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External Habit and the Cyclicality of Expected Stock Returns*

I. Introduction

Expected stock returns are related to the business cycle, as shown in papers by Fama and French (1989), Fama (1990), Kandel and Stambaugh (1990), Harrison and Zhang (1999), and others. In fact, expected returns are countercyclical: higher in recessions and lower in booms. Much of this research focused on identifying the cyclicality of expected returns without attempting to explain it. "Standard" models, with time-separable utility and exogenous endowment processes, tend to generate procyclical returns. This finding is reported in Kandel and Stambaugh (1990). This result is robust to a variety of extensions. Balvers, Cosimano, and McDonald (1990) introduce production, and Zhang (1997) introduces heterogeneous agents and incomplete markets with short-sale constraints into

We estimate an equilibrium asset pricing model in which agents' preferences have an unobserved external habit using the efficient method of moments (EMM). Given the estimated structural parameters, we examine the cyclical behavior of expected stock returns in the model. We find that the estimated structural parameters imply countercyclical expected stock returns as documented in existing empirical studies. The model, however, is still rejected at the 1% level. Detailed examination of the moment conditions in our estimation indicates that the model performs reasonably well in matching the mean of returns, but it fails to capture the higher-order moments.

the model examined by Kandel and Stambaugh (1990), both finding that expected stock returns remain procyclical.

Campbell and Cochrane (1999) show that relaxing time separability in preferences can generate countercyclical returns. The mechanism they propose is a slow-moving, nonlinear external habit. The representative agent's utility is now a function of current private consumption as well as current and past aggregate consumption. Since the habit is external, individual agents do not consider the effects of current consumption on future utility. The setup is similar in spirit to Abel's (1990) "catching up with the Joneses" framework. The habit process moves more slowly than consumption so that, in a downturn, consumption falls faster than the habit, resulting in an increase in local risk aversion. In an expansion, the opposite happens. The countercyclical effect on risk aversion results in a countercyclical pattern for risk premia and therefore countercyclical expected returns. This explanation is consistent with Black (1990), who argues that risk aversion should be higher in recessions when wealth is low. However, Ljungqvist and Uhlig (1999) show that the Campbell and Cochrane specification implies consumption bunching. This is the result of the habit moving negatively with consumption with these preferences when consumption is endogenous. More standard "catching up with the Joneses" preferences have consumption and habit moving together. In a Campbell and Cochrane world, a benevolent government counteracts the externality by inducing cycles, while in a more standard catching-up world, the benevolent government stabilizes the economy (Ljungqvist and Uhlig 2000).

In this paper, we estimate the model proposed by Campbell and Cochrane (1999). We use the efficient method of moments (EMM) proposed by Gallant and Tauchen (1996). The EMM is in the same spirit as the indirect inference method proposed by Gourieroux, Monfort, and Renault (1993) and Smith (1993). This allows us not only to pin down the structural parameters of interest but also perform a detailed assessment of the performance of the economic model in matching observed stock returns. The estimation method is based on simulation and allows us to handle the unobserved external habit with relative ease. In a similar situation, Eichenbaum and Hansen (1990) propose that an initial guess for the unobserved variable be used and then the Generalized Method of Moments (GMM) be applied. Following this approach in our case may lead to poor estimates and statistical inference. In particular, the external habit is extremely persistent. Given the small sample size, initial conditions have a strong effect on the estimates of the other parameters. Since the EMM estimator is simulation based, we can remove the effect of initial conditions on the persistent habit process by discarding a long series of simulated realizations of returns before we start to collect the observations used in our EMM estimation. Specifically, for numerical tractability, we take a two-step approach. First, we estimate the exogenous

driving processes consisting of consumption growth and dividend growth using a vector autoregression. We then estimate the structural parameters, such as the discount factor, the risk aversion coefficient, and the persistence parameter for the surplus consumption ratio, using the EMM and the exogenous driving processes estimated in the first step.

Using the estimated structural parameters, we then investigate the cyclicality of expected stock returns implied by the economic model at various holding intervals. We first provide some benchmark results on the cyclical behavior of expected returns of the S&P 500 index portfolio at various holding intervals from the quarter to 5 years. This is done in two steps. In the first step, we obtain the expected holding returns by regressing compounded stock returns on a set of information variables, such as the dividend yield, earning-price ratio, default premium, and term premium. In the second step, we regress the expected holding returns on a business cycle proxy, as in Harrison and Zhang (1999), to quantify the comovements with business cycles. To examine the cylicality of expected stock returns implied by the economic model, we obtain long series of the expected holding returns at the same holding intervals as for the data using Monte Carlo integration. We then regress the expected holding returns on the business cycle dummy variables constructed for the economic model using the methodology suggested in Rouwenhorst (1995) to quantify their cyclical behavior.

We find that the estimated subjective discount factor is slightly and statistically significantly above one. This result supports the finding reported in Kocherlakota (1990). The estimated persistence parameter for the surplus consumption ratio is statistically significantly above 0.9, indicating the existence of a slow-moving external habit. The economic model with the external habit is rejected at the 1% level (but not at the 0.1% level). Detailed examination of the moment conditions (both the scores and the conventional moments) indicates that the economic model matches reasonably well the mean stock returns but it fails to match the higher-order moments such as variance, skewness, and kurtosis. The external habit implied by the estimated structural parameters generates countercyclical expected stock returns at longer horizons, such as 1 year, 2 years, and 5 years. The result is driven by the fact that the surplus consumption ratio reacts strongly positively to the contemporaneous consumption growth rate. This implies that the external habit moves at a much slower rate than consumption. The consumption and habit differential thus decreases and agents' local risk aversion increases in recessions. High expected returns are needed to induce agents to hold stocks. Finally, the curvature parameter (the coefficient of relative risk aversion in a time separable environment) has a point estimate of 6.27, which is more than three times the value chosen by Campbell and Cochrane and outside what is generally considered to be the reasonable range for this parameter.

Consistent with the finding that the economic model can match the mean stock returns reasonably well, we also find that the expected simulated stock returns during nonrecession periods are comparable to the expected S&P 500 index returns at various holding horizons ranging from 1 quarter to 5 years. The spread in expected returns between recession and nonrecession periods implied in the economic model is much smaller than the counterpart for the S&P 500 index. This is consistent with the finding that the economic model is unable to match the variance of the observed stock returns.

The rest of the paper is organized as follows. Section II presents the model and defines the law of motion for exogenous forcing variables. Section III presents and discusses the EMM estimation results and evaluates the performance of the model. Section IV investigates the cyclicality of expected returns implied in the model, and Section V concludes the paper.

II.   Preferences with an External Habit

There is an infinitely lived representative agent who derives utility from consuming a single consumption good. Consumer's preferences depend on both the current consumption and some habit level. Let C_t be the representative agents' consumption at time t and X_t be the agent's habit level at time t. Following Campbell and Cochrane, we assume that X_t depends on economywide per-capita consumption rather than the agent's own consumption, although in equilibrium the two are equal. The agent's objective is to maximize the expected sum of discounted future utility given as follows:

$$\max E\left[\sum_{t=0}^{\infty} \beta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1 - \gamma}\right], \tag{1}$$

where β is the subjective discount factor and γ is the curvature parameter.[1]

The habit level X_t evolves as specified by Campbell and Cochrane. It is treated by individual agents as beyond their control and therefore a function of aggregate per-capita consumption. It follows a nonlinear process to keep the habit below consumption $(X_t < C_t)$. Let S_t denote the surplus consumption ratio:

$$S_t = \frac{C_t - X_t}{C_t}. \tag{2}$$

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1. Because of the presence of the external habit, the agent's local risk aversion coefficient is no longer γ but rather a function of consumption and habit level.

