BS after the CP removal and the N-point DFT operation, we obtain the ISI-free k-th subcarrier symbol at the t-th ranging time slot as

$$Y^{(2t+1)}(k) = \sum_{i=0}^{N_r-1} X_{i,r}^{(2t+1)}(k)H_{i,r}(k) + \sum_{j=0}^{N_d-1} X_{j,d}^{(2t+1)}(k)H_{j,d}(k) + W^{(2t+1)}(k) \quad (12)$$

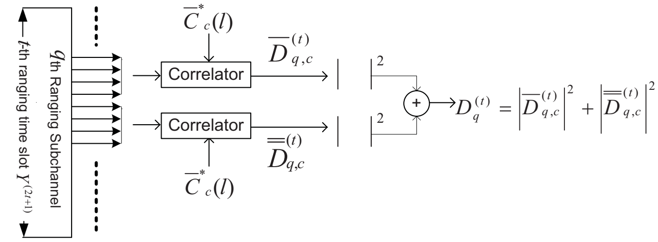

Fig. 2. The block diagram of the proposed ranging signal detector.

where

$$H_{i,\star}(k) = e^{\frac{j2\pi k d_{i,\star}}{N}} \sum_{l=0}^{L-1} h_{i,\star}(l)\ e^{\frac{-j2\pi lk}{N}} \triangleq e^{\frac{j2\pi k d_{i,\star}}{N}} \widetilde{H}_{i,\star}(k) \tag{13}$$

$$W^{(2t+1)}(k) = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} \omega((2t+1)(N+N_g)+N_g+l)\ e^{\frac{-j2\pi lk}{N}}. \tag{14}$$

The ranging code detector for every q-th ranging subchannel and t-th ranging time slot is shown in Fig. 2, where the BS correlates the first half and the second half of the frequency domain symbols with fixed-phase-compensated c-th ranging code $\bar{C}_c(l) = C_c(l)\ e^{\frac{j2\pi \mathcal{J}_q(l)\frac{d_{\max}}{2}}{N}}$ as

$$\bar{D}^{(t)}_{q,c} = \sum_{l=0}^{\frac{\gamma_r}{2}-1} Y^{(2t+1)}(\mathcal{J}_q(l))\ \bar{C}^*_c(l) \tag{15}$$

$$\bar{\bar{D}}^{(t)}_{q,c} = \sum_{l=\frac{\gamma_r}{2}}^{\gamma_r-1} Y^{(2t+1)}(\mathcal{J}_q(l))\ \bar{C}^*_c(l-\frac{\gamma_r}{2}). \tag{16}$$

The above fixed-phase-compensation is equivalent to shifting the range of the possible timing offsets from $\{0, 1, \ldots, d_{\max}-1\}$ to $\{-\frac{d_{\max}}{2}, \ldots, 0, \ldots, \frac{d_{\max}}{2}-1\}$, which in turn reduces the detrimental effect of timing offsets in ranging performance. For simplicity, we drop the ranging code index c and the ranging time slot index t. Then the decision variable D_q for the RSS detection at the q-th ranging subchannel is defined as

$$D_q = |\bar{D}_q|^2 + |\bar{\bar{D}}_q|^2 \tag{17}$$

which will be compared with predefined threshold(s) to decide "no RSS detected", "1 RSS detected", or "collision occurred".

In developing decision thresholds for our RSS detector, we consider only 0, 1 or 2 RSSs for each ranging opportunity (i.e., 0, 1, or 2 RSSs using the same ranging code, ranging subchannel, and ranging time slot), since the probability of 3 or more RSSs choosing the same ranging opportunity is small. The expressions of $\bar{D}_q$ and $\bar{\bar{D}}_q$ for 0, 1 or 2 RSSs cases are given below, where $(\cdot)$ denotes $\big(\mathcal{J}_q(l)\big)$:

$$\bar{D}_q = \begin{cases} \sum_{l=0}^{\frac{\gamma_r}{2}-1} \widetilde{W}^{(2t+1)}(\cdot), 0 \text{ RSS} \\ \sum_{l=0}^{\frac{\gamma_r}{2}-1} \Big[A_{i,r}H_{i,r}(\cdot) + \widetilde{W}^{(2t+1)}(\cdot)\Big], 1 \text{ RSS} \\ \sum_{l=0}^{\frac{\gamma_r}{2}-1} \Big[A_{i,r}H_{i,r}(\cdot) + A_{j,r}H_{j,r}(\cdot) + \widetilde{W}^{(2t+1)}(\cdot)\Big], 2 \text{ RSSs} \end{cases} \tag{18}$$

$$\bar{\bar{D}}_q = \begin{cases} \sum_{l=\frac{\gamma_r}{2}}^{\gamma_r-1} \widetilde{W}^{(2t+1)}(\cdot), 0 \text{ RSS} \\ \sum_{l=\frac{\gamma_r}{2}}^{\gamma_r-1} \Big[A_{i,r}H_{i,r}(\cdot) + \widetilde{W}^{(2t+1)}(\cdot)\Big], 1 \text{ RSS} \\ \sum_{l=\frac{\gamma_r}{2}}^{\gamma_r-1} \Big[A_{i,r}H_{i,r}(\cdot) + A_{j,r}H_{j,r}(\cdot) + \widetilde{W}^{(2t+1)}(\cdot)\Big], 2 \text{ RSSs.} \end{cases} \tag{19}$$

