

Optimal Life-Cycle Asset Allocation with Housing as Collateral

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ABSTRACT

We investigate a household's asset allocation, housing, and mortgage decisions in a calibrated life-cycle model with costly stock market entry and participation and a risky housing market featuring housing adjustment costs, mortgage collateral borrowing requirement, refinancing charge, and default penalty. Our analysis demonstrates that a household's liquid wealth is the most important determinant of both home and stock ownership. Our model can generate empirically documented hump-shaped life-cycle home and stock ownership patterns as well as explaining the negative (positive) effect of housing (mortgage) position on stock market participation and overall equity exposure. Extensive comparative static analysis suggests that ignoring mortgage refinancing cost and default option introduces significant biases in stock market participation rates and overall stockholdings.

KEY WORDS: life-cycle model, portfolio choices, housing, mortgage

JEL CLASSIFICATION CODES: D91, E21, R21

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1. Introduction

For the vast majority of U.S. households, a house is the largest and most important single asset and a mortgage is the largest consumer debt in their balance sheet. For instance, the Survey of Consumer Finances (SCF) reveals that from 1989 to 2001, the average home value ranges from 55 percent to 60 percent of a homeowner's total asset, while a mortgage accounts for 62 percent to 70 percent of the total consumer debt. Home equity accounts for half of household networth.

Households have also become more leveraged in housing investments than they were more than a decade ago. In 1989 the average mortgage balance relative to home value (the loan-to-value ratio) was 25.9 percent. In 2001, the loan-to-value ratio has climbed to 34.8 percent. Many economists attribute the steady appreciation of home value in household balance sheets and prevalent "cash-out" refinance as the driving force behind the robust consumption expenditures during the economic downturn between 2000 and 2002, when the stock market plummeted (see Case, Quigley, and Shiller (2003) and Campbell and Cocco (2005) for the effects of house price appreciation on household consumption).

From 1989 to 2001, stock ownership increased from 17.8 percent to 34.1 percent for renters and from 41.6 percent to 62.8 percent for homeowners. Among those who own stock, average dollar stock investment has increased from \$29,900 to \$38,500 for renters and from \$51,300 to \$149,000 for homeowners.¹ The fraction of networth invested in stock has increased from 20.7 percent to 37.0 percent for renters and from 12.2 percent to 26.7 percent for homeowners.

Recent empirical analysis has uncovered several important characteristics on household asset allocations and housing and mortgage choices (see for example Heaton and Lucas (2000a), Kullmann and Siegel (2003), Cocco (2005), and Yao and Zhang (2005)). First, both home ownership and stock ownership are hump-shaped in age, with the home ownership peaking around retirement age and stock ownership peaking in investors' 50s. Second, households with a low home value-networth ratio and/or a high mortgage-networth ratio are more likely to participate in stock market. Third, households with a large home value in networth hold less stock and households with a high mortgage balance relative to networth invest more in stock relative to networth or other broader definition of assets. Fourth, the effects of house value or mortgage balance as a fraction of networth on

¹All stock investment is denominated in 2001 dollars.

stockholdings relative to a narrow yet widely used definition of wealth—liquid financial assets—are much weaker and statistically inconclusive.

In this paper we investigate the interplay between a household’s asset allocation decisions and its housing and mortgage decisions in a realistic life-cycle framework. In our model, an investor buys a house by taking a self-amortizing mortgage loan at a fixed nominal rate, subject to an initial collateral requirement. The investor can refinance his mortgage contract to gain access to home equity by paying a fee. He can also default on his mortgage contract, which incurs a one-time penalty. In the event of a house sale, a current homeowner must pay an agency fee. However, an investor can avoid the risk and transaction costs associated with home ownership by renting housing services. To invest in stock, the investor pays a per-period maintenance cost to monitor his investment in addition to a one-time entry cost.

Several recent theoretical and numerical studies attempt to understand the effect of housing on an investor’s life-cycle stock investment. Cocco (2005) introduces durable housing to a standard life-cycle consumption and portfolio choice model. He assumes that the investor consumes housing services only from owning a house above a minimum size, i.e., no housing rental market is available. His study suggests that housing price risk “crowds out” stockholding. He also finds a positive correlation between mortgage debt and stockholdings and attributes the result to the effect of future labor income which encourages both leveraged purchases of housing assets and stock investment.

Yao and Zhang (2005) explicitly incorporate a rental market for housing services. They find that when indifferent between renting and owning, the homeowner holds a lower equity proportion in his networth, yet a higher equity proportion in his liquid portfolio. They also demonstrate that the investor holds a suboptimal portfolio and suffers a large utility cost by following the alternative policy of acquiring housing services either only from renting or only from owning a house. Hu (2005) studies portfolio choices for homeowners in the presence of a house rental market in a stylized five-period model. She finds that young and middle-aged households, regardless of whether they are currently homeowners, have much less stock investment than predicted by traditional models without housing.

Our paper is closely related to Cocco (2005) and Yao and Zhang (2005), but extends their studies in several important dimensions. First, we explicitly incorporate a refinancing cost, which leads to infrequent access to home equity and induces simultaneous holdings of mortgage and bond. Both Cocco (2005) and Yao and Zhang (2005) assume that the investor can costlessly access home equity by borrowing up to a fraction of home value every period. While this assumption effectively

eliminates mortgage as a separate state variable and greatly simplifies the numerical solution of the problem, it has some undesirable implications. When the lending rate is lower than the borrowing rate, as assumed in Cocco (2005), the investor will not hold any bond until he completely pays off his mortgage. For investors with a positive mortgage balance, this leads to zero liquid financial asset holdings for non-stockholders and an all-equity portfolio for stockholders. When the lending and borrowing rates are equal, as assumed in Yao and Zhang (2005), the investor is indifferent between paying down mortgage debt and holding extra investments in a riskless bond, and the portfolio of liquid asset is indeterminate.² A refinancing cost separates networth into liquid financial assets and illiquid home equity. Further, in the absence of refinancing charges, the investor can costlessly access home equity accumulated through house price appreciation or mortgage principal payments. Thus he is less inclined to invest liquid assets conservatively.

Second, we apply the collateral borrowing constraint only to the newly initiated loans, but not to the existing mortgage contracts. Both Cocco (2005) and Yao and Zhang (2005) assume that the collateral borrowing constraint needs to be satisfied in every period. By applying the collateral borrowing constraint to both new loans and ongoing loan contracts, an investor is forced to inject liquid funds to reduce mortgage balance when his home equity falls below the collateral requirement. Yet in reality mortgage loans are subject to an “initial margin” but not a “maintenance margin” requirement. As long as an investor makes his scheduled mortgage interest and principal payments, the lender cannot take possession of the borrower’s property even under a negative home equity situation. The “maintenance margin” assumption eliminates the option to default on the investor’s mortgage, which offers an important protection for homeowners from severe housing depreciation. Given a documented positive correlation between shocks to housing value and labor income (see Cocco (2005) and Ortalo-Magnè and Rady (2005)), this assumption exaggerates the reduction in available liquid assets when the investor suffers negative shocks to his income and his housing asset simultaneously, and significantly reduces the investor’s willingness to take corporate equity risk.

Third, we model the cost associated with stock investment as consisting of two components: a one-time entry cost and a per-period maintenance cost. Under a one-time entry cost, as in Cocco (2005) and Gomes and Michaelides (2005), stock ownership rate is non-decreasing in age, in contrast to the empirically observed hump-shaped stock ownership pattern. Intuitively, as argued by Vissing-Jørgensen (2002 and 2004), stock market participants have to spend time and resources to monitor

²To resolve the indeterminacy, Yao and Zhang (2005) assume that the investor always carries the maximum mortgage balance allowed. If the investor’s home equity rises above the level required by the collateral requirement, the excess equity is cashed out and invested in stock or bond.

and maintain their stock investments in each time period. When the per period maintenance cost is high enough, some stockholders with low liquid wealth may choose to liquidate their equity positions and exit from stock market.

Our model identifies liquid wealth as the most important determinant for an investor's home and stock ownership decisions. A young investor who has yet to accumulate enough wealth remains a renter and a non-stockholder. As the investor ages, he is more likely to become a homeowner first, reflecting the large benefit that home ownership offers compared to stock investment when total wealth is relatively low. A homeowner becomes a stockholder after accumulating enough liquid wealth. Our model also generates a hump-shaped life-cycle stock ownership pattern, reflecting stock market exits among elderly investors to avoid the per period maintenance when their investable liquid wealth is low.

Furthermore, a homeowner with a large illiquid home equity position may hold much less stock as a fraction of his networth than desired, even with an all equity portfolio in his liquid wealth. When facing such a liquidity constraint, the homeowner may optimally choose to pay a fee to refinance his mortgage contract and cash out a fraction of his home equity to increase his overall equity exposure.

Our numerical analysis demonstrates that the model can reconcile the documented empirical relations between a household's stock market participation and asset allocation decisions and its housing and mortgage positions. Given wealth and income, an investor with a lower house value or a higher mortgage balance is more likely to participate in the stock market. This is attributable to the fact that the investor with a larger fraction of wealth in liquid form can more easily overcome the cost associated with stock investing.

When not making adjustments to his housing and mortgage positions, the investor with a large mortgage balance also has a large amount of liquid wealth, given wealth, income, and house value. Conditional on owning stock, he optimally allocates a higher fraction of networth to equity and yet holds a lower equity proportion in liquid portfolio. This reflects the combined effects of the desire to hold an overall equity exposure close to the optimal level in the absence of borrowing restriction on one hand and the availability of liquid wealth on the other.

Using simulation analysis, Cocco (2005) also generates a positive correlation between the equity proportion in networth and the mortgage-networth ratio. However, there is an important distinction between his result and ours. In Cocco (2005), the positive relation between the equity proportion and

the mortgage–networth ratio reflects a contemporaneous correlation, instead of a causal relationship, since the mortgage balance is not a state variable under costlessly refinancing. In our model, due to mortgage refinancing charges, existing mortgage balance becomes a state variable and directly impacts an investor’s portfolio choices by changing the composition and liquidity of household wealth.

Our analysis on optimal decision rules indicates that the relations between a homeowner’s asset allocation and house value are complex, and dependent upon the homeowner’s mortgage balance. Therefore, the lack of statistical significance of the home value–networth ratio and mortgage loan–to–value ratio on the liquid asset equity proportion reported in existing empirical studies could be caused by empirical mis-specification. Future empirical models should allow more flexible forms of the interactions between households’ stock allocations and their housing and mortgage positions. Empirical analysis should also explicitly incorporate both the one–time stock market entry cost and the per period maintenance cost to correctly identify the life–cycle pattern of stock ownership.

Detailed comparative static analysis suggests that introducing a borrowing and lending spread drastically accelerates home equity accumulation by inducing faster mortgage pay-downs. This reduces the percentage of stock ownership and overall equity exposure. Further, ignoring refinancing charges introduces upward biases in both stock ownerships and the overall equity exposure in total wealth. Interestingly, imposing maintenance margin on home equity—which eliminates mortgage default—increases stock ownership rate among young renters by delaying their home purchase and inducing more liquid savings prior to home ownership. Yet it discourages equity market participation and stockholdings among homeowners by reducing their liquid wealth.

The rest of the paper is organized as follows. Section 2 discusses empirical findings on the effects of housing and mortgage decisions on the investor’s portfolio choice. Section 3 introduces the model, while section 4 discusses the numerical results. Section 5 provides concluding remarks.

2. Empirical Evidence on Household Housing and Portfolio Choice

To understand households’ housing and portfolio choices, we first describe the sample averages for key housing and portfolio choice related variables for households in the Survey of Consumer Finances between 1989 and 2001. We then review the main empirical findings on the relations between households’ asset allocations and their housing and mortgage positions.

Table 1 reports percentage of home ownership (OWNHOUSE), average house value–asset ratio (PH/ASSET) and house value–networth ratio (PH/NW), percentage of homeowners owing mortgage (OWEMORT), average mortgage–total debt ratio (MORT/DEBT), mortgage loan–to–value ratio (LTV), home equity–networth ratio (HE/NW), percentage of stock ownership (OWNSTOCK), average equity proportions in total asset (S/ASSET), networth (S/NW), and liquid financial assets (S/SB), across age groups and various years.

Home ownership rate exhibits a hump–shape over an investor’s life cycle, peaking at 84.7 percent just before retirement. The home ownership rate is lowest at 19.3 percent for the youngest age group and remains high at 77.7 percent for the oldest age group (75 years and beyond). Homeowners’ house value–networth ratio decreases in age, reflecting households’ wealth accumulation as human capital is gradually realized. The average loan–to–value ratio declines drastically as homeowners pay off their mortgage debt. The home equity in networth is slightly U–shaped in age and lowest for households reaching retirement age. In the sample period between 1989 and 2001, home ownership has increased slightly. Households also increasingly depend on leverage to finance their home ownership. The average mortgage balance relative to home value increases from around 25 percent to about 35 percent from 1989 to 2001. Consistent with the increasing loan–to–value ratio, the home equity–networth ratio declines over the same period.

Overall, stock ownership rate is much higher for homeowners than that of renters and also demonstrates a hump–shape in age, highest among age groups 35–44 for renters and 45–54 for homeowners. Among stockholders, the fraction of stock investment in total asset increases from a low 20 percent to around 30 percent for renters, and from about 5 percent to almost 20 percent for homeowners as investors age. The fraction of stock investment in networth is rather flat over life cycle, at around 30 percent for renters and 20 percent for homeowners across age groups. The share of stock investment in the liquid financial assets however shows somewhat a hump–shape for both renters and homeowners. It peaks at 42.6 percent for ages 35–44 for renters and at 47.2 percent for 45–54 for home owners. Over the sample period, there is a strong positive trend in stock ownership for both renters and homeowners, increasing from 18 percent to 34 percent for renters, and from 42 percent to 62 percent for homeowners. Similarly, stock investments as a fraction of total asset, networth, or liquid assets also exhibit a strong positive time trend for both renters and homeowners.

Up until recently, empirical studies on the relation between housing and household portfolio choices have been scarce. When examining the effect of entrepreneurial income risk on portfolio

choice and asset prices with OLS regressions using the 1992 Survey of Consumer Finances (SCF) data, Heaton and Lucas (2000a) find that mortgage balance relative to financial networth has a positive effect on the shares of stock in liquid assets, financial assets, and total assets, while the relative value of residential real estate has a negative effect.³ They interpret the positive effects of mortgage on equity proportions as “suggesting that some stockholdings are indirectly financed via mortgage debt.” The negative effect of real estate is attributed to a risk “crowding out” effect, where “households bearing more risk from other assets cut back on stockholdings,” or “a pure substitution effect,” where “households with more in other assets hold less stock.” However, their empirical model specification does not account for limited stock market participation among households in the data.

Three recent studies provide more in-depth empirical investigation on how housing influences portfolio choice. Cocco (2005) applies cross-sectional Tobit regressions to account for limited stock market participation in the 1989 wave of the Panel Study of Income Dynamics (PSID) data to examine the relation between stockholdings and housing related variables. Similar to Heaton and Lucas (2000a), he finds that mortgage debt relative to financial networth has a positive effect on share of stock in liquid assets, financial assets, and total assets, while relative house value has a negative effect.⁴ However, equity ownership and conditional equity proportions may have different determinants. The same explanatory variables could also have different effects on stock market participation versus conditional equity proportions. Tobit regression does not allow two effects to be different.

Using the PSID data from 1984 to 1999, Kullmann and Siegel (2003) examine portfolio choice as a function of household exposure to real estate risks. Their study uses Heckman’s sample selection technique to account for limited stock market participation. They also use first differencing in both dependent and explanatory variables to deal with unobservable household-specific heterogeneity in stockholdings. Kullmann and Siegel (2003) find that house value–networth ratio has a negative effect on both stock market participation and stock (or risky financial asset) holdings relative to liquid financial assets, while mortgage–networth ratio has a positive effect on stock market participation and stockholdings (or risky financial asset holdings). These results are robust to dynamic specifications

³The effects of mortgage and real estate are statistically insignificant in Heaton and Lucas (2000a) when stockholdings are measured relative to a narrow definition of wealth, i.e., liquid asset, which is defined as the sum of direct and indirect holdings of cash, bonds, bills, stocks. Financial asset is liquid asset plus housing and other real estate, proprietary businesses, pensions, and trusts. Total asset is financial asset plus capitalized labor, social security, and pension income. Financial networth is defined as financial assets minus various types of debt including mortgages and consumer loans.

