where mobiles j are not served by the same base station that serves mobile i. We note that the interference power of the signal has the property

$$E\left\{|\iota_{\text{intra},i}+\iota_{\text{inter},i}|^2\right\}=E\left\{|\iota_{\text{intra},i}|^2\right\}+E\{\iota_{\text{inter},i}|^2\} \quad (31)$$

where the intracell interference comes from the same source as the signal itself

$$E\left\{|\iota_{\text{intra},i}|^2\right\}=\frac{P}{QL}\bar{g}_{ii}\frac{\theta}{W}\left(\sum_{l=1}^{L}|h_{ii,l}|^2\right)^2\left(\frac{QL}{M}-1\right). \quad (32)$$

Each cell is loaded with Q mobiles. Thus, the interference power becomes

$$E\left\{|\iota_{\text{inter},i}|^2\right\}=\sum_{\substack{j=1\\j\neq i}}^{T}\frac{P}{QL}\bar{g}_{ij}\sum_{l=1}^{L}|h_{ii,l}|^2|h_{ij,l}|^2\frac{1}{W}\frac{L}{M}+\sum_{l=1}^{L}|h_{ii,l}|^2\sigma^2. \quad (33)$$

Let us define

$$\zeta_{ii}=\frac{1}{L}\sum_{l=1}^{L}|h_{ii,l}|^2$$
$$\zeta_{ij}=\frac{\sum_{l=1}^{L}|h_{ii,l}|^2|h_{ij,l}|^2}{\sum_{l=1}^{L}|h_{ii,l}|^2}. \quad (34)$$

Furthermore, let $g_{ij}=\bar{g}_{ij}\zeta_{ij}$. Assuming that R_i information bits are transmitted per each 2-D chip sequence of length WL, the E_b/I_0 at the mobile i can be written as

$$\Gamma_i=\frac{1}{R_i}\frac{E\left\{|s_i|^2\right\}}{E\left\{|\iota_{\text{intra},i}+\iota_{\text{inter},i}|^2\right\}} \quad (35)$$
$$=S_f\frac{g_{ii}\frac{P}{Q}}{\theta\left(\frac{QL}{M}-1\right)\frac{P}{Q}+\sum_{j\in\mathcal{B}_j}g_{ij}\frac{LP}{M}+L\nu} \quad (36)$$

where $S_f=WL/R_i$ denotes the spreading factor, and $\nu=W\sigma^2$ denotes the noise power per carrier. Thus, $\gamma_i=(R_i/W)\Gamma_i$ reduced to the SINR of mobile i in (6).

References

[1] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Commun. Mag.*, vol. 35, no. 12, pp. 126–133, Dec. 1997.

[2] N. Maeda, Y. Kishiyama, H. Atarashi, and M. Sawahashi, "Variable spreading factor-OFCDM with two dimensional spreading that prioritizes time domain spreading for forward link broadband wireless access," *IEICE Trans. Commun.*, vol. E88-B, no. 2, pp. 487–498, Feb. 2005.

[3] L.-L. Yang and L. Hanzo, "Multicarrier DS-CDMA: A multiple access scheme for ubiquitous broadband wireless communications," *IEEE Commun. Mag.*, vol. 41, no. 10, pp. 116–124, Oct. 2003.

[4] K. Zheng, G. Zeng, and W. Wang, "Performance analysis for OFDM-CDMA with joint frequency–time spreading," *IEEE Trans. Broadcast.*, vol. 51, no. 1, pp. 144–148, Mar. 2005.

[5] Y. Zhou, J. Wang, and M. Sawahashi, "Downlink transmission of broadband OFCDM systems—Part II: Effect of Doppler shift," *IEEE Trans. Commun.*, vol. 54, no. 6, pp. 1097–1108, Jun. 2006.

Simplified Frequency Offset Estimation for MIMO OFDM Systems

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***Abstract*—This paper addresses a simplified frequency offset estimator for multiple-input–multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems over frequency-selective fading channels. By exploiting the good correlation property of the training sequences, which are constructed from the Chu sequence, carrier frequency offset (CFO) estimation is obtained through factor decomposition for the derivative of the cost function with great complexity reduction. The mean square error (MSE) of the CFO estimation is derived to optimize the key parameter of the simplified estimator as well as to evaluate the estimator performance. Simulation results confirm the good performance of the training-assisted CFO estimator.**

***Index Terms*—Frequency offset estimation, frequency-selective fading channels, low complexity, multiple-input–multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM).**

I. Introduction

Carrier frequency offset (CFO) estimation is an important issue for both single-antenna and multiple-antenna orthogonal frequency-division multiplexing (OFDM) systems [1]–[7]. The numerical calculation of the maximum-likelihood (ML) CFO estimation is computationally complicated since it requires a large-point discrete Fourier transform (DFT) operation and a time-consuming line search. Therefore, many papers have proposed reduced-complexity algorithms [2], [3], [5]–[7]. In particular, the search-free approaches were proposed in [3], [6], and [7], where polynomial rooting is exploited to estimate the CFO. The solution proposed in [3] is based on computing the roots from the derivative of the cost function, whereas the solutions proposed in [6] and [7] are based on computing the roots directly from the cost function. However, both solutions still need the complicated polynomial rooting operation, which is hard to implement in practical OFDM systems [8].

In this paper, by further investigating the above search-free approaches, a simplified CFO estimator is developed for multiple-input–multiple-output (MIMO) OFDM systems over frequency-selective fading channels. With the aid of the training sequences generated from the Chu sequence [9], we propose to estimate the CFO via a simple polynomial factor. Thus, the complicated polynomial rooting operation is avoided. Correspondingly, the CFO estimator can be implemented via simple additions and multiplications. To

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optimize the key parameter of the simplified CFO estimator as well as to evaluate the estimator performance, the mean square error (MSE) of the CFO estimation is derived.

Notations: $(\cdot)_P$ denotes the remainder of the number within the brackets modulo P. $\otimes$ and $\odot$ denote the Kronecker product and Schur–Hadamard product, respectively. $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of the enclosed parameters, respectively. $\boldsymbol{x}^{(m)}$ denotes the m-cyclic-downshift version of $\boldsymbol{x}$. $\boldsymbol{F}_N$ and $\boldsymbol{I}_N$ denote the $N \times N$ unitary DFT matrix and identity matrix, respectively. $\boldsymbol{e}_N^k$ denotes the kth column vector of $\boldsymbol{I}_N$. Unless otherwise stated, $0 \leq \mu \leq N_t - 1$, $0 \leq \nu \leq N_r - 1$, $0 \leq p \leq P - 1$, and $0 \leq q \leq Q - 1$ are assumed, where $Q = N/P$ with $(N)_{2P} = 0$.