Then we have

$$D_q = \begin{cases} \Big|\sum_{l=0}^{\frac{\gamma_r}{2}-1} \widetilde{W}^{(2t+1)}(\cdot)\Big|^2 + \Big|\sum_{l=\frac{\gamma_r}{2}}^{\gamma_r-1} \widetilde{W}^{(2t+1)}(\cdot)\Big|^2, 0 \text{ RSS} \\ \Big|\sum_{l=0}^{\frac{\gamma_r}{2}-1} \big[|A_{i,r}H_{i,r}(\cdot)|e^{j\theta} + \widetilde{W}^{(2t+1)}(\cdot)\big]\Big|^2 \\ + \Big|\sum_{l=\frac{\gamma_r}{2}}^{\gamma_r-1} \big[|A_{i,r}H_{i,r}(\cdot)|e^{j\theta} + \widetilde{W}^{(2t+1)}(\cdot)\big]\Big|^2, 1 \text{ RSS} \\ \Big|\sum_{l=0}^{\frac{\gamma_r}{2}-1} \big[|A_{i,r}H_{i,r}(\cdot)|(e^{j\theta_1} + e^{j\theta_2}) + \widetilde{W}^{(2t+1)}(\cdot)\big]\Big|^2 \\ + \Big|\sum_{l=\frac{\gamma_r}{2}}^{\gamma_r-1} \big[|A_{i,r}H_{i,r}(\cdot)|(e^{j\theta_1} + e^{j\theta_2}) + \widetilde{W}^{(2t+1)}(\cdot)\big]\Big|^2, 2 \text{ RSSs,} \end{cases} \tag{20}$$

where θ_1 and θ_2 are independent and uniformly distributed over $[0, 2\pi)$ and $\widetilde{W}^{(2t+1)}(\cdot)$ is i.i.d. zero-mean complex Gaussian random variable with a variance σ_w^2. Each RSS adjusts its transmission power to compensate for the power loss due to channel and hence $|A_{i,r}H_{i,r}(\mathcal{J}_q(l))|^2 \cong P_r$, $\forall i$.

The case of "no ranging code detected", "1 ranging code detected", or "collision occurred" can be denoted by the number of detected RSSs n_{RSS} on the same ranging opportunity, which is detected according to the maximum a posteriori probability (MAP) criterion as

$$n_{\text{RSS}} = \arg\max_{n=0,1,2}\{p_n \cdot f_{D_q}(D_q \mid n \text{ RSSs})\} \tag{21}$$

where $\{p_n\}$ representing the probability of n RSSs in the same ranging opportunity can be replaced with pre-defined design values, and the f_{D_q} representing the probability density function (PDF) of D_q can be derived as

$$f_{D_q}(x) = \begin{cases} \frac{1}{4\sigma^4}\ x\ e^{-\frac{x}{2\sigma^2}}, 0 \text{ RSS} \\ \frac{1}{2\sigma^2}\left(\frac{x}{s_1^2}\right)^{\frac{1}{2}} e^{-\frac{s_1^2+x}{2\sigma^2}} I_1\left(\frac{\sqrt{x}s_1}{\sigma^2}\right), 1 \text{ RSS} \\ \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} \frac{1}{2\sigma^2}\left(\frac{x}{s_2^2}\right)^{\frac{1}{2}} e^{-\frac{s_2^2+x}{2\sigma^2}} I_1\left(\frac{\sqrt{x}s_2}{\sigma^2}\right) d\theta_1 d\theta_2, 2 \text{ RSSs,} \end{cases} \tag{22}$$

where $I_1()$ is the first order modified Bessel function of the first kind, $\sigma^2 = \frac{\gamma_r}{4}\sigma_w^2$, $s_2^2 = 4\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)s_1^2$, and $s_1^2 = \Big|\sum_{l=0}^{\frac{\gamma_r}{2}-1} A_{i,r}H_{i,r}(\mathcal{J}_q(l))\Big|^2 + \Big|\sum_{l=\frac{\gamma_r}{2}}^{\gamma_r-1} A_{i,r}H_{i,r}(\mathcal{J}_q(l))\Big|^2 \cong 2(\frac{\gamma_r}{2})^2 P_r$. See Appendix for the derivations of the PDFs.

Fig. (3) shows $p_n \cdot f_{D_q}(D_q)$ for 0, 1 and 2 RSSs, from which the detection thresholds can be determined as

$$n_{\text{RSS}} = \begin{cases} 0, & \text{if } D_q < \eta_0 \\ 1, & \text{if } \eta_1 < D_q < \eta_2 \\ 2, & \text{if } (\eta_0 \le D_q \le \eta_1) \text{ or } (D_q \ge \eta_2) \end{cases} \tag{23}$$

where $\{\eta_i\}$ are the crossover points of $\{p_n \cdot f_{D_q}(D_q \mid n \text{ RSSs})\}$. In fact, we just need to check for $n_{\text{RSS}} = 1$ case which provides detected ranging codes, associated ranging subchannels and time-slots. In other words, we just need η_1 and η_2 for the RSS detection.

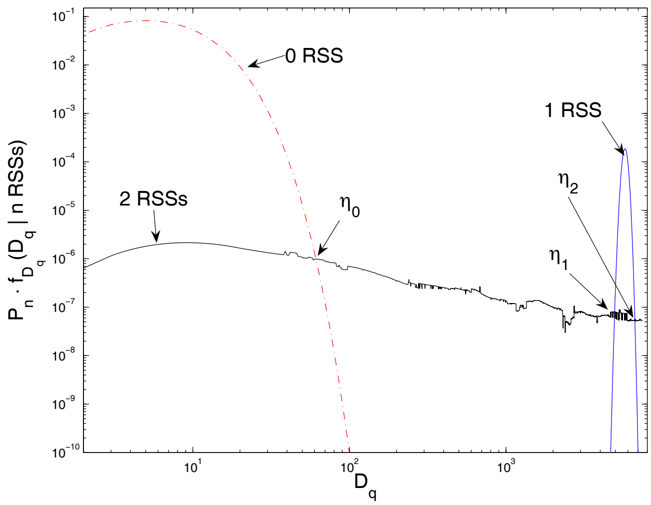

Fig. 3. The PDFs of the ranging code detection decision variable.

In an alternative approach, we may assume that there is only 0 or 1 RSS in each ranging opportunity. Then, we have

$$n_{\text{RSS}} = \begin{cases} 0, & \text{if } D_q < \eta \\ 1, & \text{else} \end{cases} \tag{24}$$

where η is the crossover point of $p_0 \cdot f_{D_q}(D_q \mid 0 \text{ RSS})$ and $p_1 \cdot f_{D_q}(D_q \mid 1 \text{ RSS})$.