⁴The effects of mortgage and housing on equity proportions are statistically significant for all three portfolio definitions in Cocco (2005).

allowing for lagged dependent variable and endogeneity in income and wealth. Kullmann and Siegel (2003) interpret the findings as being consistent with home ownership presenting a background risk.

In Kullmann and Siegel (2003), the conditional error in the stockholding equation is assumed to be a linear function of the error term in the participation equation so that the unobserved time-invariant heterogeneity effect can be removed after taking the first difference in all variables. Violation of this assumption may lead to inconsistent estimates. To provide consistent estimates on the relation between equity proportion and variables on housing asset in the presence of limited market participation and unobserved heterogeneity, Yao and Zhang (2005) apply the two-step estimation procedure proposed by Kyriazidou (1997) to the PSID data from 1984 to 2001. The coefficients of the stock market participation (selection) equation are first consistently estimated using a conditional Logit regression. The estimates are then used to construct a kernel weight to consistently estimate the equity proportion equation.

Yao and Zhang (2005) find that all housing-related variables have a significant impact on homeowners' stock market participation decisions. Home value-networth ratio affects stock market participation negatively and mortgage-networth ratio has a positive effect on homeowners' stock market participation, both at a decreasing rate. Further, home value-networth ratio has a negative effect on share of stock in networth while mortgage-networth ratio has a positive effect at a decreasing rate. However, the effects of home value-networth ratio and mortgage-networth ratio on stockholdings relative to liquid financial assets are ambiguous due to lack of statistical significance.

Both Kullmann and Siegel (2003) and Yao and Zhang (2005) find that home value-networth ratio and mortgage-networth ratio affect households' stock market participation decision in opposite directions but with similar magnitude. Given that home equity is illiquid, this finding suggests that households' stock market participation decision is largely determined by the size of households' available liquid assets relative to their networth. Indeed when liquid financial wealth is included in the Probit regression for stock market participation using pooled SCF data from 1989 to 2001, home value-networth ratio and mortgage-networth ratio are no longer significant.⁵ The effects of home value-networth ratio and mortgage-networth ratio on stockholdings in either liquid financial assets or networth, however, could not be combined into that of home equity. While the signs of the two variables on equity proportions remain opposite of each other, as in the case of stock ownership, the magnitudes in absolute values and statistical significance are very different. This indicates that not

⁵Regression results are not reported. We thank Annette Vissing-Jørgensen for the comment.

only the size of illiquid home equity but also the composition of home equity, i.e., the leverage ratio, affects a household's asset allocations.

Overall, empirical evidence suggests that house value and mortgage leverage ratio affect stock market participation through changing the amount of liquid financing assets. Further, home value–networth ratio has a significant negative effect while mortgage–networth ratio has a significant positive effect on stockholdings relative to broader definitions of wealth. The effects of home value–networth ratio and mortgage–networth ratio on a narrow definition of equity proportion in liquid financial assets are much weaker and statistically inconclusive.

3. The Economic Model

Our economic model extends the setup in Cocco (2005) and Yao and Zhang (2005) by introducing costly refinancing and default options to residential mortgage contracts, and both a one–time entry cost and a per period maintenance cost to stock market investing. Our description on the model below is focused on the extensions to the existing literature. The detailed derivation of the model and the solution algorithm are provided in the appendix.

The investors in our economy face positive per period mortality rate and live for at most $T > 0$ periods. In each time period, the investor receives a real nonfinancial income Y_t . Before retirement at age J , Y_t represents labor income given by:

$$Y_t = P_t^Y \varepsilon_t, \quad \text{and} \quad P_t^Y = \exp\{f(t)\} P_{t-1}^Y \nu_t \quad \text{for } t = 0, \dots, J - 1, \quad (1)$$

where P_t^Y is the permanent component of the investor's labor income, ν_t and ε_t are persistent and temporary shocks to labor income, respectively. We assume that $\ln \varepsilon_t$ and $\ln \nu_t$ are *i.i.d.* normal with mean $\{-0.5\sigma_\varepsilon^2, -0.5\sigma_\nu^2\}$ and variance σ_ε^2 and σ_ν^2 , respectively.⁶ $f(t)$ is a function of household age. $\ln P_t^Y$ thus follows a random walk with a deterministic drift. The growth rate of an individual labor income evolves according to

$$\Delta \ln Y_t = f(t) + \ln \varepsilon_t + \ln \nu_t - \ln \nu_{t-1}, \quad (2)$$

⁶The corresponding mean values for ε_t and ν_t are one.

with unconditional variance $\sigma_\varepsilon^2 + 2\sigma_\nu^2$. After retirement at age J , Y_t represents payments from investors' pension or retirement accounts, including social security, that replaces a constant fraction (θ) of his pre-retirement permanent labor income P_{J-1}^Y .⁷

The investor acquires housing services by either renting or owning a house. If the investor rents housing services in period $t - 1$, he can either keep renting, or buy a house and become a homeowner at period t . To rent, the investor pays a fraction (α) of the market value of the rental house to the landlord. To become a homeowner, the investor needs to pay at least a fraction (δ) of the house value as a down payment at loan initiation and finance the rest by borrowing a mortgage with a constant nominal rate of interest $r_m^n = (1 + r_m)(1 + \iota)$, where r_m is the real rate of interest and ι is the deterministic rate of inflation. The mortgage is assumed to mature at time T , such that the entire balance is paid off in the terminal period. Let H_t be the unit of housing services and P_t^H be the real price per unit of housing services at time t . A newly initiated mortgage, M_t , therefore needs to satisfy:

$$0 \leq M_t \leq (1 - \delta)P_t^H H_t. \quad (3)$$

A home buyer also incurs search and closing costs associated with identifying houses to purchase and securing mortgage and title, assumed to be proportional to the house value (φ). A homeowner also spends a fraction (ψ) of the house value on repair and maintenance in order to keep house quality constant.

At the beginning of each period, the investor incurs an exogenous moving shock, represented by D_t^m , which takes the value of 1 if the investor moves for exogenous reasons such as job-related relocation, and 0 otherwise. While a renter can move without incurring any transaction costs, a homeowner who experiences the moving shock ($D_t^m = 1$) is forced to liquidate his house through either voluntary selling or involuntary foreclosure by defaulting on mortgage payment. Selling a house entails a transaction cost—assumed to be a fraction (ϕ) of the market value of the house—which is borne by the seller. The full balance of the mortgage is due upon house sale. In case of a default, we assume that the debtor incurs a one-time cost equal to κ fraction of the house value, which captures the damage to the debtor's reputation and future access to the credit market. Upon

⁷In our numerical analysis, we implement this specification by setting the investor's permanent income in retirement at his pre-retirement level, while setting the transitory shock to his income replacement ratio, i.e., $\exp\{f(t)\}\nu_t = 1$ and $\varepsilon_t = \theta$ for $t \geq J$.

defaulting, the investor is relieved of his mortgage liability and the lender forecloses the investor's house. Let \bar{l}_t be the investor's beginning-of-period loan-to-value ratio:

$$\bar{l}_t = \frac{M_{t-1}(1+r_m)}{P_t H_{t-1}}. \quad (4)$$

Conditional on house liquidation, the investor will default if his loan-to-value ratio is sufficiently high:

$$-\kappa < 1 - \bar{l}_t - \phi. \quad (5)$$

The right-hand side of the inequality is the proceeds upon house sale after paying off mortgage debt and selling costs. The left-hand side, on the other hand, is the cost of default. Upon liquidating his house, a homeowner faces the same decision as a renter.

If a homeowner does not have to move for exogenous reasons, he needs to decide whether to stay in the same house or liquidate the house and move endogenously. If he chooses to stay in the same house, he has the option to convert illiquid home equity to liquid financial assets through a "cash-out" refinancing. However, refinancing entails a closing cost (ξ) as a percentage of the house value. The mortgage balance after refinancing also needs to satisfy the collateral constraint given by equation (3). If a homeowner chooses not to refinance, he is required to pay down his mortgage balance according to the mortgage amortization schedule:

$$\begin{aligned} M_t &= \frac{1}{1+\pi} \left[M_{t-1}(1+r_m^N) - \frac{M_{t-1}}{\sum_{j=t}^T (1+r_m^N)^{t-j-1}} \right] \\ &= M_{t-1}(1+r_m) \left[1 - \frac{r_m^N/(1+r_m^N)}{1-(1+r_m^N)^{t-T-1}} \right]. \end{aligned} \quad (6)$$

If a homeowner cannot make the scheduled principal and interest payments, then he has to either refinance his mortgage or liquidate his house.

We assume that the real gross return on housing assets (\tilde{R}_t^H) follows a stochastic (binomial) process, which can be correlated with stock returns and pre-retirement labor-income growth rate. The investor can invest in two financial assets: a riskless bond (B_t) and a risky stock (S_t). If the investor has not entered the stock market previously, he needs to pay a one-time entry cost in order to participate in the equity market. The cost is assumed to be a fraction (η) of the investor's current permanent income, P_t^Y , reflecting the opportunity cost in acquiring information about stock

investments.⁸ Following Vissing-Jørgensen (2004), we assume that the investor also faces a per period maintenance cost to invest in the stock market for the current period, which is assumed to be a fraction (ϱ) of the investor’s permanent income. The real gross return on the riskless bond is denoted $R_f = 1 + r_f$ and is assumed to be constant over time. The real gross return on the risky stock is denoted \tilde{R}_t^S and is assumed to follow a stochastic (binomial) process that can be contemporaneously correlated with labor income shocks and the house appreciation return. Borrowing is allowed only through a mortgage by collateralizing one’s house. Short sale of stock is prohibited.

The investor derives utility from consuming a numeraire good (C_t) and housing services (H_t). Intratemporal utility is assumed to be of Cobb-Douglas form, modified to allow for demographic effects such as family size. The investor’s intertemporal preference is characterized by the recursive Epstein-Zin utility function (Epstein and Zin (1989)) with γ and ζ denoting the curvature parameter and the intertemporal elasticity of substitution, respectively. The investor also has a bequest motive represented by a function of bequeathed wealth net of house liquidation cost and the unit price of housing services.

The investor maximizes his recursive lifetime utility of numeraire good consumption, housing services, and bequest, subject to the intertemporal budget constraint given the initial asset holdings, non-financial income endowment, and previous stock market participation status. To reduce the dimension of the state space, we normalize all continuous variables by the investor’s spendable resources including labor income (denoted as Q_t) or house value ($P_t^H H_t$).⁹ As a result of this normalization, the investor’s optimization problem involves the following choice variables: the stock market entry choice, D_t^e ; the stock market participation choice, D_t^p ; the house tenure choice, D_t^o ; the mortgage refinancing choice, D_t^r ; and the house liquidation decision through selling, D_t^s ; or default, D_t^d ; the numeraire good consumption–wealth ratio, $c_t = C_t/Q_t$; the house value–wealth ratio, $h_t = P_t^H H_t/Q_t$; the mortgage loan–to–value ratio after housing and mortgage choices, $l_t = M_t/P_t^H H_t$; the fraction of wealth allocated to bond, $b_t = B_t/Q_t$; the fraction of wealth allocated to stock, $s_t = S_t/Q_t$. The relevant state variables for the normalized optimization problem are stock market entry status dummy, D_{t-1}^e ; home ownership status dummy, D_{t-1}^o ; moving shock dummy, D_t^m ; beginning–of–period

⁸The proportionality assumption of stock market entry cost with respect to permanent income greatly simplifies the numerical solution. A similar assumption is made in Haliassos and Michaelides (2003) and Gomes and Michaelides (2005).

⁹We call Q_t “wealth” for the ease of discussion. The exact expression of the investor’s spendable resources dependent on his housing and mortgage positions is provided in the appendix.

total wealth–labor–income ratio, $q_t = Q_t/P_t^Y$; beginning-of-period house value–total wealth ratio, $\bar{h}_t = P_t^H H_{t-1}/Q_t$; and beginning-of-period mortgage loan–to–value ratio, $\bar{l}_t = M_{t-1}(1+r_m)/P_t^H H_{t-1}$.

4. Numerical Results

We use the following parameter values for our baseline analysis. The investor is assumed to make decisions annually starting at age 20 ($t = 0$) and lives at most for another 60 years ($T = 80$). The annual mortality rate is calibrated to the 1998 life table for the total U.S. population from the National Center for Health Statistics (Anderson (2001)). We use similar labor income and security returns processes as in the portfolio choice literature. Specifically, we assume that the investor retires at age 65. After retirement, the investor receives 68 percent of his labor income at age 64. This replacement ratio, as well as the age–dependent deterministic labor income growth rate before retirement, $f(t)$, is based on the empirical estimation of Cocco, Gomes and Maenhout (2005) by fitting a third–order polynomial to the labor income of high school graduates using the PSID data. Standard deviations of permanent and temporary shocks to labor income are set at 10 percent and 25 percent, respectively. The real risk-free rate is set at 2 percent, while the real stock return is assumed to have a mean of 6 percent, and a standard deviation of 18 percent, as in Gomes and Michaelides (2005). The deterministic rate of inflation is set at 3 percent, consistent with its historical average. We also assume that the real mortgage borrowing rate (r_m) is the same as the lending rate that the investor can earn on holding bond (r_f) under the baseline case.¹⁰

We set rental costs and housing maintenance costs at 6 percent and 1.5 percent of house value, respectively, as in Yao and Zhang (2005). We assume a mean of 0 for real house price appreciations, and a standard deviation of 11.5 percent, the latter an estimate based on individual residential houses in PSID by Cocco (2005). We set the one–time market entry cost at 10 percent, as in Haliassos and Michaelides (2003), and set the per period stock investment maintenance cost at 2 percent of an investor’s permanent income. Refinancing charge is set at 0.8 percent of house value, or 1 percent of mortgage balance for a loan with 20 percent down payment, within the range of the empirical

¹⁰In doing so, we eliminate the financial incentives of refinancing to take advantage of lower market mortgage rates, and focus instead exclusively on the consumption smoothing and portfolio allocation–driven reasons of refinancing. Also under the assumption of equal lending and borrowing rates and costly refinancing, it is optimal for the investor to borrow the maximum allowed loan amount upon mortgage initiation and pay the minimum amount required by the amortization schedule to reduce the chance of triggering refinancing cost and increase the option value of defaulting. Hence, the mortgage decision is reduced to a binary choice of whether or not to refinance. The effect of a positive spread between mortgage rate and riskfree rate is investigated in subsection 4.4.

estimate (Benett, Peach, and Peristiani (2001)). House purchase cost is set at 3 percent to take into account the additional search costs. The default penalty is set at 15 percent of house value so that the model generates overall default rate consistent with its empirical counterpart for residential mortgages (see Deng, Quigley, and Van Order (2000)).

We set the correlation between the permanent component of labor income and housing returns at 0.2, and the correlation between equity and housing returns at 0.0, as in Yao and Zhang (2005). The exogenous moving shock is calibrated to the average annual migration rate between March 2000 to March 2001 for non-housing related reasons, as reported in the Current Population Survey by the U.S. Census Bureau (2003). For preference parameters, we set the investor’s risk aversion curvature parameter at 5 as in Yao and Zhang (2005) and Cocco (2005). Time discount rate, elasticity of intertemporal substitution, and number of beneficiaries are set at $\beta = 0.93$, $\zeta = 0.5$, and $L = 8$, respectively, to generate life-cycle wealth and home ownership profiles consistent with households’ behavior in the PSID. A complete list of baseline parameter values are summarized in Table 2.

Under the assumption of costless stock market participation and zero mortgage refinancing cost, Yao and Zhang (2005) provide a detailed analysis of the optimal housing and portfolio choices for an investor who enters the current period as a renter. Conditional on stock market participation, a renter’s optimal consumption, housing tenure choice, housing expenditure, and asset allocations under the current setup are qualitatively similar to the results in Yao and Zhang (2005). In general, the renter chooses to become a homeowner when his wealth–income ratio is sufficiently high. The investor’s optimal consumption–wealth ratio and house value–wealth ratio are both decreasing in wealth–income ratio, consistent with permanent income hypothesis. The renter’s optimal stockholdings in both liquid wealth and total wealth are decreasing functions of his wealth–labor income ratio, reflecting the role of future earnings as a close substitute for explicit bond holdings. Moreover, when indifferent between renting and owning, the homeowner holds less stock as a proportion of total wealth—reflecting the substitution effect—yet a higher equity proportion in his liquid wealth, reflecting the diversification benefit afforded by low correlation between housing and equity returns.