II. Signal Model

Consider a MIMO OFDM system with N_t transmit antennas, N_r receive antennas, and N subcarriers. The training sequences for CFO estimation are the same as in [6] and [7]. Let $\boldsymbol{s}$ denote a length-P Chu sequence [9]. Then, the $P \times 1$ pilot sequence vector at the μth transmit antenna is generated from $\boldsymbol{s}$ as follows: $\tilde{\boldsymbol{s}}_\mu = \sqrt{Q/N_t}\boldsymbol{F}_P \boldsymbol{s}^{(\mu M)}$, where $M = \lfloor P/N_t \rfloor$. Define $\boldsymbol{\Theta}_q = [\boldsymbol{e}_N^q, \boldsymbol{e}_N^{q+Q}, \ldots, \boldsymbol{e}_N^{q+(P-1)Q}]$. Then, the $N \times 1$ training sequence vector at the μth transmit antenna is constructed as follows: $\tilde{\boldsymbol{t}}_\mu = \boldsymbol{\Theta}_{i_\mu} \tilde{\boldsymbol{s}}_\mu$, where $0 \leq i_\mu \leq Q - 1$, and $i_\mu = i_{\mu'}$ iff $\mu = \mu'$. For convenience, we refer to $\{\tilde{\boldsymbol{t}}_\mu\}_{\mu=0}^{N_t-1}$ as the Chu-sequence-based training sequences (CBTSs).

Let $\boldsymbol{y}_\nu$ denote the $N \times 1$ received vector at the νth receive antenna after cyclic prefix (CP) removal. Let $\boldsymbol{h}^{(\nu,\mu)}$ denote the $L \times 1$ channel impulse response vector, with L being the maximum channel length. Assume that L is shorter than the length of CP N_g. Let $\tilde{\varepsilon}$ denote the frequency offset normalized by the subcarrier frequency spacing. Define

$$\begin{aligned} \boldsymbol{y} &= \left[\boldsymbol{y}_0^T, \boldsymbol{y}_1^T, \ldots, \boldsymbol{y}_\nu^T, \ldots, \boldsymbol{y}_{N_r-1}^T\right]^T \\ \boldsymbol{h}_\nu &= \Big[\left(\boldsymbol{h}^{(\nu,0)}\right)^T, \left(\boldsymbol{h}^{(\nu,1)}\right)^T, \\ &\qquad \ldots, \left(\boldsymbol{h}^{(\nu,\mu)}\right)^T, \ldots \left(\boldsymbol{h}^{(\nu,N_t-1)}\right)^T\Big]^T \\ \boldsymbol{h} &= \left[\boldsymbol{h}_0^T, \boldsymbol{h}_1^T, \ldots, \boldsymbol{h}_\nu^T, \ldots, \boldsymbol{h}_{N_r-1}^T\right]^T \\ \boldsymbol{D}_{\bar{N}}(\tilde{\varepsilon}) &= \mathrm{diag}\left\{\left[1, e^{j2\pi\tilde{\varepsilon}/N}, \ldots, e^{j2\pi\tilde{\varepsilon}(\bar{N}-1)/N}\right]^T\right\}. \end{aligned}$$

Then, the cascaded received vector $\boldsymbol{y}$ over the N_r receive antennas can be written as [6], [7]

$$\boldsymbol{y} = \sqrt{N} e^{j2\pi\tilde{\varepsilon}N_g/N} \left\{\boldsymbol{I}_{N_r} \otimes [\boldsymbol{D}_N(\tilde{\varepsilon})\boldsymbol{S}]\right\} \boldsymbol{h} + \boldsymbol{w} \tag{1}$$

where

$$\begin{aligned} \boldsymbol{S} &= \bar{\boldsymbol{F}}^H \mathrm{diag}\left\{\left[\tilde{\boldsymbol{s}}_0^T, \tilde{\boldsymbol{s}}_1^T, \ldots, \tilde{\boldsymbol{s}}_\mu^T, \ldots, \tilde{\boldsymbol{s}}_{N_t-1}^T\right]^T\right\} \breve{\boldsymbol{F}} \\ \bar{\boldsymbol{F}} &= [\boldsymbol{\Theta}_{i_0}, \boldsymbol{\Theta}_{i_1}, \ldots, \boldsymbol{\Theta}_{i_\mu}, \ldots, \boldsymbol{\Theta}_{i_{N_t-1}}]^T \boldsymbol{F}_N \\ \breve{\boldsymbol{F}} &= \Big[\boldsymbol{e}_{N_t}^0 \otimes \boldsymbol{\Theta}_{i_0}^T, \boldsymbol{e}_{N_t}^1 \otimes \boldsymbol{\Theta}_{i_1}^T, \ldots, \boldsymbol{e}_{N_t}^\mu \otimes \boldsymbol{\Theta}_{i_\mu}^T, \\ &\qquad \ldots, \boldsymbol{e}_{N_t}^{N_t-1} \otimes \boldsymbol{\Theta}_{i_{N_t-1}}^T\Big]\left\{\boldsymbol{I}_{N_t} \otimes \left[\boldsymbol{F}_N\left[\boldsymbol{I}_L, \boldsymbol{0}_{L\times(N-L)}\right]^T\right]\right\} \end{aligned}$$

and $\boldsymbol{w}$ is an $N_r N \times 1$ vector of uncorrelated complex Gaussian noise samples with a mean of zero and an equal variance of σ_w^2.

III. Simplified CFO Estimator for MIMO OFDM Systems

By exploiting the periodicity property of CBTS, $\boldsymbol{y}$ can be stacked into the $Q \times N_r P$ matrix $\boldsymbol{Y} = [\boldsymbol{Y}_0, \boldsymbol{Y}_1, \ldots, \boldsymbol{Y}_\nu, \ldots, \boldsymbol{Y}_{N_r-1}]$ with its element given by $[\boldsymbol{Y}_\nu]_{q,p} = [((\boldsymbol{e}_{N_r}^\nu)^T \otimes \boldsymbol{I}_N)\boldsymbol{y}]_{qP+p}$. Define

$$\begin{aligned} \boldsymbol{b}_\mu &= \Big[1, e^{j2\pi(\tilde{\varepsilon}+i_\mu)/Q}, \ldots, e^{j2\pi(\tilde{\varepsilon}+i_\mu)q/Q}, \\ &\qquad \ldots, e^{j2\pi(\tilde{\varepsilon}+i_\mu)(Q-1)/Q}\Big]^T \\ \boldsymbol{B}(\tilde{\varepsilon}) &= [\boldsymbol{b}_0, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_\mu, \ldots, \boldsymbol{b}_{N_t-1}]. \end{aligned}$$