B. Timing Offset Estimation

After the ranging signal detection, timing offsets can be estimated by incorporating the detection results. Consider a collision-free ($n_{\text{RSS}} = 1$) detected RSS, say i-th RSS with corresponding ranging channel response $\widetilde{H}_{i,r}(\mathcal{J}_q(l))$. $\bar{D}_q$ and $\bar{\bar{D}}_q$ for this i-th RSS can be approximated by

$$\bar{D}_q \simeq B_1 \sum_{l=0}^{\frac{\gamma_r}{2}-1} e^{\frac{j2\pi(\epsilon_i - \frac{d_{\max}}{2})\mathcal{J}_q(l)}{N}} + \widetilde{W}_{1,i} \tag{25}$$

$$\bar{\bar{D}}_q \simeq B_2\, e^{\frac{j2\pi(\epsilon_i - \frac{d_{\max}}{2})\frac{\gamma_r}{2}}{N}} \sum_{l=0}^{\frac{\gamma_r}{2}-1} e^{\frac{j2\pi(\epsilon_i - \frac{d_{\max}}{2})\mathcal{J}_q(l)}{N}} + \widetilde{W}_{2,i} \tag{26}$$

where $B_1 \cong B_2 = A_{i,r}\widetilde{H}_{i,r}(\mathcal{J}_q(l))$, ϵ_i is the timing offset of the i-th RSS, and $\{\widetilde{W}_{1,i}, \widetilde{W}_{2,i}\}$ are i.i.d. zero-mean complex Gaussian random variables with a variance of $\sigma_w^2\gamma_r/2$. Since $B_1 \cong B_2$, the timing offset (an integer in the unit of samples[4]) for the i-th RSS can be estimated as

$$\hat{\epsilon}_i = \text{round}\Big\{\frac{N}{\pi\gamma_r} \cdot \text{angle}\Big\{\frac{\bar{\bar{D}}_q}{\bar{D}_q}\Big\} + \frac{d_{\max}}{2}\Big\}. \tag{27}$$

Since $0 \le \epsilon_i \le d_{\max}$, for occasional cases of $\hat{\epsilon}_i > d_{\max}$, we can set $\hat{\epsilon}_i = d_{\max}$. Note that the above timing estimator provides a significant complexity reduction if compared to the existing OFDMA ranging timing estimators in [9]–[11].

At the first ranging trial, all RSSs have positive timing offset values (i.e., $0 \le \epsilon_i \le d_{\max}$) due to the transmission/propagation delays in both DL and UL. To maintain positive timing offsets within the range $0 \le \epsilon_i \le d_{\max}$ (to avoid ISI[15]) in the following ranging trials while taking into account timing offset estimation errors, the BS can modify the timing offset estimate as $\hat{\epsilon}_i - \lambda$ when informing back to the RSSs. Note that λ is a pre-defined positive value (e.g., $\lambda = 5$) which can be set around the standard deviation of the adopted timing estimation.

[4]Non-integer part of the delay is absorbed in the channel response.

C. RSS Power Estimation

From the multi-user ranging signal detection results, we know the number of codes detected on each ranging subchannel in each ranging time slot. If there is only one code used on a ranging subchannel in a ranging time slot, then we can use the estimated timing offset of the corresponding RSS to improve the power estimation performance.

The power estimator for the considered RSS is given by

$$\hat{P} = \frac{(\bar{P}_q - \frac{2\sigma_w^2}{\gamma_r}) + (\bar{\bar{P}}_q - \frac{2\sigma_w^2}{\gamma_r})}{2} \tag{28}$$

where the noise variance σ_w^2 is assumed to be known (can be measured or estimated), $\bar{P}_q$ and $\bar{\bar{P}}_q$ are the average received power per subcarrier of the first half and the second half of the ranging subchannel, respectively. They are defined as

$$\bar{P}_q = \left| \frac{2}{\gamma_r} \sum_{l=0}^{\frac{\gamma_r}{2}-1} Y^{(2t+1)}(\mathcal{J}_q(l)) C_c^*(l) e^{\frac{-j2\pi\hat{\epsilon}_i\mathcal{J}_q(l)}{N}} \right|^2 \tag{29}$$

$$\bar{\bar{P}}_q = \left| \frac{2}{\gamma_r} \sum_{l=\frac{\gamma_r}{2}}^{\gamma_r-1} Y^{(2t+1)}(\mathcal{J}_q(l)) C_c^*(l - \frac{\gamma_r}{2}) e^{\frac{-j2\pi\hat{\epsilon}_i\mathcal{J}_q(l)}{N}} \right|^2 \tag{30}$$

if only one ranging code is detected on a ranging subchannel in a ranging time slot, and

$$\bar{P}_q = \left| \frac{2}{\gamma_r} \bar{D}_q \right|^2 \tag{31}$$

$$\bar{\bar{P}}_q = \left| \frac{2}{\gamma_r} \bar{\bar{D}}_q \right|^2 \tag{32}$$

if there are more than one distinct ranging codes detected on a ranging subchannel in a ranging time slot.

