

Optimal Consumption and Investment with Capital Gains Taxes

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This article characterizes optimal dynamic consumption and portfolio decisions in the presence of capital gains taxes and short-sale restrictions. The optimal decisions are a function of the investor's age, initial portfolio holdings, and tax basis. Our results capture the trade-off between the diversification benefits and tax costs of trading over an investor's lifetime. The incentive to rediversify the portfolio is inversely related to the size of the embedded gain and investor's age. Contrary to standard financial advice, the optimal equity holding increases well into an investor's lifetime in our model due to the forgiveness of capital gains taxes at death.

Taxes play an important role in the decision-making process of individuals concerning their consumption and investment plans. The taxation of returns on financial assets alters the benefits of saving for future consumption and thus affects the trade-off between current consumption and investment. Moreover, the ability of investors to defer the taxation of capital gains alters the relative valuation of stocks and bonds and thus affects the optimal portfolio composition of investors. The deferral feature of capital gains taxes also affects the trade-off investors face regarding taxes and diversification. Investors would like to maintain an optimally diversified portfolio over time,

We thank George Constantinides, Michael Gallmeyer, Richard Green, John Heaton, Burton Hollifield, Ravi Jagannathan (the editor), Alan Kraus, Anthony Lynch, Stathis Tompaides, Luis Viceira (the referee), Joseph Williams, Stan Zin, and participants at presentations at Boston College, Carnegie Mellon University, Cornell University, Michigan State University, Stanford University, the Federal Reserve Board of Governors, University of Arizona, University of British Columbia, University of Florida, University of North Carolina at Chapel Hill, University of Rochester, University of Texas at Dallas, the 1999 American Economic Association meetings in New York, the 1999 Western Finance Association meetings in Santa Monica, the 1999 NBER Summer Institute Asset Pricing Workshop in Cambridge, the Tenth Annual Conference on Financial Economics and Accounting at the University of Texas at Austin, and the 2000 Utah Winter Finance Conference for helpful comments. Financial support provided by the Teachers Insurance Annuity Association—College Retirement Equities Fund, the Carnegie Mellon University Faculty Development Fund, and the Carnegie Mellon Financial Research Center is gratefully acknowledged. Part of this article was written during the summer of 1998, while Robert M. Dammon was a visiting research professor at the University of British Columbia. Address correspondence to Chester Spatt, Graduate School of Industrial Administration, Carnegie Mellon University, Pittsburgh, PA 15213, or e-mail: cspatt@andrew.cmu.edu.

but this can conflict with their desire to delay the taxation of capital gains. The purpose of this article is to investigate the effect of taxation on the intertemporal consumption, investment, and realization decisions of investors.

In his seminal article, Constantinides (1983) examines the effect of capital gains taxes on investors' optimal intertemporal consumption and investment decisions and on equilibrium asset prices. Assuming that securities can be sold short with full use of the short-sale proceeds, and that the wash-sale rule¹ can be easily circumvented, he shows that the optimal investment and liquidation decisions are separable from the consumption decision. In particular, with symmetric taxation of long-term and short-term capital gains and losses, Constantinides (1983) shows that it is optimal to realize losses and defer gains, independent of the investor's consumption needs or optimal portfolio holdings.

While these results are appealing from a theoretical perspective, they seem to be inconsistent with the observed realization behavior of investors. Poterba (1987), using U.S. tax return data, and Odean (1998), using brokerage account data, document that investors realize substantial capital gains. If investors are able to sell short at relatively low cost, then this behavior would indeed be puzzling. However, in practice the cost of selling short for an individual investor can be quite high. For example, it is common practice for brokerage firms to not only retain the proceeds from the short sale, but to also require the investor to post additional collateral beyond the short-sale proceeds. In addition, the ability of investors to circumvent the payment of the capital gains tax by shorting a security instead of selling it outright has been severely limited by the 1997 U.S. tax legislation. Under current tax law, investors are not able to indefinitely defer the realization of a capital gain on an asset by simply holding offsetting long and short positions.²

