A Reduced Overhead OFDM Relay System with Clusterwise TMRC Beamforming

Baile Xie, *Student Member, IEEE*, Daniel Munoz, *Student Member, IEEE*, and Hlaing Minn, *Senior Member, IEEE*

***Abstract*—We consider a relay clustering system using decode and forward (DF) relaying, orthogonal frequency division multiplexing (OFDM), and transmit maximal ratio combining beamforming (TMRC-BF) at the relays. We address channel estimation at the destination for the above system. A cluster pairing scheme with corresponding pilot designs is proposed. It reduces both necessary pilot overhead and computational complexity by half. Simulation results show that if the signal to noise ratio (SNR) of pilots for channel estimation for TMRC-BF at the relays is kept above a threshold (5 dB below the SNR of the pilots sent to the destination by the cluster pairing strategy), nearly optimal channel estimation performance can be achieved.**

***Index Terms*—Relay, pilot designs, channel estimation, relay cluster pairing, TMRC-BF.**

I. INTRODUCTION

RELAY systems expand coverage ranges and provide diversity/throughput enhancements [1]–[5], yet often channel state information (CSI) is needed. Recently, relay clustering has been studied in [6], [7] to enhance system throughput and reduce outage probability, but CSI estimation with relay clustering has not been addressed in existing works on relay CSI acquisition [8]–[12]. For multiple-input single-output (MISO) systems, TMRC-BF is an effective diversity scheme to boost the transmit SNR [13]–[15]. By exploiting the knowledge of CSI at the transmitter, it coherently combines the channel gains and enhances transmission reliability. However, relay clustering combined with TMRC-BF has not been studied, and we address it here.

In this paper, we consider an OFDM-based DF relay system. TMRC-BF is used at the relays in order to improve reliability while relay clustering is employed for multiplexing gains. We develop CSI acquisition and pilot designs for the above system. In particular, we find that TMRC-BF causes the effective channel experienced by transmitted signal to be about twice as long and non-causal. Traditional CSI acquisition methods would need to significantly increase the pilot overhead to accurately estimate this effective channel. Inspired by the real-value property of the equivalent channel after TMRC-BF, we propose a new cluster pairing strategy with corresponding pilot designs. The new scheme reduces both pilot overhead and computational complexity by half.

Manuscript received July 4, 2012. The associate editor coordinating the review of this letter and approving it for publication was D. B. da Costa.

The authors are with the Department of Electrical Engineering, The University of Texas at Dallas (e-mail: {baile.xie, dan.munoz, hlaing.minn}@utdallas.edu).

Digital Object Identifier 10.1109/LCOMM.2012.101712.121450

II. SYSTEM MODEL AND PROPOSED STRATEGY

We consider a multiple relay OFDM system with a single source and single destination using time division duplexing. The DF relays ($R_k,\ k = 0, \ldots, \tilde{N}_R - 1$) each have N_{R_k} antennas, the source S has N_S antennas, and the destination (D) is with a single antenna. We use I or J to represent node (S, R_k, or D) and I_j to denote the jth antenna of node I. Each node has knowledge of the number of antennas at each node as well as the cluster structure. OFDM system has N subcarriers and a sufficient cyclic prefix (CP) is used to avoid inter-symbol interference (ISI). During the CSI acquisition and associated data transmission, the channels remain static. The sample-spaced channel impulse response (CIR) $\boldsymbol{h}_{I_j - J_m}$ between any transmit and receive antenna pair ($I_j - J_m$) has maximum length L. For instance, stacking the channels from all transmit antennas of S to R_{k_j}, we have $\boldsymbol{h}_{S-R_{k_j}} = \left[\boldsymbol{h}^T_{S_0-R_{k_j}} \boldsymbol{h}^T_{S_1-R_{k_j}} \ldots \boldsymbol{h}^T_{S_{N_S-1}-R_{k_j}}\right]^T \in \mathcal{C}^{LN_S \times 1}$ as the channel vector from S to the jth receive antenna of R_k. For a general link from all transmit antennas of node I to J_m, the time-domain received signal $\boldsymbol{y}_{J_m} \in \mathcal{C}^{N\times 1}$is

$$\boldsymbol{y}_{J_m} = (\boldsymbol{Q}_I + \boldsymbol{X}_I)\boldsymbol{h}_{I-J_m} + \boldsymbol{n}_{J_m}, \tag{1}$$

where $\boldsymbol{Q}_I = [\boldsymbol{Q}_{I_1}, \cdots, \boldsymbol{Q}_{I_j}, \cdots, \boldsymbol{Q}_{I_{N_I}}]$ is the circulant time domain pilot matrix. The first column of submatrix $\boldsymbol{Q}_{I_j} \in \mathcal{C}^{N\times L}$ contains the time domain pilot vector. $\boldsymbol{X}_I$ is the data matrix defined similar to $\boldsymbol{Q}_I \in \mathcal{C}^{N\times(LN_I)}$. $\boldsymbol{n}_{J_m}$ is zero-mean complex Gaussian noise vector with covariance matrix $\sigma^2_{J_m}\boldsymbol{I}$.