Then, $\boldsymbol{Y}$ can be expressed in the following equivalent form [6], [7]:

$$\boldsymbol{Y} = \boldsymbol{B}(\tilde{\varepsilon})\boldsymbol{X} + \boldsymbol{W} \tag{2}$$

where

$$\begin{aligned} \boldsymbol{X} &= [\boldsymbol{X}_0, \boldsymbol{X}_1, \ldots, \boldsymbol{X}_\nu, \ldots, \boldsymbol{X}_{N_r-1}] \\ \boldsymbol{X}_\nu &= \left[\boldsymbol{x}^{(\nu,0)}, \boldsymbol{x}^{(\nu,1)}, \ldots, \boldsymbol{x}^{(\nu,\mu)}, \ldots, \boldsymbol{x}^{(\nu,N_t-1)}\right]^T \\ \boldsymbol{x}^{(\nu,\mu)} &= \sqrt{P} e^{j2\pi\tilde{\varepsilon}N_g/N} \boldsymbol{D}_P(\tilde{\varepsilon} + i_\mu) \\ &\quad \times \boldsymbol{F}_P^H \mathrm{diag}\{\tilde{\boldsymbol{s}}_\mu\} \boldsymbol{\Theta}_{i_\mu}^T \boldsymbol{F}_N \left[\boldsymbol{I}_L, \boldsymbol{0}_{L\times(N-L)}\right]^T \boldsymbol{h}^{(\nu,\mu)} \end{aligned}$$

and $\boldsymbol{W}$ is the $Q \times N_r P$ matrix generated from $\boldsymbol{w}$ in the same way as $\boldsymbol{Y}$.

According to the multivariate statistical theory, the log-likelihood function of $\boldsymbol{Y}$ conditioned on $\boldsymbol{B}(\varepsilon)$ and $\boldsymbol{X}$, with ε denoting a candidate CFO, can be obtained as follows:

$$\ln p\left(\boldsymbol{Y}|\boldsymbol{B}(\varepsilon), \boldsymbol{X}\right) = -\sigma_w^{-2} \mathrm{Tr}\left\{\left[\boldsymbol{Y} - \boldsymbol{B}(\varepsilon)\boldsymbol{X}\right]\left[\boldsymbol{Y} - \boldsymbol{B}(\varepsilon)\boldsymbol{X}\right]^H\right\}. \tag{3}$$

Exploit the condition $i_\mu = i_{\mu'}$ iff $\mu = \mu'$. Then, after some straightforward manipulations, we can obtain the reformulated log-likelihood function conditioned on ε as follows:

$$\ln p(\boldsymbol{Y}|\varepsilon) = \mathrm{Tr}\left[\boldsymbol{B}^H(\varepsilon)\hat{\boldsymbol{R}}_{\boldsymbol{YY}}\boldsymbol{B}(\varepsilon)\right] \tag{4}$$

where $\hat{\boldsymbol{R}}_{\boldsymbol{YY}} = \boldsymbol{Y}\boldsymbol{Y}^H$. Direct grid searching from (4) yields the ML estimate; however, this approach is computationally quite expensive. To efficiently compute the CFO, we will subsequently propose a simplified CFO estimator for MIMO OFDM systems.

Define $z = e^{j2\pi\varepsilon/Q}$, $z_\mu = e^{j2\pi i_\mu/Q}$, and $\boldsymbol{b}(z) = [1, z, \ldots, z^q, \ldots, z^{Q-1}]^T$. Then, by exploiting the Hermitian property of $\hat{\boldsymbol{R}}_{\boldsymbol{YY}}$, the log-likelihood function in (4) can be transformed into the following equivalent form:

$$\begin{aligned} f(z) &= \boldsymbol{c}^T\left\{\left[\sum_{\mu=0}^{N_t-1} \boldsymbol{b}(z_\mu)\right] \odot \boldsymbol{b}(z)\right\} \\ &\quad + \boldsymbol{c}^H\left\{\left[\sum_{\mu=0}^{N_t-1} \boldsymbol{b}\left(z_\mu^{-1}\right)\right] \odot \boldsymbol{b}(z^{-1})\right\} \end{aligned} \tag{5}$$

where $\boldsymbol{c}$ is a $Q \times 1$ vector with its qth element given by $[\boldsymbol{c}]_q = \sum_{j-i=q}[\hat{\boldsymbol{R}}_{\boldsymbol{YY}}]_{i,j}$. It can be seen from its definition that the qth element of $\boldsymbol{c}$ corresponds to the summation of the qth upper diagonal elements of $\hat{\boldsymbol{R}}_{\boldsymbol{YY}}$. Taking the first-order derivative of $f(z)$ with respect to z yields

$$f'(z) = z^{-1}\left\{\boldsymbol{c}^T\left\{\left[\sum_{\mu=0}^{N_t-1}\boldsymbol{b}(z_\mu)\right]\odot\boldsymbol{b}(z)\odot\boldsymbol{q}\right\} - \boldsymbol{c}^H\left\{\left[\sum_{\mu=0}^{N_t-1}\boldsymbol{b}\left(z_\mu^{-1}\right)\right]\odot\boldsymbol{b}(z^{-1})\odot\boldsymbol{q}\right\}\right\} \quad (6)$$

where $\boldsymbol{q} = [0, 1, \ldots, q, \ldots, Q-1]^T$. By letting the derivative of the log-likelihood function $f'(z)$ be zero, the solutions for all local minima or maxima can be obtained. Put these solutions back into the original log-likelihood function $f(z)$, and select the maximum by comparing all the solutions obtained in the previous stage. The improved blind CFO estimator that exploits the above mathematical rule has been addressed for single-antenna OFDM systems in [3]. Although the search-free approach has a relatively lower complexity, it still requires a complicated polynomial rooting operation, which is hard to implement in practical OFDM systems. With the aid of the CBTS training sequences, we will show in the following that the polynomial rooting operation can be avoided for training-aided CFO estimation in MIMO OFDM systems.