Absent the ability to sell short without triggering a capital gains tax liability on an offsetting long position, or obtaining use of the short-sale proceeds, investors' consumption, investment, and liquidation decisions are not separable. Investors with diversification motives and/or consumption needs beyond that provided by dividends, interest, and labor income may optimally sell assets with embedded capital gains. In this article we examine the optimal consumption, investment, and liquidation decisions over an investor's lifetime in the presence of capital gains taxes and short-sale restrictions. In general, the optimal policy will depend upon the investor's current wealth, age, portfolio holdings, and the prices at which the securities in the portfolio were

¹ The wash-sale rule disallows a tax deduction for a loss on a security that is sold if the same security or "substantially identical" securities are acquired within a period of time beginning 30 days prior to the date of sale and extending 30 days after the date of sale. The wash-sale rule does not apply to securities sold at a gain.

² An investor with an offsetting long and short position at the end of a tax year has the first 30 days of the next tax year to unwind the short position (and leave it closed for at least 60 days). Otherwise the IRS will treat the position as if the investor used the long position to cover the short position, thereby generating a capital gains tax liability.

acquired (i.e., the tax bases). Since assets may be acquired at different prices over time, the dimensionality of the problem grows rapidly with the number of assets and time periods. To keep the problem manageable, simplifying assumptions are needed.

We simplify the economic environment by assuming that there are only two assets: a risky stock whose price follows a binomial process and a riskless one-period bond paying a constant interest rate. We further assume that the investor's tax basis in the stock is calculated as the weighted average purchase price.³ Although the tax-timing option is more valuable if the investor identifies each purchase with its own tax basis, the assumption that the investor uses the weighted average purchase price as the tax basis for all shares keeps the dimensionality of the state space constant over time.⁴ However, even with these simplifying assumptions, we are unable to obtain an analytical solution and therefore use numerical techniques for much of our analysis.

The optimal consumption-wealth ratio is increasing and convex in the investor's tax basis. This is because the value of the investor's tax-timing option is also increasing and convex in the investor's tax basis. For elderly investors, the consumption decision is driven primarily by the investor's bequest motive and the desire to defer the realization of capital gains to benefit from the *reset provision* at death. The reset provision of the U.S. tax code forgives the tax on any embedded capital gains or losses at the time of death and requires the tax bases of the inherited assets to be reset to the prevailing market prices. For young investors, the bequest motive and reset provision at death are relatively less important because of their lower mortality rates.

The introduction of taxes causes investors to shift their portfolios toward equity because of the favorable tax treatment afforded capital gains relative to interest income. While interest income is fully taxable when received, capital gains (and losses) are taxed only when the investor sells the asset, and escape taxation altogether at the time of death. When an investor has an embedded capital loss, he optimally realizes the loss before rebalancing his portfolio to his *unconstrained optimum*. The unconstrained optimal holding of equity is age dependent and reaches its maximum at a rather late age. This is the age at which the combined benefits of the tax-timing option and the reset

³ In some countries (e.g., Canada), the average purchase price method is required for calculating the tax basis. Under current U.S. tax law, however, investors are allowed to use either the specific share identification method or the average purchase price method. Although the specific share identification method offers more flexibility for the investor, in some contexts the average purchase price method simplifies record keeping and can be less burdensome to use in practice. For example, many mutual funds report the investor's average purchase price on account statements to facilitate tax reporting.

⁴ In a recent article, Dybvig and Koo (1996) present an intertemporal model of investment with capital gains taxes and short-sale constraints, but without intermediate consumption. In contrast to our approach, their model accounts for a separate tax basis for each asset purchase. While this approach allows investors to maximize the value of the tax-timing option, the computational burden of their numerical algorithms restricts the number of assets and time periods that can be studied. With one riskless and one risky asset, they are able to implement their numerical solution algorithms for only a small number of periods (up to four).

provision at death are highest and equity is most valuable to the investor. To maintain an optimally diversified portfolio, investors may also sell stock with embedded capital gains. The size of the gain that investors are willing to realize is increasing in the degree to which the investor is overweighted in equity and decreasing in age. We find that for young investors who are substantially overweighted in equity the sale of stock with an embedded capital gain in excess of 100% can be optimal. A welfare analysis shows that the optimal age-dependent realization policy derived in this article provides substantial benefits for investors relative to some common alternatives.