The presence of stock market participation costs further divides renters into stockholders and non-stockholders. To demonstrate how home ownership and stock market participation decisions interact, we plot the joint home ownership and stock ownership decisions as a function of the wealth–income ratio (Q_t/P_t^Y) and age in the top panel of figure 1 for an investor who does not own a house in the previous time period and has no previous stock market participation experience. To own both a

house and stock requires higher wealth relative to permanent income than to own either one alone. Except at very young ages, it takes less wealth to trigger home ownership than stock ownership. At very young ages, the investor chooses to become a stock owner before becoming a home owner. The shape of the home and stock ownership trigger bounds is primarily driven by the investor's life-cycle earnings, mobility likelihood and bequest motive.

A young investor faces a steep earnings profile and therefore desires a large house relative to his current permanent income P_t^Y . Because of the collateral borrowing constraint in the housing market, a larger house requires more wealth on hand to finance the housing down payment. A young investor also faces high exogenous mobility risk, leading to higher likelihood of triggering house liquidation cost, which reduces the incentive to own a house and increases the liquid wealth on hand necessary to induce home ownership. Yet a young investor can benefit more from paying the one-time stock market entry cost since he has a longer investment horizon to recover the initial entry cost. A young investor also invests a higher proportion of liquid wealth in equity, due to the substitution effect of life-time human capital for explicit bond holdings. By delaying home purchase, a young investor with a moderate amount of wealth can potentially earn higher average return and afford a house of desired size in the future.

From middle age onward, the stock ownership boundary increases monotonically, reflecting the reduction in investment horizon left to recoup one-time entry cost. The home ownership trigger bound, however, first becomes lower for a middle-aged investor as his income increases and exogenous mobility risk decreases. Starting at this stage of the life cycle, home ownership surpasses stock ownership as a primary objective for a household. Immediately after retirement, the uncertainty about future non-financial income is resolved, which lowers the home ownership trigger bound. Yet as an investor approaches his terminal date, he is more concerned with leaving a bequest with minimum liquidation cost. Therefore, at this stage of his life he is less likely to initiate home ownership and the wealth-permanent income trigger bound for home ownership increases sharply.¹¹

In our discussion below, we first investigate a homeowner's optimal discrete house liquidation, mortgage refinancing and default, and stock market participation decisions (figure 1b). We then focus on a homeowner's optimal asset allocation decision when he chooses to stay in the existing

¹¹This result however does not imply that all investors will revert to renting before death. The finding here says that a renter or a homeowner upon house liquidation will not initiate new home ownership at advanced ages. If an investor owns his house, he may optimally choose to stay in the existing home and does not have to liquidate.

house without refinancing his mortgage,¹² conditional on stock market entry (figure 2 to figure 4). We present the simulation results under the baseline parameter, a borrowing and lending rate spread, and comparative static analysis at the end of this section.

4.1. Optimal Housing, Mortgage, and Stock Market Participation Decisions

In the bottom panel of figure 1, we plot the discrete housing, mortgage, and stock market participation decisions as a function of beginning-of-period loan-to-value ratio (LTV) and house value-wealth ratio for a homeowner who has entered the stock market previously.¹³ For housing and mortgage decisions, the presence of collateral constraints, house liquidation cost (cost of selling or defaulting), and mortgage refinancing charge divides the state space into five distinct regions of actions: (1) the non-admissible region (N.A.)—the investor cannot be in this region, (2) the staying region (Stay)—the investor stays in the existing house without refinancing, (3) the refinancing region (Refi)—the investor stays in the existing house and cashes out a fraction of his home equity, (4) the selling region (Sell)—the investor sells his existing house, and (5) the default region (Default)—the investor defaults on his mortgage payments and the house is foreclosed by the lender.

The non-admissible region is a convex set on the upper left corner, with boundary values of incoming loan-to-value ratio and house value-wealth ratio defined by $(1 - \bar{l}_t - \phi)P_t^H H_{t-1}/Q_t = 1$. The investor is restricted from taking unsecured debt, hence the size of his home equity cannot exceed total wealth. Intuitively, in order to finance a large house relative to one's wealth, the investor has to carry a high mortgage balance.

A homeowner experiencing an exogenous moving shock ($D_t^m = 1$) has to liquidate his house by either selling or defaulting. When a homeowner's mortgage loan-to-value ratio is sufficiently high, i.e., when $\bar{l} > 1 + \kappa - \phi$, he prefers defaulting on his mortgage to selling the house. This criterion is shown in the figure as the dotted line.

When not moving for exogenous reasons, the homeowner stays in the existing house if his house value-wealth ratio does not deviate far from the optimal house value-wealth ratio that a renter with identical wealth-income ratio would have chosen. The house selling boundary is also significantly affected by an investor's mortgage balance. Before the investor's loan-to-value ratio reaches the

¹²Under the assumption that refinancing cost is proportional to house value—thus independent of the mortgage balance before and after mortgage refinancing—the optimal decision rules conditional on refinancing are invariant to the investor's beginning-of-period loan-to-value ratio.

¹³The investor's wealth-labor-income ratio is set at $Q_t/Y_t = 1.75$ and age is set at 50 in figure 1.

conditional default trigger $\bar{l}_t = 1 + \kappa - \phi$, the lower bound for selling decreases, while the upper bound increases in the mortgage loan-to-value ratio, leading to a wider range of tolerable house sizes at a higher mortgage balance. This result reflects the tradeoff between consuming the optimal amount of housing services and maintaining liquidity. Selling a house with a high loan-to-value ratio reduces liquidity because the mortgage balance is due upon home sale. This is particularly important for an investor holding a negative home equity position, who has to inject liquid financial wealth to pay off mortgage balance and brokerage fees not covered by the home sale proceeds. Unlike a margin loan in a brokerage account, a mortgage contract has no maintenance margin requirement. The liquidity provided by a large mortgage balance has a long-lasting effect in providing liquidity for consumption and stock investment.

In the absence of exogenous moving shock, the investor may choose to stay in the existing house even when his mortgage loan-to-value ratio exceeds the conditional default threshold by a large margin. Intuitively, default can be thought of as an American put option. The investor does not have to default as soon as the put option is in-the-money, i.e., $\bar{l}_t > 1 + \kappa - \phi$, since there might still be enough time value left in the option. The exact exercise boundary level of the loan-to-value ratio for mortgage default is also affected by the investor's housing position and is lower when the investor's relative house size deviates far away from its optimal level. This reflects the investor's desire to adjust his housing services, which strengthens his incentive to default strategically.¹⁴

If a homeowner decides to stay in the existing house, he can convert a fraction of illiquid home equity to liquid financial assets through mortgage refinancing when his mortgage loan-to-value ratio is too low. Since there is no interest rate uncertainty or spread between borrowing and lending rates in our baseline specification, a homeowner refinances entirely for consumption smoothing and/or portfolio rebalancing reasons. Further, because refinancing cost is independent of the size of home equity cashed out, the investor optimally chooses not to refinance immediately after his home equity exceeds 20 percent or the loan-to-value ratio falls below 80 percent. In fact, he often waits until his home equity position becomes very sizable before he refinances his mortgage.

Panel (b) in Figure 1 also illustrates how a homeowner's stock market participation decision interacts with his housing and mortgage positions. When staying in the existing house without

¹⁴In an unreported exercise, we find that when mortgage default results in an exclusion from future access to credit market, the effect of deviations from optimal house size on default become asymmetric. While the trigger level of the loan-to-value ratio for default is still lowered for a high house value-wealth ratio, it becomes higher for a low house value-wealth ratio. In the latter case, the benefit of defaulting (write-off of negative home equity), measured as a fraction of the investor's wealth, is small while the cost in losing future access to the credit market is large.

refinancing, a homeowner is likely to initiate stock ownership (“Stay w/ Stocks”) when his house value–wealth ratio is low or when his mortgage loan–to–value ratio is high. Intuitively, at a given level of wealth and labor income, the investor with a high mortgage balance or low house value has more wealth in liquid form and less in illiquid home equity. He is thus able to invest a large amount in stock to offset the stock market participation cost. Our findings are consistent with the empirical results discussed in section 2, which indicate that the mortgage–networth ratio has a significant positive effect while the home value–networth ratio has a significant negative effect on homeowners’ stock participation decisions. For the same reason, within the “Refi” region, stock ownership is initiated when the house value–wealth ratio is low.¹⁵

4.2. Optimal Asset Allocation Decisions

We now turn to the homeowner’s asset allocation decisions. In a complete market economy with unrestricted borrowing, Grossman and Laroque (1990) and Damgaard, Fuglsbjerg, and Munk (2003) demonstrate that the investor’s stockholdings as a fraction of wealth is U-shaped in his house value–wealth ratio. Intuitively, around the optimal house value–wealth ratio, the investor takes a more conservative equity position to reduce the need to trigger house liquidation. When close to the adjustment boundaries, he takes a riskier position due to option–like payoff around liquidation trigger bounds. The differences between our portfolio rules and those of earlier research, therefore, reflect the effects of borrowing constraints and incomplete market (inability to capitalize future labor income).

Figure 2 shows the fraction of an investor’s wealth in liquid financial assets ($(S_t + B_t) / (S_t + B_t + P_t^H H_t - M_t)$) as a function of the beginning–of–period house value–wealth ratio and mortgage loan–to–value ratio.¹⁶ The top panel shows that within the no-adjustment region the proportion of the investor’s liquid assets in his wealth varies substantially with his house value–wealth ratio and loan–to–value ratio. Specifically, when the loan–to–value ratio is less than 100 percent (the investor has a positive

¹⁵The decisions to stay, sell, default, refinance, and enter the stock market are also affected by the investor’s wealth–income ratio (figures not shown). At a given loan–to–value ratio, both the upper and the lower trigger bounds of the house value–wealth ratio decrease as the investor’s wealth–income ratio increases. This is consistent with the behavior that the investor spends a smaller proportion of his wealth on housing if his future labor income accounts for a smaller proportion of his lifetime wealth. At a given house value–wealth ratio, the trigger bound of the loan–to–value ratio for refinancing is higher for lower wealth–income ratio, indicating the greater need for liquidity. The trigger bound of the loan–to–value ratio for defaulting is lower for higher wealth–income ratio, reflecting the reduced marginal benefit of preserving liquidity when realized wealth dominates unrealized future labor income. Also, at a given house value–wealth ratio and mortgage loan–to–value ratio, a high wealth–income ratio increases the amount of liquid wealth relative to permanent income and encourages stock market participation.

¹⁶For figure 2 to figure 4, the investor’s wealth worth–labor–income ratio is fixed at $Q_t/P_t^Y = 7.0$, and his age at 50. The investor’s house value–wealth ratio is fixed at $P_t^H H_{t-1}/Q_t = 0.8$ for panel b of these figures.

home equity), as the house value–wealth ratio increases a larger fraction of the investor’s wealth is tied to illiquid home equity and less is available for investment in liquid financial assets. When the loan–to–value ratio is greater than 100 percent, a larger house is associated with more liquid financial assets because the investor has negative home equity. The bottom panel shows that within the “Stay” region for a given house value–wealth ratio, the investment in liquid financial assets increases in the investor’s loan–to–value ratio. However, upon refinancing a fraction of illiquid home equity is converted to liquid financial assets, leading to a higher liquid financial assets–total wealth ratio. It is worth noticing that an investor can have more than 100 percent of his networth in liquid form when he holds a negative home equity position.

Figures 3 and 4 show the investor’s stock investment as a fraction of his networth ($S_t/S_t+B_t+P_t^H H_t-M_t$) and as a fraction of his liquid assets (S_t/S_t+B_t), respectively, as a function of the beginning–of–period house value–wealth ratio and mortgage loan–to–value ratio. The investor’s asset allocation choice reflects the desire to hold an optimal overall portfolio conditional on his existing housing and mortgage positions. Specifically, at very low levels of the loan–to–value ratio or very high levels of house value–wealth ratio, most of the investor’s wealth is in illiquid home equity and his liquid wealth is small. He thus holds an all-equity liquid portfolio, yet is still underweighted in stock. Since the investor can not borrow against his liquid assets, the homeowner’s total equity exposure is limited by his liquid wealth. His networth equity proportion declines as his mortgage loan–to–value ratio decreases and/or as his house value–networth ratio increases.¹⁷ When the exposure to equity risk in his wealth becomes too low, the investor optimally triggers the refinancing and converts a portion of his home equity into liquid wealth to increase his exposure to equity risk.

At higher levels of loan–to–value ratio or lower levels of house value–wealth ratio, the investor holds an interior mix for his liquid asset portfolio. His networth equity proportion is relatively flat or slightly U-shaped in the house value–wealth ratio, similar to the findings in Grossman and Laroque (1990) and Damgaard, Fuglsbjerg, and Munk (2003). Given the house value–wealth ratio, while the investor’s networth equity proportion is still increasing in his loan–to–value ratio, it increases at a slower rate. These reflect the fact that when the loan–to–value ratio is high, the investor’s desired equity exposure can be financed by his liquid wealth on hand. Therefore the homeowner can achieve an equity exposure that is close to the unconstrained optimal level. At a given house value–

¹⁷The effect of a large home equity position, which causes an all-equity liquid portfolio, is similar to that of high future income relative to wealth on hand. At a low level of wealth–income ratio, the entire liquid assets are invested in stock, irrespective of one’s housing or mortgage positions, and the investor’s networth equity proportion is identical to the fraction of liquid assets in total wealth (figures not shown).

wealth ratio, investors with higher mortgage leverage ratio are less likely to face binding borrowing constraints in the future, which explains the slightly higher overall equity exposure.

When not holding an all-equity liquid portfolio, the investor's liquid asset equity proportion can be increasing, U-shaped, or decreasing in his house value-wealth ratio, depending on the loan-to-value ratio. At low levels of the loan-to-value ratio, the investor allocates a larger fraction of his liquid wealth to stock at a higher house value-wealth ratio. They do so to maintain the optimal equity exposure in the overall portfolio while their liquid wealth becomes smaller. At high levels of the loan-to-value ratio (greater than 100 percent), the liquid asset equity proportion decreases in the house value-wealth ratio, since a negative home equity position increases the relative size of liquid portfolio. In between, when almost the entire housing asset is mortgage-financed, the equity proportion in liquid assets is similar to that in networth and exhibits a slight U-shape in the house value-wealth ratio. Furthermore, at a given house value-wealth ratio, the investor's liquid asset equity proportion decreases in his loan-to-value ratio, reflecting a larger fraction of liquid assets in the investor's wealth at higher loan-to-value ratio.

Interestingly, at very high levels of the loan-to-value ratio—when the ratio is close to the default boundary—the investor's liquid asset equity proportion is relatively flat in his loan-to-value ratio. Since the size of his liquid portfolio becomes larger as the loan-to-value ratio increases, this leads to once again a quickly increasing networth equity proportion. This captures the effect of option-like payoffs around the default boundary, which encourages risk-taking behavior.

In summary, our numerical analysis suggests that an investor desires to have the optimal overall exposure to stocks, given his housing position. The objective is achieved by adjusting the availability of liquid asset (through refinancing) and/or the composition of his liquid asset portfolio.

Existing empirical studies document that the house value has a negative effect on households' networth equity proportion, while the mortgage balance has a positive effect on the networth equity proportion. These findings are broadly consistent with our numerical results and suggest that asset allocation decisions for a large proportion of households are affected by borrowing constraints. While studies like Heaton and Lucas (2000a) and Cocco (2005) identify a positive correlation between mortgage balance and stockholdings, our analysis establishes a theoretical causal relationship as well as offering a detailed mechanism through which housing wealth and mortgage balance interact with stock investment.