V. Theoretical Analysis

A. Variance of Timing Offset Estimation

The proposed timing estimator is given by $\hat{\epsilon} = \text{round}(\epsilon_c)$ where $\epsilon_c = \frac{N}{\pi\gamma_r}\psi + \frac{d_{\max}}{2}$ and ψ is the phase angle between two vectors $\bar{D}_q$ and $\bar{\bar{D}}_q$ defined as:

$$\psi = \text{angle}\Big\{\frac{\bar{\bar{D}}_q}{\bar{D}_q}\Big\}. \tag{33}$$

The PDF of the phase angle between two vectors perturbed by Gaussian noise was derived in [16]. Applying this result, we obtain

$$p_\psi(\psi) = \frac{1}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-[U - U\cos(\Delta\Phi - \psi)\cos(\theta)]} \cdot [1 + U + U\cos(\Delta\Phi - \psi)\cos(\theta)]\cos(\theta)d\theta \tag{34}$$

where $U = |A_{i,r}H_{i,r}(\mathcal{J}_q(l))|^2 / \sigma_w^2$ and $\Delta\Phi \approx \frac{2\pi(\epsilon_i - \frac{d_{\max}}{2})\frac{\gamma_r}{2}}{N}$ is the phase angle between the two vectors in the absence of noise. The PDF of ϵ_c is given by

$$p_{\epsilon_c}(\epsilon_c) = \frac{\pi\gamma_r}{N}\ p_\psi\Big(\frac{\pi\gamma_r}{N}(\epsilon_c - \frac{d_{\max}}{2})\Big). \tag{35}$$

Then the probability mass function (PMF) of the timing estimate $\hat{\epsilon}$ is given as

$$P_{\hat{\epsilon}}(\hat{\epsilon}) = \begin{cases} \int_{\hat{\epsilon}-0.5}^{\hat{\epsilon}+0.5} p_{\epsilon_c}(\epsilon_c)d\epsilon_c, & 0 \le \hat{\epsilon} < d_{\max} \\ \int_{d_{\max}}^{\infty} p_{\epsilon_c}(\epsilon_c)d\epsilon_c, & \hat{\epsilon} = d_{\max} \end{cases} \tag{36}$$

where we have incorporated that the timing offset is not larger than $d_{\max}$. The variance of the timing offset estimation can be readily computed by using (36).

B. Variance of Power Estimation

From (28), we know that $\hat{P} = \frac{2}{\gamma_r^2}D_q - \frac{2}{\gamma_r}\sigma_w^2$. The PDF of D_q for one RSS in one ranging opportunity case is non-central Chi-square PDF with 4 degrees of freedom as given in (22). So the mean of the power estimation is

$$E[\hat{P}] = \frac{2}{\gamma_r^2}E[D_q] - \frac{2}{\gamma_r}\sigma_w^2 = \frac{2}{\gamma_r^2}(4\sigma^2 + s_1^2) - \frac{2}{\gamma_r}\sigma_w^2 = P_r \tag{37}$$

and the variance of power estimation $\sigma_{\hat{P}}^2$ can be calculated as

$$\sigma_{\hat{P}}^2 = \frac{4}{\gamma_r^4}\ \sigma_{D_q}^2 = \frac{4}{\gamma_r^4}\ (8\sigma^4 + 4\sigma^2 s_1^2) = \frac{2\sigma_w^4}{\gamma_r^2} + \frac{2\sigma_w^2 P_r}{\gamma_r}. \tag{38}$$

C. Average Ranging Transmission Power

Each RSS picks the best ranging subchannel based on the DL channel estimation. This provides multiuser diversity gain since different RSSs have independent channels. The corresponding advantage in term of transmission power saving is investigated in the following. In our proposed method, the system has Q_r ranging subchannels and each ranging subchannel is composed of γ_r adjacent subcarriers. For analytical tractability, in this section we consider a multi-path Rayleigh fading channel with Q_r sample-spaced i.i.d. taps (having a uniform power delay profile). The proposed equi-spaced ranging subchannels can be approximately modeled as Q_r cyclically equi-spaced ranging subchannels, i.e., $\mathcal{J}_q(0) = \frac{N}{Q_r}q$. Let $H_{i,k}$ denote the channel gain of the i-th RSS ($i = 0, 1, \ldots, N_r - 1$) on the k-th ($k = 0, 1, \ldots, N-1$) subcarrier. $\{H_{i,k}\}$ are circularly-symmetric complex Gaussian random variables with zero mean and unit variance. Then the average channel power gain of the q-th ranging subchannel for the i-th RSS is

$$G_{i,q} = \frac{\sum_{k\in\mathcal{J}_q} |H_{i,k}|^2}{\gamma_r} \cong |H_{i,\frac{N}{Q_r}q}|^2, q = 0, 1, \ldots, Q_r - 1. \tag{39}$$

From (39), we find that $\{G_{i,q}, q = 0, 1, \ldots, Q_r - 1\}$ are approximately i.i.d. exponentially-distributed random variables with unit mean. The cumulative distribution function (CDF) of $G_{i,q}$ is given by

$$F_G(g) = (1 - e^{-g})\ u(g) \tag{40}$$

where $u(g)$ is the unit step function. In the proposed ranging method, the RSS picks the best ranging subchannel based on its channel power gain $G_{i,q}$ in (39). Define

$$Z_i = \max\{G_{i,q} : q = 0, 1, \ldots, Q_r - 1\}. \tag{41}$$

Then the CDF of Z_i is

$$F_{Z_i}(z) = (1 - e^{-z})^{Q_r}. \tag{42}$$

Each i-th RSS adjusts its transmission power $P_{\text{tx},i}$ such that

$$P_{\text{tx},i}Z_i = P_r \tag{43}$$

where the P_r is the target received ranging signal power per subcarrier of an RSS at the BS. From (42) and (43), we obtain the PDF of $P_{\text{tx},i}$ as

$$f_{P_{\text{tx},i}}(p) = \frac{P_r}{p^2}Q_r\ (1 - e^{-\frac{P_r}{p}})^{Q_r-1}e^{-\frac{P_r}{p}}. \tag{44}$$

Then the mean of the ranging transmission power is

$$\begin{aligned} E[P_{\text{tx},i}] &= \int_0^{P_{\max}} p\ f_{P_{\text{tx},i}}(p)\ dp \\ &= P_r Q_r \sum_{q=0}^{Q_r-1} \binom{Q_r - 1}{q}(-1)^q E_1\left(\frac{P_r\cdot(q+1)}{P_{\max}}\right) \end{aligned} \tag{45}$$

where $P_{\max}$ is the maximum allowable transmission power per subcarrier of an RSS and E_1 is the exponential integral defined by $E_1(x) = \int_x^\infty e^{-t}t^{-1}dt$.