A comparative statics analysis shows that the investor's bequest motive has an important effect on the optimal consumption and portfolio allocation decisions. As the utility derived from the bequest declines, the investor's optimal consumption-wealth ratio increases, especially for elderly investors who have the highest mortality rates. Because the tax on capital gains is forgiven at death, elderly investors retain their equity holdings and finance their higher consumption levels by selling bonds and borrowing. As a consequence, the optimal equity proportion increases dramatically at late ages. A simulation analysis of the investor's optimal consumption and portfolio allocation choices illustrates many of these important features.

A variety of alternative model specifications are also examined. When capital gains and losses are taxed at death (mandatory realization), as in the Canadian tax code, the optimal holding of equity declines for all investors. The decline is more pronounced for elderly investors, who have the highest mortality rates and benefit the most from the reset provision at death. In fact, under the mandatory capital gains realization at death, the optimal equity proportions are nearly identical across all ages. We also find that with asymmetric long-term and short-term tax rates, the optimal holding of equity increases substantially for the young and only slightly for the elderly. A lower long-term capital gains tax rate is more valuable to young investors because it reduces the tax cost of rebalancing their portfolios and allows them to exploit the asymmetry in the tax rates over a longer horizon. Finally, we find that in the presence of nonfinancial (labor) income, investors consume a larger fraction of their financial wealth and hold more equity than in the case without nonfinancial income. This is due to the fact that nonfinancial income increases the investor's total wealth and reduce its overall risk. Nonfinancial income also allows investors to rebalance their portfolios over time without the need to sell assets with embedded capital gains.

This article is organized as follows. In Section 1 we present our basic model, develop the investor's intertemporal consumption-investment problem, and discuss the numerical algorithm for obtaining a solution. In Section 2 we present our numerical solutions, perform a welfare analysis, investigate the sensitivity of the optimal decisions to various model parameters, and conduct a simulation analysis of the investor's optimal decisions over his lifetime. Section 3 analyzes the optimal portfolio and liquidation decisions

under alternative model specifications. Section 4 provides some concluding remarks.

1. The Model

The economy consists of investors living for at most T periods, where T is a positive integer. This allows us to directly consider the impact of the investor's age (and increasing mortality) upon his optimal consumption, investment, and realization behavior. Let λ_j be the single-period hazard rate for period j . We assume that $\lambda_j > 0$ for all j and that $\lambda_T = \infty$. The probability that an individual investor lives through period t ($t \leq T$) is given by the following survival function:

$$F(t) = \exp\left(-\sum_{j=0}^t \lambda_j\right) \quad (1)$$

where $0 < F(t) < 1$ for all $0 \leq t < T$, and $F(T) = 0$.

Investors in the economy derive utility from consuming a single consumption good. For simplicity, we assume that all income for consumption is derived from financial assets. (We allow for nonfinancial income in Section 3.3.) Investors can trade two assets in the financial markets: a riskless one-period bond and a risky stock. The pretax nominal return on the riskless bond is denoted r and is assumed to be constant over time. The nominal payoff to holding one share of stock from date $t - 1$ to date t is $(1 + d)P_t$, where d is a constant pretax dividend yield and P_t is the nominal stock price at date t . We assume that the pretax nominal capital gain return on the stock is serially independent and follows an exogenous binomial process. No transaction costs are incurred for trading assets. We denote by n_t the number of shares of stock held *after* trading at time t and assume that short sales are not allowed ($n_t \geq 0$). Nominal dividend and interest payments are taxed at a constant rate of τ_d .

The tax treatment of capital gains and losses is as follows. Any *realized* capital gains are subject to a constant capital gains tax rate of τ_g , while *realized* capital losses are credited at the same rate.⁵ To calculate an investor's nominal capital gain, we assume that the tax basis for shares currently held is the weighted average purchase price of those shares. Denote by P_t^* the nominal tax basis after trading at time t . The nominal tax basis follows the law of motion:

$$P_t^* = \begin{cases} \frac{n_{t-1}P_{t-1}^* + \max(n_t - n_{t-1}, 0)P_t}{n_{t-1} + \max(n_t - n_{t-1}, 0)}, & \text{if } P_{t-1}^* < P_t \\ P_t, & \text{if } P_{t-1}^* \geq P_t. \end{cases} \quad (2)$$

