Some Issues of Complexity and Training Symbol Design for OFDM Frequency Offset Estimation Methods Based on BLUE Principle

Hlaing Minn
Department of Electrical Engineering
University of Texas at Dallas
P.O. Box 830688, Mail Station EC33
Richardson, TX 75083-0688, U.S.A.
email: hlaing.minn@utdallas.edu

Poramate Tarasak, Vijay K. Bhargava
Department of Electrical and Computer Engineering
University of Victoria,
P.O. Box 3055 STN CSC
Victoria, B.C., V8W 3P6, Canada
email: ptarasak, bhargava@ece.uvic.ca

Abstract— **In this paper we present reduced complexity OFDM frequency offset estimation methods based on the best linear unbiased estimation (BLUE) principle. Firstly, reduced complexity version of methods from [6] is presented. Secondly, a method that possesses both slight performance improvement of [6] and complexity advantage of [5] is proposed. Furthermore, we present the effects of the number of identical parts contained in the training symbol on the frequency offset estimation performance. Our results indicate that an improper choice of the number of identical parts contained in the training symbol can cause significant performance degradation to the methods of [5] and [6] while a proper choice can give extra MSE improvement.**

I. INTRODUCTION

One of the main drawbacks of OFDM is its high sensitivity to frequency offsets caused by oscillator inaccuracies and Doppler shift of the mobile channel [1]. The main problem with frequency offset is that it causes loss of orthogonality among subcarriers and introduces inter subcarrier interference and hence, can degrade system performance significantly. Moreover, frequency offset also induces phase error which accumulates over successive symbols. Unless pilot tones for phase tracking are employed, this cumulative phase error can also degrade the system performance to a larger instant for a system with a larger packet length. All of them demand a highly accurate frequency offset estimation method.

Several schemes (e.g., [2] - [5]) have been proposed for OFDM frequency offset estimation. In [2], a maximum likelihood frequency offset estimator was presented based on the use of two consecutive and identical symbols. The maximum frequency offset that can be handled is $\pm 1/2$ of the subcarrier spacing. The method of [4] also applied two training symbols. The first has two identical halves and is used to estimate a frequency offset less than the subcarrier spacing while the second symbol is used to resolve the frequency offset estimation ambiguity. Recently in [5], Morelli and Mengali (M&M) presented an improved frequency offset estimation based on the best linear unbiased estimation (BLUE) principle. The M&M method uses a training symbol composed of $L > 2$ identical parts and the frequency acquisition range is $\pm L/2$. At the cost of increased complexity, the M&M method brought in some MSE performance improvement over the method of [4].

In [6], three frequency offset estimation methods based on the BLUE principle were presented: one of them has the same frequency offset estimation mean square error (MSE) performance as the M&M method while the other two methods give a slightly better MSE performance than the M&M method especially at low SNR values. The slight improvement of the methods in [6] is associated with the cost of some complexity; however, the complexity issue was not considered in [6].

Firstly, in this paper we address the complexity issue involved in the methods of [6]. In particular, we present reduced complexity versions of the methods in [6]. Secondly, we propose a frequency offset estimation method which possesses both the slight improvement of the methods from [6] and the complexity advantage of the M&M method. At the same complexity, this method has a marginal MSE improvement over the M&M method, especially at low SNR values. It should be emphasized that a slight improvement over the M&M method, even though not significant in terms of improvement gain, is not trivial since the performance of the M&M method is already very close to the CRB. Thirdly, we discuss the effects of the number of identical parts contained in the training symbol on the frequency offset estimation performance. This gives an important insight on how the training symbol should be designed in order to achieve a better MSE performance with the same amount of training overhead.

II. SIGNAL MODEL

The time-domain complex baseband samples $\{s(k)\}$ of the useful part of an OFDM signal with N subcarriers are generated by taking the N-point inverse fast Fourier transform (IFFT$_N$) of a block of subcarrier symbols $\{C_l\}$ which are from a QAM or PSK signal constellation as

$$s(k) = \frac{1}{\sqrt{N}} \sum_{l=-N_u}^{N_u} C_l \; e^{j2\pi lk/N} \; , \quad 0 \le k \le N-1 \quad (1)$$

where the number of used subcarriers is $2N_u + 1 \le N$. The useful part of each OFDM symbol has a duration of T seconds

