

# ESTIMATION OF CARRIER-FREQUENCY OFFSET AND FREQUENCY-SELECTIVE CHANNELS IN MIMO OFDM SYSTEMS USING A COMMON TRAINING SIGNAL

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**Abstract**—This paper presents a common training signal design and corresponding estimation methods for carrier frequency offset and frequency-selective channels in MIMO OFDM systems. In designing the common training signal, a training signal structure which yields low complexity estimation methods is developed while the optimality of the training signal is maintained. Frequency offset estimation is based on the best linear unbiased estimation principle while channel estimation is based on the least squares (also maximum likelihood) approach. The proposed training signal and estimation methods can be applied to systems with pilot-only training signals as well as those with pilot-data multiplexed signals. The estimation range of the frequency offset can be flexibly adjusted. The performances of the proposed methods are very close to the Cramer-Rao bounds or theoretical minimum mean square error.

## I. INTRODUCTION

The higher data rate requirement of future wireless communications systems, the significant information theoretic capacity gain of MIMO systems, and the robustness and suitability of OFDM for high data rate transmission highlight the significant potential of MIMO OFDM systems. However, MIMO OFDM inherits a high sensitivity to frequency offset error from OFDM. Hence, highly-accurate frequency synchronization is an important issue in MIMO OFDM. The estimation of increased numbers of channels in MIMO systems presents another challenge in implementing MIMO OFDM systems.

Most previous approaches address frequency offset estimation and channel estimation separately using separate training signals (see [1]-[6] and references therein). There are only a few works which address synchronization and channel estimation using a common training signal (e.g., [10] for SISO OFDM systems). Due to the training overhead saving, the approaches using a common training signal merit further investigation. For MIMO OFDM systems, [11] has recently presented a combined frequency offset and channel estimation method based on a common training signal. [11] considers a pilot-data multiplexed scheme where the number of OFDM symbols required for transmission of pilot tones has to be at least the same as the number of channel impulse response taps. The authors use sub-space-based frequency offset estimation and linear minimum mean square error channel estimation. Hence, the method from [11] is more appropriate for systems which can accommodate relatively high complexity, are insensitive to processing delay, and have knowledge of the channel covariance matrix and the noise variance.

In this paper, we consider a combined frequency offset and channel estimation in MIMO OFDM systems based on a common

training signal. We develop a common training signal and the corresponding estimation methods which have low complexity, low processing delay, low training overhead, and high performance. Our proposed designs can be applied to systems with pilot-only training signal as well as those with pilot-data multiplexed signal. Extension of frequency offset estimation range by means of multiple OFDM training symbols is also presented.

## II. SIGNAL MODEL

Consider a MIMO OFDM system with  $K$  sub-carriers,  $N_t$  transmit antennas and  $N_r$  receive antennas. The training signals from  $N_t$  transmit-antennas are transmitted over  $Q$  OFDM symbols where  $Q \in \{1, 2, \dots\}$ . The channel impulse response (CIR) for each transmit-receive antenna pair (including filters' effects) is assumed to have  $L$  taps, and is quasi-static over  $Q$  OFDM symbols. Let  $\mathbf{C}_{n,q} = [c_{n,q}[0], \dots, c_{n,q}[K-1]]^T$  be the pilot tones vector of the  $n$ -th transmit-antenna at the  $q$ -th symbol interval and  $\{s_{n,q}[k] : k = -N_g, \dots, K-1\}$  be the corresponding time-domain complex baseband training samples, including  $N_g (\geq L-1)$  cyclic prefix samples. Define  $\mathbf{S}_n[q]$  as the training signal matrix of size  $K \times L$  for the  $n$ -th transmit-antenna at the  $q$ -th symbol interval whose elements are given by  $[\mathbf{S}_n[q]]_{m,l} = s_{n,q}[l-m]$ ,  $m \in \{0, \dots, K-1\}$ ,  $l \in \{0, \dots, L-1\}$ .