The above specification indicates that the updating rule for the investor's tax basis depends upon whether there is an embedded capital gain or loss on the

⁵ The current U.S. tax code limits net deductible capital losses to \$3,000 per year. Net capital losses in excess of \$3,000 can be carried forward indefinitely. We ignore the annual limit on deductible losses in our numerical analysis.

shares currently held, and whether the investor buys or sells shares in period t . In the case of an embedded capital gain (i.e., $P_{t-1}^* < P_t$), the investor's tax basis is unchanged from the previous period (i.e., $P_t^* = P_{t-1}^*$) if the investor sells shares in period t (i.e., $n_t < n_{t-1}$). However, if the investor buys shares in period t (i.e., $n_t > n_{t-1}$), then the investor's tax basis is equal to a weighted average of the previous tax basis and the purchase price of the new shares, with the weights determined by the number of old and new shares. In the case of an embedded capital loss (i.e., $P_{t-1}^* > P_t$), the investor's tax basis after trading at date t is equal to the current stock price, P_t . This is due to the fact that, without transaction costs or wash-sale rules, it is optimal for the investor to liquidate all shares for tax purposes before rebalancing his portfolio.⁶ Hence, any shares held after trading at date t will have been bought at the current stock price.

The investor's problem is to maximize his discounted expected utility of lifetime consumption, given his initial endowment and asset holdings, subject to the intertemporal budget constraint. Since at any given time t an investor has a positive probability of death, the treatment of his terminal wealth is an issue. In this model we assume that at the time of death the investor's asset holdings are liquidated without payment of the capital gains tax and the proceeds are used to purchase an H -period annuity for the benefit of the investor's beneficiary. This forgiveness of the capital gains tax at death is consistent with the *reset provision* of the current U.S. tax code. (We later discuss the impact of removing the reset provision in Section 3.1.) We assume that the H -period annuity provides the investor's beneficiary with nominal consumption of

$$\frac{r^*(1+r^*)^H}{(1+r^*)^H-1} W_t(1+i)^{k-t} \equiv A_H W_t(1+i)^{k-t}$$

at date k , $t+1 \leq k \leq t+H$, where i is the constant rate of inflation, $r^* = [(1-\tau_d)r-i]/(1+i)$ is the after-tax real bond return, W_t is the investor's wealth at the time of death, and $A_H = [r^*(1+r^*)^H]/[(1+r^*)^H-1]$ is the H -period annuity factor.⁷ We assume that the investor and his beneficiary

⁶ Note that the law of motion for the tax basis, P_t^* , does not allow for the sale and repurchase of stock with an embedded capital gain (i.e., $P_t > P_{t-1}^*$). This is because it is never optimal to do so when the long-term and short-term capital gains tax rates are equal and constant over time, as in our model. With constant and symmetric tax rates, the future tax benefits of a higher tax basis on the shares that are repurchased can never exceed the immediate tax liability that is generated by the sale. The sale of stock with an embedded capital gain can be optimal only if the proceeds of the sale are used for consumption or to rebalance the portfolio. Our model does not prohibit either of these possibilities.

⁷ Note that the after-tax nominal annuity payments grow at the inflation rate and thus provide the beneficiary with constant *real* consumption of $A_H W_t/(1+i)^k$ for each period $t+1 \leq k \leq t+H$. We assume that annuity contracts are provided in a competitive market so that the present value of the after-tax annuity payments is equal to the amount invested. We also ignore any state and federal inheritance taxes that may be owed on the value of the investor's estate, although it would not be difficult to include such a tax in our model. However, unless the inheritance tax is proportional to the value of the estate, the model would be more difficult to solve because the optimal consumption and investment policies would no longer be homogeneous in wealth (a property of the solution we develop later).

have identical preferences and that the utility derived by the investor from his bequest is equal to the utility derived by the beneficiary. This specification allows us to examine the sensitivity of the investor's optimal consumption and investment policies to the number of periods that the investor wishes to provide consumption support to his beneficiary, with higher values for H indicating a stronger bequest motive.⁸

