A Reduced Complexity, Improved Frequency Offset Estimation for OFDM-Based WLANs

Hlaing Minn[1], *Member, IEEE* and Vijay K. Bhargava[2], *Fellow, IEEE*
[1] Dept. of Electrical Engineering, University of Texas at Dallas.
[2] Dept. of Electrical and Computer Engineering, University of British Columbia.
hlaing.minn@utdallas.edu, vijayb@ece.ubc.ca

***Abstract*— This paper presents a reduced complexity frequency offset estimation method as a modification to the non-linear least square (NLS) method from [3]. The complexity reduction is achieved by utilizing the Chirp-Z transform. At the same estimation performance, the complexity gain, defined as the complexity ratio between the NLS method and the proposed method, can be approximately from 2 to 9 depending on the implemented complexity of the NLS method. At similar complexity, the estimation performance gain, defined as the mean square error ratio between the NLS method and the proposed method, can be approximately from 1.5 dB to 8.5 dB depending on the operating SNR value. We also present a modified training structure that can give approximately from 2.5 dB to 10.5 dB estimation performance gain over the training structures of [3] and IEEE 802.11a.**

I. Introduction

OFDM has been adopted in several wireless standards such as European digital audio and video broadcasting standards (DAB, DVB), wireless local area networks (WLAN) standards HiperLAN-2 and IEEE 802.11a, and wireless metropolitan area networks standard IEEE 802.16a. OFDM increases the spectrum efficiency by allowing the subcarrier spectra to overlap. As a side effect, OFDM becomes much more sensitive to frequency offsets which destroys orthogonality among the subcarriers and causes inter-carrier interference. Several methods (e.g. [1] [2]) have been proposed for OFDM frequency offset estimation. Recently, a non-linear least square (NLS) frequency offset estimation method for OFDM-based WLANs has been proposed in [3] using the training structure adopted in the IEEE 802.11a [4]. [3] also proposed a training structure which outperforms the IEEE 802.11a training structure.

In this paper, we present a reduced complexity frequency offset estimation method as a modification to the NLS method. The NLS method is implemented by the fast Fourier transform (FFT) algorithm. The complexity is determined by the FFT size (N_1) used in the NLS method which is much larger than the FFT size used in the OFDM modulation/demodulation. The estimation accuracy of the NLS method is determined by the channel SNR and N_1. As a modification to the NLS method, the proposed reduced complexity (RC) method uses a smaller size FFT (N_2) and a chirp-Z transform (CZT) of size (N_c). When $N_1 = N_2 N_c/2$, both the NLS and the RC method will have the same resolution of the normalized frequency offset bins in the estimation and hence, the same estimation accuracy.

At the same estimation performance, the complexity gain, defined as the complexity ratio between the NLS and the proposed method, can be approximately 2, 4 and 9 for N_1 = 256, 512 and 2048, respectively. At similar complexity, the estimation performance gain, defined as the mean square error ratio between the NLS method and the proposed method, can be approximately 1.5 dB at SNR = 0 dB or 5 dB. For higher SNR values, the estimation performance gain increases and at SNR = 20 dB, it is approximately 8.5 dB.

We also present a modified training structure that can give approximately from 2.5 dB to 10.5 dB estimation performance gain over the training structure proposed in [3] while using the same training signal energy and the same time interval for the training signal.

The rest of the paper is organized as follows. Section II describes the signal model. Section III presents the proposed reduced complexity frequency offset estimation method. The modified training structure is introduced in Section IV. The complexity and the estimation performance are discussed in Section V and conclusions are given in Section VI.