For the case without exploiting multiuser diversity, i.e. the RSS randomly picks the ranging subchannel, the PDF of $P_{\text{tx},i}$ can be derived as

$$f_{P_{\text{tx},i}}(p) = \frac{P_r}{p^2}\ e^{-\frac{P_r}{p}}. \tag{46}$$

Then the mean of the ranging transmission power is

$$E[P_{\text{tx},i}] = \int_0^{P_{\max}} p\ f_{P_{\text{tx},i}}(p)\ dp = P_r\ E_1\left(\frac{P_r}{P_{\max}}\right). \tag{47}$$

VI. Computation Complexity

We first compare the computational complexities of ranging codes detection and timing estimation of the two ranging methods. The maximum possible delay $d_{\max,\text{r}}$ is set to 112. For the reference method, $\gamma_r = 144$, $N_c = 128$, so it requires $(\gamma_r + 1)N_c(d_{\max,\text{r}} + 1) = 2,097,280$ complex multiplications, $(\gamma_r - 1)N_c(d_{\max,\text{r}} + 1) = 2,068,352$ complex additions, and $N_c(d_{\max,\text{r}} + 1) = 14,464$ compare operations. For Design B, $\gamma_r = 8$, $N_c = 4$, and $Q_r = 18$. So the proposed method needs $(\frac{\gamma_r}{2} + 2)Q_rN_c = 432$ complex multiplications, $(\gamma_r - 2)Q_rN_c = 432$ complex additions, $Q_rN_c = 72$ compare operations, $Q_rN_c = 72$ angle operations, and $Q_rN_c = 72$ real addition. The overall complexity ratio is 3870 and hence the proposed method achieves a tremendous complexity saving. The complexity of the reference method increases with the maximum possible delay $d_{\max,\text{r}}$, while the complexity of the proposed method is not affected by $d_{\max,\text{r}}$. For power estimation, we only need 2 real multiplication and 1 real addition per each detected ranging code for the case of more than one distinct ranging codes detected on the same ranging subchannel in the same ranging slot, because we can use the output of the ranging code detector to estimate the power. For the case of only one ranging code detected on the same ranging subchannel in the same ranging time slot, we need $(\gamma_r + 2) = 10$ complex multiplication, $(\gamma_r - 2) = 6$ complex addition and 1 real addition for each detected ranging code.

TABLE I
SIMULATION PARAMETERS

Carrier frequency	2.5 GHz	
System channel BW	10 MHz	
Sampling frequency	11.2 MHz	
FFT size N	1024	
CP length N_g	128	
BS-to-BS distance	3 km	10 km
Maximum RSS timing offset $d_{\max,\mathrm{r}}$	112 samples	384 samples
Maximum DSS timing offset $d_{\max,\mathrm{d}}$	32 samples	
Sub-carrier spacing	10.94kHz	
No. of guard subcarriers, left	92	
No. of guard subcarriers, right	91	
No. of used subcarriers	840	
No. of OFDM symbols in one frame	48	
DL/UL Partition	29 : 18 (1 TTG)	
No. of OFDM symbols allocated for ranging	8	
Frame duration	5 ms	
OFDM symbol duration	102.9 μs	
Modulation	BPSK for RSS	
	QPSK for DSS	
No. of data subcarriers per subchannel	48	
Channel models[21]	Pedestrian B and Vehicular A	
Mobile speed in Pedestrian B (Ped B) channel	3 km/hr	
Mobile speed in Vehicular A (Veh A) channel	60km/hr	
Timing requirement	32 Samples	
Power requirement	9.4 dB[22]	
Residual normalized frequency offsets	$[-0.02, 0.02]$	

VII. SIMULATION RESULTS AND DISCUSSION

A. Simulation Parameters

For comparison, the ranging code detector and timing estimator in [9] with the ranging signals from the IEEE 802.16e are used as the reference method. The system specification is based on mobile WiMAX system profiles specified by WiMAX Forum [17], [18]. The system simulation parameters are summarized in Table I.

The system parameters are the same for the proposed method and the reference method. The difference is the ranging channel setup. Q_r is equal to 18 for Design B and 72 for Design C. For the reference method, each RSS transmits one selected CDMA code and its gradual phase rotated version[5] on the single ranging channel that consists of 144 non-contiguous subcarriers over two consecutive symbols. The number of available ranging codes for the reference method is 256. The code set is composed of four categories: initial ranging codes, periodic ranging codes, bandwidth requests codes, and handover ranging codes. We assume 128 codes are assigned for the initial ranging. Since there is no power estimator provided in [9], we assume perfect power estimation for the reference method and hence its performance presented will be slightly optimistic. The arrival of new RSSs follows Poisson arrival model, and the collided RSSs retry their ranging in the next frame. The full-buffer traffic model is assumed for DSS, i.e. the DSS always has data to transmit.

For the reference method, the initial ranging transmission power on each ranging subcarrier is set 10dB above the noise power (σ_w^2) per subcarrier. So the total ranging transmission power over 144 ranging subcarriers normalized by σ_w^2 is 10 $\log_{10}(1440)$ dB. In the proposed method, the target total received ranging signal power of an RSS is set the same as the statistical average of the total received initial ranging signal power of an RSS in the reference method, i.e., $\frac{\gamma_r P_r}{\sigma_w^2} = 1440$.

[5]To maintain phase continuity between these two OFDM ranging symbols.

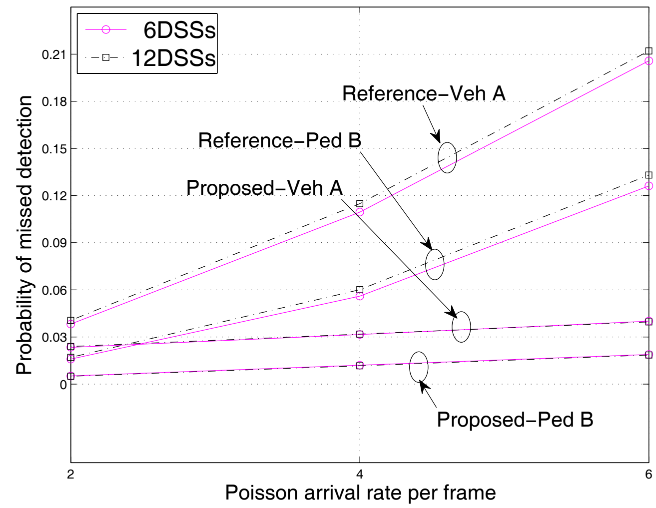

Fig. 4. The probabilities of missed detection at the first ranging frame.