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and is preceded by a cyclic prefix, which is longer than the channel impulse response, in order to avoid inter-symbol interference (ISI). Assuming that the timing synchronization eliminates the ISI, the receive filter output samples $\{r(k)\}$ taken at the sampling rate of N/T can be given by

$$r(k) = e^{j2\pi vk/N}\, x(k) + n(k) \tag{2}$$

where v is the carrier frequency offset normalized by the subcarrier spacing $1/T$, $n(k)$ is a sample of complex Gaussian noise process with zero mean and variance $\sigma_n^2 = E\{|n(k)|^2\}$ and $x(k)$ is the channel output signal component given by

$$x(k) = \frac{1}{\sqrt{N}} \sum_{l=-N_u}^{N_u} C_l\, \Gamma_l\, e^{j2\pi lk/N}\ , \quad 0 \le k \le N-1 \tag{3}$$

where Γ_l is the total frequency response at the l^{th} subcarrier, including the effects of the channel, filters, timing offset and arbitrary carrier phase factor. The signal-to-noise ratio is defined as SNR $\triangleq \sigma_x^2/\sigma_n^2$, with $\sigma_x^2 \triangleq E\{|x(k)|^2\}$. The frequency offset estimation considered is based on the training symbol $\{s(k)\}$ consisting of L identical parts as in [6].

III. Frequency Offset Estimation

The proposed frequency offset estimation is based on the correlations among the identical parts of the received training symbol. Define the correlation term as

$$R(m) = \sum_{k=0}^{N-mM-1} r^*(k)\, r(k+mM), \quad 1 \le m \le H \tag{4}$$

where $M = N/L$ is the number of the samples of each identical part of the training symbol and H is a design parameter with $1 \le H \le L-1$. Substituting (2) into (4) results in

$$R(m) = e^{j2\pi vmM/N}\, \{(L-m)E_1 + G(m) + \mathcal{N}(m)\} \tag{5}$$

where

$$E_1 \triangleq \sum_{k=0}^{M-1} |x(k)|^2 \tag{6}$$

$$G(m) \triangleq \sum_{k=0}^{N-mM-1} \{x^*(k)\tilde{n}(k+mM) + x(k+mM)\tilde{n}^*(k)\} \tag{7}$$

$$\mathcal{N}(m) \triangleq \sum_{k=0}^{N-mM-1} \tilde{n}^*(k)\, \tilde{n}(k+mM) \tag{8}$$

and $\tilde{n}(k) \triangleq n(k)e^{-j2\pi vk/N}$ is a random variable statistically equivalent to $n(k)$.

Define the following:

$$\theta_m \triangleq \frac{N}{2\pi mM}\, arg\{R(m)\}. \tag{9}$$

If $|v| < N/(2mM)$, then we have

$$\theta_m = v + \frac{N}{2\pi mM}\, arg\{(L-m)E_1 + G(m) + \mathcal{N}(m)\} \tag{10}$$

and hence, θ_m gives an estimate of v. For smaller m values, M can be designed to handle the possible maximum frequency offset, i.e., to satisfy the condition $|v| < N/(2mM)$. However, the oscillator inaccuracies and the channel Doppler shift may not guarantee the condition $|v| < N/(2mM)$ for larger m values. Hence, θ_m with larger m values are associated with an ambiguity problem and would not be suitable for use as an estimate for v. To circumvent this, [6] proposed the following. First, θ_1[1] is calculated and used as an initial estimate of v. Then the initial frequency offset compensation is performed on the received training symbol by using the initial frequency offset estimate θ_1. The frequency offset compensated received training symbol sample $\tilde{r}(k)$ can be expressed as

$$\tilde{r}(k) = r(k)e^{-j2\pi\theta_1 k/N}. \tag{11}$$

Using $\tilde{r}(k)$ in place of $r(k)$ in (4) and (9) gives

$$\tilde{R}(m) = \sum_{k=0}^{N-mM-1} \tilde{r}^*(k)\, \tilde{r}(k+mM), \quad 2 \le m \le H \tag{12}$$

$$\begin{aligned} &= e^{j2\pi(v-\theta_1)mM/N}\, \{(L-m)E_1 + \tilde{G}(m) + \tilde{\mathcal{N}}(m)\} \\ \tilde{\theta}_m &= \frac{N}{2\pi mM}\, arg\{\tilde{R}(m)\}\ , \quad 2 \le m \le H \\ &= (v-\theta_1) + \frac{N}{2\pi mM}\, arg\{(L-m)E_1 \\ &\quad + \tilde{G}(m) + \tilde{\mathcal{N}}(m)\} \end{aligned} \tag{13}$$

where $\tilde{G}(m)$ and $\tilde{\mathcal{N}}(m)$ have the same statistical behaviors as $G(m)$ and $\mathcal{N}(m)$, respectively. Since θ_1 would be close to v, $\{\tilde{\theta}_m :\ 2 \le m \le H\}$ give estimates of $(v-\theta_1)$ without any ambiguity. Now, $\{\theta_m :\ 2 \le m \le H\}$ can be given by $\theta_m = \theta_1 + \tilde{\theta}_m,\ \ 2 \le m \le H$.