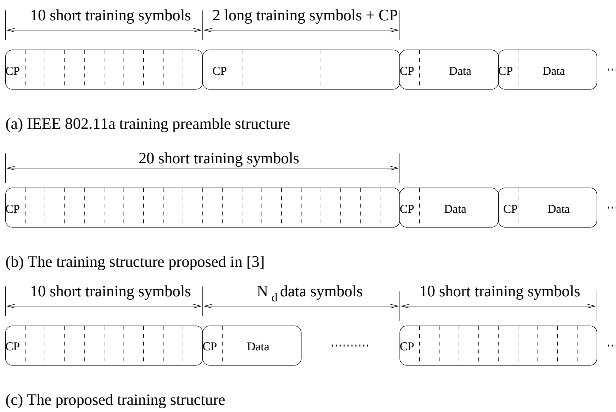

Fig. 1. The training structures for OFDM-based WLANs

II. Signal Model

The discrete-time complex baseband OFDM signal $\{s(k)\}$, sampled at the rate of N/T ($1/T$ = subcarrier spacing), are transmitted through a frequency selective fading channel. Assuming that the timing synchronization eliminates ISI, the receive-filter output samples $\{r(k)\}$ can be given by

$$r(k) = e^{j\psi} e^{j2\pi vk/N} x(k) + n(k) \tag{1}$$

where v is the carrier frequency offset normalized by the subcarrier spacing, ψ is an arbitrary carrier phase, $n(k)$ is a zero-mean complex Gaussian noise with a variance of σ_n^2, and $x(k)$ is the receive-filter output signal component.

Due to the oscillators' inaccuracies and the Doppler shift of the mobile wireless channel, frequency offsets are unavoidable at the receiver. Since OFDM is much more sensitive to the frequency offset error than single carrier systems, training signal based frequency offset estimation is usually applied at the receiver. The IEEE 802.11a wireless LAN standard [4] adopted a training preamble structure which contains 10 identical short OFDM symbols of length K = 16 samples each (the first short symbol serves as a cyclic prefix guard interval), followed by two identical long OFDM symbols of length 64 samples and the corresponding cyclic prefix guard interval of length 32 samples.

In [3], another training structure was proposed which contains $L + 1$ = 20 identical short OFDM symbols (obtained by repeating the first part of the IEEE 802.11a training structure). Both training structures contain $D = 320$ training samples. [3] reported that its proposed training structure outperforms the IEEE 802.11a training structure. Both training structures are depicted in Fig. 1(a) and (b).

III. Frequency Offset Estimation

Let us consider the training signal of [3] given by $\{s(k) : k = -K, -K+1, \ldots, D-K-1\}$ representing (L+1) identical short symbols of K samples each. The receive-filter output training samples are given by $\{x(k) : k = -K, -K+1, \ldots, D-K-1\}$. The cyclic prefix guard samples $\{x(k) : k = -K, -K+1, \ldots, -1\}$ are discarded. Let us define a matrix $\boldsymbol{X}$ of size $L \times K$ whose elements are given by

$$\begin{aligned} \boldsymbol{X}_{m,n} = x(mK+n) \quad , \quad & m = 0, 1, \ldots, L-1; \\ n \ = \ & 0, 1, \ldots, K-1. \end{aligned} \tag{2}$$

Apart from the phase terms introduced by the frequency offset, the rows of $\boldsymbol{X}$ are identical. Let $\boldsymbol{Y}$ be a matrix of size $N_1 \times K$ given by

$$\boldsymbol{Y} = \mathrm{FFT}_{N_1}\{\boldsymbol{X}\} \tag{3}$$

where FFT_{N_1} represents N_1-point one-dimensional fast Fourier transform applied to each column, N_1 is a power of 2 and $N_1 > L$. The n^{th} column of $\boldsymbol{Y}$, $\boldsymbol{Y}_n$, is the N_1 point FFT of $\boldsymbol{X}_n$, the n^{th} column of $\boldsymbol{X}$. Define

$$\begin{aligned} \boldsymbol{Z} &= \sum_{n=0}^{K-1} \boldsymbol{Y}_n \odot \boldsymbol{Y}_n^* \\ &= [z_0, \ z_1, \ \ldots, \ z_{N_1-1}]^T \end{aligned} \tag{4}$$

where $\odot$ represents an element-wise product and the superscripts * and T denote the conjugate and the transpose, respectively. Then the NLS frequency offset estimation method can be given by