Let  $s_{n,q}$  represent the 0-th column of  $\mathbf{S}_n[q]$ . Then the  $l$ -th column of  $\mathbf{S}_n[q]$  is the  $l$ -sample cyclic-shifted version of  $s_{n,q}$  denoted by  $s_{n,q}^{(l)}$ . Assume that  $K = ML_0$  where  $M=1, 2, \dots$ , and  $L_0 \geq L$ . Let  $\mathbf{h}_{n,m}$  denote the length- $L$  CIR vector corresponding to the  $n$ -th transmit antenna and  $m$ -th receive antenna. After the cyclic prefix removal at the receiver, denote the received vector of length  $K$  from the  $m$ -th receive antenna at the  $q$ -th symbol interval by  $\mathbf{r}_{m,q} = [r_{m,q}(0), r_{m,q}(1), \dots, r_{m,q}(K-1)]^T$ . Then the received vector over the  $Q$  symbol-intervals at the  $m$ -th receive antenna is given by

$$\mathbf{r}_m = \bar{\mathbf{W}}(v) \mathbf{S} \mathbf{h}_m + \mathbf{n}_m \quad (1)$$

where

$$\mathbf{r}_m = [\mathbf{r}_{m,0}^T, \mathbf{r}_{m,1}^T, \dots, \mathbf{r}_{m,Q-1}^T]^T \quad (2)$$

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_0[0] & \mathbf{S}_1[0] & \dots & \mathbf{S}_{N_t-1}[0] \\ \mathbf{S}_0[1] & \mathbf{S}_1[1] & \dots & \mathbf{S}_{N_t-1}[1] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_0[Q-1] & \mathbf{S}_1[Q-1] & \dots & \mathbf{S}_{N_t-1}[Q-1] \end{bmatrix} \quad (3)$$

$$\mathbf{h}_m = [\mathbf{h}_{0,m}^T, \mathbf{h}_{1,m}^T, \dots, \mathbf{h}_{N_t-1,m}^T]^T \quad (4)$$

$$\bar{\mathbf{W}}(v) = \text{diag}[\mathbf{W}(v), e^{j2\pi v(N_g+K)/K} \mathbf{W}(v), \dots, e^{j2\pi v(Q-1)(N_g+K)/K} \mathbf{W}(v)] \quad (5)$$

$$\mathbf{W}(v) = \text{diag}[1, e^{j2\pi v/K}, e^{j2\pi 2v/K}, \dots, e^{j2\pi(K-1)v/K}] \quad (6)$$

and  $\mathbf{n}_m$  is a length  $KQ$  vector of zero-mean, circularly symmetric, uncorrelated complex Gaussian noise samples with equal vari-

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ance of  $\sigma_n^2$ . The diagonal matrix  $\mathbf{W}(v)$  corresponds to the normalized frequency offset  $v$ , normalized by the sub-carrier spacing. We consider a system where the RF branches of all antennas use a common local oscillator and hence, there is only one common normalized carrier frequency offset  $v$  between the transmitter and the receiver.

### III. DESIGN OF A COMMON OFDM TRAINING SYMBOL FOR ESTIMATION OF FREQUENCY OFFSET AND CHANNELS

Training signals consisting of several consecutive identical sub-blocks are commonly used for frequency offset estimation in SISO OFDM systems (e.g., IEEE 802.11a, HIPERLAN-2, [1] [2]). On the other hand, optimal training signals for estimation of frequency-selective channels in MIMO OFDM systems were presented in [6] and [9]. In [12], we have recently derived general classes of optimal training signals for estimation of frequency-selective channels in MIMO OFDM. The pilot tone allocation among transmit antennas are classified as frequency division multiplexing (FDM), time division multiplexing (TDM), code division multiplexing in time-domain (CDM-T), code division multiplexing in frequency domain (CDM-F), and combinations thereof. Based on these optimal training signals for MIMO channel estimation, a common training signal for estimation of both frequency offset and MIMO frequency-selective channels will be derived. The goal is to obtain a training signal having two properties: (i) the training signal for each transmit antenna consists of several consecutive identical sub-blocks for efficient implementation of frequency offset estimator and the sub-block signal is optimal for frequency offset estimation, (ii) the training signal is optimal for estimation of MIMO frequency-selective channels.

