A Reduced Complexity Channel Estimation for OFDM Systems with Precoding and Transmit Diversity in Mobile Wireless Channels

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Abstract – **A precoding approach for reduced complexity channel estimation for OFDM systems with transmit diversity is presented. Two different OFDM schemes that employ the precoding are investigated: the first one is based on space-time coding as in [1], the second is based on a transmit diversity using an optimal design of symbols for channel estimation. The first scheme reduces the size of the inverse matrix in channel estimation to half of the existing method [1] at the expense of slightly degraded BER performance. The second does not require any matrix inversion and hence achieves much complexity saving but has a BER performance degradation at moderate and high SNR and a better BER performance at low SNR.**

Keywords: **OFDM, Precoding, Transmit diversity, Reduced complexity channel estimation, Space-time coding, MRC**

I. Introduction

In order to compensate for the frequency selective fading in OFDM, techniques such as error correcting code and diversity have to be used [1]-[5]. For OFDM systems, transmit diversity combined with Reed-Solomon code has been proposed for clustered OFDM in [2]. Application of space-time coding [6] in OFDM systems has been studied in [7] with perfect channel knowledge at the receiver. The channel estimation for OFDM systems without transmit diversity has been studied by many researchers (e.g., [8] - [10]). However, for systems with transmit diversity, the received signal is a superposition of the different transmitted signals from all transmit antennas and, consequently, the channel estimation becomes more complicated. Recently, the channel estimation for OFDM systems with transmit diversity using space-time coding has been proposed in [1]. In order to reduce the complexity associated with the matrix inverse operation, [1] also proposed a simplified approach.

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In this paper, we focus on reduced complexity channel estimation for OFDM systems with transmit diversity. We propose a channel estimation method that has less complexity than the simplified method of [1]. The proposed channel estimation employs precoding on the subcarrier symbols of each transmit antenna and makes use of the correlation between adjacent subchannel frequency responses. An optimal criterion for the training symbol of the proposed channel estimation is derived. The proposed channel estimation is applied in two schemes: the first scheme uses space-time coding as in [1] and the second scheme uses a transmit diversity scheme that satisfies an optimal design for the proposed channel estimation and maximal ratio combining (MRC) at the receiver. The rest of the paper is organized as follows. In Section II, the proposed channel estimation is presented. In Section III, the mean square error (MSE) of the proposed channel estimation is derived. The channel estimation MSE and BER performances are evaluated for the proposed schemes and [1] in Section IV. Conclusion is given in Section V.

II. System Description

Consider an OFDM system with two-branch transmit diversity and two-branch receive diversity. Let the frequency-domain subcarrier symbols for i^{th} transmit antenna at time n be $\{t_i[n,k] : k = 0, 1, \ldots, K-1\}$ for $i = 1, 2$, with K being the total number of subcarriers. Then, the DFT output frequency-domain subcarrier symbols from each receive antenna can be expressed as

$$r_j[n,k] = \sum_{i=1}^{2} H_{ij}[n,k]\ t_i[n,k] + w_j[n,k] \quad (1)$$

where $w_j[n,k]$ is the additive complex Gaussian noise with zero mean and variance σ_n^2, on the j^{th} receive antenna, that is uncorrelated for different n's, k's, or j's, and $H_{ij}[n,k]$ is the channel frequency response for the k^{th} tone at time n, corresponding to the i^{th} transmit antenna and the j^{th} receive antenna. With tolerable leakage, it can be expressed as [1]

$$H[n,k] = \sum_{l=0}^{K_0-1} h[n,l]\ W_K^{kl} \quad (2)$$

where $W_K = exp(-j2\pi/K)$ and K_0 is the total number of WSSUS channel paths. In the proposed channel estimation scheme, precoding is applied to $\{t_i[n,k]\}$ before entering IDFT block, as shown in Fig. 1. The precoded frequency-domain subcarrier symbols $\{t_{pi}[n,k]\}$ are given by

$$t_{pi}[n,k] = (-1)^{ik}\ t_i[n,k]. \quad (3)$$

Then, the frequency-domain subcarrier symbols from the j^{th} receive antenna can be expressed as

$$r_j[n,k] = \sum_{i=1}^{2} H_{ij}[n,k]\ (-1)^{ik}\ t_i[n,k] + w_j[n,k]. \quad (4)$$