We use lowercase letters to denote the natural logs of variables ($x = \log X$). As in Campbell and Cochrane, we assume that the surplus consumption ratio evolves according to

$$s_{t+1} = (1 - \phi)\bar{s} + \phi s_t + \lambda(s_t)(\bar{c}_{t+1} - \bar{c}_t - g), \tag{3}$$

where Φ determines the persistence of the surplus consumption ratio, $\bar{s}$ is the steady-state surplus consumption ratio, $\lambda(s)$ is the sensitivity function of the surplus consumption ratio to the contemporaneous per-capita consumption growth rate, $\bar{c}_t$ is the log of per-capita consumption (which equals the consumption of the agent in equilibrium), and g is the average economywide per-capita consumption growth rate.

The sensitivity function, $\lambda(s)$, takes the same form as in Campbell and Cochrane:

$$\lambda(s_t) = \begin{cases} (1/\bar{S})\sqrt{1 - 2(s_t - \bar{s})} - 1, & s_t < s_{\max} \\ 0 & s_t \geq s_{\max} \end{cases}, \tag{4}$$

where $\bar{S}$, the steady-state surplus consumption ratio, is given by $\sigma\sqrt{\gamma/(1 - \phi)}$, where σ is the standard deviation of consumption growth rate and $s_{\max}$, the upper limit for the surplus consumption ratio, is given by

$$s_{\max} = \bar{s} + \tfrac{1}{2}(1 - \bar{S}^2). \tag{5}$$

The implication of this specification is that the risk-free rate is constant when the consumption growth rate is a log normal, independently and identically distributed (i.i.d.) random variable, and the habit moves nonnegatively with consumption.

We consider a stock with dividend process D_t. The equilibrium stock price, denoted by P_t, can then be expressed as

$$P_t = E_t\left[\beta\left(\frac{C_{t+1} - X_{t+1}}{C_t - X_t}\right)^{-\gamma}(P_{t+1} + D_{t+1})\right]$$

$$= E_t\left[\beta\left(\frac{S_{t+1}}{S_t}\right)^{-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(P_{t+1} + D_{t+1})\right]. \tag{6}$$

As in Campbell and Cochrane, we model consumption and dividends as separate processes. Since the growth rates of dividends and consumption are only weakly correlated in U.S. data, it can be important to model dividends and consumption separately.

To induce stationarity, we normalize the stock price by dividends (D_t). The normalization yields

$$\tilde{P}_t = E_t\left[\beta\left(\frac{S_{t+1}}{S_t}\right)^{-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{P}_{t+1}+1)\left(\frac{D_{t+1}}{D_t}\right)\right], \qquad (7)$$

where $\tilde{P} = P_t/D_t$.

Comparing this pricing function with the one without the external habit, we have introduced the term $(S_{t+1}/S_t)^{-\gamma}$, which decreases as the growth of the surplus consumption ratio increases. Since the growth of the surplus consumption ratio enters the pricing function exactly as consumption growth, we expect the former to affect the stock price and expected stock returns in the same way as the latter does. The important issue is how the surplus consumption ratio evolves as a function of contemporaneous consumption growth. If the growth of the surplus consumption ratio is highly correlated with per-capita consumption growth, the stock price will be strongly procyclical and expected stock returns countercyclical.

The exogenous forcing variables in the model consist of the per-capita consumption growth rate ($\Delta c_t = \bar{c}_t - \bar{c}_{t-1}$) and dividend growth rate (Δd_t). We assume that the consumption and dividend growth rates follow a bivariate vector autoregressive (VAR) process with one lag;[2] that is,

$$\begin{bmatrix} \Delta c_{t+1} \\ \Delta d_{t+1} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} \Delta c_t \\ \Delta d_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \end{bmatrix}, \qquad (8)$$

where $[\varepsilon_{1t}, \varepsilon_{2t}]' \sim N(0, \Sigma)$. The sufficient state space for the preceding problem thus consists of consumption and dividend growth rates and the surplus consumption ratio of the previous period. We denote by $z_t = [\Delta c_t, \Delta d_t]'$ the vector of exogenous state variables. The one-step-ahead conditional density for z_t therefore is a Gaussian with mean

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} \Delta c_t \\ \Delta d_t \end{bmatrix}$$

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2. Campbell and Cochrane (1999) suggest that it would be better to make consumption and dividends cointegrated. They report that imposing cointegration does not significantly change their results while complicating some of calculations by introducing an additional state variable. Campbell and Cochrane assume that consumption and dividend growth rates are i.i.d., potentially correlated, and have identical means. We relax the i.i.d. and identical mean assumptions in our specification, but we do not impose cointegration.

TABLE 1   Vector Autoregression Coefficients for Exogenous Processes

Variable	Const.	Std. Errors	Slopes	Std. Errors	Var.-covar. Matrix
Δc	.0032	4.827e-4	.1720, .0267	.0709, .0128	2.849e-5 2.141e-5
Δd	.0023	2.249e-3	.5942, .0558	.4046, .0728	.2141e-4 .9264e-3

NOTE.—Δc denotes the per-capita consumption growth rate of nondurables and services and Δd represents the per-capita dividend growth rate. The bivariate VAR is estimated using quarterly data from 1947:Q2 to 1995:Q3 taken from the CITIBASE data set.

and covariance matrix Σ. We thus can represent the state space by (z_t, S_{t-1}). The stock price then can be written as

$$
\tilde{P}(z_t, S_{t-1}) = E \left(\beta \begin{Bmatrix} \dfrac{S[z_{t+1}, S(z_t, S_{t-1})]}{S(z_t, S_{t-1})} \end{Bmatrix}^{-\gamma} \exp(\Delta c_{t+1})^{-\gamma} \\ \{\tilde{P}[z_{t+1}, S(z_t, S_{t-1})] + 1\} \exp(\Delta d_{t+1}) \middle| z_t, S_{t-1} \right). \quad (9)
$$

To estimate the bivariate VAR specification for the exogenous state variables, we construct quarterly per-capita consumption growth rate of nondurables and services and per-capita dividend growth rate adjusted by the population growth (age 16 and above) using the CITIBASE data from 1947:Q2 to 1995:Q3. Table 1 presents the estimation results. We make the following observations. Consumption growth at time t is positively related to both consumption and dividend growth rates at time $t - 1$. The estimates are statistically significant at the 5% level. While the dividend growth rate is also positively related to the lagged consumption and dividend growth rates, the estimates are not statistically significant at the conventional test level. In the next section, we formally estimate the economic model with the external habit and provide a statistical assessment of its performance in matching the observed stock returns.

III.   EMM Estimation of Structural Parameters

A.   The EMM Estimator

Our goal here is to jointly estimate the Euler equation and the law of motion for the external habit for a given exogenous driving force[3]:

$$
\tilde{P}_t = E_t \left[\beta \left(\frac{S_{t+1}}{S_t} \right)^{-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (\tilde{P}_{t+1} + 1) \frac{D_{t+1}}{D_t} \right], \quad (10)
$$

$$
s_{t+1} = (1 - \phi)\bar{s} + \phi s_t + \lambda(s_t)(\Delta \bar{c}_{t+1} - g), \quad (11)
$$

3. The vector autoregressive process for the exogenous forcing variables is fixed at the point estimates reported in table 1 when we estimate the structural parameters governing preferences. Therefore, sampling errors in the first step estimation are not explicitly accounted for.

$$\lambda(s_t) = \begin{cases} (1/\bar{S})\sqrt{1 - 2(s_t - \bar{s})} - 1, & s_t < s_{\max} \\ 0, & s_t \geq s_{\max}. \end{cases} \quad (12)$$