Let us consider the training signal design using one OFDM symbol, i.e.,  $Q=1$ , which contains pilot tones only. For simplicity, the symbol index  $q$  will be omitted.

(C.1) For the training signal to have  $D$  consecutive identical sub-blocks within one OFDM symbol where  $K/D$  is an integer and  $D \in \{2, 3, \dots\}$ , the non-zero pilot tones for each transmit antenna must be located at the sub-carrier indices  $\{kD : k=0, 1, \dots, (K/D) - 1\}$ .

(C.2) Among several classes of optimal training signals presented in [12], the above condition (C.1) is satisfied by the CDM-F type pilot tone allocation if  $K/D \geq N_t L$ .

(C.3) The CDM-F pilot tone allocation requires that generally all non-zero pilot tones must have the same amplitude and the optimal pilot tones for  $k$ -th transmit antenna,  $\{c_k[n]\}$ , are given by

$$c_k[n] = c_0[n] e^{j2\pi k L_0 n / K} \quad (7)$$

$$c_0[n] = \begin{cases} b[m], & n = mD, m = 0, 1, \dots, (K/D) - 1 \\ 0, & \text{else} \end{cases} \quad (8)$$

$$|b[m]| = b_0 > 0 \quad (9)$$

where  $L_0 \geq L$ ,  $K/L_0$  is an integer equal to the number of (active) transmit antennas within one OFDM symbol,  $k=0, 1, \dots, K/L_0$ , and  $\{b[m]\}$  are constant modulus symbols.

By combining the above conditions, we have the desired OFDM training symbol given by (7)-(9) where  $K = DN_t L_0$  and  $L_0 \geq L$ . This training signal contains  $D$  consecutive identical sub-blocks and satisfies the optimality condition for MIMO channel estimation which is inherited from the CDM-F pilot tone allocation.

The optimality of the sub-block signal for frequency offset estimation is investigated in the following. In [14], we show for frequency offset estimation in SISO systems that the sub-block signal is optimal in minimizing the average CRB of the frequency offset estimation in a frequency-selective fading channel if the sub-block signal possesses a zero autocorrelation for any non-zero correlation lag (this type of signal is usually referred to as zero autocorrelation (ZAC) signal). In other words, the ZAC signals result in minimum fluctuation of the received training signal energy in a frequency-selective fading channel which in turn translates into minimum average CRB. In MIMO systems, the received training signal at a receive antenna is the superposition of channel output training signals from all transmit antennas. For MIMO systems where the channels are independent, the minimum fluctuation of the total received training signal energy is achieved if each transmit antenna's channel output signal has minimum energy fluctuation. This is readily obtained if each transmit antenna's training sub-block signal is a ZAC signal. Our training sub-block signal for  $n$ -th transmit antenna, denoted by  $\bar{s}_n = [s_n[0], s_n[1], \dots, s_n[(K/D) - 1]]^T$ , can be generated by  $K/D$  point IFFT of the corresponding  $K/D$  non-zero pilot tones denoted by  $\bar{C}_n = [c_n[0], c_n[D], \dots, c_n[K - D]]^T$ . The periodic autocorrelation of  $\bar{s}_n$  with correlation lag  $l - m$  is given by  $(\bar{s}_n^{(l)})^H \bar{s}_n^{(m)}$ . By using  $\bar{s}_n^{(l)} = F_{K/D}^{-1} W(l) \bar{C}_n$  where  $F_{K/D}$  is the  $K/D$  point FFT matrix and  $W(l) = \text{diag} \{1, e^{-j2\pi l D / K}, e^{-j2\pi l 2D / K}, \dots, e^{-j2\pi l (K/D - 1) D / K}\}$  is a diagonal matrix, it can be easily shown that  $(\bar{s}_n^{(l)})^H \bar{s}_n^{(m)} = 0$  for  $l \neq m$ . This means that our training sub-block signal for each transmit antenna is a ZAC signal, hence an optimal sub-block signal for frequency offset estimation.