Let us define the following:

$$\begin{aligned}
t_{12}[n,2m] &\triangleq t_1^*[n,2m]t_2[n,2m] \\
&\quad + t_1^*[n,2m+1]t_2[n,2m+1] \\
r_{1j}[n,2m] &\triangleq t_2^*[n,2m]r_j[n,2m] \\
&\quad - t_2^*[n,2m+1]r_j[n,2m+1] \\
r_{2j}[n,2m] &\triangleq t_1^*[n,2m]r_j[n,2m] \\
&\quad + t_1^*[n,2m+1]r_j[n,2m+1] \\
w_{1j}[n,2m] &\triangleq t_2^*[n,2m]w_j[n,2m] \\
&\quad - t_2^*[n,2m+1]w_j[n,2m+1] \\
w_{2j}[n,2m] &\triangleq t_1^*[n,2m]w_j[n,2m] \\
&\quad + t_1^*[n,2m+1]w_j[n,2m+1]
\end{aligned}$$

where x^* denotes the complex conjugate of x and $m = 0,\ 1,\ \ldots,\ M-1$ and $M = K/2$.

If the channel frequency responses are such that

$$H_{ij}[n,2m] = H_{ij}[n,2m+1], \quad (5)$$

then the following are obtained:

$$\begin{aligned}
r_{1j}[n,2m] &= H_{1j}[n,2m]\ t_{12}^*[n,2m] + w_{1j}[n,2m] \quad (6) \\
r_{2j}[n,2m] &= H_{2j}[n,2m]\ t_{12}[n,2m] + w_{2j}[n,2m]. \quad (7)
\end{aligned}$$

Assuming that (5) holds, the channel impulse response estimate can be obtained, using a similar approach to [1], by minimizing the following MSE cost function,

$$C\left(\{\tilde{h}_{ij}[n,l]; i=1,\ 2;\ j=1,2\}\right) = \sum_{m=0}^{M-1} \left| r_{ij}[n,2m] - \sum_{l=0}^{K_0-1} \tilde{h}_{ij}[n,l]\ W_K^{2ml}\ t_{12}^i[n,2m] \right|^2 \quad (8)$$

where

$$t_{12}^i[n,2m] \triangleq \begin{cases} t_{12}^*[n,2m], & i=1 \\ t_{12}[n,2m], & i=2. \end{cases} \quad (9)$$

The channel impulse response estimate can then be given by

$$\tilde{\mathbf{h}}_{ij}[n] = \mathbf{Q}^{-1}[n]\ \mathbf{p}_{ij}[n]. \quad (10)$$

where

$$\mathbf{Q}[n] \triangleq \begin{pmatrix} q[n,0] & q[n,-1] & \ldots & q[n,-K_0+1] \\ q[n,1] & q[n,0] & \ldots & q[n,-K_0+2] \\ \vdots & \vdots & \ddots & \vdots \\ q[n,K_0-1] & q[n,K_0-2] & \ldots & q[n,0] \end{pmatrix}$$

$$\begin{aligned}
\tilde{\mathbf{h}}_{ij}[n] &\triangleq (\tilde{h}_{ij}[n,0],\ \tilde{h}_{ij}[n,1],\ \ldots,\ \tilde{h}_{ij}[n,K_0-1])^T \\
\mathbf{p}_{ij}[n] &\triangleq (p_{ij}[n,0],\ p_{ij}[n,1],\ \ldots,\ p_{ij}[n,K_0-1])^T \\
q[n,l] &\triangleq \sum_{m=0}^{M-1} t_{12}[n,2m]\ t_{12}^*[n,2m]\ W_K^{-2ml} \\
p_{ij}[n,l] &\triangleq \sum_{m=0}^{M-1} r_{ij}[n,2m]\ t_{12}^{i^*}[n,2m]\ W_K^{-2ml}.
\end{aligned}$$

Following [1], the complexity involved in $\mathbf{Q}^{-1}$ can be further reduced and the channel estimation performance can be further improved by using only significant channel taps. During the training block, the significant taps can be identified by finding the l's with large $\sum_{i=1}^{2} |h_{ij}[1,l]|^2$ for each receive antenna. Suppose that J significant taps are used and let $\{l_m : m = 1,\ 2,\ \ldots,\ J;\ (0 \le l_1 < l_2 < \ldots < l_J \le K_0 - 1)\}$ be the J significant channel tap indexes for the receive antenna j. Then, the simplified channel estimation is given by