The investor's problem can now be represented as follows:

$$\max_{C_t, n_t, B_t} E \left\{ \sum_{t=0}^T \beta^t \left[F(t) u \left(\frac{C_t}{(1+i)^t} \right) + [F(t-1) - F(t)] \right. \right. \\ \left. \left. \times \sum_{k=t+1}^{t+H} \beta^{k-t} u \left(\frac{A_H W_t}{(1+i)^t} \right) \right] \right\} \quad (3)$$

s.t.

$$W_t = n_{t-1}[1 + (1 - \tau_d)d]P_t + B_{t-1}[1 + (1 - \tau_d)r], \quad t = 0, \dots, T, \quad (4)$$

$$C_t = W_t - \tau_g G_t - n_t P_t - B_t, \quad t = 0, \dots, T - 1, \quad (5)$$

$$n_t \geq 0, \quad t = 0, \dots, T - 1, \quad (6)$$

$$n_T = 0, B_T = 0, \quad (7)$$

given the initial bond and stock holdings, B_{-1} and n_{-1} , and the initial tax basis, P_{-1}^* . In Equation (3), $F(-1)$ is set equal to one to indicate that the investor has survived up to period 0, $u(\cdot)$ denotes the investor's utility function, C_t is the investor's nominal consumption at date t , B_t is his nominal investment in bonds at date t , and β is the subjective discount factor for utility. The expression inside the square brackets in Equation (3) is the investor's probability weighted utility at date t . The first term measures the investor's utility of consumption in period t weighted by the probability of living through period t , while the second term is the investor's utility of his bequest weighted by the probability of dying in period t .

Equation (4) defines the investor's beginning-of-period wealth, W_t , as the value of the investor's portfolio holdings *before* trading at date t , including the after-tax interest and dividend income, but prior to capital gains taxes. Equation (5) is the investor's time- t budget constraint, where G_t is the *realized* nominal capital gain (or loss) at date t and is given by

$$G_t = \{I(P_{t-1}^* > P_t)n_{t-1} + [1 - I(P_{t-1}^* > P_t)] \max(n_{t-1} - n_t, 0)\} \\ \times (P_t - P_{t-1}^*), \quad (8)$$

⁸ Since the utility the investor derives from his bequest is equal to the discounted utility of the consumption that the bequest provides for his beneficiary, the investor would like to leave a bequest that equates his own marginal utility of consumption with that of his beneficiary. For a given level of the bequest, a larger value for H will imply that the beneficiary receives less consumption each period, thereby increasing his marginal utility of consumption. Thus, to equate his own marginal utility with that of his beneficiary, the investor will need to leave a larger bequest for higher values of H .

where $I(P_{t-1}^* > P_t)$ is an indicator function that takes the value of one if there is an embedded capital loss (i.e., $P_{t-1}^* > P_t$) and zero otherwise. This formulation exploits the fact that the investor optimally sells all shares with an embedded capital loss to benefit from the tax rebate in the absence of transaction costs and wash-sale rules. Hence, with an embedded capital loss on the n_{t-1} shares held coming into period t , $I(P_{t-1}^* > P_t) = 1$ and $G_t = n_{t-1}(P_t - P_{t-1}^*) < 0$. Since there are no wash-sale rules, the sale of stock with an embedded capital loss does not prevent the investor from repurchasing stock in period t ($n_t > 0$) to rebalance his portfolio. With an embedded capital gain, $I(P_{t-1}^* > P_t) = 0$ and the investor pays a capital gains tax only on those shares that are actually sold in period t . In this case, the total realized capital gain in period t is $G_t = \max(n_{t-1} - n_t, 0)(P_t - P_{t-1}^*) \geq 0$, where $\max(n_{t-1} - n_t, 0)$ is the number of shares sold in period t .