The relays are divided into M clusters, $\{C_l\ :\ l = 0, \ldots, M-1\}$. Each cluster forwards a subset of source data to D but not source pilots. Each relay sends its pilots to the destination. All clusters forward different subsets of source data simultaneously to the destination. Within each relay cluster, the relays transmit the same data using TMRC-BF to improve reliability. The relays perform error checks on decoded data (e.g., using cyclic redundancy check) to prevent error propagation. If errors are found, that relay will not transmit. The other relays in the cluster will still transmit. The destination will receive the data and pilots and perform its estimation and decoding the same as if all relays transmitted.

In the following, Section II-A explains how the relays obtain the channel estimates for TMRC-BF. Section II-B then shows the change in channel length due to TMRC-BF. In Section II-C, we design pilots for the TMRC-BF channel and propose a new coding scheme for TMRC-BF pilot transmissions to reduce the overhead.

A. Channel Estimation for TMRC-BF at the Relays

To perform TMRC-BF, the knowledge of $\{\boldsymbol{h}_{R_{k_j}-D}\}$ is required at $\{R_k\}$. For that, the destination will transmit $L^\dagger$

pilot tones to the relays. Building on the pilot designs in [16], the data and pilots are disjoint and $L^\dagger = 2^{\lceil \log_2(L) \rceil}$ pilots transmitted by the destination are cyclically equi-spaced in frequency and equi-powered to meet the optimal pilot conditions from [16]. R_k obtains estimate of $\boldsymbol{h}_{R_{k_j}-D}$ as

$$\hat{\boldsymbol{h}}_{R_{k_j}-D} = \left(\boldsymbol{Q}_D^H \boldsymbol{Q}_D\right)^{-1} \boldsymbol{Q}_D^H \boldsymbol{z}_{R_{k_j}}, \ \forall\, j \tag{2}$$

where $\boldsymbol{Q}_D \in \mathcal{C}^{N\times L}$ is the circulant matrix containing the pilots sent by D and $\boldsymbol{z}_{R_{k_j}} \in \mathcal{C}^{N\times 1}$ is the received signal vector at R_{k_j}. The MSE of $\hat{\boldsymbol{h}}_{R_{k_j}-D} \in \mathcal{C}^{L\times 1}$ is

$$\mathrm{MSE}_{\hat{\boldsymbol{h}}_{R_{k_j}-D}} = \sigma^2_{R_{k_j}} \operatorname{Tr}\left[(\boldsymbol{Q}_D^H \boldsymbol{Q}_D)^{-1}\right]. \tag{3}$$

Under the optimal condition $\boldsymbol{Q}_D^H \boldsymbol{Q}_D \propto \boldsymbol{I}$ [16], (2) and (3) can be straightly simplified. The frequency domain channel estimate is obtained as $\hat{\boldsymbol{H}}_{R_{k_j}-D} = \boldsymbol{F}_L \hat{\boldsymbol{h}}_{R_{k_j}-D}$ where $\boldsymbol{F}_L$ is the first L columns of the unitary DFT matrix $\boldsymbol{F}$.

B. Relay Clustering with TMRC-BF

With TMRC-BF, the effective channel length almost doubles, leading to pilot overhead increase for traditional estimation solutions. To see this, consider how TMRC-BF is implemented: The frequency domain transmitted pilots or data are multiplied by $\hat{H}^*_{R_{k_j}-D}[i]$, the complex conjugate of the estimate of the corresponding channel gain $H_{R_{k_j}-D}[i]$. The signal is then converted to the time domain with a CP insertion of L_{CP} ($\geq L-1$) samples. Then $\{R_{k_j}\}$ from cluster C_l simultaneously transmit their signals to the destination. Thus, the effective channel gain on subcarrier i, assuming negligible channel estimation error for $\hat{H}^*_{R_{k_j}-D}[i]$, is $H_{l,T}[i] = \sum_{R_{k_j}\in C_l} |H_{R_{k_j}-D}[i]|^2$, which is a real value. From the following Fourier transform relationship