$$\bar{\mathbf{h}}_{ij}[n] = \bar{\mathbf{Q}}^{-1}[n]\ \bar{\mathbf{p}}_{ij}[n] \quad (11)$$

where

$$\bar{\mathbf{Q}}[n] \triangleq \begin{pmatrix} q[n,0] & q[n,l_1-l_2] & \ldots & q[n,l_1-l_J] \\ q[n,l_2-l_1] & q[n,0] & \ldots & q[n,l_2-l_J] \\ \vdots & \vdots & \ddots & \vdots \\ q[n,l_J-l_1] & q[n,l_J-l_2] & \ldots & q[n,0] \end{pmatrix}$$

$$\bar{\mathbf{h}}_{ij}[n] \triangleq (\tilde{h}_{ij}[n,l_1],\ \tilde{h}_{ij}[n,l_2],\ \ldots,\ \tilde{h}_{ij}[n,l_J])^T$$
$$\bar{\mathbf{p}}_{ij}[n] \triangleq (p_{ij}[n,l_1],\ p_{ij}[n,l_2],\ \ldots,\ p_{ij}[n,l_J])^T.$$

As mentioned in [1], a time correlation of $h_{ij}[n,l_m]$ can be used to further improve the channel estimation but is not considered in this paper.

III. Performance Analysis

In the previous section, a reduced complexity channel estimation for OFDM systems with transmit diversity is proposed with the assumption that (5) holds. In this section, the performance of the proposed method in a multipath fading channel not satisfying (5) is presented. For simplicity, the time index n is omitted in the following analysis. Define the following:

$$\Delta H_{ij}[2m] \triangleq H_{ij}[2m] - H_{ij}[2m+1]$$
$$\Delta H_{e1}[2m] \triangleq \Delta H_{2j}[2m] - \Delta H_{1j}[2m]\ t_1[2m+1]\ t_2^*[2m+1]$$
$$\Delta H_{e2}[2m] \triangleq \Delta H_{1j}[2m] - \Delta H_{2j}[2m]\ t_1^*[2m+1]\ t_2[2m+1]$$
$$X_1[l_0] \triangleq \sum_{m=0}^{M-1} t_{12}[2m]\ \Delta H_{e1}[2m]\ W_M^{-ml_0}$$
$$\mathbf{X}_1 \triangleq [X_1[0],\ X_1[1],\ \ldots,\ X_1[K_0-1]]^T$$

The MSE of the channel impulse response estimate can be given by

$$\begin{aligned} MSE &\triangleq \frac{1}{K_0}\ E\left\{\|\tilde{\mathbf{h}}_{1j}-\mathbf{h}_{1j}\|^2\right\} \\ &= \frac{1}{K_0}\ \left\{Trace\left\{2\ \sigma_n^2\ \mathbf{Q}^{-1^H}\right\}\right. \\ &+ \left. Trace\left\{\mathbf{Q}^{-1}E\{\mathbf{X}_1\ \mathbf{X}_1^H\}\mathbf{Q}^{-1^H}\right\}\right\} \end{aligned} \quad (12)$$

From (12), an optimal solution can be obtained when $\mathbf{Q}$ is diagonal matrix, i.e., $q[n,l]=0$ for $l \neq 0$. This optimal condition is achieved when either of the following two conditions are satisfied: (i) $t_i[n,2m] = t_i[n,2m+1]$, $i = 1,\ 2;\ m = 0,\ 1,\ \ldots,\ M-1$, and (ii) $t_1[n,k] = t_2[n,k]$, $k = 0,\ 1,\ \ldots,\ K-1$. With optimal condition, and assuming $|t_i|^2 = 1$, the MSE can be given by

$$MSE = \frac{\sum_{l=1}^{K_0-1} \left|1-W_K^l\right|^2\ P[l]}{2\ K_0} + \frac{\sigma_n^2}{K} \quad (13)$$

where $P[l]$ is the power delay profile of the channel corresponding to each transmit and receive antenna pairs. The channel frequency response estimate MSE of the proposed method under optimal condition can also be given by

$$MSE\{\tilde{H}_{ij}[m]\} = \frac{1}{2}\sum_{l=0}^{K_0-1}\left|1-W_K^l\right|^2 P[l] + \frac{K_0}{K}\sigma_n^2. \quad (14)$$

Similarly, the MSE of the channel frequency response estimate of [1] at its optimal condition can be shown to be equal to

$$MSE\{\tilde{H}_{ij}[m]\} = \sigma_n^2\ K_0/K. \quad (15)$$

Hence, the proposed method has a larger channel frequency response estimation MSE than [1] due to the assumption of (5). The extra term in the MSE expression also indicates the dependence of the proposed channel estimation MSE performance on the channel power delay profile. We have also validated the MSE expressions by simulation but the results are not shown in this paper.