Assume that $P \geq L$, the channel taps remain constant during the training period, and the channel energy is mainly concentrated in the first M taps, with $M < L$. Then, we have (see Appendix A for details)

$$\boldsymbol{c}^H\left\{\left[\sum_{\mu=0}^{N_t-1}\boldsymbol{b}\left(z_\mu^{-1}\right)\right]\odot\boldsymbol{b}(z^{-1})\odot\boldsymbol{q}\right\} = z^{-Q}\kappa(\iota)\cdot\boldsymbol{c}^T\left\{\left[\sum_{\mu=0}^{N_t-1}\boldsymbol{b}(z_\mu)\right]\odot\boldsymbol{b}(z)\odot\boldsymbol{q}\right\} \quad (7)$$

where $\kappa(\iota) = \iota[\boldsymbol{c}]_\iota^*/[(Q-\iota)[\boldsymbol{c}]_{Q-\iota}]$ with $1 \leq \iota \leq Q-1$, and the parameter ι denotes the index of the upper diagonal of $\hat{\boldsymbol{R}}_{\boldsymbol{YY}}$. From (7), it immediately follows that $f'(z)$ can be decomposed as follows:

$$f'(z) = z^{-(Q+1)}\left[z^Q - \kappa(\iota)\right]\cdot\boldsymbol{c}^T\left\{\left[\sum_{\mu=0}^{N_t-1}\boldsymbol{b}(z_\mu)\right]\odot\boldsymbol{b}(z)\odot\boldsymbol{q}\right\}. \quad (8)$$

Define $\tilde{z} = e^{j2\pi\tilde{\varepsilon}/Q}$. Assume that $N_t < Q$. Then, with (29) as shown in Appendix A, we have (see Appendix B for details)

$$\boldsymbol{c}^T\left\{\left[\sum_{\mu=0}^{N_t-1}\boldsymbol{b}(z_\mu)\right]\odot\boldsymbol{b}(\tilde{z})\odot\boldsymbol{q}\right\} > 0, \qquad f'(\tilde{z}) = 0. \quad (9)$$

It follows from (8) and (9) that $z = \tilde{z}$ is one of the roots of both $f'(\tilde{z}) = 0$ and $z^Q - \kappa(\iota) = 0$. Unlike $f'(\tilde{z}) = 0$, the roots of $z^Q - \kappa(\iota) = 0$ can be calculated without the polynomial rooting operation. Therefore, by solving the simple polynomial equation $z^Q - \kappa(\iota) = 0$, the CFO estimate can be efficiently obtained as follows:

$$\hat{\varepsilon} = \arg\max_{\varepsilon\in\{\varepsilon_q\}_{q=0}^{Q-1}}\left\{f(z)|z = e^{j2\pi\varepsilon/Q}\right\} \quad (10)$$

where $\varepsilon_q = \arg\{\kappa(\iota)\}/(2\pi) + q - Q/2$. It can be calculated that the main computational complexity of the simplified CFO estimator is $4N_rNQ + 8Q^2$. Compared with the CFO estimator in [6] and [7], whose main computational complexity is $4N_rN\log_2 N + 9Q^3 + 64/3(Q-1)^3$, the complexity of the simplified CFO estimator is generally lower. Furthermore, since the polynomial rooting operation is avoided, the simplified CFO estimator can be implemented via simple additions and multiplications, which is more suitable for practical OFDM systems. Note that ι is a key parameter for the proposed CFO estimator. We will show in the following how to determine the optimal ι.

IV. Performance Analysis and Parameter Optimization

To optimize ι and to evaluate the estimation accuracy, we first derive the MSE of the simplified CFO estimator. Invoking the definition of $\hat{\boldsymbol{R}}_{\boldsymbol{YY}}$, we can readily obtain

$$\hat{\boldsymbol{R}}_{\boldsymbol{YY}} \doteq N_rP\sigma_x^2\boldsymbol{B}(\tilde{\varepsilon})\boldsymbol{B}^H(\tilde{\varepsilon}) + \hat{\boldsymbol{R}}_{\boldsymbol{YW}} + \hat{\boldsymbol{R}}_{\boldsymbol{WW}} \quad (11)$$

where $\sigma_x^2 = \mathrm{E}[|[\boldsymbol{x}^{(\nu,\mu)}]_p|^2]$, $\hat{\boldsymbol{R}}_{\boldsymbol{YW}} = \boldsymbol{B}(\tilde{\varepsilon})\boldsymbol{X}\boldsymbol{W}^H + \boldsymbol{W}\boldsymbol{X}^H\boldsymbol{B}^H(\tilde{\varepsilon})$, and $\hat{\boldsymbol{R}}_{\boldsymbol{WW}} = \boldsymbol{W}\boldsymbol{W}^H$. Assume that

$$\mathrm{E}\left\{\left[\boldsymbol{h}^{(\nu,\mu)}\right]_\iota^*[\boldsymbol{W}]_{i,j}\right\} = 0$$
$$\mathrm{E}\left\{\left[\boldsymbol{h}^{(\nu,\mu)}\right]_\iota^*\left[\boldsymbol{h}^{(\nu',\mu')}\right]_{l'}\right\} = 0, \qquad \forall(\nu,\mu) \neq (\nu',\mu').$$

Then, for $i \neq j$ and $i' \neq j'$, it can be concluded directly from their definitions that

$$\mathrm{E}\left[[\hat{\boldsymbol{R}}_{\boldsymbol{YW}}]_{i,j}\right] = \mathrm{E}\left[[\hat{\boldsymbol{R}}_{\boldsymbol{WW}}]_{i,j}\right] = \mathrm{E}\left[[\hat{\boldsymbol{R}}_{\boldsymbol{YW}}]_{i,j}^*[\hat{\boldsymbol{R}}_{\boldsymbol{WW}}]_{i',j'}\right] = 0 \quad (12)$$

$$\mathrm{E}\left[\left|[\hat{\boldsymbol{R}}_{\boldsymbol{YW}}]_{i,j}\right|^2\right] = 2N_tN_rP\sigma_x^2\sigma_w^2$$
$$\mathrm{E}\left[\left|[\hat{\boldsymbol{R}}_{\boldsymbol{WW}}]_{i,j}\right|^2\right] = N_rP\sigma_w^4. \quad (13)$$

Invoking the definition of $\boldsymbol{c}$, we have

$$[\boldsymbol{c}]_\iota = N_rP\sigma_x^2(Q-\iota)\tilde{z}^{-\iota}\sum_{\mu=0}^{N_t-1}z_\mu^{-\iota} + \alpha_\iota + \beta_\iota \quad (14)$$

where $\alpha_\iota = \sum_{j-i=\iota}[\hat{\boldsymbol{R}}_{\boldsymbol{YW}}]_{i,j}$, and $\beta_\iota = \sum_{j-i=\iota}[\hat{\boldsymbol{R}}_{\boldsymbol{WW}}]_{i,j}$. It immediately follows from its definition that $\kappa(\iota)$ can be expressed as

$$\kappa(\iota) = \frac{\tilde{z}^\iota + \zeta_\iota}{\tilde{z}^{-Q+\iota} + \eta_\iota} = \frac{\tilde{z}^Q + \xi_\iota}{|\tilde{z}^{-Q+\iota} + \eta_\iota|^2} \quad (15)$$

where

$$\zeta_\iota = \{\alpha_\iota^* + \beta_\iota^*\}\Big/\left[N_rP(Q-\iota)\sigma_x^2\sum_{\mu=0}^{N_t-1}z_\mu^\iota\right]$$
$$\eta_\iota = \{\alpha_{Q-\iota} + \beta_{Q-\iota}\}\Big/\left[N_rP\iota\sigma_x^2\sum_{\mu=0}^{N_t-1}z_\mu^\iota\right]$$
$$\xi_\iota = \tilde{z}^{Q-\iota}\zeta_\iota + \tilde{z}^\iota\eta_\iota^* + \zeta_\iota\eta_\iota^*.$$