$$H_{R_{k_j}-D}[i] H^*_{R_{k_j}-D}[i] \leftrightarrow \underbrace{h_{R_{k_j}-D}(n) \circledast h^*_{R_{k_j}-D}(-n)}_{g_{R_{k_j}-D}(n)}, \tag{4}$$

where $\circledast$ represents convolution, we have a non-causal equivalent channel $g_{R_{k_j}-D}(n)$ with $n \in [-L+1, -L+2, \ldots, L-1]$. Applying the cyclic convolution interpretation of the time domain OFDM signals and the effective CIR, this non-causal CIR of length $L_T = 2L-1$ can be rewritten as a causal CIR of length N as $\tilde{\boldsymbol{g}}_{R_{k_j}} \triangleq [g_{R_{k_j}-D}(0), \ldots, g_{R_{k_j}-D}(L-1), 0, \ldots, 0, g_{R_{k_j}-D}(-L+1)), \ldots, g_{R_{k_j}-D}(-1)]^T$. Then the effective causal CIR vector of C_l is $\tilde{\boldsymbol{g}}_l = \sum_{R_{k_j}\in C_l} \tilde{\boldsymbol{g}}_{R_{k_j}}$. Define

$$\boldsymbol{H}_{l,T} \triangleq \boldsymbol{F}\tilde{\boldsymbol{g}}_l = \boldsymbol{F}_T \boldsymbol{h}_{l,T}, \tag{5}$$

where $\boldsymbol{F}_T$ ($\boldsymbol{h}_{l,T}$ resp.) is composed of the first L and the last $L-1$ columns of $\boldsymbol{F}$ (elements of $\tilde{\boldsymbol{g}}_l$ resp.). Then the received signal at the destination corresponding to C_l, if channel estimation errors are negligible, can be given as

$$\boldsymbol{y}_D = (\boldsymbol{Q}_{C_l} + \boldsymbol{X}_{C_l})\boldsymbol{h}_{l,T} + \boldsymbol{n}_D, \tag{6}$$

where $\boldsymbol{Q}_{C_l} = \sqrt{N}\boldsymbol{F}^H \boldsymbol{\Lambda}_l \boldsymbol{F}_T$, and $\boldsymbol{\Lambda}_l$ is a diagonal matrix with the $N\times 1$ frequency domain pilot vector (see Section II-C for pilot design) on the diagonal. $\boldsymbol{X}_{C_l}$ represents the source data subset transmitted by C_l. (6) is similar to (1) but includes transmissions from all relays in cluster C_l.

C. Relay Cluster Pilot Design and Channel Estimation at D

Due to the application of TMRC-BF at the relays, changes to the pilot design are needed to properly account for the increase in the equivalent channel length.

1) Pilot Design for TMRC-BF Relay Clusters: We treat each cluster as a virtual antenna and apply a design from [16] (e.g., frequency division multiplexing (FDM), or code division multiplexing in the frequency domain (CDM-F)) with L_T replacing the role of L. Each cluster reserves the same M disjoint sets of comb-type subcarriers for pilot transmission while the other subcarriers are used for data. In FDM, each cluster transmits pilots on one set of subcarriers and different clusters use different sets. In CDM-F, each cluster uses all M sets for pilots that are coded across frequency to distinguish channels of different clusters. The optimal pilot condition $\boldsymbol{Q}_{C_l}^H \boldsymbol{Q}_{C_k} = E_{\mathrm{av}}\boldsymbol{I}\delta_{l,k}$ [16], where E_{av} is the average transmit pilot energy per antenna and $\delta_{l,k}$ is the Kronecker delta function, suggests each pilot set contain at least L_T cyclically equi-spaced and equi-powered pilot tones. However, as the second hop effective channel length for each cluster increases to $L_T = 2L-1$, which is not a power of 2, the optimal condition cannot be met with the minimal pilot overhead of ML_T tones. One option is that each set employs $L_T^\dagger = 2^{\lceil \log_2(L_T) \rceil}$ cyclically equi-spaced, equi-powered pilots to ensure the optimal pilot condition. The other option is each set uses L_T approximately cyclically equi-spaced, equi-powered pilots. The first and last two designs of Table I give examples for the first and second options, respectively.