For systems with L-transmit diversity, the average SNR at the receiver, assuming $|t_i|^2 = 1$, is defined as

$$SNR = \frac{E\left\{\sum_{i=1}^{L}|H_{ij}[n,k]|^2\right\}}{\sigma_n^2}. \quad (16)$$

From (14), it can be observed that the channel estimation MSE mainly depends on σ_n^2 which, in turn, indicates that the channel estimation MSE performance evaluation by computer simulation depends on how to model $\{E\{|H_{ij}[n,k]|^2\}\}$. For a SNR value, if $\{E\{|H_{ij}[n,k]|^2\}\}$ are modeled very small, σ_n^2 will be very small and, consequently, the obtained channel estimation MSE will be very small. Hence, a suitable simulation model for channel estimation performance evaluation should be adopted. There are two possible approaches: the first one is to model $E\left\{\sum_{i=1}^{L}|H_{ij}[n,k]|^2\right\} = 1$, and the second is to model $E\left\{|H_{ij}[n,k]|^2\right\} = 1$. Due to the similar reasoning of the MSE's dependence on channel power gain and to be comparable with different number of diversity branches, the second approach of $E\left\{|H_{ij}[n,k]|^2\right\} = 1$ is adopted in this paper.

IV. Performance Evaluation by Computer Simulation

IV.1 Proposed Channel Estimation with Space-Time Code

The proposed channel estimation is applied in two schemes: the first scheme uses space-time coding as in [1]. As shown in Fig. 1(a), at the receiver, the channel estimates are precoded before entering space-time Viterbi decoding so that the same space-time decoding as in [1] can be used. In other words, the combined effect of precoding at the transmitter and multipath channel response can be treated as the effective channel response for space-time coding/decoding. The precoded channel estimates are given by

$$\tilde{H}_{p,ij}[n,k] = (-1)^{ik}\ \tilde{H}_{ij}[n,k]. \quad (17)$$

Then, the space-time Viterbi decoding for the system using proposed channel estimation uses the metrics

$$\left\| \mathbf{r}[n,k] - \tilde{\mathbf{H}}_p[n,k]\ \tilde{\mathbf{t}}[n,k] \right\|^2 \quad (18)$$

where $\| * \|$ denotes Euclidean norm, and

$$\mathbf{r}[n,k] \triangleq \begin{pmatrix} r_1[n,k] \\ r_2[n,k] \end{pmatrix}$$

$$\tilde{\mathbf{H}}_p[n,k] \triangleq \begin{pmatrix} \tilde{H}_{p,11}[n,k] & \tilde{H}_{p,21}[n,k] \\ \tilde{H}_{p,12}[n,k] & \tilde{H}_{p,22}[n,k] \end{pmatrix}$$

and $\tilde{\mathbf{t}}[n,k]$ is the estimated signal vector, defined as

$$\tilde{\mathbf{t}}[n,k] \triangleq \begin{pmatrix} \tilde{t}_1[n,k] \\ \tilde{t}_2[n,k] \end{pmatrix}.$$

From (14) - (15), [1] has a better channel estimation MSE at its optimal condition (i.e., for training symbol). Hence, the proposed channel estimation in the first scheme will have a larger channel estimation MSE than [1].

By observing that a matrix inversion is not needed for the training symbol, we may consider a hybrid scheme for which the channel estimation of [1] is used only for the training symbol in order to yield a better channel estimation and the proposed channel estimation is applied to the data symbols in order to reduce the complexity.

IV.2 Proposed Channel Estimation with Optimal Signal Design

To achieve comparable MSE performance, the proposed channel estimation is applied in the second scheme, as shown in Fig. 1(b), where the transmit diversity scheme that satisfies the optimal condition for the proposed channel estimation is used, i.e., $t_1[n,k] = t_2[n,k]$ before precoding. Similar to the first scheme, the combined effect of precoding at the transmitter and multipath channel response can be treated as the effective channel response for MRC detection. The MRC output is given by

$$r_{MRC}[n,k] = r_1[n,k]\ \alpha_1[n,k] + r_2[n,k]\ \alpha_2[n,k] \quad (19)$$

where

$$\alpha_1[n,k] \triangleq (\tilde{H}_{p,11} + \tilde{H}_{p,21})^*$$
$$\alpha_2[n,k] \triangleq (\tilde{H}_{p,12} + \tilde{H}_{p,22})^*.$$

The decision is to choose the signal point t from the signal constellation points that minimizes the metric $\|r_{MRC}[n,k] - t\|$.