From (15), we can see that the MSE of the estimated CFO is highly related to the variances of ζ_ι, η_ι, and ξ_ι. By invoking their definitions, the variances of ζ_ι and η_ι can be directly calculated as follows:

$$\mathrm{var}\{\zeta_\iota\} = \frac{2N_t\gamma^{-1} + \gamma^{-2}}{N_rP(Q-\iota)\left|\sum_{\mu=0}^{N_t-1}z_\mu^\iota\right|^2}$$
$$\mathrm{var}\{\eta_\iota\} = \frac{2N_t\gamma^{-1} + \gamma^{-2}}{N_rP\iota\left|\sum_{\mu=0}^{N_t-1}z_\mu^\iota\right|^2} \quad (16)$$

where $\gamma = \sigma_x^2/\sigma_w^2$. When $\gamma \gg 1$, i.e., the signal-to-noise ratio (SNR) is large enough, we have

$$\mathrm{var}\{\zeta_\iota\} \gg \mathrm{var}\left\{\zeta_\iota \eta_\iota^*\right\}, \quad \mathrm{var}\{\eta_\iota\} \gg \mathrm{var}\left\{\zeta_\iota \eta_\iota^*\right\}. \tag{17}$$

Accordingly, the variance of ξ_ι can be approximated as follows:

$$\begin{aligned}\mathrm{var}\{\xi_\iota\} &\doteq \mathrm{var}\{\zeta_\iota\} + \mathrm{var}\{\eta_\iota\} + \mathrm{E}[\tilde{z}^{Q-2\iota}\zeta_\iota\eta_\iota] + \mathrm{E}\left[\tilde{z}^{-Q+2\iota}\zeta_\iota^*\eta_\iota^*\right] \\ &= \frac{2\left[N_t Q + \rho(\iota)\right]\gamma^{-1} + Q\gamma^{-2}}{N_r P \iota (Q-\iota)\left|\sum_{\mu=0}^{N_t-1} z_\mu^\iota\right|^2}\end{aligned} \tag{18}$$

where $\rho(\iota)$ is given in (19), shown at the bottom of the page. Note that $\rho(\iota)$ is a nonlinear function with respect to ι and z_μ. When $\mathrm{var}\{\xi_\iota\} \ll 1$, which is a reasonable assumption for the practical systems, it immediately follows from (15) that

$$\begin{aligned}\hat{\varepsilon} &= \tilde{\varepsilon} + \frac{1}{2\pi}\arg\{1 + e^{-j2\pi\tilde{\varepsilon}}\xi_\iota\} \\ &\doteq \tilde{\varepsilon} + \frac{1}{2\pi}\Im\{e^{-j2\pi\tilde{\varepsilon}}\xi_\iota\}.\end{aligned} \tag{20}$$

Then, the MSE of the estimated CFO can be readily obtained as follows:

$$\mathrm{MSE}\{\hat{\varepsilon}\} \doteq \frac{1}{8\pi^2}\mathrm{var}\{\xi_\iota\} \doteq \frac{2\left[N_t Q + \rho(\iota)\right]\gamma^{-1} + Q\gamma^{-2}}{8\pi^2 N_r P \iota (Q-\iota)\left|\sum_{\mu=0}^{N_t-1} z_\mu^\iota\right|^2}. \tag{21}$$

It can be seen from (21) that $\mathrm{MSE}\{\hat{\varepsilon}\}$ depends on ι for fixed $\{i_\mu\}_{\mu=0}^{N_t-1}$, N_t, N_r, P, Q, and γ. To obtain better estimator performance, we can optimize the parameter ι based on (21).

V. Simulation Results

Numerical results are provided to verify the analytical results as well as to evaluate the performance of the proposed CFO estimator. The considered MIMO OFDM system has a bandwidth of 20 MHz and a carrier frequency of 5 GHz with $N = 1024$ and $N_g = 80$. Each of the channels is with six independent Rayleigh fading taps, whose relative average powers and propagation delays are $\{0, -0.9, -4.9, -8.0, -7.8, -23.9\}$ dB and $\{0, 4, 16, 24, 46, 74\}$ samples, respectively. The other parameters are given as follows: $P = 64$, $Q = 16$, $N_t = 3$, $N_r = 2$, and $\tilde{\varepsilon} \in (-Q/2, Q/2)$.

Figs. 1 and 2 present the MSE of the proposed CFO estimator as a function of ι with $\{i_\mu\}_{\mu=0}^{N_t-1} = \{3, 5, 11\}$ and $\{3, 7, 14\}$, respectively. The solid and dotted curves are the results from the analysis and Monte Carlo simulations, respectively. It can be observed that the results from the analysis agree quite well with those from the simulations, except when the actual MSE of the estimate is very large. It can also be observed that $\mathrm{MSE}\{\hat{\varepsilon}\}$ achieves its minimum for $\iota = 6, 8, 10$ with $\{i_\mu\}_{\mu=0}^{N_t-1} = \{3, 5, 11\}$ and for $\iota = 7, 9$ with $\{i_\mu\}_{\mu=0}^{N_t-1} = \{3, 7, 14\}$. These observations imply that we can obtain the optimum value of the parameter ι from the analytical results after $\{i_\mu\}_{\mu=0}^{N_t-1}$ is determined.

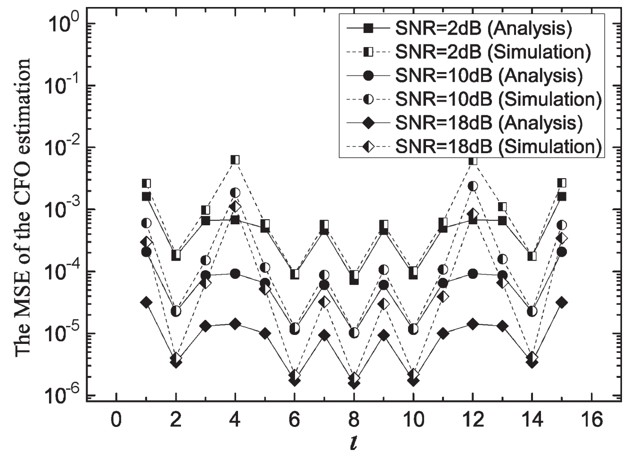

Fig. 1. MSE of the proposed CFO estimator as a function of ι with $\{i_\mu\}_{\mu=0}^{N_t-1} = \{3, 5, 11\}$.