### IV. ESTIMATION OF FREQUENCY OFFSET AND CHANNELS USING ONE OFDM TRAINING SYMBOL

At each receive antenna, after the CP removal, the received training signal contains  $D$  sub-blocks which are identical in the absence of frequency offset and noise. For this type of received training signal, the best linear unbiased estimation (BLUE) methods (e.g., [1] [2]) show excellent performance (very close to CRB) and they have low implementation complexity. Hence, we adopt the BLUE method from [2] in this paper. The frequency offset estimate from  $m$ -th receive antenna is given by

$$\hat{v}_m = \frac{D}{2\pi} \mathbf{w}^T \boldsymbol{\phi}_m \quad (10)$$

where

$$\boldsymbol{\phi}_m = [\phi_m(1), \phi_m(2), \dots, \phi_m(D - 1)]^T \quad (11)$$

$$\phi_m(l) \triangleq [\arg\{R_m(l)\} - \arg\{R_m(l - 1)\}]_{2\pi}, \quad (12)$$

$$1 \leq l \leq D - 1$$

$$\mathbf{w} = \frac{\mathbf{C}_{\boldsymbol{\phi}_m}^{-1} \mathbf{1}}{\mathbf{1}^T \mathbf{C}_{\boldsymbol{\phi}_m}^{-1} \mathbf{1}}. \quad (13)$$

Here,  $\mathbf{1}$  is an all ones column vector of length  $D - 1$ .  $\mathbf{C}_{\boldsymbol{\phi}_m}$  is the covariance matrix of  $\boldsymbol{\phi}_m$  and its detailed expression is given in [2].  $R_m(l)$  is a correlation term defined as

$$R_m(l) = \sum_{k=0}^{K-lK/D-1} r_m^*(k) r_m(k + lK/D), \quad 0 \leq l \leq D - 1. \quad (14)$$

The final frequency offset estimate is simply given by the average of estimates from all receive antennas as

$$\hat{v} = \frac{1}{N_r} \sum_{m=0}^{N_r-1} \hat{v}_m. \quad (15)$$

The frequency offset estimation range is  $\pm D/2$  sub-carrier spacing. Note that since  $K = DN_t L_0$  and  $L_0 \geq L$ , the estimation range depends on  $K/(N_t L_0)$ . For a system with a large number of transmit antennas and a very large delay spread (very large  $L$ ), the above estimation range may not be sufficient to account for transmit and receive local oscillators mismatch and the channel Doppler shift. We will tackle this possible problem later in this paper.

The final frequency offset estimate is used in frequency offset compensation on the training signal and the data signal already received (i.e., those in buffer). It is also used to correct the receiver local oscillator's frequency for next incoming signal. This oscillator frequency correction may be performed immediately after the frequency offset estimation or on packet by packet basis. The frequency offset compensated received training signal from  $m$ -th receive antenna is given by

$$\hat{r}_m = \mathbf{W}^H(\hat{v}) r_m. \quad (16)$$

After the frequency offset compensation is performed, the least-square-type channel estimation at  $m$ -th receive antenna is performed as

$$\hat{h}_m = (\mathbf{S}^H \mathbf{S})^{-1} \mathbf{S}^H \hat{r}_m. \quad (17)$$

Since the training signal is designed to be optimal for MIMO OFDM channel estimation, it satisfies the following [12]:

$$(\mathbf{S}^H \mathbf{S})^{-1} = (1/E_{av}) \mathbf{I}. \quad (18)$$

Hence, the channel estimation is simplified to

$$\hat{h}_m = \frac{1}{E_{av}} \mathbf{S}^H \hat{r}_m. \quad (19)$$

## V. ESTIMATION OF FREQUENCY OFFSET AND CHANNELS USING $Q$ OFDM TRAINING SYMBOLS

To extend the frequency offset estimation range, we can use  $Q$  OFDM training symbols instead of only one OFDM training symbol. We choose  $Q$  such that both  $M_t = N_t/Q$  and  $\bar{D} = \frac{K}{M_t L_0}$  are integers. Then, we partition  $N_t$  transmit antennas into  $Q$  groups; each has  $M_t$  transmit antennas. In  $q$ -th OFDM symbol, only one group of transmit antennas (with indices  $n = [qM_t, \dots, (q+1)M_t - 1]$ ) transmit training signals. In this case,  $\mathbf{S}_n[q]$  in (3) is a zero matrix for all  $n \notin [qM_t, \dots, (q+1)M_t - 1]$ . Then, the single-symbol-based methods described in the previous section can be applied to estimate frequency offset and channels for  $M_t$  transmit antennas in the current transmit antenna group. The training signal design discussed in the previous section is applied to each symbol interval for the  $M_t$  active transmit antennas and  $D$  is now replaced with  $\bar{D}$ . The estimation range now is  $\pm \bar{D}/2$  which is  $Q$  times that of single-symbol-based method. Note that  $\bar{D} = QD$ . The frequency offset estimation from  $m$ -th receive antenna at  $q$ -th OFDM symbol interval is

$$\hat{v}_{mq} = \frac{\bar{D}}{2\pi} \mathbf{w}_{mq}^T \phi_{mq} \quad (20)$$

where

$$\phi_{mq} = [\phi_{mq}(1), \phi_{mq}(2), \dots, \phi_{mq}(\bar{D}-1)]^T \quad (21)$$

$$\phi_{mq}(l) \triangleq [\arg\{R_{mq}(l)\} - \arg\{R_{mq}(l-1)\}]2\pi, \quad 1 \leq l \leq \bar{D}-1 \quad (22)$$

$$\mathbf{w}_{mq} = \frac{\mathbf{C}_{\phi_{mq}}^{-1} \mathbf{1}}{\mathbf{1}^T \mathbf{C}_{\phi_{mq}}^{-1} \mathbf{1}}. \quad (23)$$

Here,  $\mathbf{1}$  is an all-ones column vector of length  $\bar{D}-1$ .  $\mathbf{C}_{\phi_{mq}}$  is the covariance matrix of  $\phi_{mq}$ .  $R_{mq}(l)$  is a correlation term defined as

$$R_{mq}(l) = \sum_{k=0}^{K-lK/\bar{D}-1} r_{mq}^*(k) r_{mq}(k+lK/\bar{D}), \quad 0 \leq l \leq \bar{D}-1. \quad (24)$$

The final frequency offset estimate is simply given by the average of estimates from all receive antennas over  $Q$  symbols as

$$\hat{v} = \frac{1}{N_r Q} \sum_{q=0}^{Q-1} \sum_{m=0}^{N_r-1} \hat{v}_{mq}. \quad (25)$$

If a smaller complexity is preferred,  $\hat{v}$  can be calculated as the average of  $\{\hat{v}_{m0}\}$  only. For a given set of channel gains, the snapshot CRB of  $\hat{v}$ , after skipping details, is given by

$$\text{CRB}_{|h} = \frac{3\sigma_n^2}{2\pi^2(1-1/\bar{D}^2)E_{av} \sum_{m=0}^{N_r-1} (\mathbf{h}_m^H \mathbf{h}_m)} \quad (26)$$

In the above equation, we have used the following property of the training signal [12]:

$$\mathbf{S}_n^H \mathbf{S}_n = E_{av} \mathbf{I}, \quad n = 0, 1, \dots, N_t - 1 \quad (27)$$

$$\text{where } \mathbf{S}_n = [\mathbf{S}_n^T[0], \mathbf{S}_n^T[1], \dots, \mathbf{S}_n^T[Q-1]]^T \quad (28)$$

$$E_{av} = \frac{1}{N_t} \sum_{n=0}^{N_t-1} \sum_{q=0}^{Q-1} \sum_{k=0}^{K-1} |s_{n,q}[k]|^2. \quad (29)$$

Define the training signal to noise ratio as

$$\text{SNR} = \frac{N_t E_{av}}{K \sigma_n^2 Q}. \quad (30)$$

Then the snap-shot CRB of (26) is simplified to

$$\text{CRB}_{|Z} = \frac{\alpha}{Z} \quad (31)$$

$$\text{where } \alpha = \frac{3 \text{SNR}^{-1}}{2\pi^2 K Q N_r (1-1/\bar{D}^2)} \quad (32)$$

$$Z = \frac{\sum_{m=0}^{N_r-1} \sum_{n=0}^{N_t-1} \sum_{l=0}^{L-1} |h_{n,m}[l]|^2}{N_r N_t}. \quad (33)$$