Because the external habit is not observed, to apply the GMM as in Eichenbaum and Hansen (1990), one needs to jointly estimate the initial surplus consumption ratio (s_0) and the other structural parameters. Using the S&P 500 value-weighted returns and the returns of 10 decile portfolios in combination with the risk-free rate as the moment conditions, we find that the curvature parameter (γ) estimate in general is quite small (less than 1.0 in most cases). It gets larger when the initial surplus consumption is fixed at the steady-state level, but it is still usually less than 3.0. All the models have a high p-value, consistently above 10%, implying that the model with an external habit cannot be rejected at conventional test levels. However, this may simply reflect that the model is poorly estimated when the surplus consumption ratio process is highly persistent and the sample size is too small. Indeed, the estimated persistence parameter (ϕ) is consistently above 0.98, implying strong persistence in the surplus consumption ratio process. The high persistence parameter estimate coupled with a relatively small sample size thus render the GMM estimation unreliable in this case.[4]

A simulation-based generalization of GMM proposed in Bansal et al. (1994) and Gallant and Tauchen (1996), the EMM, is well suited to the problem at hand. It allows us to remove the effect of the initial value of the surplus consumption ratio when the habit process is highly persistent and the sample size is small. This is achieved by first simulating a long series of exogenous variables, the consumption and dividend growth rates, then recursively computing the surplus consumption ratio and the stock returns starting from an initial guess for the surplus consumption ratio. We discard the first several thousand simulated stock returns before we collect data for our EMM estimation. The idea of the EMM is to use the expectation under the structural model of the scores from an auxiliary model (called a *score generator*) as the vector of moment conditions. The scores are the derivatives of the log likelihood function of the auxiliary model with respect to its parameters. The estimator is defined as follows.

Let $\{\tilde{y}_t, \tilde{x}_{t-1}\}_{t=1}^n$ denote the observed data set (in our case, $\{\tilde{y}_t\}$ represents the S&P 500 index return series), where $\tilde{x}_{t-1} = (\tilde{y}_{t-L}, \ldots, \tilde{y}_{t-1})'$, $L \geq 1$. Let $f(\tilde{y}_t \mid \tilde{x}_{t-1}, \theta)$, $\theta \in \Theta \subset \Re^{l_\theta}$, be the one-step conditional density function which is the auxiliary model that generates scores for the economic model to match. Let $\rho \in \varrho \subset \Re^{l_\theta}$ be the vector of structural parameters to be estimated, and $\{\hat{y}_t(\rho), \hat{x}_{\tau-1}(\rho)\}_{\tau=1}^n$ be the counterpart

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4. The GMM results are not reported to save space and are available upon request. Interestingly, the results are broadly consistent with the findings reported in Ferson and Constantinides (1991) for a model with an internal habit formation using quarterly data.

of the observed data simulated from the structural model, given the structural parameters ρ. The simulated data can be obtained by numerically solving equations (10), (11), and (12) jointly for a given exogenous driving force. We choose to implement the parameterized expectations algorithm proposed by Den Haan and Marcet (1990) and applied in Bansal et al. (1993) in connection with the simulated method of moments estimation. Specifically, we parameterize the right-hand side of equation (10) as follows:

$$E_t\left[\beta\left(\frac{S_{t+1}}{S_t}\right)^{-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{P}_{t+1}+1)\frac{D_{t+1}}{D_t}\right] = \exp[h(z_t, S_{t-1}; \psi)], \quad (13)$$

where $h(z_t, S_{t-1}; \psi)$ is a polynomial in state variables $(z_t, S_{t-1})'$ and ψ is a vector of parameters for the polynomial. In our application, we find the following quadratic polynomial to work very well:

$$h(z_t, S_{t-1}; \psi) = \psi_0 + \psi_1 \Delta c_t + \psi_2 \Delta d_t + \psi_3 S_{t-1} + \psi_4 \Delta c_t^2 + \psi_5 \Delta d_t^2 + \psi_6 S_{t-1}^2. \tag{14}$$

We first estimate the auxiliary parameter θ using quasi-maximum likelihood estimation:

$$\tilde{\theta}_n = \arg\max_{\theta \in \Theta} \frac{1}{n}\sum_{t=1}^{n} \ln f(\tilde{y}_t \mid \tilde{x}_{t-1}, \theta). \tag{15}$$

This is achieved by solving the following set of first-order conditions:

$$\frac{1}{n}\sum_{t=1}^{n}(\partial/\partial\theta)\ln f[\tilde{y}_t \mid \tilde{x}_{t-1}, \tilde{\theta}_n] = 0. \tag{16}$$

The basic idea of EMM estimation is that, under the null hypothesis that the structural model is correctly specified, there exists a true parameter vector $\rho^0 \in \varrho$ such that the simulated data $\{y_t(\rho^0), \hat{x}_{\tau-1}(\rho^0)\}_{\tau=1}^{n}$ also satisfy the previous first-order conditions.

We therefore introduce the moment conditions as follows:

$$m_n(\rho, \tilde{\theta}_n) = \frac{1}{N}\sum_{\tau=1}^{N}(\partial/\partial\theta)\ln f[\hat{y}_\tau(\rho) \mid \hat{x}_{\tau-1}(\rho), \tilde{\theta}_n]. \tag{17}$$

The EMM estimator of the structural parameter vector is then defined analogously to the GMM estimator as follows:

$$\hat{\rho}_n = \arg\min_{\rho \in \varrho} m_n'(\rho, \tilde{\theta}_n)(\tilde{I}_n)^{-1}m_n(\rho, \tilde{\theta}_n) \tag{18}$$

where $(\tilde{I}_n)^{-1}$ is the weighting matrix and is given by the outer-product-of-gradient formula

$$\tilde{I}_n = \frac{1}{n}\sum_{t=1}^{n}[(\partial/\partial\theta)\ln f(\tilde{y}_t\,|\,\tilde{x}_{t-1},\tilde{\theta}_n)][(\partial/\partial\theta)\ln f(\tilde{y}_t\,|\,\tilde{x}_{t-1},\tilde{\theta}_n)]'. \quad (19)$$

Gallant and Tauchen (1996) show that the EMM estimator just defined is consistent and asymptotically normal, and the normalized objective value

$$\tilde{s}_n(\hat{\rho}_n) = nm'_n(\hat{\rho}_n,\tilde{\theta}_n)(\tilde{I}_n)^{-1}m_n(\hat{\rho}_n,\tilde{\theta}_n) \quad (20)$$

has an asymptotic chi-square distribution with degrees of freedom $l_\theta - l_\rho$, where l_θ and l_ρ are the numbers of parameters in auxiliary and economic models, respectively.

Because the conditional density function, which serves as the auxiliary model, is usually unknown, in practice, it needs to be estimated. We employ the Gallant and Tauchen (1989) seminonparametric (SNP) method to estimate $f(\tilde{y}_t\,|\,\tilde{x}_{t-1},\theta)$ along with the auxiliary parameter vector, θ. Next, we provide the empirical estimation results.

B. Empirical Results

SNP density estimate of real stock returns.   In their baseline formulation, Campbell and Cochrane (1999) choose the sensitivity function $\lambda(s)$ so that the risk-free rate is constant and the habit moves nonnegatively with consumption. In an alternative formulation, they allow the risk-free rate to vary with the state variable, for instance, let the risk-free rate be a linear function of $(s_t - \bar{s})$. However, Campbell and Cochrane show that adding interest rate variation in this way has very little effect on the stock market results. Following Campbell and Cochrane, we focus on the stock returns.[5] Specifically, the series we choose to match for our economic model is the quarterly real returns of the S&P 500 index portfolio. The data spans the period from the second quarter of 1947 to the third quarter of 1995. The ex post real returns are obtained by adjusting the nominal returns of the S&P 500 index portfolio by inflation. We estimate the conditional density function for the real returns using the SNP method, which we briefly discuss next.