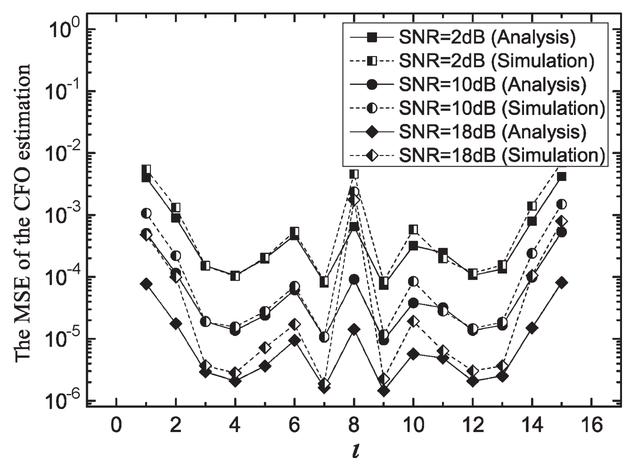

Fig. 2. MSE of the proposed CFO estimator as a function of ι with $\{i_\mu\}_{\mu=0}^{N_t-1} = \{3, 7, 14\}$.

The performance comparison between the proposed CFO estimator ($\iota = 7$ with $\{i_\mu\}_{\mu=0}^{N_t-1} = \{3, 7, 14\}$) and the estimator in [6], [7], and [10] is shown in Fig. 3. To substantiate that the training sequences generated from the Chu sequence help to improve the estimation accuracy, the performance of the proposed estimator with random sequences (RSs), whose elements are randomly generated, is included. The extended Miller and Chang bound (EMCB) [11], [12], which is obtained by averaging the snapshot Cramer–Rao bound over independent channel realizations, is also included as a performance benchmark, as follows:

$$\mathrm{EMCB}_\varepsilon = \mathrm{E}\left\{\frac{N\sigma_w^2}{8\pi^2 \boldsymbol{h}^H \boldsymbol{\mathcal{X}}^H \boldsymbol{\mathcal{B}}\left[\boldsymbol{I}_{N_r N} - \boldsymbol{\mathcal{X}}(\boldsymbol{\mathcal{X}}^H\boldsymbol{\mathcal{X}})^{-1}\boldsymbol{\mathcal{X}}^H\right]\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{X}}\boldsymbol{h}}\right\} \tag{22}$$

$$\rho(\iota) = \begin{cases} 2\iota\Re\left\{\left(\sum_{\mu=0}^{N_t-1} z_\mu^{2\iota}\right)\left(\sum_{\mu=0}^{N_t-1} z_\mu^{-\iota}\right)^2\right\} \Big/ \left|\sum_{\mu=0}^{N_t-1} z_\mu^\iota\right|^2, & 1 \le \iota \le Q/2 \\ 2(Q-\iota)\Re\left\{\left(\sum_{\mu=0}^{N_t-1} z_\mu^{2\iota}\right)\left(\sum_{\mu=0}^{N_t-1} z_\mu^{\iota}\right)^2\right\} \Big/ \left|\sum_{\mu=0}^{N_t-1} z_\mu^\iota\right|^2, & Q/2 < \iota \le Q-1 \end{cases} \tag{19}$$

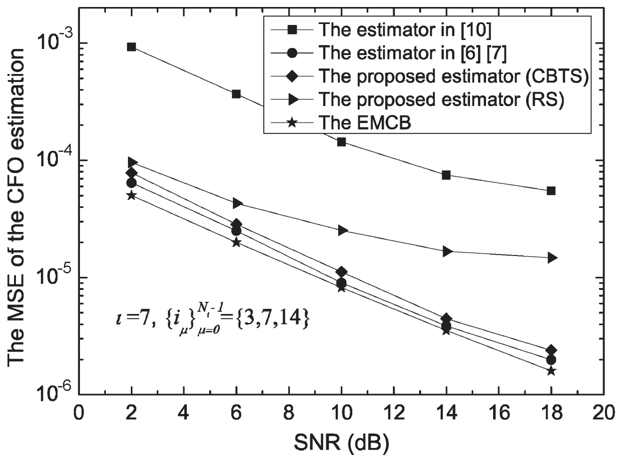

Fig. 3. MSE of the different CFO estimators as a function of SNR.

where $\boldsymbol{\mathcal{X}} = \boldsymbol{I}_{N_r} \otimes \boldsymbol{S}$, and $\boldsymbol{\mathcal{B}} = \boldsymbol{I}_{N_r} \otimes \mathrm{diag}\{[N_g, N_g+1, \ldots, N_g+N-1]^T\}$. We resort to Monte Carlo simulation for its evaluation. It can be observed that the performance of the proposed estimator with CBTS is far better than that in [10] and slightly worse than that in [6] and [7], and its performance also approaches the EMCB, which verifies its high estimation accuracy. It can also be observed that the performance of the proposed CFO estimator with CBTS is far better than that with RS, which should be attributed to the good correlation property of CBTS.

VI. Conclusion

In this paper, we have presented a low-complexity CFO estimator for MIMO OFDM systems with the training sequences generated from the Chu sequence. The MSE of the CFO estimation has been developed to evaluate the estimator performance and to optimize the key parameter. By exploiting the optimized parameter from the estimation MSE, our CFO estimator with CBTS yields good performance.

Appendix A

This Appendix presents the proof of (7). It immediately follows that the polynomials on the two sides of (7) are both $(Q-1)$-degree. Therefore, to validate the relationship (7), we only need to prove that the corresponding polynomial coefficients are pairwise equal.

By ignoring the noise items, $\hat{\boldsymbol{R}}_{\boldsymbol{YY}}$ can be expressed as follows:

$$\hat{\boldsymbol{R}}_{\boldsymbol{YY}} = \boldsymbol{B}(\tilde{\varepsilon})\hat{\boldsymbol{R}}_{\boldsymbol{XX}}\boldsymbol{B}^H(\tilde{\varepsilon}) \tag{23}$$

where $\hat{\boldsymbol{R}}_{\boldsymbol{XX}} = \boldsymbol{XX}^H$. Define

$$\varpi_{\mu,\mu'} = (i_\mu - i_{\mu'})/Q \quad \boldsymbol{s}_\mu = \sqrt{N_t/Q}\boldsymbol{F}_P^H\tilde{\boldsymbol{s}}_\mu.$$