If we consider collision-free condition only, i.e., using $p_0 \cdot f_{D_q}(D_q \mid 0 \text{ RSS})$ and $p_1 \cdot f_{D_q}(D_q \mid 1 \text{ RSS})$ with $p_0 = 0.90$ and $p_1 = 0.10$, the threshold normalized by σ_w^2 can be obtained as $\eta = 1475$. For the case with collision, the PDFs of $\{p_n \cdot f_{D_q}(D_q \mid n \text{ RSSs})\}$ with $p_0 = 0.8941$, $p_1 = 0.1004$ and $p_2 = 0.0054$ are shown in Fig. 3. The crossover points of these curves give the thresholds normalized by σ_w^2 as $\eta_0 = 65$, $\eta_1 = 4925$ and $\eta_2 = 6690$. The p_n is designed by assuming 32 RSSs initiating the ranging simultaneously. We use one threshold $\eta = 1475$ in our simulations since p_2, the probability of 2 RSSs in the same ranging opportunity, is small.

The probabilities of missed detection P_{miss} is defined as $E[\frac{D_m}{N_r}]$ where D_m is the number of RSSs whose transmitted ranging signals are not detected at the BS and N_r is the total number of RSSs. The probabilities of false alarm P_{false} is defined as $E[\frac{D_a}{N_{\text{total}} - N_r}]$ where D_a is the number of ranging signals which are detected at the BS but are not transmitted from any RSSs. The true timing offsets for RSSs and DSSs are taken randomly from the interval $[0, d_{\max,\mathrm{r}}]$ and $[0, d_{\max,\mathrm{d}}]$, respectively. The frequency offsets (normalized by the subcarrier spacing) of the RSSs are modeled as i.i.d. uniform random variables over the range of $[-0.02, 0.02]$.

B. Simulation Results and Discussions

The proposed designs outperform the reference method. Due to the space limitation, the discussions and simulation results of Design A are omitted, but can be found in [19]. We mainly present the simulation results for Design B on Ped B and Veh A channels, which use the same simulation environments as in WiMAX system evaluation [20]. The simulation results of timing offset estimation are presented for both Design B and Design C on both Ped B and Veh A channels.

Fig. 4 and Fig. 5, respectively, show the probabilities of missed detection P_{miss} and false alarm P_{false} in medium cell size environment on both Ped B and Veh A channels. The proposed method performs much better than the reference method. The reference method's performances substantially degrade as the Poisson arrival rate increases.

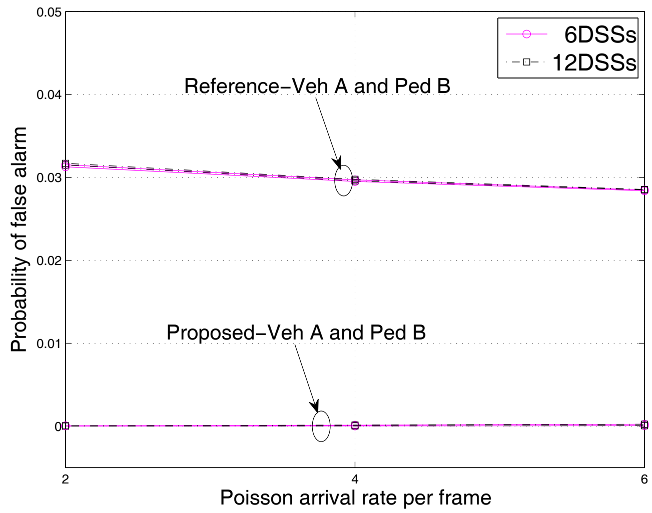

Fig. 5. The probabilities of false alarm at the first ranging frame.

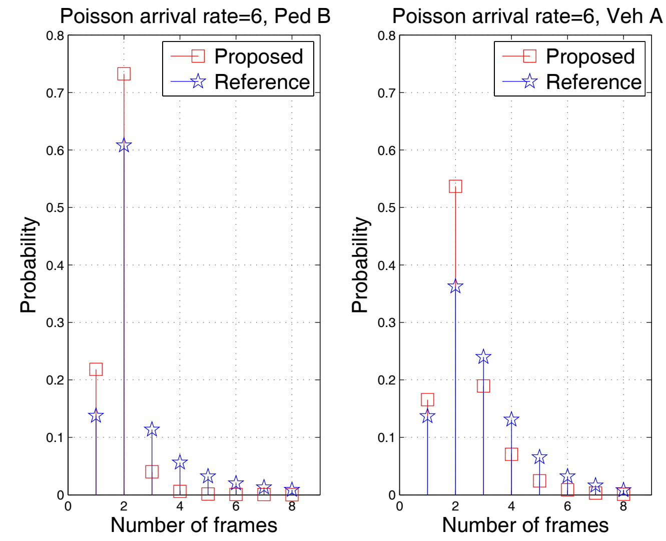

Fig. 6. The probability distribution of the number of frames required for a successful ranging.

Fig. 6 shows the probability distribution of the number of frames required for an RSS to complete a successful ranging. In Ped B channel with the Poisson arrival rate of 6 per frame, the simulation results show that about 95% of the RSSs in the proposed method (74% in the reference method) can finish the ranging process in two frames. In Veh A channel, the percentage of RSSs that can finish ranging in two frames is smaller due to the fast time varying channel. About 70% of the RSSs in the proposed method (50% in the reference method) can finish the ranging process in two frames.