The SNP method is based on the notion that a Hermite expansion can be used as a general-purpose approximation to a density function. This basic approach can be adapted to the estimation of the conditional density of the return series, denoted $\{y_t\}$, which has a Markovian structure, where the conditional density of y_t given the entire history $\{y_{t-1}\}_{l=1}^{\infty}$

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5. The implications for the risk-free rate are discussed later.

depends only on L lags from the most recent past. Collecting these lags in a single vector gives a L vector denoted as x_{t-1}:

$$x_{t-1} = (y_{t-L}, \cdots, y_{t-2}, y_{t-1})'.$$

The SNP approximation of a density takes the following form:

$$f(y_t \mid x_{t-1}, \theta) = \frac{1}{\xi} [P(z_t, x_{t-1})]^2 n(y_t \mid \mu_{x_{t-1}}, \sigma_{x_{t-1}}), \qquad (21)$$

where ξ is a scalar that makes the density integrate to one,[6] $z_t = \sigma_{x_{t-1}}^{-1}(y_t - \mu_{x_{t-1}})$ is an innovation, $P(z_t, x_{t-1})$ denotes a polynomial in z_t of degree K_x whose coefficients are polynomials of degree K_x in x_{t-1}, $n(y_t \mid \mu_{x_{t-1}}, \sigma_{x_{t-1}})$ is a normal distribution with mean $\mu_{x_{t-1}}$ (the location function) and standard deviation $\sigma_{x_{t-1}}$ (the scale function) whose values depend on x_{t-1}. The constant term of the polynomial is put to one to obtain a unique representation. This normalization means that the leading term of the entire expansion is $n(y_t \mid \mu_{x_{t-1}},)\sigma_{x_{t-1}}$.[7] The location function $\mu_{x_{t-1}}$ is given by an autoregression,

$$\mu_{x_{t-1}} = b_0 + Bx_{t-1}. \qquad (22)$$

It is assumed to depend on $L_\mu \leq L$ lags. The scale function $\sigma_{x_{t-1}}$ is given by

$$\sigma_{x_{t-1}} = \rho_0 + P \mid e_{t-1}^* \mid, \qquad (23)$$

where $e_{t-1}^* = [(y_{t-L_r} - \mu_{x_{t-L_r-1}}), \ldots, (y_{t-1} - \mu_{x_{t-2}})]$, and $\mid \cdot \mid$ denotes elementwise absolute value. The scale function depends on L_r lagged (unnormalized) innovations $(y_t - \mu_{x_{t-1}})$ and $(L_\mu + L_r) \leq L$ lagged y_t in total. This is an ARCH-type process akin to that proposed by Nelson (1991).

The Hermite polynomial $P(z_t, x_{t-1})$ is given by

$$P(z_t, x_{t-1}) = \sum_{\alpha=0}^{K_z} \left(\sum_{\beta=0}^{K_x} a_{\alpha\beta} x_{t-1}^\beta \right) z_t^\alpha, \qquad (24)$$

where $\alpha \leq K_z$ and $\beta \leq K_x$ are nonnegative integers. It is assumed that the polynomial depends on $L_p \leq L$ lags of y from x.

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6. Therefore, $\xi = 1 / \int [P(s,x)]^2 \Phi(s) ds$.

7. The vector θ of $f(y_t \mid x_{t-1}, \theta)$ consists of the coefficients of the polynomial plus $\mu_{x_{t-1}}$ and $\sigma_{x_{t-1}}$ and is estimated by maximum likelihood. Equivalent to maximum likelihood, but more stable numerically, is to estimate θ in a sample of size n by minimizing $s_n(\theta) = -\frac{1}{n} \sum_{t=1}^n \log[f(y_t \mid x_{t-1}, \theta)]$. If the number of parameters $p\theta$ grows with the sample size n, then the true density, its derivatives, and its moments are estimated consistently, as shown in Gallant and Nychka (1987).

When K_z is positive, the resulting density function is a modification of the Gaussian due to the multiplication by the polynomial $\{P(z_t)\}^2$. When K_x is positive, the shape of the density depends on x_{t-1}. Thus, all moments can depend on x_{t-1} and the density can approximate any form of conditional heterogeneity (Gallant and Tauchen 1989). The shape modifications are rich enough to accurately approximate densities from a large class that includes densities that have fat tails, thin tails, or are skewed.[8]

For notational convenience, hereafter, the hierarchical SNP structure is denoted as SNP $(L_\mu, L_r, L_p, K_z, K_x)$. To illustrate, consider first the model with $L_\mu = 4, L_r = 4, L_p = 0, K_z = 4$, and $K_x = 0$. The polynomial is of the form

$$P(z_t) = \sum_{\alpha=0}^{4} a_\alpha z_t^\alpha, \tag{25}$$

where the $a_0, a_1, \ldots, a_4$ are the polynomial coefficients with the constant term $a_0 = 1$ to achieve a unique representation. Both $\mu_{x_{t-1}}$ and $\sigma_{x_{t-1}}$ are linear in $y_{t-1}, \ldots, y_{t-4}$. The model has 14 free parameters: the 4 free polynomial parameters, the intercept and 4 slope parameters in $\mu_{x_{t-1}}$, and the intercept and 4 slope parameters of $\sigma_{x_{t-1}}$.

Now consider $L_p = 1$ and $K_x = 1$ but everything else the same. The polynomial becomes

$$P(z_t, x_{t-1}) = \sum_{\alpha=0}^{4} (a_{0\alpha} + a_{1\alpha} y_{t-1}) z_t^\alpha. \tag{26}$$

The normalization is $a_{00} = 1$. The polynomial has 9 free parameters, yielding 19 free parameters in total.

To select the optimal SNP model for the real returns, the following strategy is adopted. We start with a VAR process, SNP(10100), and gradually expand L_μ until a certain model selection criterion reaches a minimum. Three model selection criteria are calculated for each SNP fit: the Schwarz criterion, $s_n \neq (p_\theta/2n)\ln(n)$; the Hannan-Quinn criterion, $s_n + (p_\theta/n)\ln\{\ln(n)\}$; and the Akaike criterion, $s_n + p_\theta/n$. We then introduce ARCH by increasing L_r, introduce non-Gaussian ARCH by increasing K_z, and finally bring in general nonlinear processes by increasing L_p and K_x.

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8. Fenton and Gallant (1996) assess the qualitative behavior of SNP in finite samples using the Marron and Wand (1992) test suite. For each of the 15 densities proposed by Marron and Wand, Fenton and Gallant generate samples at sizes 400, 900, 1,600, 2,500, and 5,625. To each, they fit a kernel and SNP. For both the kernel and SNP, they compute a Riemann sum of the absolute value of the distance between the estimated density and the true density on 1,024 points evenly spaced over the interval $[-3, 3]$. Their results suggest that the SNP estimator for a univariate series is both qualitatively and asymptotically similar to the kernel estimator.

TABLE 2   **SNP Estimation of the Density Function**

Model	p_θ	s_n	Schwarz	H-Q	Akaike
SNP(10100)	3	1.4143	1.4550	1.4399	1.4297
SNP(11100)	4	1.3985	1.4528	1.4328	1.4191
SNP(12100)	5	1.3979	1.4658	1.4407	1.4237
SNP(11120)	6	1.3675	1.4489	1.4189	1.3984
SNP(11140)	8	1.3531	1.4617	1.4217	1.3944
SNP(11121)	9	1.3507	1.4729	1.4277	1.3971
SNP(11141)	13	1.3130	1.4895	1.4244	1.3800
SNP(20100)	4	1.4109	1.4652	1.4452	1.4315

Note.—The ex post real returns of the S&P 500 index portfolio is constructed using quarterly nominal returns of the S&P 500 index portfolio and inflation. The data spans the period from 1947:Q2 to 1995:Q3. The return series is taken from the CRSP tape and the inflation series is from the CITIBASE. p_θ is the number of parameters in the SNP model. s_n is the objective value. Schwarz, H-Q, and Akaike are the Schwarz, Hannan and Quinn, and Akaike model selection criterion, respectively.