As for the lack of robust empirical conclusion on the effects of households' housing and mortgage positions on liquid asset equity proportions, a simple reduced form specification may have difficulty capturing the interactions between asset allocations and housing and mortgage positions, given the complexity of the predicted theoretical relations. The presence of sample selection and unobserved heterogeneity in data further complicates the empirical estimation.¹⁸

4.3. Simulation Analysis

We now examine the investor's life-cycle housing and asset allocation choices using a simulation analysis. We first simulate permanent and transitory shocks to labor income, exogenous moving shocks, and stock and housing returns based on a serially uncorrelated Markov process with two outcomes for each variable. We then use the optimal decision rules from our state-space solution to calculate the investor's optimal consumption, housing, mortgage, and portfolio choices. Home ownership and stock market entry status, wealth-labor income ratio, house value-wealth ratio, and loan-to-value ratio are updated each period to determine the investor's optimal decisions for the next period. The time series profiles of the optimal decisions are generated by repeating the calculation from $t = 0$ (age 20) to $t = 60$ (age 80) for 50,000 simulation paths.

Figure 5 shows the home ownership rate (panel a), refinancing rate (panel b), default rate (panel c), and the average loan-to-value ratio for refinanced and defaulted mortgages (panel d). Consistent with empirical evidence, the percentage of home ownership is hump-shaped in age. It rapidly increases from zero to over 50 percent at the investor's early 30s. By early 40s, almost all investors own a house. After age 76, the percentage of home ownership decreases, reflecting the fact that some investors sell their houses in anticipation of bequeathing their wealth to their beneficiaries. While the simulated life-cycle home ownership rate resembles the overall pattern of the observed home ownership rate, it is higher than its empirical counterpart. The increase in home ownership rate among young households is much faster than it is in the data.¹⁹

¹⁸If the sample selection effect is not adequately controlled, estimated regression coefficients in the equity proportion equations are likely to inherit signs in the stock market participation equation. This biases against finding a positive (negative) effect for home value-networth ratio (mortgage-networth ratio) on liquid asset equity proportion, while reinforces the negative (positive) effect of home value-networth ratio (mortgage-networth ratio) on networth equity proportion. We thank Annette Vissing-Jørgensen for this comment.

¹⁹Heterogeneity in home ownership benefits among investors in reality can lead to some investors being lifetime renters. For example, rent controls in New York City provide an incentive for some residents to remain as renters throughout their life time. Tax rates also vary across different wealth and income brackets. Young and low-income investors with lower marginal tax rates have less incentive to become a homeowner since the benefit of mortgage interest deductability is small. These features are absent from our model.

The refinancing rate reaches the peak of about 2.6 percent for investors in early 30s. With costly refinancing, an investor who has just purchased a house is less likely to refinance because he has not accumulated enough home equity through mortgage payment and/or house price appreciation. Investors in their mid-to-advanced ages are also less likely to refinance because of their large liquid asset accumulation, which allows the investors to smooth consumption and hold the desired amount of stock investment without incurring a refinancing charge. Young homeowners face steep income profiles and are more likely to be liquidity-constrained upon a bad income shock due to large mortgage payment obligations and a low level of liquid wealth. They also have high desired equity exposure (because of their high present value of labor income) yet are limited by the amount of liquid asset on hand for stock investment. Hence they are more likely to refinance to smooth intertemporal consumption and/or to increase stockholdings when their houses appreciate in value.

The default rate peaks at around 1.6 percent in the investor's early 40s and goes down to zero in their early 70s. New homeowners will not default because of the mortgage down payment provides protection for the lender's interest in the property. The desire to maintain liquidity also reduces the incentive to exercise the default option. The default rate is lower for a senior homeowner since the investor on average reduces his mortgage balance over time with principal amortization. Overall, the predicted (low) level of default rate is consistent with the observed rate reported in Deng, Quigley, and Van Order (2000) for residential mortgages.

The average mortgage balance relative to house value for refinanced mortgages decreases from about 65 percent at the investor's late 20s to about 35 percent at late 50s. Investors cash out the excess illiquid home equity only when their loan-to-value ratio falls well below the collateral requirement (their home equity substantially exceeds minimum home equity requirement of 20 percent) due to refinancing cost. The downward drift in the refinancing trigger bound in the loan-to-value ratio reflects liquid wealth accumulation over time so that the investor does not need to tap into his home equity as often for consumption smoothing and stock investment. The mortgage balance relative to the house value for defaulted mortgage averages around 120 percent, significantly higher than the conditional default boundary ($\bar{l}_t = 1 + \kappa - \phi = 109\%$), reflecting the time value of the option to default, and the cost of lost liquidity associated with exercising the default option.

Figure 6 shows the average numeraire good consumption and income (panel a), house value-income ratio and wealth-income ratio (panel b), loan-to-value ratio (panel c), and home equity-income ratio and liquid wealth-income ratio (panel d) across different ages in the life-cycle. Before

retirement, the numeraire good consumption tracks the shape of labor income. It exhibits a hump shape and peaks in the investor's 50s. The decline accelerates after retirement when the investor stops receiving labor income. Both the average house value–income ratio and the wealth–income ratio for homeowners are also hump-shaped and peaking just at retirement. Young investors accumulate wealth slowly while renting until early 30s, at which point some of them purchase their first home. For homeowners, the accumulated wealth increases steadily from early 30s until retirement. Investors experiencing bad income shocks have little wealth on hand and keep renting. After retirement, homeowners' wealth decreases as they draw down their wealth accumulated during working years to supplement retirement income to pay for housing and numeraire good consumption expenses. The house value–income ratio for homeowners starts at around 3 and reaches the peak value of 4 around retirement, and declines until the investor reaches the terminal date thereafter.

As the investor ages, he gradually accumulates home equity by making scheduled mortgage principal and interest payments. The average mortgage balance relative to house value starts at 80 percent at the investor's late 20s and declines to zero at age 80. Overall the simulated mortgage loan–value ratio decreases more slowly than what is observed empirically. One possible explanation is that the benefit of paying down mortgages is smaller in the baseline model without a positive spread between lending and mortgage borrowing rates.

Consistent with the behavior of the loan–value ratio, the average home equity–income ratio gradually increases. The homeowners' liquid financial wealth–income ratio tracks the wealth–income ratio closely. It increases faster than the home equity before retirement. In retirement stage, however, homeowners draw down liquid savings first to defer mortgage refinancing and home selling costs. Home equity is accessed only as a last resort.

Figure 7 shows the fraction of investors owning stock (panel a), the average networth equity proportion (panel b), the average liquid asset equity proportion (panel c), and the average amount of stockholding (panel d). Young investors in their early 20s are deterred by stock market entry and maintenance costs because of insufficient liquid wealth. Stock ownership begins in the investor's mid-20s as renters accumulate wealth by saving a fraction of their income for precautionary reasons as well as future house down payments. The transition to stock ownership then slows down temporarily in the late 20s and early 30s as some investors become homeowners and convert a significant fraction of liquid wealth to illiquid home equity.

As investors age and accumulate more liquid financial wealth, the stock market participation rate quickly increases, reflecting the large lifetime benefit of stock market participation. It reaches 99 percent at age 53, and peaks at close to 100 percent around retirement. After age 70, the stock ownership rate starts to decline. By the terminal date, 34 percent of investors have exited from the stock ownership market, an indication of the investor's desire to hold safer financial assets and to avoid per period maintenance cost associated with stock investment. While the predicted peak level of stock ownership rate (close to full ownership) is higher than the observed counterpart, our model still offers a significant improvement over existing studies in generating a lower stock market participation rate as well as exits from stock market.

The homeowner's equity proportion in networth is hump-shaped. It starts at around 65 percent in investors' late 20s and peaks just below 80 percent between ages 40 and 50. It then gradually declines to about 26 percent at age 80. A homeowner's liquid wealth is entirely concentrated in stock until his early 40s and declines gradually afterwards. Overall, the behavior of the homeowner's equity proportion reflects the changing patterns of human capital and home equity over life-cycle. Young homeowners have a large fraction of wealth in home equity, which reduces their liquid wealth. Yet they also have a high present value of lifetime nonfinancial income that serves as a close substitute for bond. Therefore young homeowners concentrate their liquid asset investment in stock. As they grow older and accumulate more liquid wealth, both the present value of remaining labor income and the proportion of total wealth held in illiquid home equity are reduced, leading to declining equity proportions in both the networth and liquid wealth.

Both the average liquid wealth equity proportion and average networth equity proportion are higher than their observed counterparts in the SCF data. Empirically, the homeowners' liquid wealth equity proportion is rather flat in age, and only declines slightly from the peak around 47 percent between age 45 to 50 to about 40 percent for the age group 75 and beyond. For the networth equity proportion, except for the youngest age group, the observed average stock investment in networth stays within a narrow range between 19 percent and 23 percent with the peak reached between ages 45 and 55. The predicted decline after retirement is much more dramatic than observed in the data.

Panel (d) shows that the average dollar amount of stock investment increases steadily until the investor reaches retirement. It then declines gradually as liquid assets are drawn down to supplement retirement income and the investor becomes more conservative in his liquid wealth portfolio.

This pattern is consistent with the observed investors' behavior in the SCF data. Empirically the stockholding peaks slightly later in life, between ages 65 and 75.

4.4. Borrowing and Lending Rate Spread

The above analysis is based on the assumption that the borrowing rate paid by investors on their mortgage balance is the same as the lending rate received from their bond holding. Given the potential for default and institutional costs in loan underwriting and monitoring, mortgage lenders may require a positive spread between the borrowing rate and the lending rate as a compensation. We now examine the effects when the borrowing rate is explicitly set to be 1.5 percent above the lending rate.²⁰

Figure 8 shows the simulated home ownership rate and average loan-to-value ratio (panel a), stock ownership rate (panel b), and the average equity proportion in liquid financial assets (panel c) and networth (panel d) among stockholders in the presence of a positive credit spread. Overall, the life-cycle home ownership pattern resembles that in the baseline case. However, investors have a much stronger incentive to pay down mortgage balance with a positive spread than in the baseline case. While the mean loan-to-value ratio stays at around 80 percent from age 30 to age 40 in the baseline model, it decreases drastically from 80 percent to 40 percent in the model with a spread. While the mean loan-to-value ratio goes down to about 50 percent by age 70 in the baseline model, it is reduced to 1 percent with a positive credit spread, reflecting the incentives to pay down debt when borrowing cost is higher.

Investors' stock ownership is significantly reduced when facing a higher borrowing rate. The stock market entry begins in the investor's late 20s, and then grows steadily at about 2.5 percent per year until peaking at slightly above 85 percent around retirement. It then drops to 62 percent at the terminal date. Consistent with the rapidly declining loan-to-value ratio over the life-cycle, a large fraction of the investor's networth is saved in home equity, reducing the amount of liquid wealth for stock investment. The average stock ownership across all age groups is about 50 percent, similar to that reported in 2001 SCF data. The rapidly increasing home equity position also leads to a declining homeowner's networth equity proportion over his lifetime. This is in contrast to the baseline case in which the equity proportion in networth exhibits a slight hump-shape. The diversification benefit

²⁰To maintain similar home ownership incentives, we also raise the rental cost by the same amount, i.e. set α to 7.5percent. Other parameter values are held the same as the baseline model.

associated with a large home equity position allows the investor to concentrate his liquid asset in stockholdings relative to the baseline case.

4.5. Comparative Static Analysis

In this section we provide comparative static analysis to demonstrate how different features of the model affect investors' housing and asset allocation decisions. Specifically, we examine five alternative specifications including an expected utility with constant relative risk aversion ($\zeta = 1/\gamma$),²¹ zero per period stock investment cost ($\varrho = 0$), zero mortgage refinancing charges ($\psi = 0.0$), a collateral borrowing constraint that is applicable both at loan initiation and for ongoing loans ($l_t \leq \delta$), and a model without home ownership ($D_t^o = 0$). Our discussions focus on simulated household wealth accumulation, home and stock ownership, and portfolio choices for various model specifications.

Table 3 reports the average household wealth–income ratio before receiving labor income, the percentage of home ownership, and the percentage of stock ownership across different ages for the baseline model and the alternative specifications.²² Compared to the baseline case with Epstein-Zin preferences, investors with time–additive CRRA preferences accumulate more wealth due to lower intertemporal rate of substitution ($\zeta = 0.2$). They are less inclined to become homeowners earlier. Their stock ownership increases much faster and reaches 100 percent much earlier than under the baseline case.

Omitting per period stock investment cost, as in Cocco (2005) and Yao and Zhang (2005), drastically increases stock ownership. It also increases average wealth accumulation as equity investment generates a higher mean return. Interestingly, young investors postpone home ownership slightly relative to the baseline case in order to enter the stock market earlier and to invest in stock more heavily. Furthermore, with only the one–time stock market entry cost, the model cannot generate stock market exit.

Eliminating mortgage refinancing charges reduces the costs to access liquid home equity and increases an investor's incentive to purchase a house early. 74 percent of investors own their homes at age 30, about 20 percent higher than 62 percent in the baseline case. Stock ownership rate is also

²¹When the elasticity of intertemporal substitution is the reciprocal of the risk aversion coefficient, the recursive Epstein-Zin preference is reduced to the time–additive constant relative risk aversion preference.

²²The home ownership rate in the $D_t^o = 0$ model is zero. For this case, we only report household wealth–income ratio and the percentage stock ownership.

higher than the baseline case at young ages. This can be attributed to the larger fraction of wealth in liquid form when investors can costlessly access home equity.

The presence of a minimum home equity requirement, an assumption made in Cocco (2005) and Yao and Zhang (2005), severely discourages home ownership by eliminating mortgage default option and forcing investors to inject liquid wealth to maintain home equity position in a housing market down turn (the collateral constraint becomes a maintenance margin requirement). Young investors wait much longer and accumulate more wealth prior to becoming a homeowner. Relative to the baseline case, stock market entry starts earlier and increases at a faster rate at younger ages as renters invest their liquid wealth accumulated for house down payment. However, stock market participation grows much slower for homeowners, and by the investors' mid-30s, the baseline model surpasses the model with minimum home equity retirement in stock ownership. At age 40, the stock market participation rate is 69 percent with maintenance margin requirement, 9 percent lower than the baseline case.

Finally, when the investor can only rent housing services, as implicitly assumed in models without housing and mortgage decisions, the stock ownership is much higher among young investors. For example, at age 30, more than one third of investors are stockholders, relative to the baseline case of 11 percent. This reflects that a renter's entire wealth is in liquid form and can be used for stock investment.

Table 4 presents the average liquid financial assets–networth ratio, and the average equity proportion in liquid financial assets and in networth for homeowners participating in stock market for the baseline and five alternative models.²³ Under the time-additive CRRA preferences, young homeowners accumulate much more liquid financial assets in networth than in the baseline case. For instance, at age 30, homeowners on average hold 67 percent of networth in liquid financial assets under the CRRA utility preferences compared to 42 percent in the baseline case. The difference is reduced over time as the investor ages. Conditional on participating in the stock market, the investor holds a higher equity proportion in his networth before age 45 but a lower equity proportion afterwards. The equity proportion in liquid financial assets is slightly lower than in the baseline case.

Without per period stock market participation cost, young homeowners accumulate more liquid wealth than in the baseline case. They hold slightly more stock as a fraction of his total wealth to take

²³Since there is no home ownership in the $D_t^o = 0$ model, we only report the networth equity proportions for renters. The equity proportion in liquid financial assets is the same as the equity proportion in networth in this case.

advantage of better investment opportunities in equity than in the baseline case. The conditional equity proportions are quite similar to the baseline case after age 30.

In the absence of mortgage refinancing costs, an investor will always cash out excess equity beyond the minimum requirement. By doing so they increase the fraction of liquid wealth as well as the value of default option. Thus, liquid wealth accounts for a higher proportion of the investor's total wealth than in the baseline case. The smaller illiquid home equity also leads to a higher equity proportion in wealth. The equity proportion in liquid financial assets, however, decreases much faster than in the baseline case as the investor ages, reflecting the larger liquid wealth available for investment in financial assets than under the baseline case.

When the collateral constraint becomes a maintenance margin requirement, young home owners have a slightly larger fraction of total networth in liquid form than in the baseline case. This is because investors buy a smaller home in order to meet future home equity requirements. The fraction of homeowners' networth allocated to equity is also slightly higher at this stage than in the baseline model. However, investors in the baseline model accumulate liquid wealth at a faster rate since they do not need to allocate liquid wealth to satisfy mortgage maintenance margin requirements. This leads to a higher networth equity proportion for most of a homeowner's life time in the baseline model. The equity proportion in liquid financial assets in the baseline model is slightly lower, reflecting lower diversification benefit of home equity for stockholdings.