2) Pilot Design for TMRC-BF Relays with Cluster Pairing: Recall that the effective channel $\boldsymbol{H}_{l,T}$ in (5) is real valued due to TMRC-BF ideally. Exploiting this fact to reduce overhead, we propose grouping the clusters into $\lfloor M/2 \rfloor$ pairs so that the combined effective channel of each pair spans two dimensions in the complex space. (For odd M, the last cluster represents an additional virtual antenna.) Regard each cluster pair as a virtual antenna and apply the pilot design from Section II-C1. Each relay's N_{R_k} antennas belong to the same cluster. Within a cluster pair n consisting of C_{2n} and C_{2n+1}, C_{2n} sends pilot a_i while C_{2n+1} transmits ja_i on subcarrier i where $|a_i|$ is constant for all pilot tone i of the cluster pair. This gives

$$\boldsymbol{Q}_{C_{2n+1}} = j\boldsymbol{Q}_{C_{2n}} \tag{7}$$

and under optimal condition we have $\boldsymbol{Q}_{C_{2n}}^H \boldsymbol{Q}_{C_{2l}} = E_{\mathrm{av}}\boldsymbol{I}\delta_{n,l}$. With perfect CSI at the relays, the corresponding frequency domain received signal on pilot subcarrier i at D is

$$\begin{aligned} Y_D[i] &= a_i(H_{2n,T}[i] + jH_{2n+1,T}[i]) + N[i] \\ &\triangleq a_i H_{n,\mathrm{pair}}[i] + N[i], \ \text{for } n = 1, \cdots, \lfloor M/2 \rfloor \end{aligned} \tag{8}$$

where $N[i]$ is Gaussian noise. Note that $\boldsymbol{h}_{n,\mathrm{pair}} = \boldsymbol{F}_T^H \boldsymbol{H}_{n,\mathrm{pair}}$ has length L_T. The total number of pilot tones used now reduces from ML_T to $\lceil M/2 \rceil L_T$ after pairing (or $ML_T^\dagger$ to $\lceil M/2 \rceil L_T^\dagger$). The last three designs in Table I show examples of the proposed cluster pair pilot designs. They reduce pilot overhead by half if compared to existing designs under the same system parameters.

3) Channel Estimation and Complexity: With perfect CSI at the relays, the effective channel for a relay cluster to the destination will be real-valued, while in practice errors in CSI

TABLE I
EXAMPLES OF PILOT SETS FOR TMRC-BF

Conditions	cluster n	Pilot Set ($\mathcal{J}_l$ = pilot tone index set for C_l and $c_l[k \in \mathcal{J}_l]$ = corresponding pilot sequence)
$N = 32$, $L = 2$, $N_R = 4$, FDM type, 2 relays per cluster using $L_T^\dagger$ pilot tones	0	$\mathcal{J}_0 = \{2, 10, 18, 26\}$, $c_0[k \in \mathcal{J}_0] = \{a_1, a_2, a_3, a_4\}$
	1	$\mathcal{J}_1 = \{3, 11, 19, 27\}$, $c_1[k \in \mathcal{J}_1] = \{b_1, b_2, b_3, b_4\}$
Same conditions as above, CDM-F type	0	$\mathcal{J}_0 = \{2, 3, 10, 11, 18, 19, 26, 27\}$, $c_0[k \in \mathcal{J}_0] = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$
	1	$\mathcal{J}_1 = \{2, 3, 10, 11, 18, 19, 26, 27\}$, $c_1[k \in \mathcal{J}_1] = \{a_1, -a_2, a_3, -a_4, a_5, -a_6, a_7, -a_8\}$
Same conditions as above, 1 cluster pair	0	$\mathcal{J}_0 = \{2, 10, 18, 26\}$, $c_0[k \in \mathcal{J}_0] = \{a_1, a_2, a_3, a_4\}$
	1	$\mathcal{J}_1 = \{2, 10, 18, 26\}$, $c_1[k \in \mathcal{J}_1] = \{ja_1, ja_2, ja_3, ja_4\}$
$N = 32$, $L = 3$, $N_R = 8$, FDM type, 2 cluster pairs, 2 relays per cluster using L_T pilot tones	0	$\mathcal{J}_0 = \{3, 9, 15, 21, 27\}$, $c_0[k \in \mathcal{J}_0] = \{a_1, a_2, a_3, a_4, a_5\}$
	1	$\mathcal{J}_1 = \{3, 9, 15, 21, 27\}$, $c_1[k \in \mathcal{J}_1] = \{ja_1, ja_2, ja_3, ja_4, ja_5\}$
	2	$\mathcal{J}_0 = \{4, 10, 16, 22, 28\}$, $c_0[k \in \mathcal{J}_0] = \{b_1, b_2, b_3, b_4, b_5\}$
	3	$\mathcal{J}_1 = \{4, 10, 16, 22, 28\}$, $c_1[k \in \mathcal{J}_1] = \{jb_1, jb_2, jb_3, jb_4, jb_5\}$
$N = 32$, $L = 3$, $N_R = 8$, CDM-F type, 2 cluster pairs, 2 relays per cluster using L_T pilot tones	0	$\mathcal{J}_0 = \{3, 4, 9, 10, 15, 16, 21, 22, 27, 28\}$ $c_0[k \in \mathcal{J}_0] = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}$
	1	$\mathcal{J}_1 = \{3, 4, 9, 10, 15, 16, 21, 22, 27, 28\}$ $c_1[k \in \mathcal{J}_1] = \{ja_1, ja_2, ja_3, ja_4, ja_5, ja_6, ja_7, ja_8, ja_9, ja_{10}\}$
	2	$\mathcal{J}_2 = \{3, 4, 9, 10, 15, 16, 21, 22, 27, 28\}$ $c_0[k \in \mathcal{J}_2] = \{a_1, -a_2, a_3, -a_4, a_5, -a_6, a_7, -a_8, a_9, -a_{10}\}$
	3	$\mathcal{J}_3 = \{3, 4, 9, 10, 15, 16, 21, 22, 27, 28\}$ $c_1[k \in \mathcal{J}_3] = \{ja_1, -ja_2, ja_3, -ja_4, ja_5, -ja_6, ja_7, -ja_8, ja_9, -ja_{10}\}$
$\lvert a_i\rvert$ = constant $\forall i$ within each case and $\lvert a_i\rvert = \lvert b_i\rvert$.		