The frequency offset estimator based on the BLUE principle can then be given by [7]

$$\hat{v} = \sum_{m=1}^{H} w_m\, \theta_m \tag{14}$$

where w_m is the m^{th} component of the weighting vector

$$\boldsymbol{w} = \frac{\boldsymbol{C_\theta}^{-1}\,\mathbf{1}}{\mathbf{1}^T\,\boldsymbol{C_\theta}^{-1}\,\mathbf{1}}. \tag{15}$$

Here, $\boldsymbol{C_\theta}$ is the covariance matrix of $\boldsymbol{\theta} \triangleq [\theta_1,\ \theta_2,\ \ldots,\ \theta_H]^T$ and $\mathbf{1}$ is an all ones column vector of length H. The above frequency offset estimation based on the BLUE principle requires $\boldsymbol{C_\theta}$. Three methods (Method A, B and C) were presented in [6] for obtaining the required (approximate) value of $\boldsymbol{C_\theta}$. Reduced complexity version of these methods will be discussed in the following section.

[1] In circumventing the ambiguity problem, instead of θ_1, it would be slightly better to use θ_k which is associated with the largest weighting value among those $\{\theta_l\}$ that do not have ambiguity problem

IV. Reduced Complexity Version of Previous Methods

After re-arranging, (14) can be expressed as

$$\hat{v} = \frac{N}{2\pi M}\left(arg\{R(1)\} + \sum_{m=2}^{H} arg\{\tilde{R}(m)\}\ \frac{w_m}{m}\right). \quad (16)$$

By observing the following relationship

$$\tilde{R}(m) = e^{-jm\ arg\{R(1)\}}\ R(m), \quad (17)$$

$arg\{\tilde{R}(m)\}$ can be obtained without calculating $\tilde{R}(m)$ as

$$arg\{\tilde{R}(m)\} = [arg\{R(m)\} - m \cdot arg\{R(1)\}]_{2\pi}. \quad (18)$$

By observing the above relationship, reduced complexity version of the methods from [6] can be implemented by using (16) and (18) rather than the direct approach using (11)-(14).

V. Proposed Method (Method D)

Rather than directly calculating $arg\{\tilde{R}(m)\}$, using the phase differential of correlation terms with respect to $R(1)$, namely $arg\{R(m)\} - m \cdot arg\{R(1)\}$, can save extra complexity. A larger complexity saving would be achieved if the phase differential of adjacent correlation terms are used as in the M&M method. Hence, in the proposed method, we use the phase differential of adjacent correlation terms as follows.

$$\begin{aligned} \phi_m &\triangleq [arg\{R(m)\} - arg\{R(m-1)\}]_{2\pi}, \quad 1 \le m \le H \\ \hat{v} &= \frac{1}{2\pi/L} \sum_{m=1}^{H} w_m\ \phi_m \end{aligned} \quad (19)$$

where w_m is the m^{th} component of the weighting vector $\boldsymbol{w}$ in (15) with $\boldsymbol{C_\theta}$ replaced by $\boldsymbol{C_\phi}$ which is described below.

Using high SNR approximation as in [5], we have

$$\phi_m \simeq 2\pi v/L + \gamma_I(m) - \gamma_I(m-1), \quad 1 \le m \le H, \quad (20)$$

$$\gamma_I(m) = Im\left\{\frac{1}{(N-mM)\sigma_s^2} \sum_{k=mM}^{N-1}[x(k)\tilde{n}^*(k-mM) + x^*(k-mM)\tilde{n}(k) + \tilde{n}(k)\tilde{n}^*(k-mM)]\right\}, \quad (21)$$

for $0 \le m \le H$. The element of $\boldsymbol{C_\phi}$ can be given by

$$\begin{aligned} \boldsymbol{C_\phi}(m,n) = E\{&\gamma_I^*(m)\gamma_I(n) - \gamma_I^*(m-1)\gamma_I(n) \\ &- \gamma_I^*(m)\gamma_I(n-1) + \gamma_I^*(m-1)\gamma_I(n-1)\} \end{aligned} \quad (22)$$

where for $0 \le p \le H$ and $0 \le q \le H$, and the expectation term can be expressed as (23) at the bottom of the page.