IV.3 Simulations, Results and Discussions

The system parameters used in the simulation are the same as [1]: 128 subchannels, 4 guard-subchannels on each end, $1/160\mu s$ subchannel spacing, 40 μs guard interval, two-ray (TR) and GSM-typical urban (TU) channel models with delay spread of $1.06\mu s$ and 40 Hz Doppler frequency, 16-state space-time code with four PSK. The number of significant taps used in all channel estimation methods is 7.

The channel estimation MSE performances are shown in Fig. 2 for both TR and GSM-TU channel models. When comparing [1] and the first scheme both using space-time coding, it is found that the latter has a slightly worse MSE performance for both channel as expected from (14)-(15). The MSE of hybrid scheme is also plotted, which is almost the same as the first scheme: just marginally better but not noticeable in the figure.

The MSE performances of the proposed channel estimation with optimal signal design (i.e., the second scheme) and [1] are similar. At low SNR region, the second scheme has a slightly better MSE performance for both channels. At high SNR region, [1] has a slightly better MSE performance than the second scheme for both channels.

The reason may be explained as follows. The MSE of [1] is contributed by the noise, the decision-directed reference data error and the channel estimation leakage due to non-sample spaced channel taps, while that of proposed channel estimation is contributed by the noise, the decision-directed reference data error, the channel power delay profile and the channel estimation leakage. The effect of noise on MSE of [1] is larger than that of the proposed method with optimal signal design because of the non-optimality of data symbols for the channel estimation of [1]. At very low SNR region, noise effect is much more dominant than the others and hence [1] has a slightly worse MSE. At high SNR region, the noise effect becomes less dominant than the others and hence [1] tends to achieve a slightly better MSE. The slightly worse MSE of the second scheme in both channels may be attributed to the larger reference data error, as can be seen from its BER performance in Figs. 3 and 4.

When comparing [1] and the first scheme, both using space-time coding, the latter has a slightly worse (but comparable) BER performance than the former does, for both channels. This is due to the slightly worse channel estimation performance of the latter and is the price paid for the reduced complexity. It reduces the size of the matrix, the inverse of which has to be found for channel estimation at every OFDM symbol, to half of that in [1]. The BER performance of the hybrid scheme is almost the same as the first scheme.

Although the MSE performances of the second scheme and [1] are similar, there are considerable differences in their BER performances. The second scheme has a better BER performance at very low SNR region but [1] has a much better BER performance at high SNR region. The reason is that, at very low SNR region, space-time code loses its coding gain and hence [1] has a worse BER performance while, at high SNR region, non-error correction capability of the second scheme causes it to have a worse BER performance.

V. Conclusion

A new channel estimation based on precoding technique has been proposed for OFDM systems with transmit diversity, which aims at greatly reducing the complexity in the matrix inversion needed for every OFDM symbol. In particular, the proposed scheme using precoding combined with space-time coding, as in [1], shows a comparable BER performance to [1] while achieving complexity saving, i.e., the matrix inverse size is reduced by half. Also, the other scheme with transmit diversity based on the optimal design of precoded symbols, combined with MRC at the receiver, attains much receiver complexity saving at the expense of some BER performance degradation at moderate and high SNR but has a better BER performance at low SNR.

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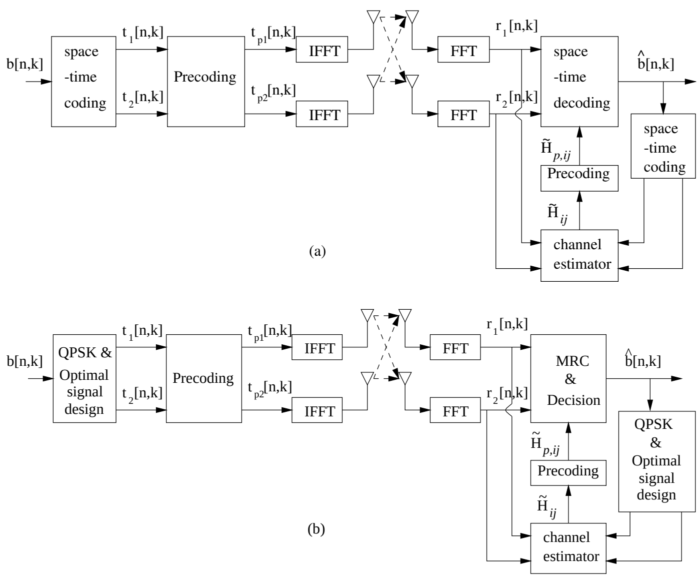