$$\hat{v} = \frac{N}{KN_1} mod[\arg \max_m \{z_m\}] \tag{5}$$

where $mod[\ ]$ is a modulo operation for the range $(N_1/2, N_1/2]$, (i.e., if $0 \leq m \leq N_1/2$, $mod[m]$ equals m and otherwise it equals $m - N_1$). The normalized frequency offset estimation range is $[-\frac{N}{2K} + \frac{N}{KN_1}, \frac{N}{2K}]$. The accuracy of this method depends on the frequency spacing of the FFT bins, $N/(KN_1)$. To achieve a reliable estimate, N_1 must be sufficiently large. For example, for $N = 64$ and $K = 16$, [3] used $N_1 = 512$. A larger FFT point N_1 gives a better accuracy at the expense of a larger complexity. To improve the accuracy and reduce the complexity, we propose the following reduced complexity (RC) method. First, we apply the NLS method with N_2-point FFT where N_2 is a power of 2 and $L < N_2 < N_1$, (i.e., N_1 is replaced with N_2 in (3)-(5)). Let $\hat{v}_{|N_2}$ denote the estimate obtained from (5) with N_2-point FFT. Then

$$\hat{v}_{|N_2} = \frac{N}{KN_2} mod[\arg \max_m \{z_m\}]. \tag{6}$$

Define the following:

$$\hat{v}_0 = \hat{v}_{|N_2} - \frac{N}{KN_2}. \tag{7}$$

We apply the N_c-point CZT (N_c is a power of 2 and $N_c > L$) to each column of $\boldsymbol{X}$ as follows:

$$\tilde{\boldsymbol{Y}} = \mathrm{CZT}_{N_c}\{\boldsymbol{X}\}. \tag{8}$$

The CZT efficiently evaluates the Z-transform of a finite duration sequence along certain general contours in the Z-plane [5]. If the contour is on a unit circle, the CZT evaluates the DFT of the sequence over the contour. Using $exp(j2\pi K\hat{v}_0/N)$ as the starting point of the contour, $exp(-j4\pi/(N_2N_c))$ as the ratio of the two adjacent points on the contour, the N_c-point CZT gives the N_c equally spaced DFT points over the range $[exp\{j2\pi K\hat{v}_0/N\}, exp\{j2\pi K(\hat{v}_0+2N/(KN_2))/N\})$ on the unit circle. Define the following:

$$\tilde{\boldsymbol{Z}} = \sum_{n=0}^{K-1} \tilde{\boldsymbol{Y}}_n \odot \tilde{\boldsymbol{Y}}_n^* \tag{9}$$

$$= [\tilde{z}_0,\ \tilde{z}_1,\ \ldots,\ \tilde{z}_{N_c-1}]^T. \tag{10}$$

Then the RC frequency offset estimate is given by

$$\hat{v} = \hat{v}_0 + \frac{2N}{KN_2N_c} mod[\arg \max_m \{\tilde{z}_m\}]. \tag{11}$$

The frequency offset estimation range of the RC method is [- $\frac{N}{2K}$, $\frac{N}{2K} + \frac{N(N_c-2)}{KN_2N_c}$] which is approximately the same as that of the NLS method.

IV. Modified Training Structure

The modified training structure is presented in Fig. 1(c). Total number of training samples are the same as the training signal of [3] or IEEE 802.11a. In fact, the modified training structure contains the same 20 identical short OFDM symbols as in the training signal of [3]. The difference from [3] is that the first 10 short symbols and the last 10 short symbols are separated by N_d OFDM data symbols. Hence, if $N_d = 0$, the modified training structure is equivalent to the training signal of [3]. When $N_d = 0$, the first short symbol serves as a cyclic prefix and the remaining 19 short symbols can be used in the estimation. When $N_d > 0$, only 18 short symbols can be used in the estimation since the first and the 11 th short symbols serve as cyclic prefixes. The first short symbol is discarded as before. In the frequency offset estimation, the N_d OFDM data symbols between the training signals and the 11 th short training symbol (cyclic prefix) are replaced with zeros. Then both the NLS method and the RC method can be applied to the zero-masked received training vector.