The impact of home ownership on stock ownership and portfolio choices is best illustrated by contrasting the results with and without home ownership. Without home equity tying up his wealth, the investor holds an all equity portfolio in their networth until age 40 conditional on owning stock, while in the baseline case the equity proportion in networth ranges from 63 percent at age 28 to 77 at age 40. The liquid asset equity proportion in the baseline model is higher than in the case without home ownership. This reflects the diversification benefit from housing assets for stock investment in the presence of home ownership.

Overall, our comparative static analysis highlights two important effects. First, the transition to home ownership affects stock market participation by influencing the investor's wealth accumulation while renting. Second, the illiquidity of home equity has an important effect on portfolio compositions conditional on stock market participation. On one hand, a larger home equity reduces networth equity proportion. On the other hand, it increases equity proportion in the investor's liquid portfolio.

5. Conclusion

We introduce a dynamic life-cycle model to reconcile the documented empirical relations between households' portfolio choices and housing and mortgage decisions. Our model incorporates many realistic features including stochastic labor income process, stock market entry and maintenance costs, and housing market frictions such as collateral borrowing requirement, liquidation cost, refinancing charge, and default penalty.

Our state space analysis on the optimal decision rules reveals that the model can qualitatively explain the documented empirical relations between household asset allocations and their housing and mortgage choices. In our model, given wealth and labor income, the investor with a lower housing value-networth ratio and/or a higher loan-to-value ratio has more liquid wealth. He is thus in a better position to take advantage of opportunities offered by the stock market because the cost associated with stock market participation is outweighed by the benefit to the investor holding a large liquid wealth. Further, the investor with a lower housing value-networth ratio and/or a higher loan-to-value ratio also invests a higher proportion of wealth in stock, because with a large portion of his wealth in liquid assets he can better buffer negative shocks to his wealth and income.

Conversely, the investor with a high housing value-networth ratio or a low loan-to-value ratio has a large portion of his wealth tied to illiquid home equity. He reduces the fraction of networth invested in stock when borrowing on margin accounts or against future labor income is not possible. However, the equity proportion in an investor's liquid wealth increases with the size of his illiquid home equity, reflecting the diversification benefit of home equity for stock. Further, an investor might trigger refinancing charge in order to convert a fraction of his illiquid home equity into liquid form to supplement consumption or to increase equity exposure.

Our simulation analysis further reveals that young homeowners not only have relatively low networth, they also have most of their wealth in illiquid form. This explains the lack of stock market participation and the low networth equity proportion for investors in this age group. However, both the predicted networth equity proportion and liquid wealth equity proportion are still higher than their observed counterparts. While various frictions on the housing market help explaining important empirical relations between stock investment and housing and mortgage decisions, it remains a challenge to generate empirically plausible equity proportions.

6. Appendix: Derivation of the Model and Solution Algorithm

The investor's wealth process and intertemporal budget constraint depend on the investor's home and stock ownership statuses at time $t - 1$ and his housing, mortgage, and stock market entry and participation choices at time t . Let D_t^e take the value of 1 if the investor has already entered the stock market at t or decides to enter for the first time at t , and 0 otherwise, D_t^p take the value of 1 if the investor participates in the stock market at t and 0 otherwise, D_t^o take the value of 1 if the investor owns a house at time t and 0 otherwise, D_t^r take the value of 1 if the investor refinances his mortgage at time t and 0 otherwise, D_t^s take the value of 1 if the investor sells his house at time t and 0 otherwise, and D_t^d take the value of 1 if the investor defaults on his mortgage at time t and 0 otherwise. If the investor is a renter at time $t - 1$, then his spendable resource evolves according to:

$$Q_t = B_{t-1}R_f + S_{t-1}\tilde{R}_t^S + P_{t-1}^Y \exp\{f(t)\}\nu_t\varepsilon_t. \quad (7)$$

If the investor is a homeowner at time $t - 1$, then his spendable resource also includes home equity (net of liquidation cost associated with either selling or default):²⁴

$$Q_t = B_{t-1}R_f + S_{t-1}\tilde{R}_t^S + P_{t-1}^Y \exp\{f(t)\}\nu_t\varepsilon_t + P_{t-1}^H H_{t-1}\tilde{R}_t^H \max\{1 - \bar{l}_t - \phi, -\kappa\}. \quad (8)$$

Let $l_t = M_t/P_t^H H_t$ be the investor's mortgage loan-to-value ratio after housing and mortgage adjustments. The investor's intertemporal budget constraint can then be written as:

1. If the investor is a renter at time $t - 1$ ($D_{t-1}^o = 0$), or if he is a homeowner at time $t - 1$ but decides to liquidate his house ($D_{t-1}^o = 1$ and $D_t^s + D_t^d = 1$), and rent for the current period ($D_t^o = 0$):

$$Q_t = C_t + B_t + S_t + P_t^H H_t \alpha + (1 - D_{t-1}^e) D_t^e \eta P_t^Y + D_t^p \varrho P_t^Y. \quad (9)$$

2. If the investor is a renter at time $t - 1$ ($D_{t-1}^o = 0$), or if he is a homeowner at time $t - 1$ but decides to liquidate his house ($D_{t-1}^o = 1$ and $D_t^s + D_t^d = 1$), and own a house for the current period ($D_t^o = 1$):

$$Q_t = C_t + B_t + S_t + P_t^H H_t (1 - l_t + \psi + \varphi) + (1 - D_{t-1}^e) D_t^e \eta P_t^Y + D_t^p \varrho P_t^Y. \quad (10)$$

²⁴For ease of discussion, we simply refer to Q_t as the investor's wealth.

3. If the investor is a homeowner at time $t - 1$ ($D_{t-1}^o = 1$), and decides to stay in existing house without refinancing his mortgage ($D_t^o = 1$ and $D_t^s = D_t^d = D_t^r = 0$):

$$Q_t = C_t + B_t + S_t + P_t^H H_{t-1} [(\bar{l}_t - l_t + \psi) + \max\{1 - \phi - \bar{l}_t, -\kappa\}] + (1 - D_{t-1}^e) D_t^e \eta P_t^Y + D_t^p \varrho P_t^Y, \quad (11)$$

$$\text{with } l_t \leq \bar{l}_t \left[1 - \frac{r_m^N / (1 + r_m^N)}{1 - (1 + r_m^N)^{t-T-1}} \right].$$

4. If the investor is a homeowner at time $t - 1$ ($D_{t-1}^o = 1$), and decides to stay in existing house and refinance his mortgage ($D_t^o = 1$, $D_t^s = D_t^d = 0$, and $D_t^r = 1$):

$$Q_t = C_t + B_t + S_t + P_t^H H_{t-1} [(\bar{l}_t - l_t + \psi + \xi) + \max\{1 - \phi - \bar{l}_t, -\kappa\}] + (1 - D_{t-1}^e) D_t^e \eta P_t^Y + D_t^p \varrho P_t^Y,$$

with $l_t \leq 1 - \delta$. Since the investor will refinance only when he has more home equity than required by collateral borrowing constraint, i.e. $\bar{l}_t < 1 - \delta$, under the assumption that $\delta - \phi > -\kappa$, we can further simplify the wealth process when refinancing to:

$$Q_t = C_t + B_t + S_t + P_t^H H_{t-1} (1 - l_t + \psi + \xi - \phi) + (1 - D_{t-1}^e) D_t^e \eta P_t^Y + D_t^p \varrho P_t^Y. \quad (12)$$

The vector of sufficient state variables consists of the beginning-of-period stock ownership dummy, home ownership dummy, moving shock dummy, total wealth, permanent labor income, price per unit of housing services, size of the existing house, and outstanding mortgage balance. We represent the vector of state variables by $X_t \equiv \{D_{t-1}^e, D_{t-1}^o, D_t^m, Q_t, P_t^Y, P_t^H, H_{t-1}, M_{t-1}(1 + r_m)\}$.

We assume that the investor has the recursive Epstein–Zin preference (Epstein and Zin (1989)), generalized to incorporate mortality risks, bequest motives, and demographic effects. This specification allows the separation of risk aversion and elasticity of intertemporal substitution. The investor's problem at time t then has the following iterative representation:

$$V_t(X_t) = \max_{A(t)} \left\{ (1 - \beta) \left[\lambda_t u(C_t, H_t; N_t)^{1-1/\zeta} + (1 - \lambda_t) b(Q_t, P_t^H; L)^{1-1/\zeta} \right] + \beta \lambda_t E_t \left[V_{t+1}(X_{t+1})^{1-\gamma} \right]^{\frac{1-1/\zeta}{1-\gamma}} \right\}^{\frac{1}{1-1/\zeta}}, \quad (13)$$

where β is the time discount factor, λ_t is the survival probability at time t conditional on living up to time $t - 1$, ζ is the elasticity of intertemporal substitution, γ is the risk aversion coefficient,

$u(C_t, H_t; N_t)$ is a function which aggregates numeraire good consumption and housing services into utility at time t , and $b(Q_t, P_t^H; L)$ is a bequest function, subject to labor income process (equation 1), mortgage collateral constraint and amortization schedule (equations 3 and 6), borrowing and short sale constraints, and intertemporal budget constraint (equations 7–12), given the initial stock market entry status $D_{(-1)}^e$, home ownership status $D_{(-1)}^o$, exogenous moving shock D_0^m , after labor income wealth (net of house liquidation cost if applicable) Q_0 , permanent income P_0^Y , housing price P_0^H , housing stock $H_{(-1)}$, and mortgage value $M_{(-1)}$. Due to the presence of stock market entry cost and house liquidation cost, time $t - 1$ stock market entry decision (D_{t-1}^e) and housing choices (D_{t-1}^o and H_{t-1}), are also state variables at time t . $A(t) = \{D_t^e, D_t^o, D_t^r, D_t^s, D_t^d, C_t, H_t, l_t, S_t, B_t\}$ is the vector of choice variables at period t .

As to intratemporal preferences, we assume that the investor aggregates housing and numeraire consumptions using the Cobb-Douglas function:

$$u(C_t, H_t; N_t) = \left[N_t \left(\left(\frac{C_t}{N_t} \right)^{1-\omega} \left(\frac{H_t}{N_t} \right)^\omega \right)^{1-1/\zeta} \right]^{\frac{1}{1-1/\zeta}} = N_t^{\frac{1}{\zeta-1}} C_t^{1-\omega} H_t^\omega, \quad (14)$$

where ω measures the relative importance of housing services versus numeraire good consumption, and N_t measures the effective size of the family, capturing the economies of scales in household consumption. We assume that the proceeds from the bequest are equally divided among L beneficiaries and used to pay for the beneficiaries' numeraire good consumption and housing rental costs. Further, the beneficiaries' numeraire good and housing-service expenditures are set at the fixed proportion of $(1-\omega)/\omega$, the optimal proportion when the beneficiaries have the Cobb-Douglas utility function when renting.²⁵ Specifically, the investor's bequest function is represented by:

$$b(Q_t, P_t^H; L) = \left[N_t \left(\frac{Q_t}{N_t} (1-\omega)^{1-\omega} \left(\frac{\omega}{\alpha P_t^H} \right)^\omega \right)^{1-1/\zeta} \right]^{\frac{1}{1-1/\zeta}} = L^{\frac{1}{\zeta-1}} Q_t (1-\omega)^{1-\omega} \left(\frac{\omega}{\alpha P_t^H} \right)^\omega. \quad (15)$$

We simplify the investor's optimization problem by normalizing the investor's continuous consumption and asset allocation choices by his wealth Q_t . Let $c_t = C_t/Q_t$ be the consumption-wealth ratio; $h_t = P_t^H H_t/Q_t$ the house value-wealth ratio; $b_t = B_t/Q_t$ the fraction of the investor's wealth invested in bonds after trading; and $s_t = S_t/Q_t$ the fraction of wealth allocated to stocks after trading. By assuming a Cobb-Douglas utility function, proportional housing purchase, maintenance and liq-

²⁵This specification of bequest function preserves homogeneity needed for normalization to reduce the dimension of the state space.

liquidation costs, proportional mortgage down payment, initiation, refinancing and default costs, and proportional stock market entry and per period maintenance costs, we ensure that the numeraire good consumption, housing services, mortgage, and portfolio rules are independent of the investor's wealth level, Q_t . With the above normalization, the relevant state variables for the investor's problem can be written as $x_t = \{D_{t-1}^e, D_{t-1}^o, D_t^m, q_t, \bar{h}_t, \bar{l}_t\}$, where $q_t = Q_t/P_t^Y$ is the investor's wealth-income ratio, $\bar{h}_t = P_t^H H_{t-1}/Q_t$ is the homeowner's beginning-of-period house value-wealth ratio and $\bar{l}_t = M_{t-1}(1+r_m)/P_t^H H_{t-1}$ is the homeowner's beginning-of-period loan-to-value ratio. Depending on the investor's home ownership status and the realization of the moving shock at the beginning of the period, we can further specify the state and choice variables as follows. If the investor rents his housing services in the previous period ($D_{t-1}^o = 0$), or if he is a homeowner who experiences a moving shock ($D_{t-1}^o = D_t^m = 1$), then:

$$x_t \equiv \{D_{t-1}^e, q_t\} \quad \text{and} \quad a_t \equiv \{D_t^e, D_t^p, D_t^o, c_t, h_t, l_t, s_t, b_t\}.$$

If the investor owns his residence in the previous period and does not have to move for exogenous reasons ($D_{t-1}^o = 1$ and $D_t^m = 0$), then:

$$x_t \equiv \{D_{t-1}^e, q_t, \bar{h}_t, \bar{l}_t\} \quad \text{and} \quad a_t \equiv \{D_t^e, D_t^p, D_t^o, D_t^r, D_t^s, D_t^d, c_t, h_t, l_t, s_t, b_t\}.$$

Define $G_{t+1} = Q_{t+1}/Q_t$ as the real (gross) growth rate of the investor's wealth. If the investor is a renter at time $t - 1$, then:

$$G_{t+1} = b_t R_f + s_t \tilde{R}_{t+1}^S + \exp\{f(t+1)\} \nu_{t+1} \varepsilon_{t+1} / q_t. \quad (16)$$

If he is a homeowner at time $t - 1$, then:

$$G_{t+1} = b_t R_f + s_t \tilde{R}_{t+1}^S + \exp\{f(t+1)\} \nu_{t+1} \varepsilon_{t+1} / q_t + h_t \tilde{R}_{t+1}^H \max\{1 - \bar{l}_{t+1} - \phi, -\kappa\}. \quad (17)$$

Similarly, the budget constraint in equation 9–12 can be normalized as:

$$1 = c_t + b_t + s_t + h_t \alpha + (1 - D_{t-1}^e) D_t^e \eta + D_t^p \varrho, \quad (18)$$

$$1 = c_t + b_t + s_t + h_t (1 - l_t + \psi + \varphi) + (1 - D_{t-1}^e) D_t^e \eta + D_t^p \varrho, \quad (19)$$

$$1 = c_t + b_t + s_t + \bar{h}_t [(\bar{l}_t - l_t + \psi) + \max\{1 - \phi - \bar{l}_t, -\kappa\}] + (1 - D_{t-1}^e) D_t^e \eta + D_t^p \varrho, \quad (20)$$

$$1 = c_t + b_t + s_t + \bar{h}_t (1 - l_t + \psi + \xi - \phi) + (1 - D_{t-1}^e) D_t^e \eta + D_t^p \varrho. \quad (21)$$

Define $v_t(x_t) = V_t(X_t)P_t^{H\omega}/Q_t$ to be the normalized value function. The investor's problem now can be restated as follows:

$$v_t(x_t) = \max_{a(t)} \left\{ (1 - \beta) \left[\lambda_t \left(N_t^{\frac{1}{\zeta-1}} c_t^{1-\omega} h_t^\omega \right)^{1-1/\zeta} + (1 - \lambda_t) \left(L^{\frac{1}{\zeta-1}} (1 - \omega)^{1-\omega} (\omega/\alpha)^\omega \right)^{1-1/\zeta} \right] \right. \\ \left. + \beta \lambda_t E_t \left[\left(v_{t+1}(x_{t+1}) G_{t+1} / \left(\tilde{R}_{t+1}^H \right)^\omega \right)^{1-\gamma} \right]^{\frac{1-1/\zeta}{1-\gamma}} \right\}^{\frac{1}{1-1/\zeta}} \quad (22)$$

s.t.

$$c_t > 0, \quad h_t > 0, \quad l_t \geq 0, \quad b_t \geq 0, \quad s_t \geq 0 \quad (23)$$

and equations 16 to 21.