will yield complex channel gains. Two approaches can be considered. The first does not force the channel estimate to be real, and the estimator similar to (2) can be applied on $\boldsymbol{y}_D$ using the pilot matrix $\boldsymbol{Q}_{C_l}$ according to the design in Section II-C1. Its MSE takes the same form as in (3).

The second approach forces the estimate to be real, and is adopted for the cluster pairing pilot scheme. Using an LS estimator for $\boldsymbol{h}_{n,\text{pair}}$ based on $\boldsymbol{y}_D$, we obtain the frequency domain equivalent channel vector of the cluster pair n as

$$\hat{\boldsymbol{H}}_{n,\text{pair}} = \boldsymbol{F}_T(\boldsymbol{Q}_{C_{2n}}^H \boldsymbol{Q}_{C_{2n}})^{-1} \boldsymbol{Q}_{C_{2n}}^H \boldsymbol{y}_D. \tag{9}$$

The channel estimate of each cluster inside cluster pair n is respectively obtained as the real and imaginary part of (9):

$$\hat{\boldsymbol{H}}_{2n,T} = \text{Re}[\hat{\boldsymbol{H}}_{n,\text{pair}}] \quad \& \quad \hat{\boldsymbol{H}}_{2n+1,T} = \text{Im}[\hat{\boldsymbol{H}}_{n,\text{pair}}]. \tag{10}$$

The corresponding channel estimation MSE per cluster is given by (19) (see the Appendix for details). Note that the estimator (10) can also be used in tandem with TMRC-BF without cluster pairing and the MSE will be similar to (19) except without the terms in (14) corresponding to C_{2n+1}.

Note that the term $\boldsymbol{F}_T(\boldsymbol{Q}_{C_{2n}}^H \boldsymbol{Q}_{C_{2n}})^{-1} \boldsymbol{Q}_{C_{2n}}^H$ in (9) can be precomputed. The channel estimation for each cluster requires a multiplication of $N \times N$ matrix and $N \times 1$ vector. For M relay clusters, the complexity is $\boldsymbol{O}(MN^2)$ for the existing scheme [16] versus $\boldsymbol{O}(\lceil M/2 \rceil N^2)$ for the proposed scheme. The computational complexity is reduced by half.