V. Complexity and Estimation Performance

In the NLS method, the trial values of v correspond to N_1 equally spaced points on the unit circle with the resolution of the trial normalized frequency offset points in the estimation being $N/(KN_1)$. The RC method first obtains the initial estimate $\hat{v}_0$ from the trial values corresponding to N_2 equally spaced points on the unit circle ($N_2 < N_1$) with the resolution of $N/(KN_2)$. Then by using N_c-point CZT, the RC method finds the estimate from the trial values corresponding to the N_c equally spaced points over an arc on the unit circle. The arc length corresponds to the estimation range of $[\hat{v}_0, \hat{v}_0 + 2N/(KN_2))$. The final frequency resolution is $2N/(KN_2N_c)$. When $N_1 = N_2N_c/2$, both methods have the same frequency resolution in the estimation and hence, the same estimation performance. But the estimation complexities are different. Similarly, if the complexities are kept the same, both methods will have different estimation performance.

The complexities of both methods in terms of the numbers of real multiplication (#RM), real addition (#RA), and comparison (#Cmp) are given below. For the NLS method,

$$\begin{aligned}
\#\mathrm{RM} &= 4KN_1(0.5\, log_2(N_1) + 1) + 1 \\
\#\mathrm{RA} &= 2KN_1(1.5\, log_2(N_1) + 1) + N_1(K-1) \\
\#\mathrm{Cmp} &= N_1 - 1.
\end{aligned}$$

For the RC method,

$$\begin{aligned}
\#\mathrm{RM} &= [4KN_2(0.5\, log_2(N_2) + 1) + 1] \\
+4 &\quad [\quad 2N_2 + K(N_2 + J + 2N_c + J\, log_2(J)) + 1] \\
\#\mathrm{RA} &= [2KN_2(1.5\, log_2(N_2) + 1) + N_2(K-1)] \\
+2 &\quad [\quad 2N_2 + K(N_2 + J + 2N_c + 3J\, log_2(J)) \\
&- \quad N_c + 1] \\
\#\mathrm{Cmp} &= N_2 + N_c - 2
\end{aligned}$$

where $J = 2^{\lceil log_2(N_2+N_c-1)\rceil}$ and $\lceil \cdot \rceil$ is the ceiling operation.

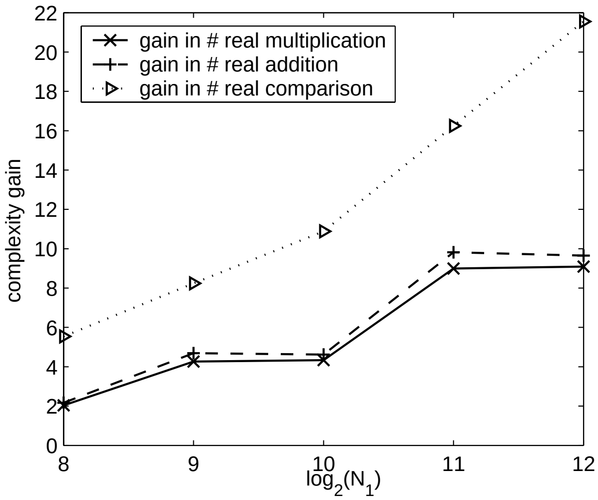

Fig. 2. The complexity gain of the proposed RC method over the NLS method

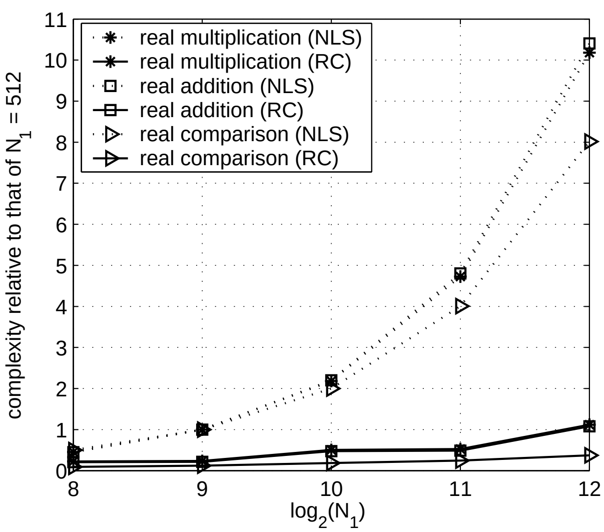

Fig. 3. The estimation complexities of the NLS and RC methods ($N_2 N_c = 2N_1$) relative to that of the NLS with N_1 = 512