The preferred models are then subject to a battery of diagnostic tests to determine the goodness of fit. The diagnostic tests entail checking for predictability in the residuals from each SNP model. Residuals are examined for both short-term and long-term predictability of the mean (residual levels) and variance (squared residuals). For the long-term tests, the residuals and their squares are projected onto annual dummy variables. For the short-term tests, the residuals and their squares are projected onto a space formed by the linear, quadratic, and cubic terms of past variables (three lags are used). If an SNP model is the true density, the residuals should be orthogonal to these regressors. Therefore, for a given SNP specification, the smaller the R^2 of the regressions, the better the SNP model approximates the true density.

Table 2 presents the SNP estimation results along with the model selection criteria. Both the Schwarz and Hannan-Quinn preferred model is SNP(11120). It is a non-Gaussian ARCH model with mean and variance functions depending linearly on x_{t-1} with one lag. The model also has a quadratic polynomial in the innovations. The Akaike preferred model is SNP(11141). It is a full nonlinear process with a quartic polynomial whose coefficients are linear in x_{t-1} with one lag. We choose the SNP(11141) as our final fit based on the diagnostic tests discussed previously.[9] The model has 13 parameters and a saturation ratio of 15.[10]

Estimates of the structural parameters. We use the bivariate VAR(1) for consumption and dividend growth rates discussed in Section II as our exogenous driving force. The structural parameters that we are interested in estimating consist of the discount factor (β), the curvature parameter

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9. The results of diagnostic tests are not reported and are available upon request.

10. The saturation ratio is defined as the total number of observations divided by the number of parameters estimated. Portnoy (1985) gives the maximum number of parameters in a linear regression as a function of sample size such that asymptotic normality of a linear function of the parameters is preserved. However, to our knowledge, no similar studies have been done in a nonlinear analysis such as ours.

TABLE 3   EMM Estimates of Structural Parameters

Parameter	Estimate	Std. Error	t-Ratio
β	1.0071	.0040	253.14
γ	6.2740	.1400	44.809
ϕ	.9119	.0015	597.66
$\tilde{s}_n(\hat{\rho})$	$\chi^2(10) = 25.585$	p-value $= .0043$	

Note.—The selected auxiliary model is SNP(11141). β is the discount factor. γ is the curvature parameter. ϕ is the persistence parameter. $\tilde{s}_n(\hat{\rho})$ is the normalized objective value evaluated at the estimated structural parameter values. The p-value is the probability for the hypothesis test that the economic model is correctly specified under the null.

(γ), and the persistence of the surplus consumption ratio (ϕ). Thus, the ρ vector can be represented by $\rho = (\beta, \gamma, \phi)'$.

Because the series to be matched is stock returns, we create a simulated stock return series from the model by taking the following steps. First, we simulate a long series of the exogenous variables including the consumption and dividend growth rates. We then recursively calculate the corresponding surplus consumption ratio and the stock prices using equations (10), (11), and (12), starting from the steady-state surplus consumption ratio, $\bar{s}$, which is a function of the structural parameters, ρ. Finally, we calculate the simulated stock returns as follows:

$$r_i = \frac{(\tilde{P}_i + 1)\frac{D_i}{D_{i-1}}}{\tilde{P}_{i-1}} - 1, \quad i = 1, 2, \cdots, N, \tag{27}$$

where N is the length of simulation, chosen to be 30,000 in our estimation. The first 8,000 simulations are discarded before we collect observations for our estimation to remove possible transient effects of initial conditions. It is important to use a large N to get reliable estimates when the persistence parameter is close to one.

Table 3 reports the EMM estimates for the structural parameters along with their numerical Wald standard errors and t-ratios. The point estimate for the discount factor is 1.007 with a standard deviation of 0.004. This is higher than the discount factor used in Campbell and Cochrane (0.971). While most research restricts the discount factor to be between 0 and 1, it is possible for competitive equilibria to prevail even when β is greater than 1 (see Kocherlakota 1990). The curvature parameter is found to be 6.274 with a standard deviation of 0.14. This is much larger than the value of 2.0 used in Campbell and Cochrane and beyond the "reasonable" range based on the existing literature, for instance, Hansen and Singleton (1982, 1983). The estimated persistence parameter of the surplus consumption ratio is 0.912 with a standard deviation of 0.0015, smaller than the value used by Campbell and Cochrane (0.97). All three estimates are very highly statistically significant, as shown by large t-ratios for the estimates.
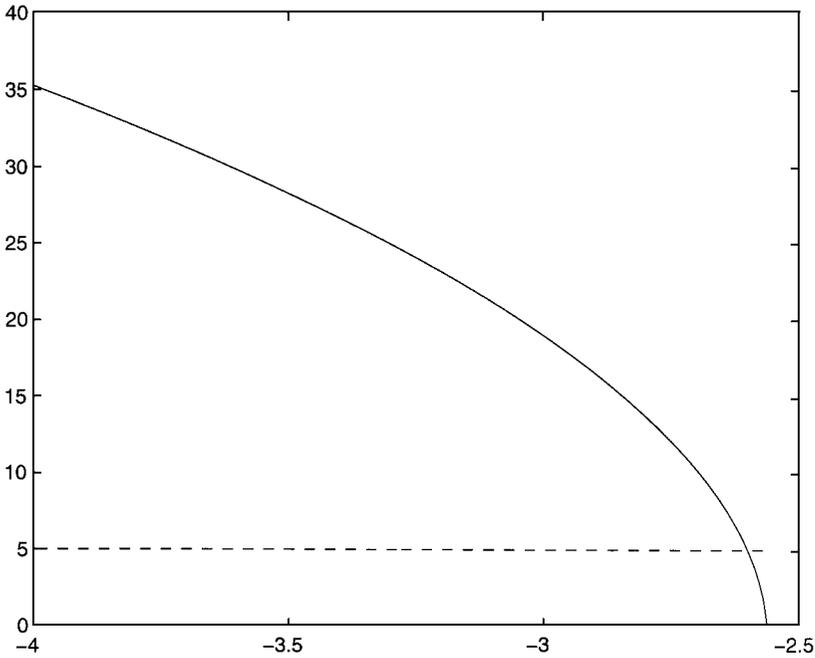
FIG. 1.—The sensitivity function (λ) versus the surplus consumption ratio. The vertical axis is λ and the horizontal axis is the logarithm of surplus consumption ratio, s_t. The dashed line corresponds to $\lambda = 5.0$.

The implied steady-state surplus consumption ratio $(\bar{S})$ is 0.047, which implies a steady-state habit level of 95.3% of consumption. The upper limit for the surplus consumption ratio is 0.077, which implies a minimum habit level of 92.3% of consumption. In figure 1, we plot the sensitivity function for the estimated model. It shows that values taken by the sensitivity function λ is above 15 around the steady-state surplus consumption ratio ($\bar{s} = -3.064$) and is above 5.0 almost everywhere, except for the tiny region around the upper limit for the surplus consumption ratio ($s_{max} = -2.564$). As we discussed in Section II, the cyclicality of expected stock returns is sensitive to how the surplus consumption ratio reacts to contemporaneous consumption growth rate. If the surplus consumption ratio reacts strongly to contemporaneous consumption growth rate (λ takes large values), then expected stock returns are countercyclical. Otherwise, they are procyclical. The sensitivity function for our parameter estimates thus implies that the economic model is likely to generate countercyclical expected stock returns.