Fig. 7 presents the advantage of the proposed method over the reference method in terms of the average number of frames (related to the average latency at the network entry) required to complete a successful ranging versus the Poisson arrival rate per frame in the presence of 6 DSSs and 12 DSSs on both Ped B and Veh A channels. Both methods are insensitive to DSS interference, but the average number of ranging frames required for the reference method increases with the Poisson arrival rate per frame, while the corresponding performance of the proposed method is robust to RSS interference.

The standard deviations of the timing offset estimation versus the Poisson arrival rate per frame for both Design B and Design C are presented in Fig. 8 for Ped B channel, and in Fig. 9 for Veh A channel with 6 DSSs and 12 DSSs. The simulation results show that both methods are robust to the DSS interference. The proposed timing estimation method performs better in Veh A channel than in Ped B channel due to the smaller coherence loss of the adjacent subcarrier channel gains. The theoretical lower bound of the standard deviation of the proposed timing offset estimation (evaluated by using (36)) is also included.

Fig. 10 and Fig. 11 compare the ranging transmission energies normalized by the noise energy contained on one subcarrier in one OFDM symbol duration. Fig. 10 shows the average normalized ranging transmission energy per ranging frame for an RSS. Fig. 11 presents the average total normalized ranging transmission energy required for an RSS to finish its ranging process. The proposed method cuts down the ranging energy consumption at an RSS to approximately $\frac{2}{5}$ of that required in the reference method. In addition to the per-frame energy saving, the proposed method requires a smaller number of frames to complete the ranging process which yields further energy saving. For example, for the Poisson arrival rate equal to 6, the proposed method requires around $\frac{1}{4}$ of the ranging energy used in the reference method. We also include the theoretical lower bound of the average normalized transmission energy for the proposed method as given in (45). The close match between simulation and theoretical results also validates our theoretical analysis of the ranging transmission power in Section V-C. The theoretical lower bound of the average normalized transmission energy for the case of no multiuser diversity as given in (47) is about 12 times more than that of the proposed method. This illustrates the multiuser diversity gain of the proposed method.

The simulation results demonstrate that the proposed method substantially outperforms the reference method in all considered aspects. The smaller number of ranging frames required to complete a successful ranging can be translated into smaller latency at the network entry, larger ranging transmission energy saving for a successful ranging, and significant complexity saving. Furthermore, the difference of the average number of ranging frames required for a successful ranging becomes larger when the number of the RSSs increases. So in the hot spot service areas, the advantages of the proposed method will be more significant. On the average, each RSS gets benefits of one frame delay reduction, corresponding throughput gain, and associated energy saving. When taking into account all users, the system-wide advantage would be quite appreciable. We have assumed perfect power estimation for the reference method. In practical scenarios without this assumption, the proposed method will yield even larger gains than those illustrated in this paper.

VIII. Conclusions

We have presented new signal designs and algorithms for initial uplink ranging process in small, medium and large

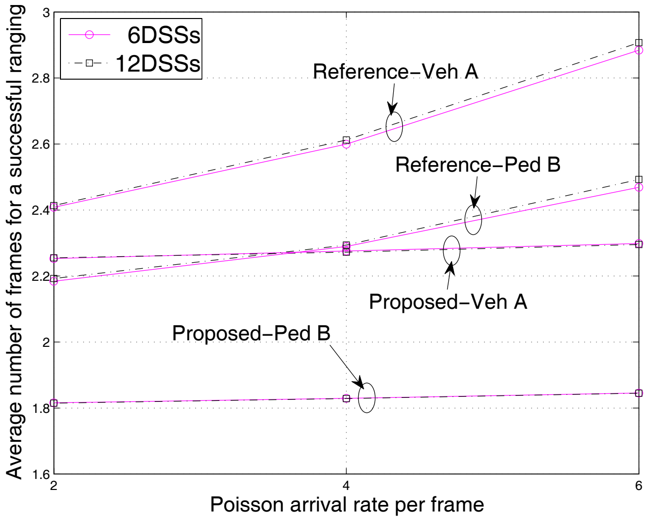

Fig. 7. Average number of ranging frames required for a successful ranging.

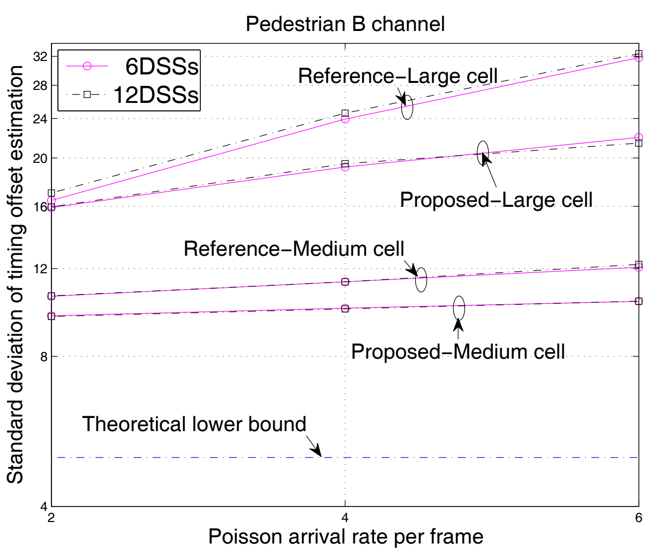

Fig. 8. Timing offset estimation performance in pedestrian B channel.