III. SIMULATION RESULTS AND DISCUSSIONS

In the simulations, $N = 64$, each channel is Rayleigh fading with an exponential power delay profile, decreasing 3 dB per tap. SNR_{D-R_k} is the SNR of the pilots sent by the destination to estimate $\boldsymbol{h}_{R_k-D}$ at the relays, and SNR_{R_k-D} is the SNR of the pilots transmitted by the relays using TMRC-BF. Fig. 1 shows the channel estimation MSEs. $N_R = 6$ relays are split into two clusters. Channel length is $L = 8$. We first plot the simulation MSE for a pilot design that only uses L pilots, which experiences a severe error floor. Other pilot designs use $L_T^\dagger$ pilots. Compared with the one without pairing, the proposed cluster pairing design achieves 3 dB SNR gain at low or medium SNR_{R_k-D} because its parameter space is L_T *real* channel taps as opposed to L_T *complex* channel taps. Better noise suppression is achieved. If SNR_{D-R_k} is not high enough, there is an error floor at high SNR_{R_k-D}. However, even with a SNR_{D-R_k} of value 5 dB below SNR_{R_k-D}, no noticeable performance loss of the cluster pairing scheme is observed. The analytical MSE from (19) is also plotted, which agrees well with the simulation result of the pairing scheme.

In Fig. 2, we use $N_R = 4$ relays with 2 antennas each which are split into 4 clusters in order to test two pilot designs (CDM-F and FDM) with and without cluster pairing as well as real versus complex estimator. Here $L = 3$, each antenna uses L_T

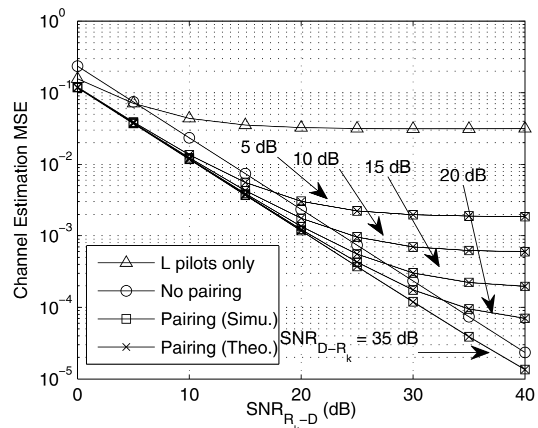

Fig. 1. Channel estimation MSE of the equivalent cluster channel gains with and without pairing.

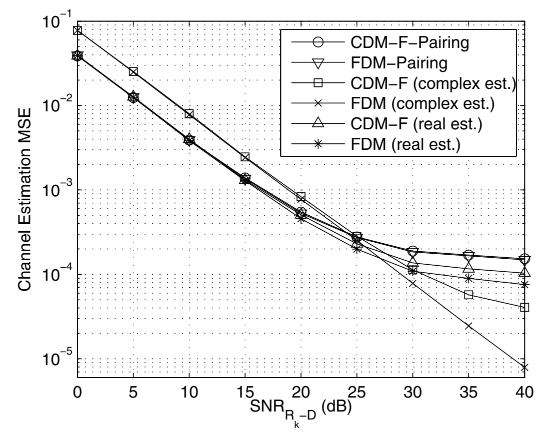

Fig. 2. Channel estimation MSE comparing CDM-F and FDM pilot designs as well as real versus complex estimator.

pilot tones approximately cyclically equi-spaced in frequency. For $\mathrm{SNR}_{D-R_k} = 10$ dB, the real estimator performs better at low to medium SNR_{R_k-D} due to the reduction in the estimation parameter space. At high SNR_{R_k-D}, the complex estimator outperforms the others, but uses twice the pilot overhead if compared with the pairing case. The FDM design performs slightly better than the CDM-F design because the FDM design uses null tones to eliminate interference while CDM-F uses coding which is reliant on the properties of the used cyclically equi-spaced pilot tones. The designs without pairing have a slightly lower error floor than those with pairing because the terms corresponding to C_{2n+1} in (14) will be zero, but with a penalty of twice the overhead. The floors can be removed by a proper setting of SNR_{D-R_k} as shown in Fig. 1.

IV. Conclusions

In this letter we address channel estimation for OFDM-based DF relay clusters system with TMRC-BF used at the relays. Based on the findings of channel length extension and the real value property of the equivalent channel after TMRC-BF, we propose a cluster pairing scheme with corresponding pilot designs. The proposed scheme reduces both the pilot overhead and the computational complexity by half. Simulation results suggest that, if the pilot SNR in acquiring the CSI required for TMRC-BF is not smaller than 5 dB below the pilot SNR during the TMRC-BF, the proposed cluster pairing method maintains nearly optimal MSE performance. We also observe that FDM design slightly outperforms CDM-F design.