We assume that agents' preferences can be expressed as follows:

$$u\left(\frac{C_t}{(1+i)^t}\right) = \frac{\left(\frac{C_t}{(1+i)^t}\right)^{1-\gamma}}{1-\gamma}, \tag{9}$$

where γ is the relative risk aversion coefficient. Note that the summation appearing in the second term of the objective function can be rewritten as follows:

$$\begin{aligned} \sum_{k=t+1}^{t+H} \beta^{k-t} u\left(\frac{A_H W_t}{(1+i)^t}\right) &= \frac{\beta(1-\beta^H)u\left(\frac{A_H W_t}{(1+i)^t}\right)}{1-\beta} \\ &= \frac{\beta(1-\beta^H)\left(\frac{A_H W_t}{(1+i)^t}\right)^{1-\gamma}}{(1-\beta)(1-\gamma)}. \end{aligned}$$

Letting X_t denote the vector of state variables at date t , we can write the Bellman equation for the above maximization problem as follows:

$$\begin{aligned} V_t(X_t) = \max_{C_t, n_t, B_t} \left\{ \frac{e^{-\lambda_t} \left(\frac{C_t}{(1+i)^t}\right)^{1-\gamma}}{1-\gamma} + \frac{(1-e^{-\lambda_t})\beta(1-\beta^H)\left(\frac{A_H W_t}{(1+i)^t}\right)^{1-\gamma}}{(1-\beta)(1-\gamma)} \right. \\ \left. + e^{-\lambda_t} \beta E_t[V_{t+1}(X_{t+1})] \right\} \tag{10} \end{aligned}$$

for $t = 0, \dots, T - 1$, subject to Equations (2) and (4)–(8). The sufficient state variables for the investor's problem at date t consist of the stock price at date t , the tax basis before trading at date t , the stock holdings before trading at date t , and the total wealth before trading at date t . We represent the vector of state variables as follows:

$$X_t = [P_t, P_{t-1}^*, n_{t-1}, W_t]'. \tag{11}$$

The above problem can be simplified by using beginning-of-period wealth, W_t , as the numeraire. Let $s_t = n_{t-1}P_t/W_t$ be the fraction of beginning-of-period wealth invested in equity *prior* to trading in period t , $f_t = n_tP_t/W_t$ be the fraction of beginning-of-period wealth allocated to equity *after* trading in period t , $b_t = B_t/W_t$ be the fraction of beginning-of-period wealth allocated to bonds *after* trading in period t , $p_{t-1}^* = P_{t-1}^*/P_t$ be the investor's basis-price ratio applicable to trading in period t , $g_t = P_t/P_{t-1} - 1$ be the pretax nominal capital gain return on the stock from period $t - 1$ to period t , and

$$\begin{aligned} R_{t+1} &= \frac{n_t[1 + (1 - \tau_d)d]P_{t+1} + [1 + (1 - \tau_d)r]B_t}{n_tP_t + B_t} \\ &= \frac{f_t[1 + (1 - \tau_d)d](1 + g_{t+1}) + [1 + (1 - \tau_d)r]b_t}{f_t + b_t} \end{aligned} \quad (12)$$

be the gross nominal return on the investor's portfolio from period t to period $t + 1$ after payment of the tax on dividends and interest, but prior to the payment of capital gains taxes. Using this notation, Equation (4) can be written as a linear dynamic wealth equation:

$$W_{t+1} = R_{t+1}(f_t + b_t)W_t. \quad (13)$$

Similarly, the budget constraint in Equation (5) can be written as follows:

$$c_t = 1 - \tau_g\delta_t - f_t - b_t \quad (14)$$

where $c_t = C_t/W_t$ is the consumption-wealth ratio for period t ,

$$\begin{aligned} \delta_t &= G_t/W_t \\ &= \{I(p_{t-1}^* > 1)s_t + [1 - I(p_{t-1}^* > 1)]\max(s_t - f_t, 0)\}(1 - p_{t-1}^*) \end{aligned} \quad (15)$$

is the fraction of beginning-of-period wealth that is taxable as *realized* capital gains in period t , and p_{t-1}^* is given by

$$p_{t-1}^* = \begin{cases} \frac{[s_{t-1}p_{t-2}^* + \max(f_{t-1} - s_{t-1}, 0)]/(1 + g_t)}{s_{t-1} + \max(f_{t-1} - s_{t-1}, 0)}, & \text{if } p_{t-2}^* < 1 \\ \frac{1}{1 + g_t}, & \text{if } p_{t-2}^* \geq 1. \end{cases} \quad (16)$$