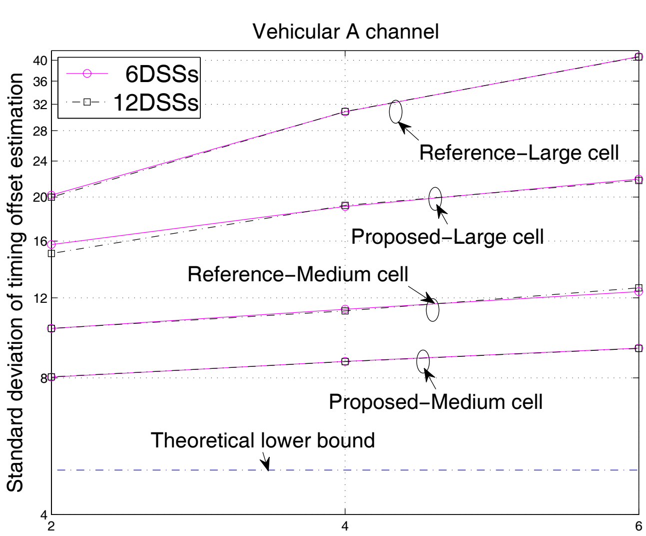

Fig. 9. Timing offset estimation performance in vehicular A channel.

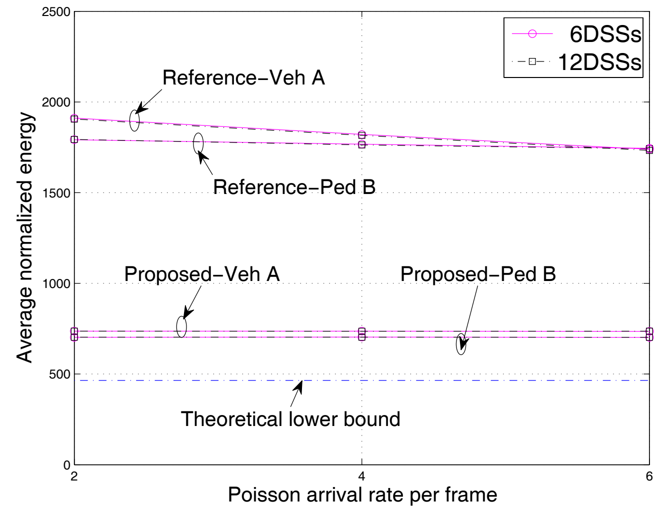

Fig. 10. Average normalized ranging transmission energy per frame.

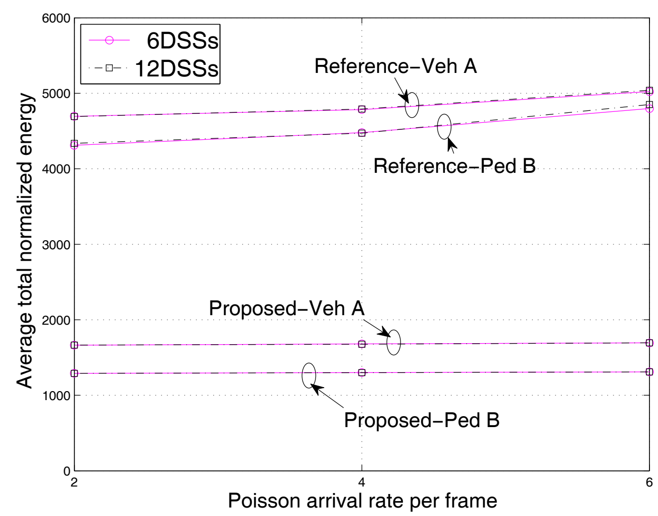

Fig. 11. Average total normalized ranging transmission energy required for a successful ranging.

cell-size OFDMA systems. Several reasons can be ascribed to the advantages of our proposed approach. First of all, our ranging signal designs provide more robustness against multiple access interference (MAI), while the reference method is vulnerable to MAI because the orthogonality of the frequency domain CDMA codes is distorted in the presence of multipath distortion. Secondly, our proposed method exploits multiuser diversity to achieve large energy saving and smaller latency in network entry. Third, the ranging signal designs facilitate efficient low complexity multiuser ranging signal detector, timing and power estimator. The analytical and simulation results corroborate that the proposed method achieves significant gains in ranging signal detection, synchronization, substantial energy saving at the subscriber stations, and huge complexity reduction over the existing methods while using the same amount of time and frequency resources.

APPENDIX

In this Appendix, we present the derivations of the PDFs of D_q in (22). For zero RSS in one ranging opportunity case, we know from (20) that the D_q has a central Chi-square PDF with 4 degrees of freedom as given in (22). Next, consider two RSSs in one ranging opportunity case. According to the (20), $|A_{i,r}H_{i,r}(\mathcal{J}_q(l))|(e^{j\theta_1}+e^{j\theta_2})+\widetilde{W}^{(2t+1)}(\mathcal{J}_q(l)) = B(e^{j\theta_1}+e^{j\theta_2})+W = B\cdot\alpha e^{j\beta}+W$, where $\alpha = \sqrt{(\cos\theta_1+\cos\theta_2)^2+(\sin\theta_1+\sin\theta_2)^2} = 2\cos\left(\frac{\theta_1-\theta_2}{2}\right)$, $\beta = \tan^{-1}\left(\frac{\sin\theta_1+\sin\theta_2}{\cos\theta_1+\cos\theta_2}\right)$. The PDF $f_{D_q}(D_q|(\theta_1,\theta_2))$ is the non-central Chi-square PDF with 4 degrees of freedom and the non-centrality parameter s_2^2 is given by

$$s_2^2 = \left|\sum_{l=0}^{\frac{\gamma_r}{2}-1} A_{i,r}H_{i,r}(\mathcal{J}_q(l))\cdot\alpha\right|^2 + \left|\sum_{l=\frac{\gamma_r}{2}}^{\gamma_r-1} A_{i,r}H_{i,r}(\mathcal{J}_q(l))\cdot\alpha\right|^2 = 4\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)s_1^2 \quad (48)$$

where s_1^2 is defined below (22). Then the PDF $f_{D_q}(D_q)$ for two RSSs in one ranging opportunity can be obtained by

$$f_{D_q}(D_q) = \int_0^{2\pi}\int_0^{2\pi} f_{D_q}(D_q|(\theta_1,\theta_2))f_{\theta_1}(\theta_1)f_{\theta_2}(\theta_2)d\theta_1 d\theta_2 \quad (49)$$

which yields (22). Applying a similar approach, we can easily obtain the PDF for one RSS case.

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