The economic model is nonetheless rejected according to the chi-square statistic ($\chi^2(10) = 25.585$). The rejection is not overwhelming (the corresponding p-value is 0.0043; we thus reject the model at the 1% but not at the 0.1% level) compared with tests of other consumption-based

TABLE 4   *t* **Tests on Various Scores**

	Score	Adj. Std. Error	*t*-Ratio
Mean function b_0	.4384	1.0042	.437
b_1	1.3141	1.0855	1.211
Variance function r_0	−3.4395	1.1113	−3.095
r_1	−2.4692	1.1352	−2.175
Hermite polynomial $a(1)$	−.3127	.5546	−.564
$a(2)$	3.6624	1.6867	2.171
$a(3)$	−2.1406	1.5683	−1.365
$a(4)$	10.883	2.6153	4.161
$a(5)$	−3.2638	2.2656	−1.441
$a(6)$	12.813	6.7345	1.903
$a(7)$	−1.5308	7.1918	−.213
$a(8)$	57.563	15.703	3.666
$a(9)$	−10.669	16.732	−.638

Note.—The selected auxiliary model is SNP(11141). The scores are evaluated using simulated returns when the structural parameters are set at the estimated values.

equilibrium asset pricing models using GMM. This justifies why Campbell and Cochrane find the model performing reasonably well in terms of explaining some of the stylized facts on the stock market.

The model fails dramatically with regard to the implications for the risk-free rate. By specifying i.i.d. consumption growth and a particular functional form for the evolution of the surplus consumption ratio, Campbell and Cochrane impose a constant risk-free rate on their model. Considering the relatively small amount of variation in the data, especially relative to equity returns, this is a very reasonable assumption. We adopted the same functional form for the evolution of s_t but allowed for predictability in consumption growth as well as covariation with dividend growth. This resulted in a time varying risk-free rate. We did not use data on the risk-free rate during estimation. The mean risk-free rate implied by our point estimates is −22.94% with a standard deviation of 12.05%. This result is due in large part to the high estimate for the curvature parameter. Using Campbell and Cochrane's value of $\gamma = 2$ along with our other point estimates results in an average risk-free rate of −8.99%.

We can further investigate which dimensions the model fails to match by examining the score functions of the auxiliary model. Table 4 presents the normalized mean SNP scores along with the adjusted standard errors and the corresponding *t*-ratios for the estimated model. The results show that the model does a reasonable job in matching the mean function of the return series but it fails to match the variance and higher-order moment functions, as indicated by the high adjusted *t*-ratios for the variance function and some of the Hermite polynomial coefficients. The performance of the model can also be evaluated according to the conventional moments of returns. In table 5, we present the comparison of four moments of the simulated stock returns and the real returns of S&P 500

TABLE 5   **Summary Statistics of Stock Returns: A Comparison**

Moment	Real Returns	Simulated Returns
Mean	.0216	.0182
Standard error	.0768	.0461
Skewness	−.6450	.2837
Kurtosis	4.4236	3.2869

NOTE.—The real returns are quarterly ex post real returns of the S&P 500 index portfolio from 1947:Q2 to 1995:Q3. The simulated returns are obtained by setting the structural parameters at the estimated values. The summary statistics of simulated returns are based on 30,000 realizations.

index portfolio. We have the following observations. The economic model matches the mean stock returns reasonably well though not perfectly. The variance of the simulated stock returns, however, is lower than its observed counterpart. The distribution of the simulated stock returns is skewed in the opposite direction from the observed returns. The economic model also fails to create the magnitude of the kurtosis observed in the real stock returns. While the real stock returns exhibit excessive kurtosis, the simulated stock returns show almost the same kurtosis as a normal distribution. Next, we discuss the robustness of our parameter estimates.

   Robustness of parameter estimates. A number of recent studies (Andersen, Chung, and Sorensen, 1999; Liu and Zhang, 1996) document that EMM estimation may be sensitive to the selection of auxiliary models. To address the issue on robustness, we re-estimate our structural model using an alternative auxiliary model, SNP(11121), in the EMM estimation. The alternative auxiliary model is also a full nonlinear process and has a quadratic Hermite polynomial whose coefficients are linear in x_{t-1} with one lag.

   Table 6 presents the results of our EMM estimation with the alternative auxiliary model SNP(11121). The point estimates for all three parameters are larger than the ones documented earlier for the preferred auxiliary model SNP(11141). The estimate for the discount factor goes up from 1.0071 to 1.0136, and the curvature parameter estimate increases from 6.274 to 8.731. The estimate for the persistence parameter, however,

TABLE 6   **EMM Estimates of Structural Parameters**

Parameter	Estimate	Std. Error	*t*-Ratio
β	1.0136	.0054	186.15
γ	8.7309	.5341	16.346
ϕ	.9294	.0926	10.035
$\tilde{s}_n(\hat{\rho})$	$\chi^2(6) = 18.015$	*p*-value = .0062	

NOTE.—The selected auxiliary model is SNP(11121). β is the discount factor. γ is the curvature parameter. ϕ is the persistence parameter. $\tilde{s}_n(\hat{\rho})$ is the normalized objective value evaluated at the estimated structural parameter values. *p*-value is the probability for the hypothesis test that the economic model is correctly specified under the null.

is only slightly larger (from 0.912 to 0.929). The estimates also are less efficient than the ones estimated with our preferred auxiliary model. The standard errors of all three parameter estimates are larger than those estimated with the preferred SNP model while the t-ratios of the parameter estimates are all smaller than that estimated with the preferred SNP model. The structural model is again rejected at the 1% test level according to the chi-squared test with a p-value of 0.0062. While this is consistent with our early finding when the preferred SNP model is used, it also seems to indicate that the economic model fits the data better, as reflected in the higher p-value. A possible explanation is the following. The simpler auxiliary model does not capture the higher-order moment characteristics of the return data as well as the more complicated preferred SNP model.[11] Consequently, it is relatively easier for the structural model to match the scores of the simpler model than to match the scores of the more complicated model.

IV.   Cyclicality of Expected Stock Returns

In this section, we first document some benchmark results on the cyclical behavior of the expected stock returns on the S&P 500 index portfolio. We then investigate the cyclicality of the expected returns implied in our economic model using the structural parameters estimated in the previous section.

A.   Cyclicality of Expected Stock Returns: Empirical Evidence

Many empirical studies[12] have documented that stock returns can be predicted by means of publicly available information, such as time series data on financial and macroeconomic variables with an important business cycle component. While a wide range of variables have been used for predicting stock returns, four variables—the dividend yield, the earnings-price ratio, the default premium, and the term premium—are consistently found to be important predictors. We therefore choose these four variables as our predictors for expected stock returns of the S&P 500 index portfolio.

For notational convenience, we denote $r(t, t + \tau)$ as the return of holding a stock purchased at time t for τ periods, dy_t as the dividend yield, ep_t as the earnings-price ratio, dp_t as the default premium, and tp_t as the term premium. Let $X_t = [dy_t, ep_t, dp_t, tp_t]'$. Following Fama and

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11. Fenton and Gallant (1996) document that high-order polynomials of the Hermite expansion are needed to accommodate highly skewed distributions. Similar results are also reported in Liu and Zhang (1996).

12. An incomplete list includes Campbell (1987), French, Schwert, and Stambaugh (1987), Fama and French (1989), Balvers et al. (1990), Breen, Glosten, and Jagannathan (1989), Cochrane (1991), Bekaert and Hodrick (1992), Ferson and Harvey (1993), Glosten, Jagannathan, and Runkle (1993), Pesaran and Timmermann (1995), among others.

TABLE 7   **Regression of Real S&P Returns on Dividend Yield (*dy*), Earning Price Ratio (*ep*), Default Premium (*dp*), and Term Premium (*tp*)**

Variable	Quarterly	Annual	Two Years	Five Years
Constant	−.0828	−.2947	−.4466	−1.3538
	(.0267)	(.0860)	(.1373)	(.2237)
dy	14.885	53.230	101.616	269.383
	(5.505)	(21.393)	(33.415)	(45.430)
ep	−3.7041	−10.596	−20.338	−46.077
	(2.127)	(7.679)	(11.906)	(22.957)
dp	.0084	−.0027	−.0445	−.0375
	(.0159)	(.0368)	(.0430)	(.1253)
tp	.0099	.0308	.0244	.0365
	(.0042)	(.01445)	(.0235)	(.0436)
$\bar{R}^2$	.0830	.2493	.3414	.6843

Note.—The data set is quarterly from 1947:Q1 to 1993:Q4. The default premium is measured by the difference between the average yield of Baa and Aaa corporate bonds, and the term premium is measured by the difference between the average yield of Aaa corporate bonds and 3-month Treasury bills. The real returns are created by adjusting the nominal S&P 500 value weighted returns for inflation. Numbers in parentheses are standard errors.