Substituting Equation (14) into Equation (13) allows us to rewrite the dynamic wealth equation as follows:

$$W_{t+1} = R_{t+1}(1 - \tau_g\delta_t - c_t)W_t. \quad (17)$$

The linearity of the dynamic wealth equation and the assumption of constant relative risk-averse preferences ensure that our two-asset model has the property that the consumption and portfolio decision rules, $\{c_t, f_t, b_t\}$, are independent of wealth, W_t . Furthermore, with the above transformation, the

relevant state variables for the investor's problem become $x_t = \{s_t, p_{t-1}^*\}$. Defining $v_t(x_t) = V_t(X_t)/[W_t/(1+i)^t]^{1-\gamma}$ to be the normalized value function and $w_{t+1} = W_{t+1}/[W_t(1+i)]$ to be the gross *real* growth rate in wealth from period t to period $t + 1$, the investor's optimization problem can now be stated as follows:

$$v_t(x_t) = \max_{c_t, f_t, b_t} \left\{ \frac{e^{-\lambda_t} c_t^{1-\gamma}}{1-\gamma} + \frac{(1 - e^{-\lambda_t})\beta(1 - \beta^H)A_H^{1-\gamma}}{(1 - \beta)(1 - \gamma)} + e^{-\lambda_t} \beta E_t[v_{t+1}(x_{t+1})w_{t+1}^{1-\gamma}] \right\}, \quad t = 0, \dots, T - 1, \quad (18)$$

s.t.

$$w_{t+1} = \frac{R_{t+1}}{1+i}(1 - \tau_g \delta_t - c_t), \quad t = 0, \dots, T - 1, \quad (19)$$

$$f_t \geq 0, \quad t = 0, \dots, T - 1 \quad (20)$$

where R_{t+1} is given by Equation (12), δ_t is given by Equation (15), and p_{t-1}^* is given by Equation (16).

The above problem can be solved numerically using backward recursion. To do this we discretize the lagged endogenous state variables, $x_t = \{s_t, p_{t-1}^*\}$, into a (501×501) grid. At the terminal date T , the investor's value function takes the value

$$v_T = \frac{\beta(1 - \beta^H)A_H^{1-\gamma}}{(1 - \beta)(1 - \gamma)} \quad (21)$$

at all points in the state space. The value function at date T is then used to solve for the optimal decision rules and value function for all points on the grid at date $T - 1$. Bilinear interpolation is used to calculate the value function for points in the state space that lie between the grid points. The procedure is repeated recursively for each time period until the solution for date $t = 0$ is found.

2. Numerical Results

In this section we present our numerical results for the optimal consumption–investment problem in the presence of taxes and short-sale restrictions. In the numerical analysis, we assume that the investor makes decisions annually starting at age 20 ($t = 0$) and lives for at most another 80 years until age 100 ($T = 80$). The annual mortality rate is calibrated to match the life expectancy table of the U.S. population.⁹ The annual mortality rates are less

⁹The mortality rates are calculated from the life expectancies in the 1980 Commissioners Standard Ordinary Mortality Table. The annualized mortality rate at time t , $\text{Prob}(t)$, is computed as follows: $\text{Prob}(t) = 1 - L(t)/[L(t+1) + 1]$, where $L(t)$ is the life expectancy at time t . We combine the resulting male and female mortality rates to reflect the different survival likelihoods implied by these rates.

than 1% through age 56, increase to 7.4% at age 80, 17.6% at age 90, 25.7% at age 95, and approach 100% at age 100.

We assume that the nominal pretax interest rate on the riskless bond is $r = 6\%$ per year, the nominal dividend yield on the stock is $d = 2\%$ per year, and the annual inflation rate is $i = 3.5\%$. The nominal capital gains return on the stock is assumed to follow a binomial process with an annual mean and standard deviation of $\bar{g} = 7\%$ and $\sigma = 20.7\%$ (corresponding to a 20% standard deviation for *real* stock returns), respectively. We assume that the tax rate on dividends and interest is $\tau_d = 36\%$ and the tax rate on capital gains and losses is $\tau_g = 36\%$. The investor is assumed to have power utility with an annual subjective discount factor of $\beta = 0.96$ and a risk aversion parameter of $\gamma = 3.0$. We initially set $H = \infty$ in the bequest function, indicating that the investor wishes to provide his beneficiary with a perpetual consumption stream from the bequest. We refer to these parameter values as the base-case scenario.