Then, with the assumptions that $P \geq L$ and the channel taps remain constant during the training period, we can readily obtain

$$[\hat{\boldsymbol{R}}_{\boldsymbol{XX}}]_{\mu,\mu'} = \frac{N}{N_t}\sum_{\nu=0}^{N_r-1}\sum_{l=0}^{L-1}\sum_{l'=0}^{L-1}\left\{\left[\boldsymbol{h}^{(\nu,\mu)}\right]_l\left[\boldsymbol{h}^{(\nu,\mu')}\right]_{l'}^*\left[\boldsymbol{A}^{(\mu,\mu')}\right]_{l,l'}\right\} \tag{24}$$

where $[\boldsymbol{A}^{(\mu,\mu')}]_{l,l'} = (\boldsymbol{s}_\mu^{(l)})^T\boldsymbol{D}_P(\varpi_{\mu,\mu'}Q)(\boldsymbol{s}_{\mu'}^{(l')})^*$. With the aim of complexity reduction, $[\hat{\boldsymbol{R}}_{\boldsymbol{XX}}]_{\mu,\mu'}$ is replaced with its expected value. Exploiting the good correlation property of CBTS, which is inherited from the Chu sequence, we obtain

$$[\hat{\boldsymbol{R}}_{\boldsymbol{XX}}]_{0,0} = [\hat{\boldsymbol{R}}_{\boldsymbol{XX}}]_{1,1} = \cdots = [\hat{\boldsymbol{R}}_{\boldsymbol{XX}}]_{N_t-1,N_t-1}. \tag{25}$$

Let $[\boldsymbol{s}]_p = e^{j\pi\upsilon p^2/P}$, with υ being coprime with P. Define $p_{\mu,l} = \mu M + l$. Then, we have

$$\begin{aligned}\left[\boldsymbol{A}^{(\mu,\mu')}\right]_{l,l'} =& (-1)^{\upsilon(p_{\mu,l}-p_{\mu',l'})+1}e^{j\pi\upsilon\left(p_{\mu,l}^2-p_{\mu',l'}^2\right)/P}\\ &\times e^{-j\pi(P-1)\left[\upsilon(p_{\mu,l}-p_{\mu',l'})-\varpi_{\mu,\mu'}\right]/P}\sin(\pi\varpi_{\mu,\mu'})\\ &/\sin\left\{\pi\left[\upsilon(p_{\mu,l}-p_{\mu',l'})-\varpi_{\mu,\mu'}\right]/P\right\}.\end{aligned}$$

It immediately follows that

$$\left|\left[\boldsymbol{A}^{(\mu,\mu')}\right]_{l,l'}\right|_{p_{\mu,l}-p_{\mu',l'}\neq 0} \ll \left|\left[\boldsymbol{A}^{(\mu,\mu')}\right]_{l,l'}\right|_{p_{\mu,l}-p_{\mu',l'}=0}. \tag{26}$$

With the assumption that the channel energy is mainly concentrated in the first M taps, $|[\hat{\boldsymbol{R}}_{\boldsymbol{XX}}]_{\mu,\mu'}|_{\mu\neq\mu'}$ can be made very small with CBTS, which yields

$$\left|[\hat{\boldsymbol{R}}_{\boldsymbol{XX}}]_{\mu,\mu'}\right|_{\mu\neq\mu'} \ll [\hat{\boldsymbol{R}}_{\boldsymbol{XX}}]_{\mu,\mu}. \tag{27}$$

Then, we have $\hat{\boldsymbol{R}}_{\boldsymbol{XX}} \doteq N_rP\sigma_x^2\boldsymbol{I}_{N_t}$, where $\sigma_x^2 = \mathrm{E}[|[\boldsymbol{x}^{(\nu,\mu)}]_p|^2]$. It immediately follows that $\hat{\boldsymbol{R}}_{\boldsymbol{YY}} \doteq N_rP\sigma_x^2\boldsymbol{B}(\tilde{\varepsilon})\boldsymbol{B}^H(\tilde{\varepsilon})$. By invoking the definition of $[\boldsymbol{c}]_q$, we can further obtain

$$[\boldsymbol{c}]_q \doteq N_rP\sigma_x^2(Q-q)\tilde{z}^{-q}\sum_{\mu=0}^{N_t-1}z_\mu^{-q}, \qquad 1 \leq q \leq Q-1 \tag{28}$$

where $\tilde{z} = e^{j2\pi\tilde{\varepsilon}/Q}$. By substituting the above result into (7), the polynomial coefficient corresponding to z^{-q} at both sides of (7) can be calculated to be

$$N_rP\sigma_x^2q(Q-q)\tilde{z}^q\sum_{\mu=0}^{N_t-1}\sum_{\mu'=0}^{N_t-1}(z_\mu/z_{\mu'})^q \tag{29}$$

where we have utilized the following property: $z_\mu^Q = 1$. This completes the proof.

Appendix B

This Appendix presents the proof of (9). With CBTS and (29), we can readily obtain

$$\begin{aligned}&\boldsymbol{c}^T\left\{\left[\sum_{\mu=0}^{N_t-1}\boldsymbol{b}(z_\mu)\right]\odot\boldsymbol{b}(\tilde{z})\odot\boldsymbol{q}\right\}\\ &\qquad = N_rP\sigma_x^2(\boldsymbol{q}\odot\boldsymbol{\Psi})^H\left[(Q-\boldsymbol{q})\odot\boldsymbol{\Psi}\right]\end{aligned} \tag{30}$$

where $\boldsymbol{\Psi} = [\Psi(0), \Psi(1), \ldots, \Psi(q), \ldots, \Psi(Q-1)]^T$, $\Psi(q) = e^{-j2\pi qi_0/Q}\sum_{\mu=0}^{N_t-1}e^{j2\pi qi_\mu/Q}$. From the definition of $\boldsymbol{\Psi}$, we have

$$\boldsymbol{\Psi} = \boldsymbol{\Phi}\boldsymbol{1}_{N_t} \tag{31}$$

where $\boldsymbol{\Phi} = [\boldsymbol{\phi}_0, \boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_{N_t-1}]$, $\boldsymbol{\phi}_\mu = [1, e^{j2\pi(i_\mu-i_0)/Q}, \ldots, e^{j2\pi q(i_\mu-i_0)/Q}, \ldots, e^{j2\pi(Q-1)(i_\mu-i_0)/Q}]^T$. Since $\boldsymbol{\Phi}$ is a Vandermonde matrix, it is of full rank (rank N_t) with $N_t < Q$. Consequently, $\boldsymbol{\Psi}$ cannot be the all-zero vector, and then, $\boldsymbol{c}^T\{[\sum_{\mu=0}^{N_t-1}\boldsymbol{b}(z_\mu)]\odot\boldsymbol{b}(\tilde{z})\odot\boldsymbol{q}\} > 0$. From (6), it immediately follows that $f'(\tilde{z}) = 0$, which completes the proof.