Appendix

We derive the channel estimation MSE for TMRC-BF with our proposed cluster pairing scheme. The channel estimation error at the relays is $\tilde{\varepsilon}_{R_{k,j}} = \hat{\boldsymbol{H}}_{R_{k,j}-D} - \boldsymbol{H}_{R_{k,j}-D}$, with $\tilde{\varepsilon}_{R_{k,j}} \sim \mathcal{N}(0, \sigma^2_{R_{k,j}} \boldsymbol{F}_L (\boldsymbol{Q}_D^H \boldsymbol{Q}_D)^{-1} \boldsymbol{F}_L^H)$. Then, $\boldsymbol{h}_{n,\mathrm{pair}}$ is

$$\boldsymbol{h}_{n,\mathrm{pair}} = \boldsymbol{F}_T^H \left[\boldsymbol{H}_{2n,T} + j\boldsymbol{H}_{2n+1,T} + \boldsymbol{\Omega}_{2n} + j\boldsymbol{\Omega}_{2n+1}\right] \quad (11)$$

where $\boldsymbol{\Omega}_l = \sum_{R_{k,j} \in C_l} \mathrm{diag}\{\tilde{\varepsilon}^*_{R_{k,j}}\} \boldsymbol{H}_{R_{k,j}-D}$. Rewriting the estimator in (10) gives

$$\hat{\boldsymbol{H}}_{2n,T} = \boldsymbol{H}_{2n,T} + \mathrm{Re}[\boldsymbol{\Omega}_{2n}] + j\mathrm{Im}[\boldsymbol{\Omega}_{2n+1}] + \boldsymbol{n}_{\mathrm{Re},D}, \quad (12)$$

$$\text{where} \quad \boldsymbol{n}_{\mathrm{Re},D} = \mathrm{Re}\left[\boldsymbol{F}_T (\boldsymbol{Q}^H_{C_{2n}} \boldsymbol{Q}_{C_{2n}})^{-1} \boldsymbol{Q}^H_{C_{2n}} \boldsymbol{n}_D\right]. \quad (13)$$

Then the error correlation matrix for (12) is given by

$$\boldsymbol{\mathcal{C}}_{\boldsymbol{\varepsilon}_{2n}} = E[\boldsymbol{n}_{\mathrm{Re},D} \boldsymbol{n}^H_{\mathrm{Re},D}] + \frac{1}{4}\Big(\boldsymbol{F}_T \boldsymbol{R}_{2n} \boldsymbol{F}_T^H + \boldsymbol{F}_T^* \boldsymbol{R}_{2n}^* \boldsymbol{F}_T^\top + \boldsymbol{F}_T \boldsymbol{R}_{2n+1} \boldsymbol{F}_T^H + \boldsymbol{F}_T^* \boldsymbol{R}^*_{2n+1} \boldsymbol{F}_T^\top\Big), \quad (14)$$

$$\text{where} \quad \boldsymbol{R}_l \triangleq \frac{1}{N} \sum_{R_{k,j} \in \mathcal{I}_{C_l}} E[\boldsymbol{\mathcal{E}}^H_{R_{k,j}} \boldsymbol{\mathcal{C}}_{\boldsymbol{h}_{R_{k,j}}} \boldsymbol{\mathcal{E}}_{R_{k,j}}] \quad (15)$$

and $\boldsymbol{\mathcal{E}}_{R_{k,j}} = \sqrt{N} \boldsymbol{F}_L^H \mathrm{diag}\{\tilde{\varepsilon}_{R_{k,j}}\} \boldsymbol{F}_T$. The matrix $\boldsymbol{\mathcal{E}}_{R_{k,j}}$ is an $L \times L_T$ circulant time-domain matrix with L non-zero values in each row. The expectation of $\boldsymbol{\mathcal{E}}^H_{R_{k,j}} \boldsymbol{\mathcal{C}}_{\boldsymbol{h}_{R_{k,j}}} \boldsymbol{\mathcal{E}}_{R_{k,j}}$ will be a diagonal matrix when the optimal pilot condition $(\boldsymbol{Q}_D^H \boldsymbol{Q}_D)^{-1} \propto \boldsymbol{I}$ is met. The first L diagonal elements of $\boldsymbol{R}_l$ where $R_{k,j} \in C_l$ are

$$\sum_{l=0}^{L-p} \sigma^2_{\varepsilon_{R_{k,j}},l} \sigma^2_{h_{R_{k,j}},l+p}, \; p = 0, 1, \ldots, L-1, \quad (16)$$

where $\sigma^2_{\varepsilon_{R_{k,j}},l}$ is the lth element of the diagonal correlation matrix of $\boldsymbol{\varepsilon}_{R_{k,j}}$, the time domain channel estimation error. The last $L-1$ diagonal elements are

$$\sum_{l=p+1}^{L-1} \sigma^2_{\varepsilon_{R_{k,j}},l} \sigma^2_{h_{R_{k,j}},l-p-1}, \; p = L-2, L-3, \ldots, 0. \quad (17)$$