The numerator of  $Z$  is the sum of central chi-square random variables  $\{|h_{n,m}[l]|^2 : l = 0, \dots, L-1, n = 0, \dots, N_t-1, m = 0, \dots, N_r-1\}$ .  $Z$  can be well approximated by a Gamma random variable (see [13], [14]). Then, the average CRB of  $\hat{v}$  is given by

$$\text{CRB} = \int_0^\infty \text{CRB}_{|Z} p(Z) dZ \simeq \frac{\alpha}{E[Z] - \sigma_z^2/E[Z]}. \quad (34)$$

We assume that the channels are independent and have the same power delay profiles, i.e.,  $E[|h_{n,m}[l]|^2] = \sigma_l^2$ ,  $l = 0, \dots, L-1$ ,  $\forall n, m$ . For the SNR defined in (30) to be the average received SNR at a receive antenna, we must have  $\sum_{l=0}^{L-1} \sigma_l^2 = 1$ . Then the CRB of  $\hat{v}$  becomes

$$\text{CRB} = \frac{3 \text{SNR}^{-1}}{2\pi^2 K Q (1-1/\bar{D}^2) (N_r - \sum_{l=0}^{L-1} \sigma_l^4 / N_t)}. \quad (35)$$

After the frequency offset compensation is performed, the least-square type channel estimation for a transmit antenna  $n \in [qM_t, \dots, (q+1)M_t - 1]$  (which is active at  $q$ -th OFDM symbol) and  $m$ -th receive antenna is performed as

$$\hat{h}_{n,m} = \frac{1}{E_{av}} \mathbf{S}_n^H[q] \hat{r}_{mq}, \quad 0 \leq q \leq Q-1, 0 \leq m \leq N_r-1. \quad (36)$$

At perfect frequency recovery, the channel estimation MSE for each channel tap is given by

$$\text{MSE}_h = \frac{\sigma_n^2}{LN_t} \text{tr}\{(\mathbf{S}^H \mathbf{S})^{-1}\} = \frac{N_t}{KQ\text{SNR}}. \quad (37)$$

Note that we can also partition  $N_t$  transmit antennas into  $Q$  groups with unequal number of transmit antennas. For example, if  $N_t=6$ ,  $K = 64$ ,  $L = L_0 = 16$ , and  $Q = 2$ , then we can assign 2 transmit antennas in the first symbol and the remaining 4 transmit antennas in the second symbol. The frequency offset estimation can be based on the first symbol only and the estimation range is  $\pm 1$  sub-carrier spacing.

For pilot-data multiplexed scheme, frequency offset estimation would be affected by the data tones' interference on the pilots since the received pilots and data are no longer orthogonal in the presence of a frequency offset. A modified scheme to alleviate the data interference is described below. Data tones closer to a pilot tone cause larger interference on the pilot. For every pilot tone with sub-carrier index  $p$ , the data tones at sub-carrier indices  $[p-1]_N$  and  $[p+1]_N$  are set to be the same where  $[\cdot]_N$  denotes modulo- $N$  operation. Since data tone interferences to the left and to the right in the sub-carrier domain are almost anti-symmetric [15], the above modified scheme almost cancels the largest interference term coming from the two data sub-carriers adjacent to the pilot.

## VI. SIMULATION RESULTS AND DISCUSSIONS

We have evaluated the estimation methods presented in this paper for MIMO OFDM systems in frequency selective fading channels. The simulation parameters are as follows: the number of sub-carriers  $K = 256$ , channel length  $L = 16$ ,  $L_0 = L$ , and the number of transmit antennas  $N_t = 4$ . We simulated one-symbol scheme ( $Q = 1$ ) with  $v = 0.3$ , and two-symbol scheme ( $Q = 2$ ) with  $v = 2.6$  (to illustrate a larger estimation range) for the number of receive antennas  $N_r = 1$  and  $N_r = 2$ . The two-symbol scheme partitions  $N_t = 4$  transmit antennas into two groups ( $Tx1, Tx2$ ) and ( $Tx3, Tx4$ ). During the first OFDM training symbol interval, only ( $Tx1, Tx2$ ) transmit training signals while during the second interval only ( $Tx3, Tx4$ ) transmit. The frequency offset estimation is based on all received signal over two symbols.