The above problem can be solved numerically using backward recursion. We first discretize the wealth–labor–income ratio, $q_t = Q_t/P_t^Y$, into a grid of 320 over the interval $[0.1, 100]$ equally spaced in the logarithm of the ratio, the house value–wealth ratio, $\bar{h}_t = P_t^H H_{t-1}/Q_t$, into an equally spaced grid of 160 over the interval $[0, 16]$, and the loan-to-value ratio, $\bar{l}_t = M_{t-1}(1+r_m)/P_t^H H_{t-1}$, into an equally spaced grid of 160 over the interval $[0, 2.0]$, respectively. At the terminal date T , $\lambda_T = 0$, and the investor's value function is a constant:

$$v_T(x_T) = \left\{ (1 - \beta) \left[\left(L^{\frac{1}{\zeta-1}} (1 - \omega)^{1-\omega} \left(\frac{\omega}{\alpha} \right)^\omega \right)^{1-1/\zeta} \right] \right\}^{\frac{1}{1-1/\zeta}} = (1 - \beta)^{\frac{1}{1-1/\zeta}} L^{\frac{1}{\zeta-1}} (1 - \omega)^{1-\omega} \left(\frac{\omega}{\alpha} \right)^\omega \quad (24)$$

at all points in the state space. The value function at date T is then used to solve for the optimal decision rules for all points on the state space grid at date $T - 1$. A three–dimension B-spline interpolation is used to approximate the value function for points that lie between grid points in the state space. The procedure is repeated recursively for each time period until the solution for date $t = 0$ is found.

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Table 1
Summary Statistics for Housing and Portfolio Choices from the Survey of Consumer Finances (SCF), 1989–2001

Variables	Age Groups							Year				
	<25	25-34	35-44	45-54	55-64	65-75	>75	1984	1989	1994	1999	2001
OWNHOUSE	0.193	0.522	0.723	0.803	0.847	0.835	0.777	0.709	0.703	0.711	0.729	0.743
PH/ASSET	0.675	0.657	0.602	0.553	0.533	0.553	0.588	0.599	0.612	0.591	0.555	0.547
PH/NW	1.886	1.892	1.349	0.982	0.768	0.668	0.626	0.982	1.111	1.161	1.063	1.031
OWEMORT	0.772	0.899	0.875	0.805	0.557	0.307	0.116	0.624	0.611	0.639	0.657	0.670
MORT/DEBT	0.655	0.779	0.759	0.696	0.560	0.454	0.351	0.619	0.653	0.664	0.686	0.692
LTV	0.519	0.596	0.489	0.364	0.210	0.095	0.032	0.259	0.300	0.335	0.357	0.348
HE/NW	0.585	0.515	0.495	0.490	0.506	0.548	0.586	0.576	0.564	0.521	0.473	0.466
OWNSTOCK	0.183	0.330	0.308	0.308	0.291	0.149	0.177	0.178	0.251	0.292	0.321	0.341
	0.334	0.558	0.614	0.602	0.530	0.392	0.333	0.416	0.459	0.493	0.595	0.628
STOCKS (\$100K)	0.095	0.169	0.255	0.483	0.447	1.147	0.776	0.299	0.191	0.239	0.468	0.385
	0.236	0.340	0.596	1.094	1.350	1.428	1.333	0.513	0.539	0.713	1.082	1.490
S/ASSET	0.232	0.217	0.260	0.243	0.288	0.311	0.289	0.169	0.197	0.246	0.283	0.288
	0.049	0.101	0.130	0.171	0.185	0.187	0.186	0.095	0.113	0.136	0.183	0.202
S/NW	0.318	0.293	0.334	0.292	0.314	0.321	0.294	0.207	0.240	0.313	0.348	0.370
	0.110	0.211	0.207	0.230	0.219	0.201	0.190	0.122	0.154	0.198	0.260	0.267
S/SB	0.448	0.385	0.426	0.421	0.415	0.396	0.348	0.291	0.349	0.403	0.436	0.485
	0.383	0.440	0.461	0.472	0.462	0.413	0.395	0.319	0.363	0.445	0.504	0.534
NETWORTH (\$100K)	0.161	0.408	0.537	0.843	0.715	0.861	0.779	0.560	0.496	0.522	0.592	0.610
	0.730	1.297	2.130	3.223	3.674	3.394	2.930	2.553	2.297	2.283	2.853	3.704
INCOME (\$100K)	0.232	0.367	0.402	0.422	0.334	0.250	0.215	0.322	0.326	0.343	0.355	0.373
	0.347	0.599	0.713	0.753	0.614	0.414	0.317	0.595	0.542	0.554	0.597	0.691
NW/INC	1.028	1.152	1.305	1.607	1.780	2.132	2.924	1.378	1.439	1.651	1.523	1.505
	2.944	2.290	3.082	4.276	6.176	8.132	9.296	4.780	4.888	4.997	5.356	5.755
NO. of OBS.	2519	10692	16187	14782	10030	8393	6150	11278	13106	14689	14510	15170

Table 1 reports percentage of home ownership (OWNHOUSE), average of house value–asset ratio (PH/ASSET), house value–networth ratio (PH/NW), percentage of homeowners owing mortgage (OWEMORT), average households’ mortgage–total debt ratio (MORT/DEBT), mortgage loan-to-value ratio (LTV), percentage of stock ownership (OWNSTOCK), average equity proportion in total asset (S/ASSET), networth (S/NW) and liquid financial assets (S/SB), across age groups and various years. Households with negative value for networth and income, and households in the top and bottom 1% of the sample for the ratio variables are dropped to limit the bias of outliers.

Table 2
Baseline Parameter Values

Parameter	Symbol	Value
Risk Aversion Coefficient	γ	5
Elasticity of Intertemporal Substitution	ζ	0.500
Housing Preference	ω	0.200
Discount Factor	β	0.930
Age of Retirement	J	65
Replacement Ratio	θ	0.682
Number of Beneficiaries	L	8
Down Payment Requirement	δ	0.200
Renting Cost	α	0.060
House Selling Cost	ϕ	0.060
House Purchase Cost	φ	0.030
Mortgage Initiation Cost	ξ	0.008
Mortgage Default Penalty	κ	0.150
Housing Maintenance and Depreciation	ψ	0.015
Stock Market Initial Entry Cost	η	0.100
Stock Market Per Period Maintenance Cost	ϱ	0.020
Inflation Rate	ι	0.030
Real Riskfree Rate	r_f	0.020
Real Mean Stock Return	μ_S	0.060
Mean Housing Return	μ_H	0.000
Standard Deviation of Permanent Shock to Labor Income	σ_ν	0.100
Standard Deviation of temporary Shock to Labor Income	σ_ε	0.250
Standard Deviation of Stock Return	σ_S	0.180
Standard Deviation of Housing Return	σ_H	0.115
Correlation of Housing Return and Permanent Shock to Labor Income	$\rho_{H,\nu}$	0.200
Correlation of Housing Return and Temporary Shock to Labor Income	$\rho_{H,\varepsilon}$	0.000
Correlation of Housing Return and Stock Return	$\rho_{H,S}$	0.000

Table 3
Summary Statistics for Wealth Accumulation, Home and Stock Ownerships from Simulations

Age	$Q_t/P_t^Y - 1$				Home Ownership				Stock Ownership								
	EZ	$\zeta = 1/\gamma$	$\varrho = 0$	$\psi = 0$	$l_t \leq \delta$	$D_t^0 = 0$	EZ	$\zeta = 1/\gamma$	$\varrho = 0$	$\psi = 0$	$l_t \leq \delta$	EZ	$\zeta = 1/\gamma$	$\varrho = 0$	$\psi = 0$	$l_t \leq \delta$	$D_t^0 = 0$
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
22	0.17	0.43	0.18	0.17	0.17	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.00	0.00	0.00
24	0.26	0.69	0.30	0.26	0.26	0.26	0.00	0.00	0.00	0.01	0.00	0.00	0.05	0.68	0.00	0.01	0.01
26	0.35	0.90	0.44	0.35	0.36	0.35	0.06	0.00	0.00	0.20	0.00	0.03	0.25	0.89	0.01	0.08	0.08
28	0.44	1.10	0.61	0.43	0.47	0.46	0.33	0.02	0.00	0.51	0.00	0.05	0.50	0.97	0.05	0.19	0.20
30	0.56	1.32	0.80	0.55	0.62	0.60	0.62	0.12	0.07	0.74	0.00	0.11	0.70	0.99	0.14	0.31	0.34
32	0.74	1.55	1.00	0.73	0.78	0.77	0.82	0.33	0.32	0.86	0.25	0.24	0.83	1.00	0.28	0.36	0.49
34	0.97	1.82	1.19	0.93	0.97	0.97	0.92	0.56	0.63	0.94	0.48	0.39	0.91	1.00	0.42	0.42	0.63
36	1.23	2.12	1.42	1.17	1.20	1.19	0.97	0.77	0.84	0.96	0.66	0.54	0.94	1.00	0.56	0.52	0.75
38	1.52	2.48	1.71	1.43	1.46	1.43	0.98	0.88	0.93	0.98	0.83	0.68	0.96	1.00	0.68	0.60	0.84
40	1.83	2.88	2.03	1.71	1.74	1.69	0.99	0.94	0.97	0.99	0.94	0.78	0.98	1.00	0.77	0.69	0.90
45	2.64	3.96	2.85	2.43	2.54	2.37	1.00	0.98	1.00	1.00	1.00	0.93	0.99	1.00	0.92	0.87	0.97
50	3.50	5.14	3.71	3.19	3.40	3.08	1.00	0.99	1.00	1.00	1.00	0.98	1.00	1.00	0.97	0.96	0.99
55	4.34	6.30	4.55	4.05	4.23	3.78	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	0.99	0.99	1.00
60	5.16	7.39	5.36	4.94	5.05	4.49	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
65	5.60	7.96	5.79	5.43	5.51	4.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
70	4.86	7.08	5.03	4.70	4.79	4.13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
72	4.66	6.77	4.83	4.52	4.60	3.93	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00
74	4.51	6.48	4.68	4.39	4.46	3.79	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	0.98	1.00
76	4.41	6.21	4.57	4.31	4.36	3.70	1.00	1.00	1.00	0.98	1.00	0.95	0.99	1.00	1.00	0.95	1.00
78	4.35	5.95	4.51	4.28	4.30	3.66	0.93	0.94	0.93	0.91	0.93	0.86	0.96	1.00	0.96	0.86	0.97
80	4.31	5.67	4.48	4.26	4.27	3.65	0.81	0.84	0.80	0.26	0.81	0.67	0.86	1.00	0.74	0.66	0.69

Table 3 reports average wealth-income ratio before labor income ($Q_t/P_t^Y - 1$), and the rates of home and stock ownerships from simulations under various parameters.

Table 4
Summary Statistics for Homeowners' Portfolio Choices from Simulations

Age	$S_t+B_t/S_t+B_t+P_t^H H_t-M_t$			$S_t/S_t+B_t+P_t^H H_t-M_t$			S_t/S_t+B_t												
	EZ	$\zeta = 1/\gamma$	$\psi = 0$	EZ	$\zeta = 1/\gamma$	$\psi = 0$	EZ	$\zeta = 1/\gamma$	$\psi = 0$	$\rho = 0$	$\psi = 0$	$\zeta = 1/\gamma$	$\rho = 0$	$\psi = 0$	$\zeta = 1/\gamma$	$\rho = 0$	$\psi = 0$	$D_t^o = 0$	
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
24	—	—	0.33	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
26	0.39	0.59	0.41	—	—	0.36	—	—	—	—	—	—	—	—	—	—	—	—	—
28	0.38	0.64	0.49	—	—	0.41	—	—	—	—	—	—	—	—	—	—	—	—	—
30	0.42	0.67	0.56	0.49	—	0.50	0.49	—	—	—	—	—	—	—	—	—	—	—	—
32	0.51	0.71	0.62	0.60	0.55	0.60	0.55	0.78	0.71	0.76	0.74	0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00
34	0.60	0.74	0.66	0.67	0.55	0.67	0.55	0.78	0.71	0.76	0.74	0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00
36	0.67	0.76	0.71	0.72	0.57	0.72	0.57	0.81	0.75	0.76	0.76	0.69	1.00	1.00	1.00	1.00	1.00	1.00	1.00
38	0.71	0.79	0.74	0.76	0.60	0.76	0.60	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
40	0.74	0.80	0.77	0.79	0.63	0.79	0.63	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
45	0.78	0.81	0.79	0.84	0.69	0.79	0.69	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	0.78	0.79	0.78	0.86	0.70	0.86	0.70	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
55	0.75	0.77	0.75	0.87	0.70	0.87	0.70	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
60	0.72	0.74	0.72	0.88	0.69	0.88	0.69	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
65	0.68	0.71	0.68	0.88	0.66	0.88	0.66	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
70	0.61	0.64	0.61	0.86	0.61	0.86	0.61	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
72	0.59	0.61	0.59	0.85	0.58	0.85	0.58	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
74	0.55	0.58	0.55	0.84	0.54	0.84	0.54	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
76	0.48	0.52	0.50	0.82	0.48	0.82	0.48	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
78	0.42	0.46	0.43	0.82	0.42	0.82	0.42	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00
80	0.40	0.42	0.41	0.56	0.40	0.56	0.40	0.81	0.75	0.76	0.76	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 4 reports the homeowners' average liquid wealth–networth ratio ($S_t+B_t/S_t+B_t+P_t^H H_t-M_t$), network equity proportion ($S_t/S_t+B_t+P_t^H H_t-M_t$), and the liquid wealth equity proportion (S_t/S_t+B_t) for homeowners who participate in the stock market.

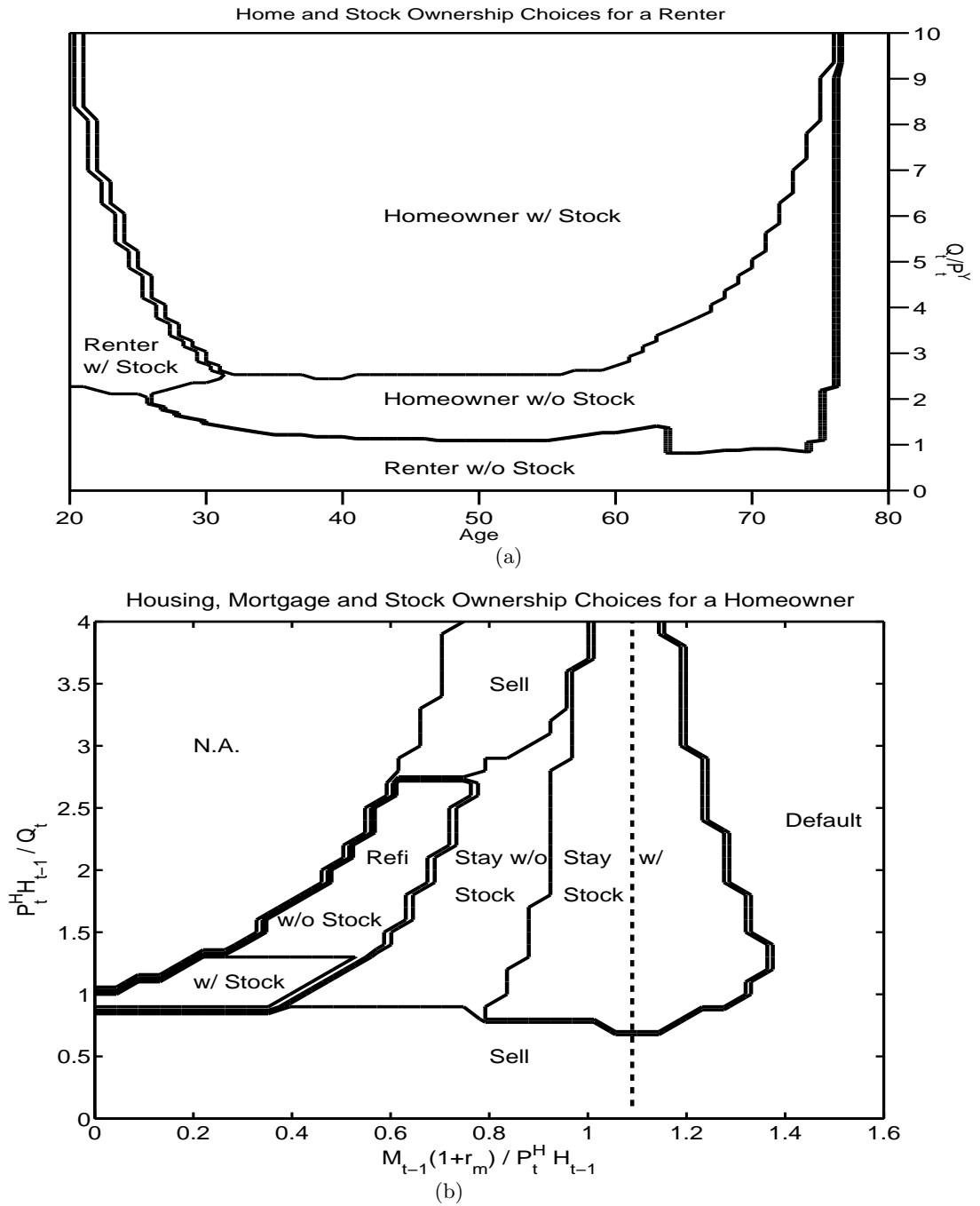


Figure 1. Panel a plots the optimal housing and stock market participation decision of an investor coming into the period as a renter without stocks, as a function of wealth–labor-income ratio. Panel b plots the optimal housing and mortgage decision of an investor coming into the period as a homeowner who previously have entered the stock market, as a function of the beginning-of-period house value–wealth ratio and loan–to–value ratio for an investor at age 50. The investor’s networth–labor-income ratio is fixed at $Q_t/P_t^Y = 1.75$ in panel b.