The variance of the BLUE in this case is given by

$$var\{\hat{v}\} = \left(\frac{L}{2\pi}\right)^2 \frac{1}{\mathbf{1}^T\ \boldsymbol{C_\phi}^{-1}\ \mathbf{1}}. \quad (24)$$

Since $\boldsymbol{C_\phi}$ is of full rank for $1 \le H \le L-1$, we can readily find that the variance (24) achieves the minimum at $H = L-1$. The SNR value required in $\boldsymbol{C_\phi}$ would be replaced by the designed value SNR_w. Our simulation results (not shown) reveal that the proposed method is insensitive to the SNR mismatch ($SNR_w \neq SNR$). The proposed method and the M&M method have the same estimator structure but different covariance matrix and hence, different weighting vector $\boldsymbol{w}$. The best setting is $H = L/2$ for the M&M method while $H = L-1$ for the proposed method.

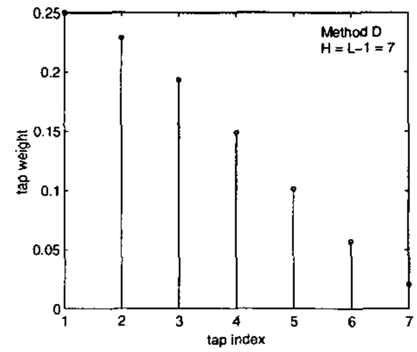

Fig. 1. The weighting values $\{w_m\}$ for the proposed method with $H = 7$ and $L = 8$.

VI. Computational Complexity

In Table I, the computational complexity of different methods are presented in terms of the number of equivalent real multiplication (ERM), equivalent real addition (ERA), phase computation ($arg\{.\}$) and modulo 2π operation. As discussed previously, reduced complexity version of the methods from [6] using (16) and (18) achieves some complexity saving over the direct implementation using (11)-(14). The proposed method and the M&M method have the same minimum complexity among the considered methods for $H \le L/2$. For $L/2 < H \le L-1$, the proposed method has the minimum complexity.

VII. Simulation Results and Discussions

A. Simulation Parameters

Simulations have been carried out to evaluate the estimation performance of the proposed methods. The simulation parameters are the same as those in [5], [6]: $N = 1024$, $2N_u + 1 = 861$, 40 cyclic prefix samples, $L = 8$ and $v = 1.6$. The channel considered is a multipath Rayleigh fading channel with 25 paths, the path delays of 0, 1, ..., 24 samples and an exponential power delay profile with the power of i^{th} path equal to $exp(-i/5)$. Two scenarios are considered: a quasi-static channel and a time-varying channel having a classical Doppler spectrum with a normalized maximum Doppler frequency $f_dT = 0.025$. In Fig. 1, the weighting values of the

$$E\{\gamma_I^*(p)\gamma_I(q)\} = \frac{1/SNR}{(N-pM)(N-qM)} \times \begin{cases} N - max(p,q)M + \frac{(N-pM)\ \delta[p-q]}{2\ SNR} & \text{if } p+q \ge L \\ min(p,q)M + \frac{(N-pM)\ \delta[p-q] - N\ \delta[p]\ \delta[p-q]}{2\ SNR} & \text{else.} \end{cases} \quad (23)$$

TABLE I

COMPUTATIONAL COMPLEXITY OF DIFFERENT FREQUENCY OFFSET ESTIMATORS

	Methods from [6]		Proposed Method	M& M
	Direct Implementation	Reduced Complexity Version		
# ERM	4NH-2MH(H+1)+6H-5	4NH-2MH(H+1)+2H-1	4NH-2MH(H+1)+H	4NH-2MH(H+1)+H
# ERA	2NH-MH(H+1)-H -1	2NH-MH(H+1)-2	2NH-MH(H+1)-1	2NH-MH(H+1)-1
# arg{ . }	H	H	H	H
# modulo 2π		H	H	H
Best setting of H	L-1 (Method A, B) L/2 (Method C)		L-1	L/2