The MSEs of frequency offset estimation using the BLUE method are shown in Fig.1. The corresponding CRBs are included as references. Fig.1 shows that two-symbol-scheme gives a 3 dB SNR advantage in frequency offset estimation performance for both  $N_r = 1$  and  $N_r = 2$ . This is simply due to the use of twice the total training signal energy. We also observe from Fig.1 that  $N_r = 2$  has a 3dB SNR advantage over  $N_r = 1$  which is simply due to the diversity provided by two receive antennas. The frequency offset estimation range extension using  $Q$  training symbols is also confirmed in the simulation results.

The channel estimation MSEs obtained from simulation and the ideal MSE (the minimum MSE in the absence of frequency offset) are shown in Fig.2. The 3 dB SNR advantage of the two-symbol-based channel estimation is simply due to the use of twice the total training signal energy for each transmit antenna.

The frequency offset estimation results for pilot-data multiplexed schemes are presented in Figs. 3 and 4, respectively, for

conventional and modified schemes.<sup>1</sup> In the simulation, the ratio of total pilot energy to data energy is 5dB. Due to data signal interference, MSE performance is degraded<sup>2</sup> but the modified scheme gives an appreciable improvement. The channel estimation results for pilot-data multiplexed schemes are shown in Fig.5. For channel estimation, the effect of data signal interference are negligible since residual frequency offset is small.

## VII. CONCLUSIONS

We have presented a common training signal design and estimation methods for carrier frequency offset and frequency-selective channels in MIMO OFDM systems. The proposed training signals are optimal for MIMO channel estimation. They contain several identical sub-blocks to yield low complexity, high performance frequency offset estimation. The sub-block signals are optimal for frequency offset estimation in frequency-selective fading channels. In the proposed methods, the best linear unbiased estimation method is applied in frequency offset estimation while maximum likelihood approach is adopted in channel estimation. The performances of the proposed methods using the proposed training signal are very close to the CRB or the minimum MSE of the estimation for pilot-only schemes. Due to data interference, the estimation performances in pilot-data multiplexed schemes suffer some degradation. This paper also presents a modified pilot-data multiplexed scheme which appreciably alleviates the data interference effect on the estimation performance.

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<sup>1</sup>The CRB for pilot-data multiplexed scheme is calculated as  $\text{CRB} = 1/\text{Tr}[\mathbf{K}_r^{-1} \frac{\partial \mathbf{K}_r}{\partial v} \mathbf{K}_r^{-1} \frac{\partial \mathbf{K}_r}{\partial v}]$  where the covariance matrices of channel and data are embedded in  $\mathbf{K}_r$ , the covariance matrix of the observation vector  $\mathbf{r}$ . Due to space limitation, it is omitted.

<sup>2</sup>Performance could be improved by utilizing decision-directed data in the estimation in addition to pilots but this approach is not considered in this paper.