Fig. 1. Proposed reduced complexity channel estimation for OFDM systems with transmit diversity in two schemes: (a) 1^{st} scheme using space-time coding, (b) 2^{nd} scheme using optimal signal design and MRC reception

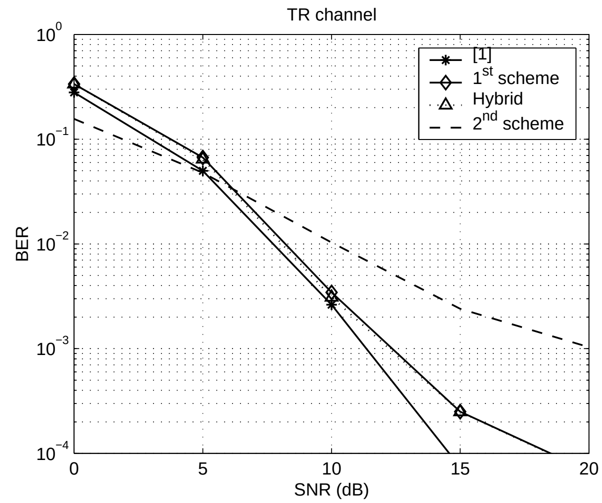

Fig. 3. BER performances of OFDM systems with transmit diversity using different channel estimation methods in TR channel.

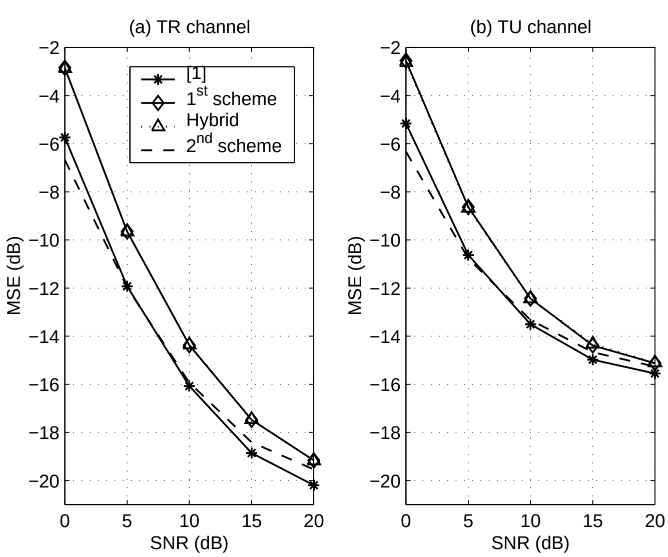

Fig. 2. Channel estimation MSE performances for OFDM systems with transmit diversity: (a) TR channel and (b) GSM-TU channel.

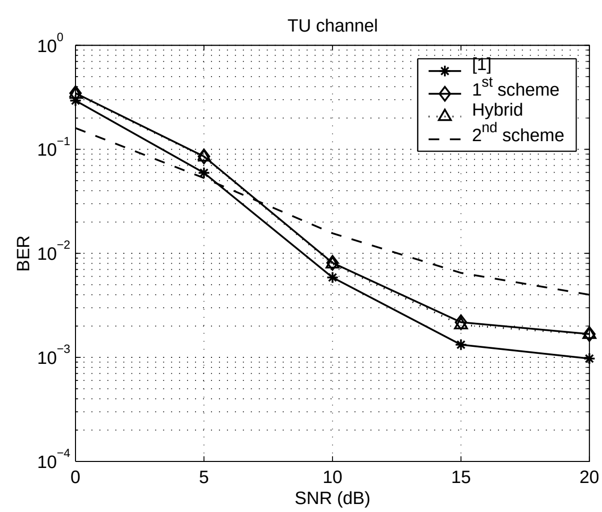

Fig. 4. BER performances of OFDM systems with transmit diversity using different channel estimation methods in GSM-TU channel.