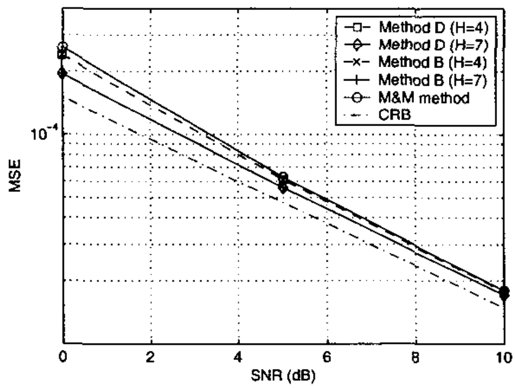

Fig. 2. The frequency estimation MSE performance comparison in a quasi-static multipath Rayleigh fading channel ($L = 8$)

proposed method are presented for $H = L - 1 = 7$. The adjacent phase differential of correlation terms with smaller correlation distances give a more reliable estimate.

B. MSE Performance in a Quasi-static Multipath Rayleigh Fading Channel

Fig. 2 shows the simulation results for the frequency offset estimation MSE performance of the proposed method, Method B of [6] and the M&M method in a quasi-static multipath Rayleigh fading channel. For the M&M method, $H = L/2$, which gives the minimum variance, is used. Method B of [6] and the proposed method have the same MSE performance. The reason is that although the estimators are of different form, both use the BLUE principle and the same approximation in calculating the weighting values, resulting in the same result. For high SNR values, all methods have virtually the same MSE performance. As the SNR value becomes smaller, the proposed method with $H = 4$ achieves slightly better MSE performance than the M&M method while keeping the same complexity as the M&M method. The proposed method with $H = L - 1$ achieves a slight additional improvement over the proposed method with $H = L/2$. Also included for comparison in the figure is the CRB given by [5]. The proposed method with $H = 7$ have the MSE performance quite close to CRB for all considered SNR values.

C. Effect of the Number of Identical Parts L

From the BLUE variance and the simulation results from [6], the best value of H has been observed to be the largest

TABLE II

EFFECT OF L, THE NUMBER OF IDENTICAL TRAINING PARTS, ON THE BLUE VARIANCE

Method	Variance Ratio	SNR (dB)				
		0	5	10	15	20
B, D	$\frac{L=8}{L=4}$	0.8571	0.9175	0.9408	0.9486	0.9512
	$\frac{L=64}{L=4}$	0.7619	0.8733	0.9163	0.9308	0.9356
	$\frac{L=512}{L=4}$	0.7515	0.8694	0.9148	0.9302	0.9352
C, [5]	$\frac{L=8}{L=4}$	0.9524	0.9524	0.9524	0.9524	0.9524
	$\frac{L=64}{L=4}$	0.9377	0.9377	0.9377	0.9377	0.9377
	$\frac{L=512}{L=4}$	0.9375	0.9375	0.9375	0.9375	0.9375

one among the allowable values, i.e., $H = L - 1$ for the proposed method and Method A, B from [6] and $H = L/2$ for Method C from [6] and the M&M method. Table II presents the BLUE variances computed from the ratio of two variances with different values of L. For Method B and D, it is intractable to obtain a close form expression of the BLUE variance. Hence, we evaluate it by computer simulation. For Method C and M&M method, the ratio of the variances is $(1 - 1/L_1^2)/(1 - 1/L_2^2)$.

Same as in Method C and M&M method, a larger L value gives a smaller BLUE variance in Method B and D. Unlike Method C and M&M method, for fixed L values, different SNR values yields different BLUE variance ratios in Method B and D. For Method B and D, the improvement with a larger value of L is slightly greater at a lower SNR value than at higher SNR value. At low SNR values, it is also observed that a larger L value brings in more improvement for Method B and D than for Method C and M&M method. This theoretical investigation suggests that the larger the value of L is, the better the estimation performance will be, although the improvement is marginal. However, it will be seen in the following that this theoretical implication is not fully complied with the simulation results.