In Fig. 2, the complexity gain, defined as the complexity ratio between the NLS and the proposed RC method, is plotted for several values of N_1, the number of FFT points in the NLS method. To keep the same estimation accuracy as the NLS method with N_1 = 256, 512, 1024, 2048, 4096, the RC method uses N_2 = 32, 32, 64, 128, 128, and N_c = 16, 32, 32, 32, 64, respectively (i.e., satisfying $N_2 N_c = 2N_1$). The complexity gain is approximately 2, 4 and 9 for N_1 = 256, 512 and 2048, respectively. Fig. 3 presents the complexities of both methods relative to that of the NLS method with N_1 = 512. The values

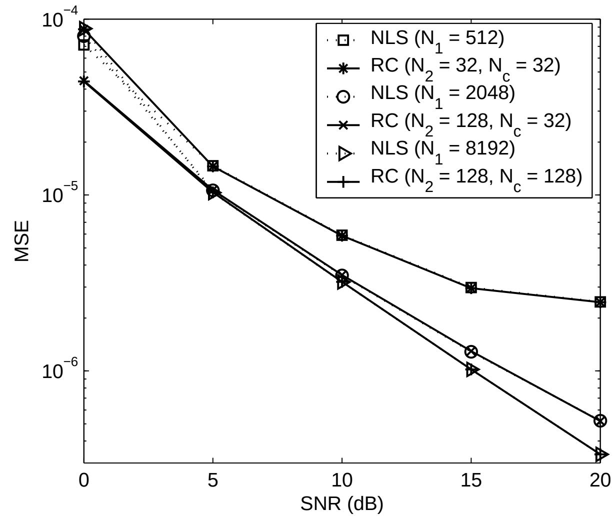

Fig. 4. The comparison of estimation MSE performance of the proposed RC method and the NLS method

of N_2 and N_c are the same as in Fig. 2. It can be observed that to achieve the same estimation performance as the NLS method with N_1 = 4096, the reduced complexity method requires an amount of complexity similar to that of the NLS with N_1 = 512 while the NLS with N_1 = 4096 requires more than 10 times the complexity of the NLS with N_1 = 512.

The estimation mean square error (MSE) performances are evaluated by simulation. The simulation parameters are $N = 64$, the IEEE 802.11a short training symbol, and v = 0.4. The multipath Rayleigh fading channel with 8 sample-spaced taps is considered. The channel power delay profile is with a -3 dB per tap decaying factor. Fig. 4 presents the MSE performance of the NLS method and the RC method. When $N_2 N_c = 2N_1$, both methods have essentially the same MSE performance. The RC method can achieve both estimation performance improvement and complexity reduction at the same time. For example, the RC method with N_2 = 128 and N_c = 32 has a smaller complexity (see Fig. 3) and a better MSE performance than the NLS method with N_1 = 512 (see Fig. 4).

Fig. 5 presents the MSE performance obtained with the training structure of [3] (N_d = 0) and the proposed structure ($N_d \neq 0$) by using the NLS method. As N_d increases, the improvement of the proposed training structure becomes larger except at very low SNR. The MSE floor of N_d = 8 and 12 is due to the frequency resolution using N_1 = 2048. By using N_1 = 8192, the above floor is removed. For very low SNR, a large value of N_d (say 12)

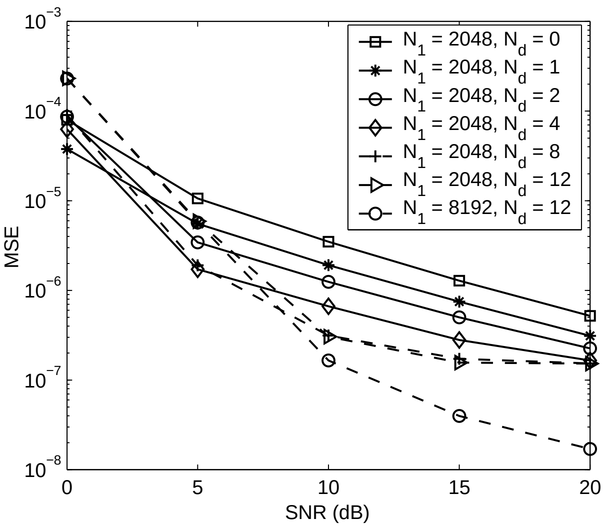

Fig. 5. The estimation MSE performance obtained with the existing training structure ($N_d = 0$) and the proposed modified training structure ($N_d \neq 0$)