French (1989), to get expected returns, we regress the holding returns on these four variables, X_t, known at time t:

$$r(t, t + \tau) = \alpha(\tau) + \beta(\tau)X_t + \varepsilon_{t, t+\tau}. \tag{28}$$

We use quarterly data from the first quarter of 1947 to the fourth quarter of 1993. The data set consists of real S&P 500 value-weighted returns, S&P 500 dividend yield, S&P 500 earnings-price ratio, the default premium measured by the difference between the average yield of Baa and Aaa corporate bonds, and the term premium measured by the difference between the average yield of Aaa corporate bonds and 3-month Treasury bill rates. The real returns are created by adjusting the nominal returns for the consumer price index. The term premium measure used here follows Kandel and Stambaugh (1990) and is slightly different from that in Fama and French (1989), in which the term premium is defined as the difference between the average yield of the long-term government bonds and short-term Treasury bills. The S&P 500 value-weighted returns and 3-month treasury bill rates are from CRSP database and the rest of series are from CITIBASE.

In table 7 we present the results for the preceding regression for $\tau =$ 1, 4, 8, and 20, which correspond to quarterly, annual, 2-year, and 5-year stock returns. The Newey-West (1987) estimate for the variance-covariance matrix is employed to correct for the heteroscedasticity and serial correlation of the error terms. The following features emerge. The dividend yield is consistently positive and statistically significant at the 1% level at all horizons reported. The earnings-price ratio, on the other hand, is consistently negative and statistically significant at the 5% level at a 5-year
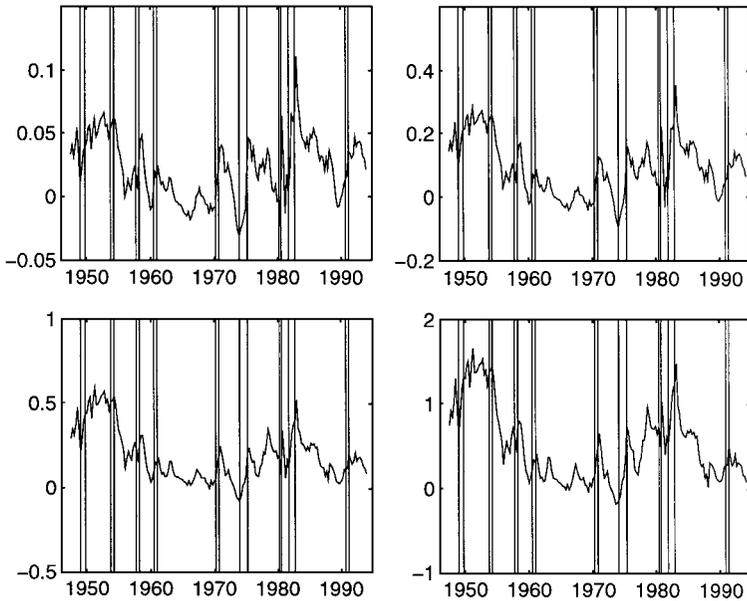
Fɪɢ. 2.—The expected stock returns at (A) quarterly, (B) annual, (C) 2-year, and (D) 5-year horizons. The vertical grid lines are NBER business-cycle peaks and troughs.

horizon. The default premium is positive at a quarterly horizon and negative at all longer horizons (annual, 2, and 5 years) but not statistically significant. Finally, the term premium is consistently positive and statistically significant at short horizons (quarterly and annually) but not at long horizons (2 and 5 years).

Given the estimates of $\alpha(\tau)$ and $\beta(\tau)$, the expected stock returns predicted by the linear model, denoted $r(t, \hat{t} + \tau)$, are obtained from

$$r(t, \hat{t} + \tau) = \hat{\alpha}(\tau) + \hat{\beta}(\tau)X_t. \tag{29}$$

Figure 2 shows the expected stock returns along with the National Bureau of Economic Research (NBER) classification of business-cycle turning points for 1 quarter, 1 year, 2 year, and 5 year holding periods. It shows that the expected returns often reach the peak when the economy is in a recession. The phenomenon becomes more pronounced as the horizon increases.

To explore how expected stock returns are related to business cycles quantitatively, we estimate the following simple linear models as in Harrison and Zhang (1999):

$$r(t, \hat{t} + \tau) = \theta_0 + \theta_1 D_t + u_{t+1} \tag{30}$$

TABLE 8   Regression of Expected Stock Returns on the Business-Cycle Proxy

Horizon	Variable	Parameter Estimate	Standard Error	T for H0: $\beta = 0$	Prob > $|T|$
Data					
Quarterly	Constant	.0201	.0020	10.220	.0001
	DB	.0102	.0047	2.157	.0323
1 year	Constant	.0848	.0070	12.112	.0001
	DB	.0435	.0169	2.573	.0109
2 years	Constant	.1768	.0120	14.699	.0001
	DB	.0759	.0291	2.611	.0098
5 Years	Constant	.4650	.0344	13.531	.0001
	DB	.2761	.0831	3.323	.0011
Simulated Returns					
Quarterly	Constant	.0182	.0001	141.69	.0001
	D1	−.0070	.0004	−18.75	.0001
1 year	Constant	.0711	.0002	315.73	.0001
	D1	.0098	.0007	14.88	.0001
2 years	Constant	.1459	.0004	411.82	.0001
	D1	.0340	.0010	32.75	.0001
5 years	Constant	.4101	.0007	610.82	.0001
	D1	.0854	.0020	43.45	.0001
Quarterly	Constant	.0178	.0001	144.26	.0001
	D2	−.0103	.0006	−17.60	.0001
1 year	Constant	.0720	.0002	330.90	.0001
	D2	.0053	.0010	5.19	.0001
2 years	Constant	.1485	.0003	428.23	.0001
	D2	.0300	.0016	18.27	.0001
5 years	Constant	.4163	.0007	627.14	.0001
	D2	.0832	.0031	26.51	.0001

NOTE.—DB is the business cycle dummy variable defined based on the NBER business-cycle turning point classification. It takes value 1 when the economy is between a peak and a trough and takes value 0 otherwise. D1 is the first dummy variable defined for the economic model. It takes value 1 if the consumption growth rate is negative for 2 consecutive quarters and takes value 0 otherwise. D2 is the second dummy variable defined for the economic model. It takes value 1 if both consumption growth rate and dividend growth rate are negative for 2 consecutive quarters and takes value 0 otherwise.

where D_t is the dummy variable created according to the NBER classi-fication of business-cycle turning points.[13] It takes value one if period t is in a recession (between a peak and a trough) and zero otherwise.

The top panel of table 8 presents the regression results for the expected stock returns with different holding periods. The estimated coefficients of the dummy variable are consistently positive, indicating that the expected stock returns are higher in recessions than in nonrecession periods at all horizons. Furthermore, the statistical significance of the recession dummy increases as the horizon expands. At a quarterly horizon, the es-timate is positive and significant at the 3% level. However, at the 5-year

13. The qualitative features of the cyclical behavior of the expected stock returns are unchanged when other business-cycle proxies, such as the consumer confidence index and industrial product growth, are used. See Harrison and Zhang (1999).

horizon, it is positive and statistically significant at the 0.2% level. The forecasting power of the business cycle variable also increases from about 1.9% to 5.1% as the horizon increases from a quarter to 5 years.