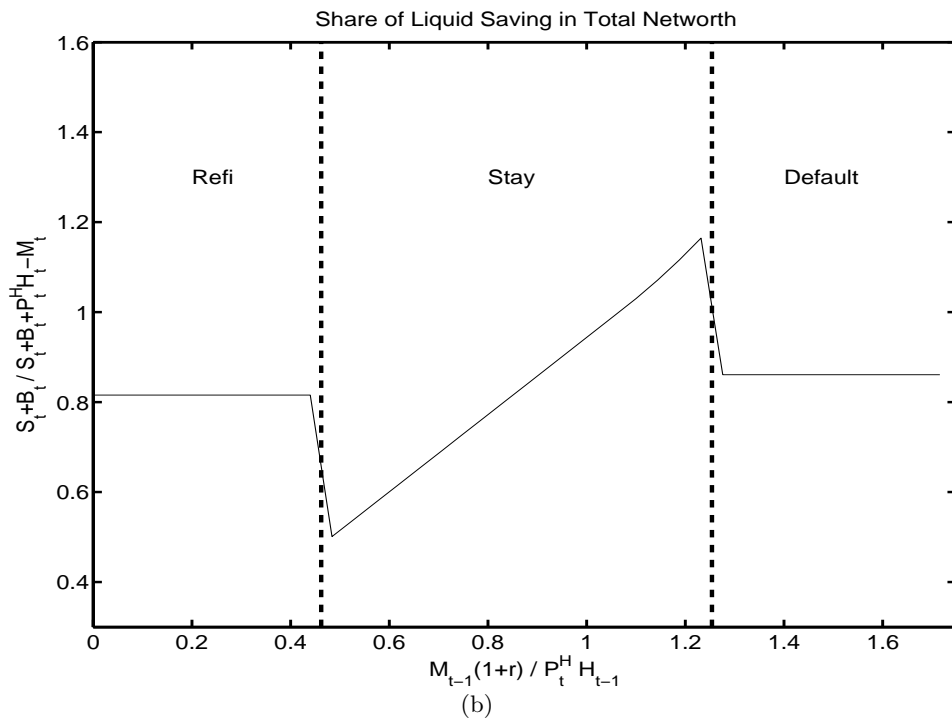
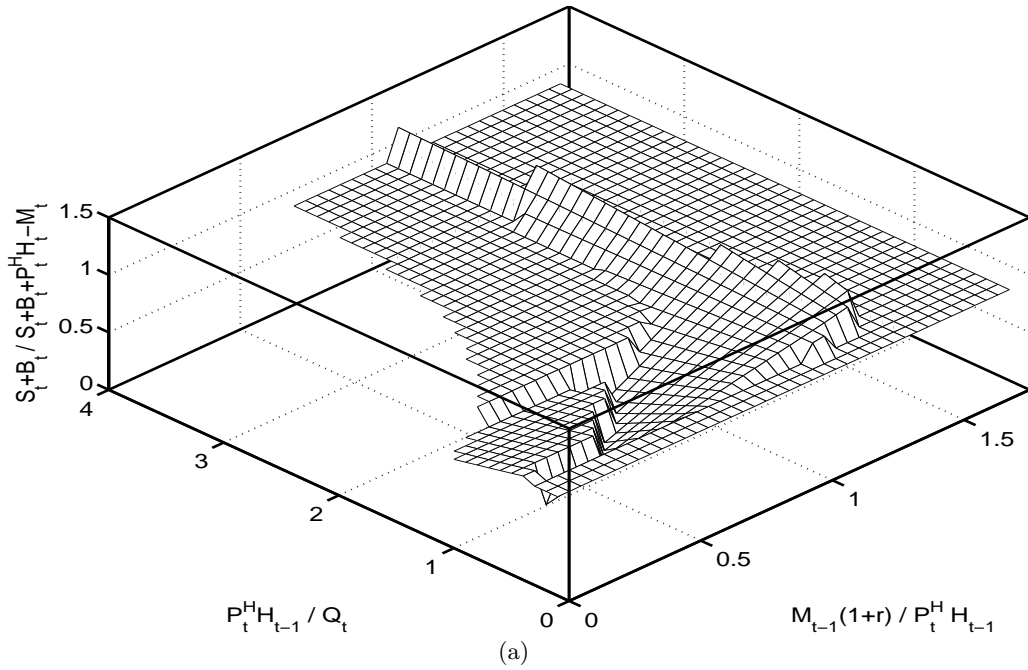


Figure 2. The figure plots the optimal liquid investment as a function of the beginning-of-period housing value-wealth ratio and loan-to-value ratio for an investor at age 50. The investor's wealth worth-labor-income ratio is fixed at $Q_t/P_t^Y = 7.0$. The investor's house value-wealth ratio is fixed at $P_t^H H_{t-1}/Q_t = 0.8$ for panel b.

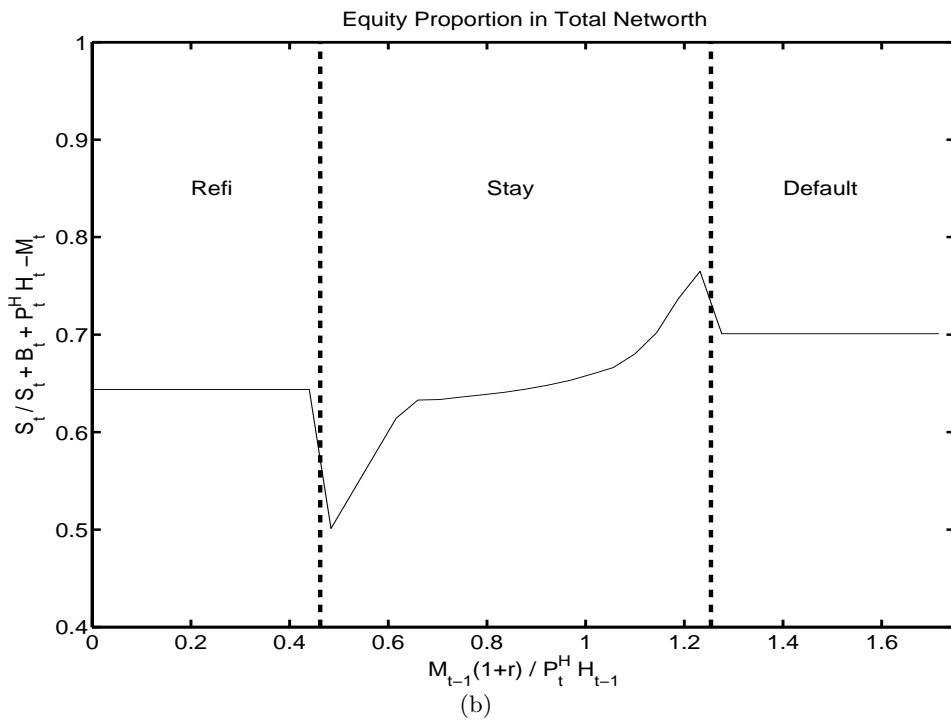
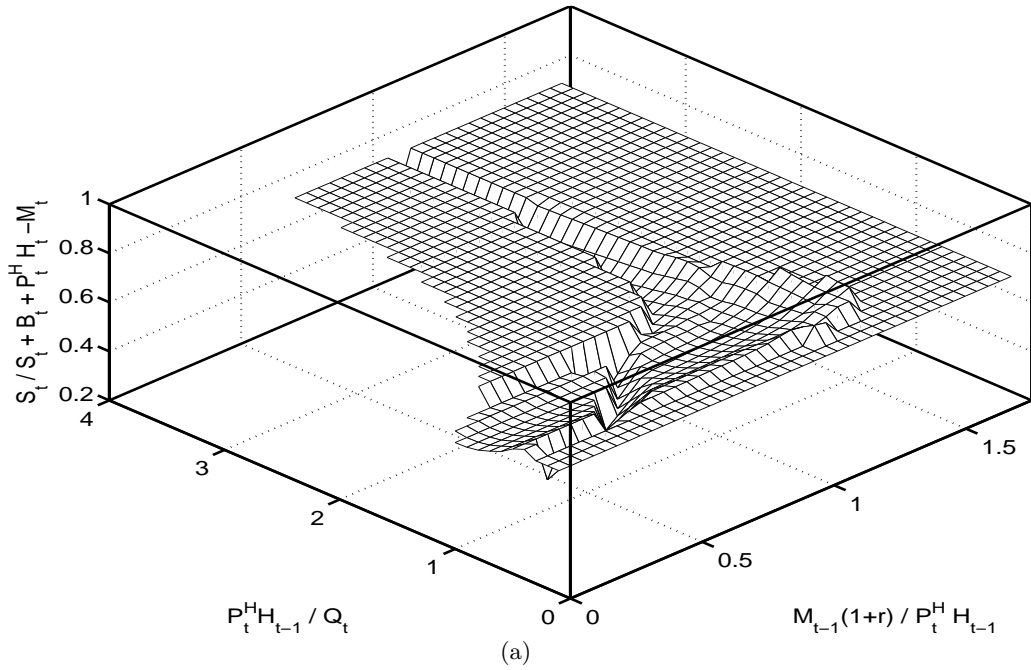


Figure 3. The figure plots the optimal equity proportion in total network as a function of the beginning-of-period housing value-wealth ratio and loan-to-value ratio for an investor at age 50. The investor's wealth worth-labor-income ratio is fixed at $Q_t/P_t^Y = 7.0$. The investor's house value-wealth ratio is fixed at $P_t^H H_{t-1}/Q_t = 0.8$ for panel b.

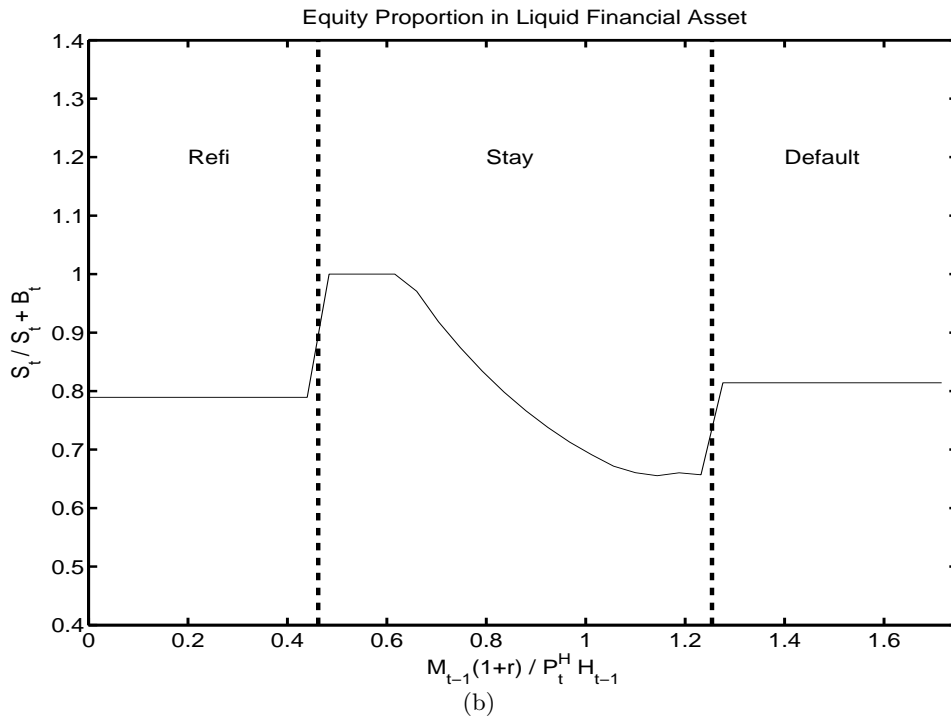
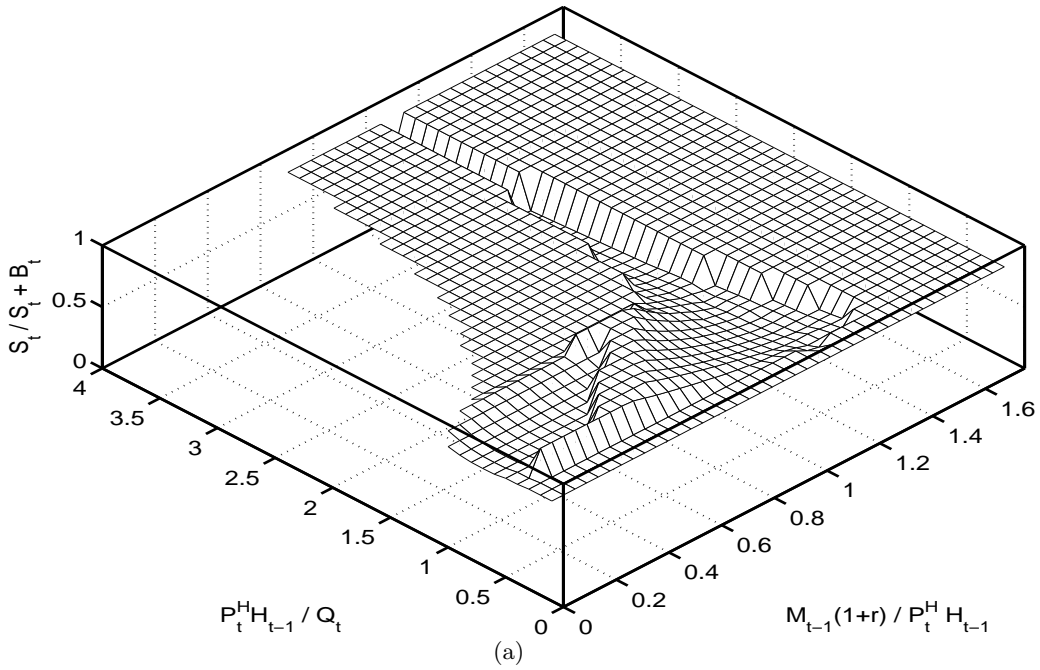


Figure 4. The figure plots the optimal equity proportion in liquid wealth as a function of the beginning-of-period housing value-wealth ratio and loan-to-value ratio for an investor at age 50. The investor's wealth worth-labor-income ratio is fixed at $Q_t/P_t^Y = 7.0$. The investor's house value-wealth ratio is fixed at $P_t^H H_{t-1}/Q_t = 0.8$ for panel b.