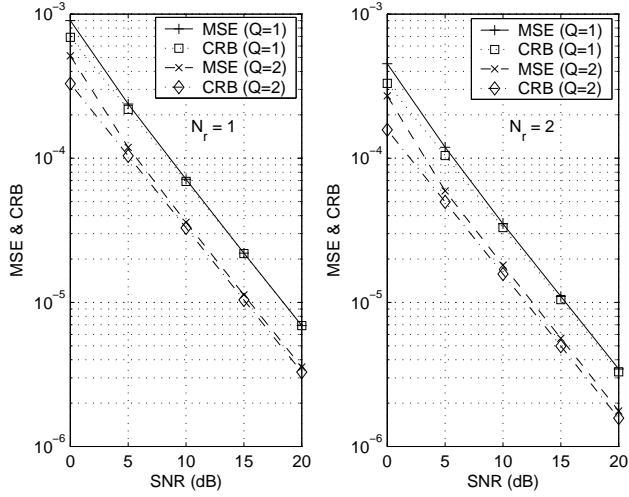


Fig. 1. Frequency offset estimation performance for the pilot-only scheme

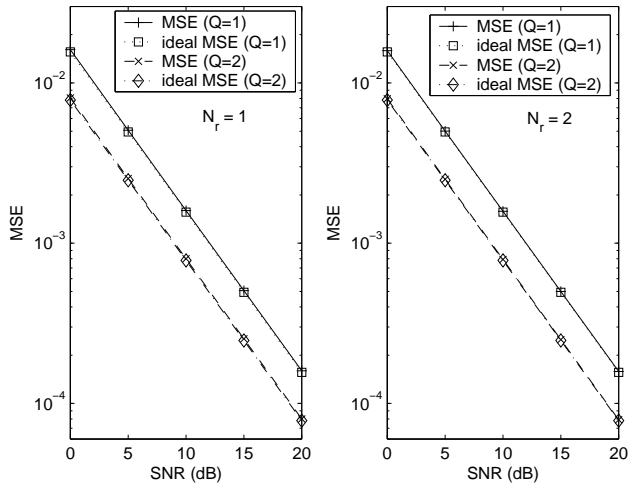


Fig. 2. Channel estimation performance for the pilot-only scheme

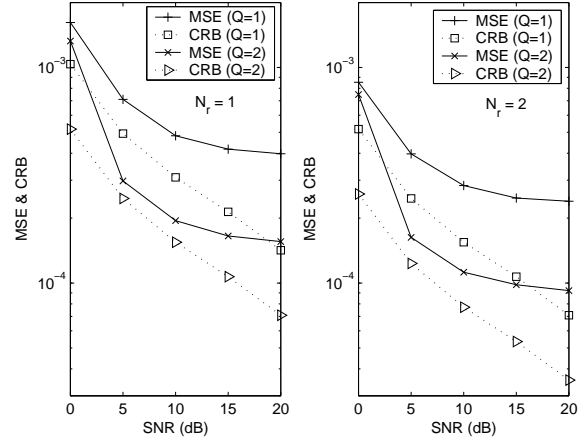


Fig. 3. Frequency offset estimation performance for the (conventional) pilot-data multiplexed scheme

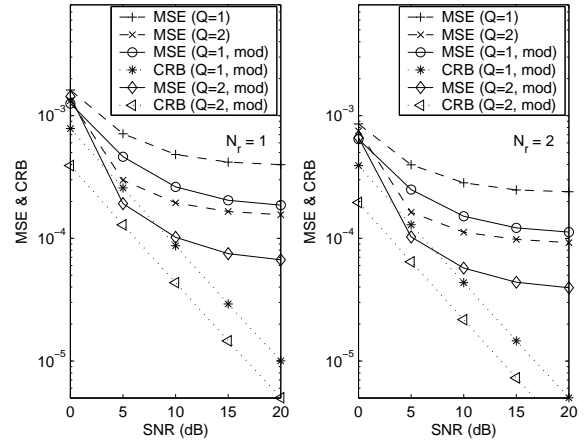


Fig. 4. Frequency offset estimation performance for the modified pilot-data multiplexed scheme (MSE performances of the conventional pilot-data multiplexed scheme are included as references.)

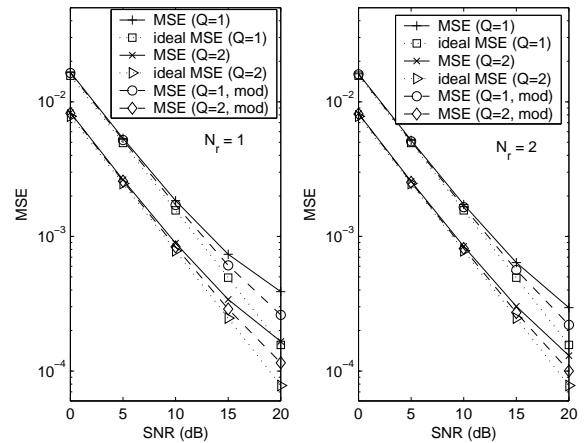


Fig. 5. Channel estimation performance for the pilot-data multiplexed schemes

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