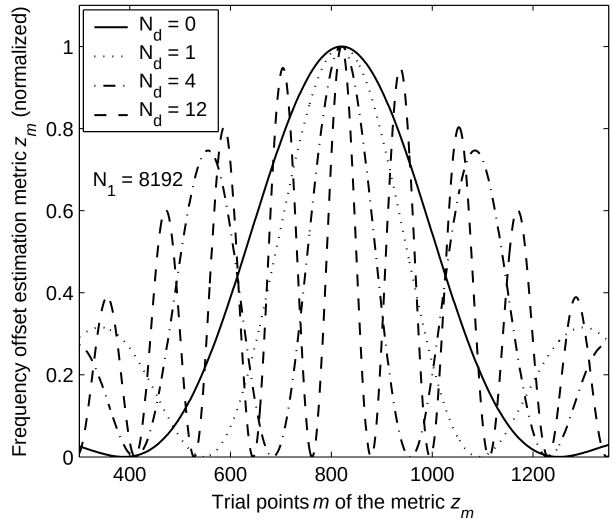

Fig. 6. The frequency offset estimation metric trajectories of the modified training structure with different values of N_d

gives a worse performance. This effect can be explained by the estimation metric trajectory plotted in Fig. 6. As N_d increases, the metric trajectory around the maximum point (which corresponds to the frequency offset) becomes sharper, hence giving a better estimation accuracy. But as N_d becomes quite large (say 12), the peaks of adjacent side lobes become comparable to that of the main lobe. These large side lobes bring in more vulnerability to estimation errors in very low SNR conditions. For the considered system parameters, N_d = 4 has adjacent side-lobes with peak about 0.75 times the main lobe peak. Hence, for other system parameters, the value of N_d can easily be chosen from the metric trajectories with maximum side-lobe peaks around 0.75 (or less) times the main lobe peak.

Note that only 18 short symbols are used in the estimation based on the modified structure while 19 short symbols are used in that of [3]. Hence, the estimation complexity associated with the modified structure is accordingly reduced if the same estimation method is used. Further complexity reduction can be achieved by using the proposed RC method.

VI. Conclusions

We have presented a reduced complexity, improved frequency offset estimation for OFDM-based WLANs. By applying CZT, the proposed method reduces the complexity of FFT-based NLS method. Depending on the required estimation accuracy, the complexity can be reduced accordingly. For the same estimation performance as the NLS method, the complexity reduction of the proposed method is larger at a higher estimation accuracy determined by a better resolution of the trial frequency offset points. Both estimation performance improvement and complexity reduction can be achieved at the same time. A modified training structure is also proposed. Depending on the SNR and the parameter of the modified training structure, the estimation MSE can be reduced by an amount approximately from 2.5 dB to 10.5 dB if compared to the existing training signal structures. The modified training structure provides an estimation performance improvement and a complexity reduction while requiring the same training energy and the same training time interval as the existing training signal structures.

References

[1] P.H. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction," *IEEE Trans. Commun.*, Vol. 42, No. 10, Oct. 1994, pp. 2908-2914.

[2] M. Morelli and U. Mengali, "An improved frequency offset estimator for OFDM applications," *IEEE Commun. Letters*, Vol. 3, No. 3, Mar. 1999, pp. 75-77.

[3] J. Li, G. Liu and G. B. Giannakis, "Carrier frequency offset estimation for OFDM-based WLANs," *IEEE Signal Processing Letters*, Vol. 8, No. 3, Mar. 2001, pp. 80-82.

[4] IEEE LAN/MAN Standards Committee, "Wireless LAN medium access control (MAC) and physical layer (PHY) specifications: High-speed physical layer in the 5 GHz band," *IEEE Standard 802.11a*, 1999.

[5] A.V. Oppenheim and R.W. Schafer, "Discrete-Time Signal Processing," *Prentice Hall, 1989.*