B. Expected Stock Returns and the External Habit

To obtain expected stock returns implied by the economic model, we use a numerical simulation method discussed next. We denote by $r_\tau^e[z(k), S(k)], \tau = 1, \ldots, \infty$, the expected return of holding the stock for τ periods starting from the state $[z(k), S(k)]$. Let $\pi(z_\tau \mid z_{\tau-1})$ be the transition matrix for the exogenous state variables (z_τ), as defined by equation (8). Given the law of motion for the stock price as defined by equations (13) and (14) and the transition matrix for the exogenous state variables, the expected holding returns implied by the economic model can then be obtained using the following Monte Carlo integration. We denote by $\{z(k)\}, k = 1, \ldots, K + N$, a long series of simulated exogenous state variables starting from the unconditional mean. Let $\{S(k)\}, k = 1, \ldots, K + N$ be the corresponding surplus consumption ratio computed using equations (11) and (12) starting from the steady-state value $\overline{S}$. We drop the first N observations to remove possible transient effects. Let $\{z_i^j(k)\}_{i=1}^\tau, j = 1, 2, \ldots, J$, be the J simulated realization path of the exogenous state variables starting from $z = z(k)$. Each path consists of τ simulated values for the exogenous state variables using the transition matrix $\pi(\cdot \mid \cdot)$. In other words, $z_1^j(k)$ is a random draw from $\pi(z' \mid z(k))$, $z_2^j(k)$ is a random draw from $\pi(z' \mid z_1^j(k))$, and so forth. We first compute the surplus consumption ratio, $\{S_i^j(k)\}_{i=1}^\tau, j = 1, \ldots, J$, starting from $S = S(k)$. We then recursively compute the stock price, $\tilde{P}_i^j[z_i^j(k), S_{i-1}^j(k)]$ for $i = 1, \ldots, \tau$ and $j = 1, \ldots, J$. The expected returns of holding the stock for τ periods corresponding to state $[z(k), S(k)]$ can be approximated as follows:

$$1 + r_\tau^e[z(k), S(k)] \approx$$

$$\frac{1}{J} \sum_{j=1}^J \frac{\tilde{P}_1^j + 1}{\tilde{P}[z(k), S(k)]} \frac{\tilde{P}_2^j + 1}{\tilde{P}_1^j} \cdots \frac{\tilde{P}_\tau^j + 1}{\tilde{P}_{\tau-1}^j} \exp(\Delta d_1^j) \cdots \exp(\Delta d_\tau^j).$$

In our application, the length of the simulated series is set at $K = 20{,}000$ with the initial $N = 8{,}000$ observations discarded to remove possible transient effects. To be consistent with our empirical investigation, we compute the expected returns for $\tau = 1, 4, 8,$ and 20, which corresponds to 1-quarter, 1-year, 2-year, and 5-year holding periods, respectively. The number of replications in computing the expected returns (J) is set at 20,000.

To investigate the cyclical behavior of the expected stock returns, we first construct dummy variables representing business cycles in the economic model. We then apply the similar business cycle regressions as in

the previous subsection to the expected simulated stock returns. Following Rouwenhorst (1995), we define a recession as an episode starting at a period in which the consumption growth rate is negative for 2 consecutive quarters. An alternative and more restrictive definition of a recession is that both the consumption and dividend growth rates are negative for 2 consecutive quarters. The dummy variable then takes value 1 if the economy is in a recession and 0 otherwise. We denote by D1 the business cycle dummy corresponding to the first definition and by D2 to the second definition. In the bottom panel of table 8, we present the results of regressing the expected simulated returns at various horizons on the business-cycle dummy variables.

Comparing to the expected returns of the S&P 500 index portfolio, we have the following observations. First, the magnitude of the expected simulated returns are comparable to the observed counterpart at all four holding horizons during nonrecession periods. For instance, the average expected returns of the S&P 500 index during nonrecession periods are 2.01%, 8.48%, 17.68%, and 46.5% at 1 quarter, 1 year, 2 years, and 5 years, respectively. The average expected simulated returns during nonrecession periods are 1.82%, 7.11%, 14.59%, and 41.01%, for the same set of holding horizons, respectively, when the less stringent definition of recession (D_1) is used. Similar results are also found for the more stringent definition of recession (D_2) as demonstrated in the bottom half of the lower panel in table 8. This is consistent with our earlier finding that the model with the external habit does a reasonable job matching the mean of stock returns. Second, the expected simulated returns are procyclical at short holding horizon, such as 1 quarter, and countercyclical at all other longer horizons, such as 1 year, 2 years, and 5 years. The countercyclical expected returns at longer horizons are consistent with the cyclical behavior of the expected returns of the data.

A possible explanation is as follows. First, consumption growth tends to drive the expected stock returns procyclically. This is because consumption growth is positively serially correlated. A low consumption growth rate now (a recession) is likely to be followed by a low consumption growth rate next period. This pushes up the current stock price according to the equilibrium stock pricing function and leads to a lower expected stock return. The surplus consumption ratio, on the other hand, exerts its influence on the expected returns via its impact on the growth rate of the surplus consumption ratio, which in turn depends on the sensitivity of the surplus consumption ratio to contemporaneous consumption growth rate. If the surplus consumption ratio is not sensitive to the consumption growth rate or the external habit level moves at about the same rate as consumption and in the same direction, then the growth rate of the surplus consumption ratio for the next period is not affected significantly. As a result, the expected stock returns basically are affected by the consumption growth rate, which is procyclical. On the other hand,

if the surplus consumption ratio reacts strongly to the contemporaneous consumption growth rate, then when the economy is in a recession, the current surplus consumption ratio decreases significantly. The growth rate of the surplus consumption ratio thus increases and the stock price goes down. The expected stock returns are higher. When the latter effect dominates the former, the overall effect moves the expected stock return countercyclically.

Furthermore, the spread of the expected returns between recession and nonrecession periods is much smaller for the simulated returns than for the observed returns. For instance, the spread for the simulated returns is about 1%, 3%, and 8% at 1 year, 2 years, and 5 years, respectively. The spread for the observed returns, however, is more than 4%, 7%, and 27%, respectively, for the same holding horizons. This is consistent with our earlier finding that the economic model fails to match the variance of observed stock returns. The finding suggests that, while we have limited success in replicating the cyclical behavior of expected stock returns by introducing the external habit, it remains an important challenge for economic models to match the large spread in expected returns between recession and nonrecession periods observed in the data.

V.   Conclusions

In this paper, we extend the Campbell and Cochrane (1999) study by formally estimating an equilibrium asset pricing model in which agents' preferences have an unobserved external habit, using the efficient method of moments proposed by Bansal et al. (1994) and Gallant and Tauchen (1996).We find that the estimated subjective discount factor to be slightly greater than one, supporting the earlier finding by Kocherlakota (1990). The surplus consumption ratio process is very persistent, as reflected by a larger than 0.9 persistence parameter in the law of motion for the surplus consumption ratio. Our estimation results suggest that the surplus consumption ratio reacts strongly positively to the contemporaneous consumption growth rate. This implies that the external habit moves at a slower rate than consumption. The model with the external habit therefore generates countercyclical expected stock returns at all longer holding periods, such as 1 year, 2 years, and 5 years. It also generates expected stock returns comparable to the expected returns of the S&P 500 index portfolio for nonrecession periods. This is consistent with the limited success of the Campbell and Cochrane study in replicating some of the stylized facts of the stock market using the model with the external habit.

The model, however, fails to create the large spread of expected returns between the recession and nonrecession periods observed in the S&P 500 index returns. Further, the model is rejected at the 1% (but not at the 0.1%) significance level according to the chi-square test. Detailed examination of

the score functions of the auxiliary model and the conventional moments indicates that, while the model does a reasonable job in matching the mean of the real returns, it fails to match the higher-order moments. The model implies negative risk-free rates using our point estimates, indicating the sensitivity of the specification of the law of motion for the surplus consumption ratio to small perturbations of other dimensions of the model, such as allowing for predictability of consumption growth and covariation with dividend growth.

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