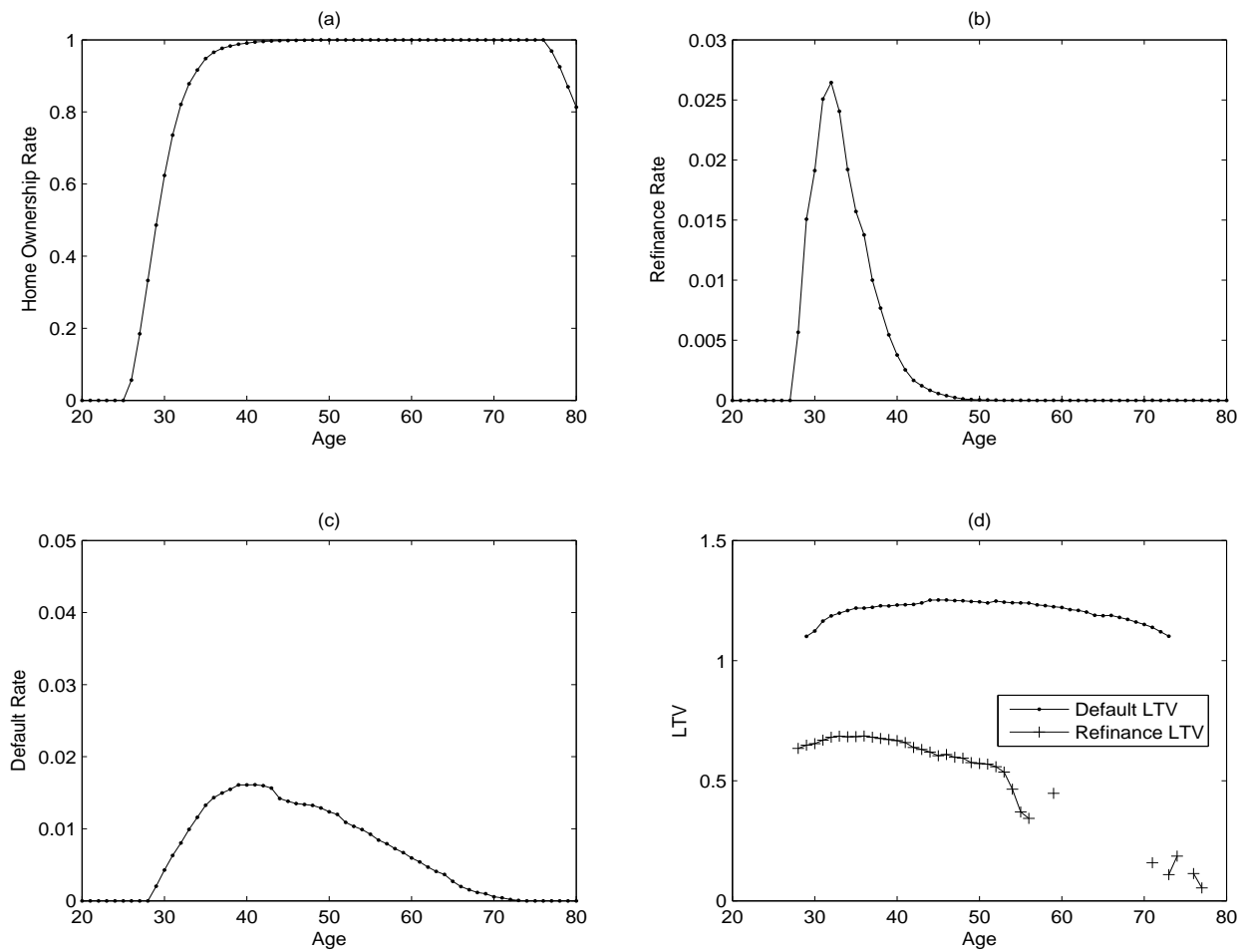


Figure 5. The figure plots the percentage of home ownership (panel a), percentage of households refinancing their mortgage (panel b), percentage of households defaulting on their mortgage (panel c), and the average loan-to-value ratio for refinanced and defaulted mortgages (panel d) as a function of age. The results are calculated based on 50,000 simulations using the baseline parameter values.

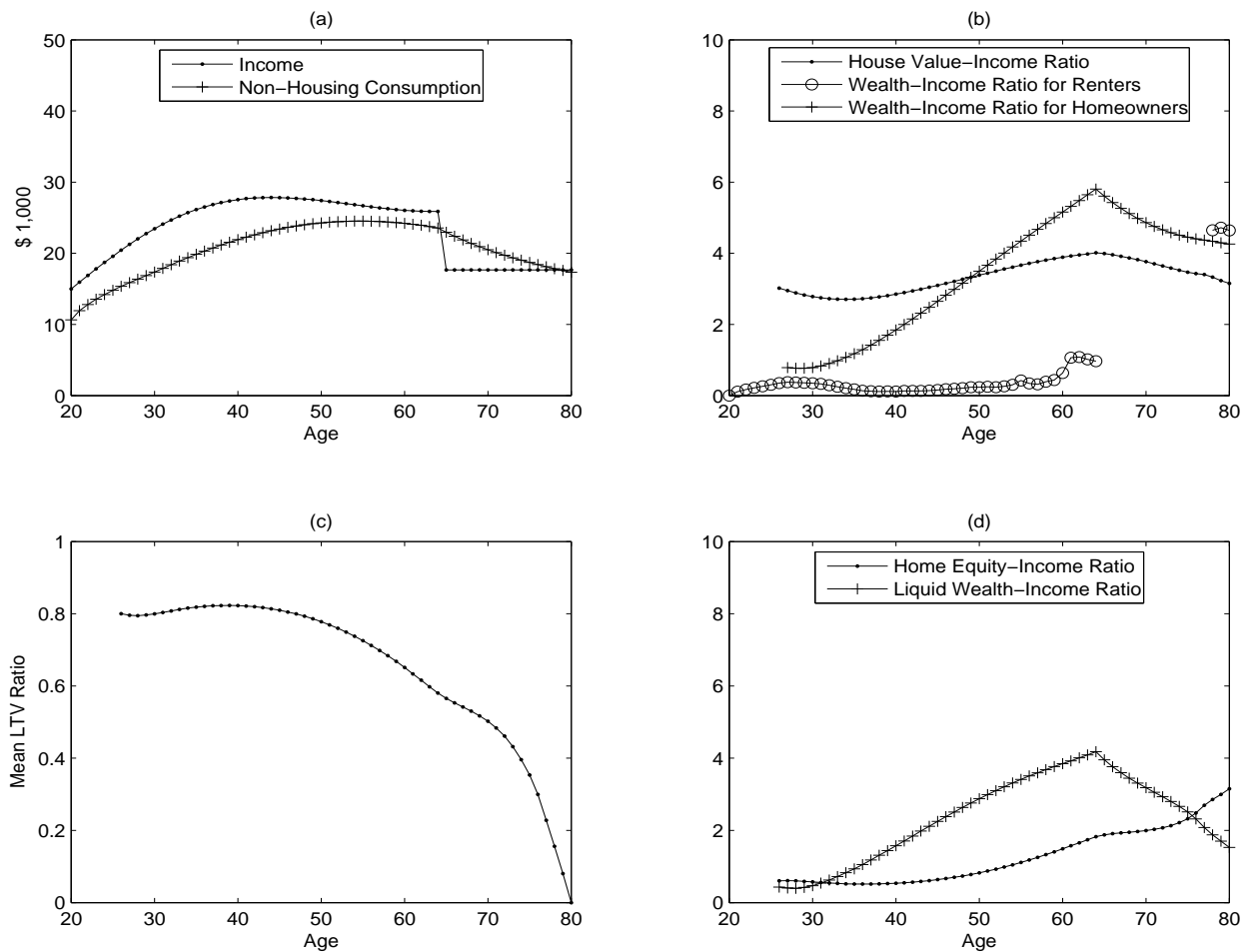


Figure 6. The figure plots the average households' numeraire good consumption and labor incomes (panel a), the average house value-income ratio and networth-income ratio (panel b), the average loan-to-value ratio (panel c), and the average home equity-income ratio and liquid wealth-income ratio (panel d) as a function of age. The results are calculated based on 50,000 simulations using the baseline parameter values.

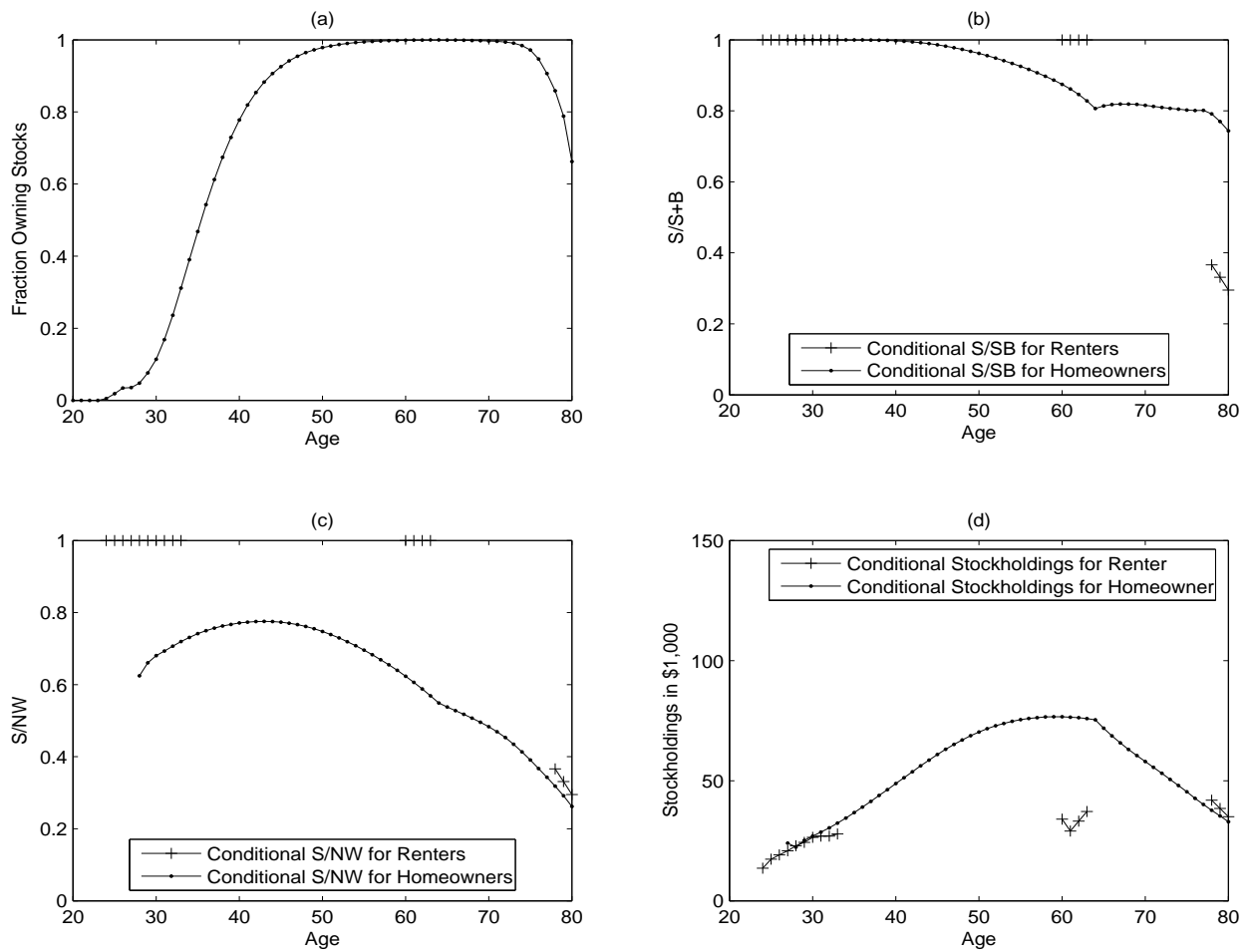


Figure 7. The figure plots the percentage of households owning stocks (panel a), the average equity proportion in liquid wealth (panel b), the average stock investment in networth (panel c), and the average equity investment in dollars for stockholders (panel d). The results are calculated based on 50,000 simulations for the baseline parameter values.

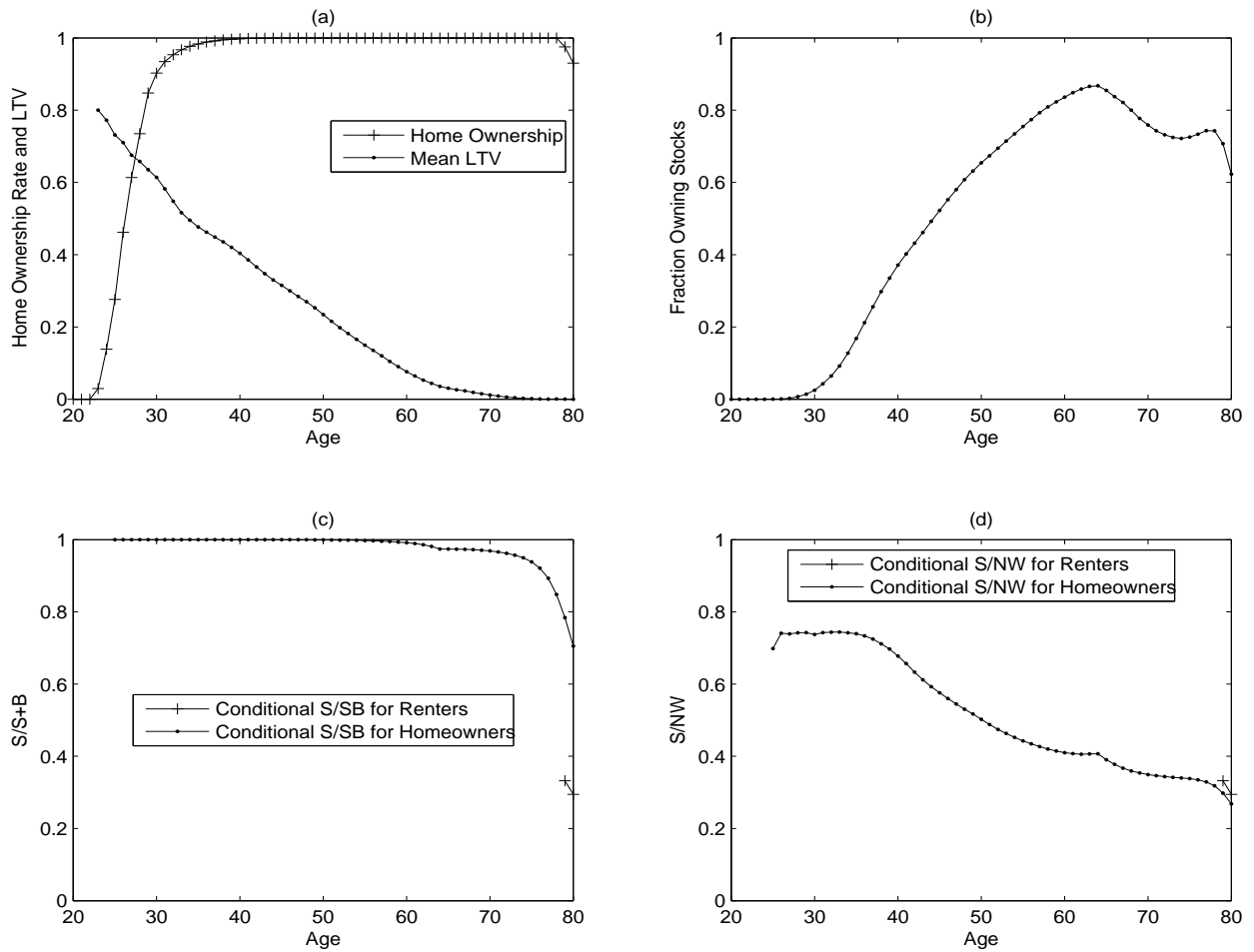


Figure 8. The figure plots the percentage of households owning homes (panel a), the average loan-to-value ratios (panel b), the average stock investment in liquid wealth (panel c), and the average stock investment in networth for stockholders (panel d), when there is a 1 percent credit spread between borrowing and lending rate. The results are calculated based on 50,000 simulations.