References

[1] P. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction," *IEEE Trans. Commun.*, vol. 42, no. 10, pp. 2908–2914, Oct. 1994.

[2] D. Huang and K. B. Letaief, "Carrier frequency offset estimation for OFDM systems using null subcarriers," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 813–823, May 2006.

[3] F. Gao and A. Nallanathan, "Blind maximum likelihood CFO estimation for OFDM systems via polynomial rooting," *IEEE Signal Process. Lett.*, vol. 13, no. 2, pp. 73–76, Feb. 2006.
[4] X. Ma, M. K. Oh, G. B. Giannakis, and D. J. Park, "Hopping pilots for estimation of frequency-offset and multi-antenna channels in MIMO OFDM," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 162–172, Jan. 2005.
[5] F. Simoens and M. Moeneclaey, "Reduced complexity data-aided and code-aided frequency offset estimation for flat-fading MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 5, no. 6, pp. 1558–1567, Jun. 2006.
[6] Y. Jiang, X. Gao, X. You, and W. Heng, "Training sequence assisted frequency offset estimation for MIMO OFDM," in *Proc. IEEE ICC*, Istanbul, Turkey, Jun. 2006, vol. 12, pp. 5371–5376.
[7] Y. Jiang, H. Minn, X. Gao, X. You, and Y. Li, "Frequency offset estimation and training sequence design for MIMO OFDM," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1244–1254, Apr. 2008.
[8] W. H. Press, *Numerical Recipes in C++: The Art of Scientific Computing*. Cambridge, U.K.: Cambridge Univ. Press, 2002.
[9] D. Chu, "Polyphase codes with good periodic correlation properties," *IEEE Trans. Inf. Theory*, vol. IT-18, no. 4, pp. 531–532, Jul. 1972.
[10] G. L. Stuber, J. R. Barry, S. Mclaughlin, Y. Li, M. A. Ingram, and T. G. Pratt, "Broadband MIMO-OFDM wireless communications," *Proc. IEEE*, vol. 92, no. 2, pp. 271–294, Feb. 2004.
[11] F. Gini and R. Reggiannini, "On the use of Cramer-Rao-like bounds in the presence of random nuisance parameters," *IEEE Trans. Commun.*, vol. 48, no. 12, pp. 2120–2126, Dec. 2000.
[12] H. Minn, X. Fu, and V. K. Bhargava, "Optimal periodic training signal for frequency offset estimation in frequency-selective fading channels," *IEEE Trans. Commun.*, vol. 54, no. 6, pp. 1081–1096, Jun. 2006.

Performance Analysis of M-QAM Scheme Combined With Multiuser Diversity Over Nakagami-m Fading Channels

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***Abstract*—In this paper, we consider an M-ary quadrature amplitude modulation (M-QAM) scheme combined with multiuser diversity over Nakagami-m fading channels. Assuming that delayed but error-free signal-to-noise ratio (SNR) feedback is available, we derive closed-form formulas for the average transmission rate and the average bit error rate (BER), which are also shown to be generalizations of many previous results. Through numerical studies and simulations, we check the validity of our analysis. In addition, we investigate the impact of the Nakagami fading parameter m and feedback delay on system performance.**

***Index Terms*—Average bit error rate (BER), average transmission rate, feedback delay, M-ary quadrature amplitude modulation (M-QAM), multiuser diversity, Nakagami-m fading.**

I. Introduction

The demand for wireless communication services has tremendously been increasing, but the available radio spectrum is scarce. Accordingly, the problem of enhancing spectral efficiency is a key issue in the design of future wireless networks. To solve this problem, multiuser diversity has been studied for a wireless network with multiple users [1], [2]. Multiuser diversity comes from independent channel variations between different users and can be exploited by allocating the channel to the user with the best channel condition. By utilizing multiuser diversity, we can maximize the total information-theoretic capacity of a wireless network [1], [2]. Another popular approach to enhance spectral efficiency is to apply M-ary quadrature amplitude modulation (M-QAM) schemes that adapt the transmission rate to a time-varying fading channel [3]–[5].

In this paper, we consider an M-QAM scheme combined with multiuser diversity over Nakagami-m fading channels. We use the Nakagami-m model because it represents a broad class of fading channels [3], [6]. Assuming that delayed but error-free signal-to-noise ratio (SNR) feedback is available, we derive closed-form formulas for the average transmission rate and the average bit error rate (BER). Note that the average transmission rate and the average BER are frequently used for performance analysis in the literature (e.g., [1]–[4] and [7]).

Our results are generalizations of previous works in [2] and [3]. In [3], Alouini and Goldsmith analyze the performance of an M-QAM scheme over Nakagami-m fading channels. They derive closed-form formulas for the spectral efficiency and the average BER under perfect channel estimation and without feedback delay. They also analyze the impact of feedback delay on the average BER, but multiuser diversity is not considered. In [2], Ma *et al.* consider a wireless network with multiple users over Rayleigh fading channels and derive closed-form formulas for the average transmission rate and the average BER of an M-QAM scheme combined with multiuser diversity, assuming that delayed but error-free SNR feedback is available.

The remainder of this paper is organized as follows. We describe the system characteristics in Section II. We derive closed-form formulas for the average transmission rate and the average BER in Sections III and IV, respectively. Numerical studies based on our analysis and simulation results are given in Section V. Finally, we give our conclusions in Section VI.

II. System Description

A. Basic Assumptions

We consider a downlink transmission from a base station (BS) to N mobile stations (MSs) with a constant transmission power. The basic assumptions regarding system modeling are the following.

1) Gray-coded finite M_n-ary QAM modes with $M_n := 2^n (n = 1, \ldots, J-1)$ are used for transmission without forward error correction.
2) The received complex envelopes at MSs are independent and identical wide-sense stationary (WSS) random processes.
3) The signal at each MS is perturbed by additive white Gaussian noise (AWGN) with zero mean and variance N_0, which is independent of the received complex envelope.
4) Perfect channel estimation is possible at each MS, and the estimated SNR is transmitted through a delayed but error-free feedback path from each MS to the BS with a delay time τ.

Under these assumptions, the received signal $r_n(t)$ of the nth MS at time t can be expressed as $r_n(t) = g_n(t)b(t) + w_n(t)$, where $g_n(t)$ is the channel gain, i.e., the received complex envelope, between the BS and the nth MS, $b(t)$ is the signal broadcast from the BS, and $w_n(t)$ represents the AWGN in the channel of the nth MS [2].

We define the instantaneous SNR $\gamma_n(t)$ of the nth MS at time t by $|g_n(t)|^2/N_0$. We denote the average received SNR $\mathrm{E}[\gamma_n(t)]$ by $\overline{\gamma}$.

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