From (13), we have

$$E[\boldsymbol{n}_{\mathrm{Re},D} \boldsymbol{n}^H_{\mathrm{Re},D}] = \frac{\sigma_D^2}{2} \mathrm{Re}\left[\boldsymbol{F}_T (\boldsymbol{Q}^H_{C_{2n}} \boldsymbol{Q}_{C_{2n}})^{-1} \boldsymbol{F}_T^H\right]. \quad (18)$$

After substituting (16), (17), and (18) into (14), the MSE can be found using

$$\mathrm{MSE}_{\hat{\boldsymbol{H}}_{2n,T}} = \mathrm{Tr}[\boldsymbol{\mathcal{C}}_{\varepsilon_{2n}}]. \quad (19)$$

References

[1] T. Abe, H. Shi, T. Asai, and H. Yoshino, "A relaying scheme for MIMO wireless networks with multiple source and destination pairs," *2005 Asia-Pacific Conf. on Commun.*

[2] B. Khoshnevis, W. Yu, and R. Adve, "Grassmannian beamforming for MIMO amplify-and-forward relaying," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1397–1407, Oct. 2008.

[3] A. Talebi and W. A. Krzymien, "Cooperative MIMO multiple-relay system with optimized beamforming and power allocation," *IET Commun.*, vol. 4, no. 14, pp. 1677–1686, Mar. 2010.

[4] E. Park, K. J. Lee, and I. Lee, "Joint MMSE transceiver design for MIMO amplify-and-forward relay systems with multiple relays," *2009 IEEE VTC – Fall*.

[5] A. El-Keyi and B. Champagne, "Adaptive training-based collaborative MIMO beamforming for multiuser relay networks," *2009 IEEE VTC – Spring*.

[6] D. B. da Costa and S. Aissa, "Performance analysis of relay selection techniques with clustered fixed-gain relays," *IEEE Signal Proc. Lett.*, vol. 17, no. 2, pp. 201-204, Feb. 2010.

[7] S. Vakil, M. Dong, and B. Liang, "Effect of cluster size selection on the throughput of multi-hop cooperative relay," *2010 IEEE VTC – Fall*.

[8] F. Gao, T. Cui, and A. Nallanathan, "Optimal training design for channel estimation in decode-and-forward relay networks with individual and total power constraints," *IEEE Trans. Signal Process.*, vol. 56, no. 12, pp. 5937–5949, Dec. 2008.

[9] F. Gao, R. Zhang, and Y. C. Liang, "Optimal channel estimation and training design for two-way relay networks," *IEEE Trans. Commun.*, vol. 57, no. 10, pp. 3024–3033, Oct. 2009.

[10] J. Ma, P. Orlik, J. Zhang, and G. Y. Li, "Pilot matrix design for interim channel estimation in two-hop MIMO AF relay systems," in *Proc. 2009 IEEE ICC*, pp. 1–5.

[11] D. Neves, C. Ribeiro, A. Silva, and A. Gameiro, "Channel estimation schemes for OFDM relay-assisted systems," in *Proc. 2009 IEEE VTC – Spring*, pp. 1–5.

[12] Z. Zhang, W. Zhang, and C. Tellambura, "Cooperative OFDM channel estimation in the presence of frequency offsets," *IEEE Trans. Veh. Technol.*, vol. 58, no. 7, pp. 3447–3459, Sep. 2009.

[13] D. B. da Costa and S. Aissa, "Cooperative dual-hop relaying systems with beamforming over Nakagami-m fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 3950–3954, Aug. 2009.

[14] T. A. Thomas, X. Zhuang, and F. W. Vook, "Channel estimation for per-subcarrier maximal ratio transmission in OFDM," *2006 IEEE VTC – Fall*.

[15] N. Yang, M. Elkashlan, J. Yuan, and T. Shen, "On the SER of fixed gain amplify-and-forward relaying with beamforming in Nakagami-m fading," *IEEE Commun. Lett.*, vol. 14, no. 10, pp. 942–944, Oct. 2010.

[16] H. Minn and N. Al-Dhahir, "Optimal training signals for MIMO OFDM channel estimation," *IEEE Trans. Wireless Commun.*, vol. 5, no. 5, pp. 1158–1168, May 2006.