Computer simulation of estimation MSE performance with different L values are presented in Fig. 3 in a quasi-static multipath Rayleigh fading channel. For SNR=0 dB, $L = 32$, 64 and 128 give almost the same MSE performance where $L = 64$ has a marginally better MSE performance. As SNR value increases, $L = 16$, 32, 64, and 128 give almost the same MSE performance, and at SNR=25 dB, $L = 16$ and 32 are just marginally better. Notably, $L = 512$ gives a substantial performance degradation. This inconsistence with theoretical implication comes from the fact that the variance of BLUE assumes that the total energy of the received training samples is constant. However, from (3), it is observed that the pilot tones are affected by the sub-channel responses. When L is

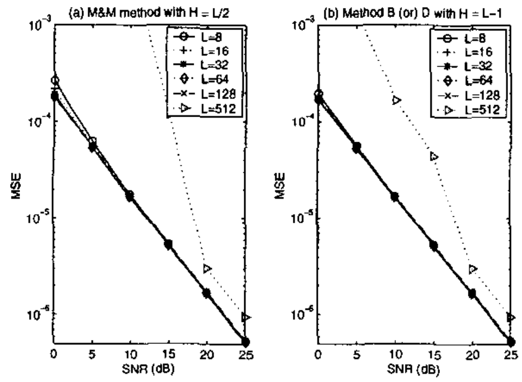

Fig. 3. The frequency estimation MSE performance with different values of L in a quasi-static multipath Rayleigh fading channel

large, the number of non-zero pilot tones becomes small. This causes high fluctuation of the receive training energy at different snap-shops. When the training signal is in deep fade, the estimation performance will be seriously affected and therefore degrade the overall performance.

From the simulation results, it is clear that an improper choice of L value can lead to a significant performance degradation while a proper choice can give a slight performance improvement. Based on the simulation results, our suggestion for a suitable choice of L value would be around N/K where K is the number of effective sample-spaced channel taps.

D. MSE Performance in a Time-Varying Multipath Rayleigh Fading Channel

Fig. 4 shows the estimation MSE performance of the proposed method with $H = L-1$, $H = L/2$ and the M&M method with $H = L/2$. Both methods experience some performance degradation due to the distorted repetitive structure of the training symbol caused by time-varying channel. With the same complexity as M&M, method, Method D with $H = L/2$ achieves a slightly better MSE performance particularly at low SNR values. With some added complexity, Method D with $H = L-1$ brings in additional slight improvement on estimation MSE at moderate and low SNR values.

VIII. Conclusions

Reduced complexity versions of OFDM frequency offset estimation methods based on BLUE principal from [6] are presented. The added complexity, which is the cost for a slight MSE performance improvement of those methods from [6] over the M&M method [5], can be reduced by using the reduced complexity version presented in this paper although the M&M method still has complexity advantage. Another frequency offset estimation method based on BLUE principle is also presented which possesses both the slight MSE performance advantage of the methods from [6] and the complexity advantage of the M&M method. At the same complexity, this proposed method achieves a slight (marginal) MSE performance improvement over the M&M method at low

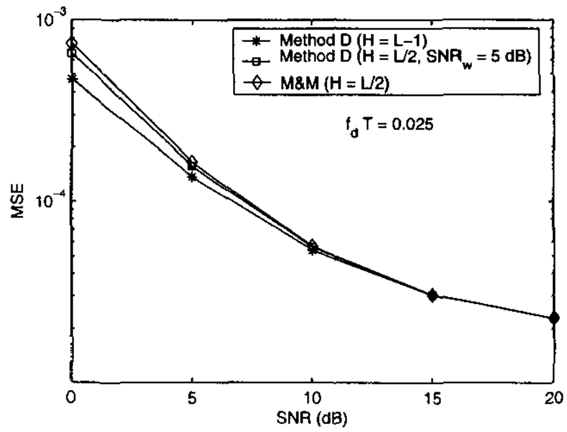

Fig. 4. The frequency estimation MSE performance comparison in a time-varying multipath Rayleigh fading channel ($L = 8$)

SNR values. With added complexity (i.e., $L/2 < H \leq L-1$), the proposed method can achieve a slight additional MSE performance improvement. If complexity is not a concern, the proposed method with $H = L-1$ can be chosen. The proposed method can give a trade-off for complexity and MSE performance by setting the value of H within $1 \leq H \leq L-1$ while the M&M method can give the trade-off by setting the value of H within $1 \leq H \leq L/2$. The number of identical parts, L, contained in the training symbol can have some impact on the MSE performance. A suitable choice of L value would be around N/K or less where K is the number of effective sample-spaced channel taps and N is the total number of subcarriers (or FFT points).

Acknowledgment

This work was supported, in part, by the Clark Foundation Grant, the University of Texas at Dallas, awarded to Dr. Hlaing Minn, and in part, by the Natural Sciences and Engineering Research Council (NSERC) of Canada under a grant awarded to Professor Vijay Bhargava.

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