Because the dividend yield is assumed to be a constant fraction of the contemporaneous stock price, the pretax expected return on the stock is given by $(1 + \bar{g})(1 + d) - 1$. Using our base-case parameter values, this produces a pretax equity risk premium (above the riskless interest rate) of 3.14%, well below the historical average. Our choice of a relatively low pretax equity risk premium is motivated by our desire to generate optimal portfolios that have interior mixes of both bonds and stocks. The problem with using the historical average equity risk premium to calibrate the model is that it generates an optimal portfolio that is composed entirely of equity, unless investors are extremely risk averse. Heaton and Lucas (1997) demonstrate this result for a model without taxes. In the presence of taxes, generating interior portfolio mixes is even more difficult because the value of the tax-timing option further increases the returns to equity on an after-tax basis.¹⁰

We derive numerical results for our model using the base-case parameters. To isolate the effects of optimal tax timing, we also derive numerical results for a model without taxes and for a model with *forced* capital gain and loss realizations each period. The optimal consumption policy is examined in Section 2.1, while the optimal investment and liquidation policies are examined in Section 2.2. A welfare analysis of the optimal realization policy is presented in Section 2.3. In Section 2.4 we examine the sensitivity of the optimal investment and liquidation policies to various model parameters and in Section 2.5 we conduct a simulation analysis to study the time-series profile of the investor's optimal decisions.

¹⁰ One might expect that these same features of the tax code that increase the attractiveness of equity ownership in our setting would deepen the well-known equity premium puzzle. However, our model, and the resulting portfolio allocations, are based on a partial equilibrium analysis. Bossaerts and Dammon (1994) make the point that while the tax-timing option increases the expected after-tax return on equity, it also alters the covariance structure of after-tax equity returns with consumption. They test a consumption-based asset-pricing model with optimal tax timing and find that while it fits the data better than the traditional no-tax model, the tax-based model is still unable to adequately explain the cross-sectional differences in asset returns.

2.1 Optimal consumption policy

We begin our discussion of the optimal consumption policy by first considering the case without taxes. In the absence of taxes (and transaction costs), the optimal consumption-wealth ratio varies across age, but is independent of the investor's portfolio composition and the embedded capital gain or loss on the portfolio. Given the base-case parameters, the optimal consumption-wealth ratio gradually decreases with age, from about 3.1% per year at age 20 to about 2.9% per year at age 99 (the year prior to imminent death). The relationship between the optimal consumption-wealth ratio and age depends primarily on the investor's bequest motive. As an investor ages, and his mortality rate increases, the bequest motive plays a more important role in the investor's consumption decision. A strong bequest motive will produce a declining consumption-wealth ratio as the investor ages, reflecting his desire to provide his beneficiary with a higher level of consumption from the bequest. In contrast, a weak bequest motive will produce an increasing consumption-wealth ratio as the investor ages. The bequest motive we have chosen for our model is relatively neutral across ages in the no-tax case, as evidenced by the relatively flat (although slightly decreasing) consumption-wealth ratio.

With capital gains taxes, the optimal consumption-wealth ratio will not only depend upon age, but also will depend upon the magnitude of the embedded capital gain (or loss) on the investor's portfolio. The larger the total embedded capital gain, the higher is the embedded capital gains tax liability and the less wealthy the investor is on an *after-tax* basis. Thus holding the investor's pretax wealth constant, a larger embedded capital gain will produce a larger reduction in the investor's optimal consumption-wealth ratio.

With *forced* capital gain and loss realizations each period (and constant relative risk-averse preferences), the relationship between the optimal consumption-wealth ratio and the magnitude of the embedded capital gain is linear. This is due to the fact that, without a deferral option, an embedded capital gain results in an immediate tax liability that reduces the investor's after-tax disposable wealth dollar for dollar.¹¹ Using our base-case parameters, with $\tau_d = \tau_g = 36\%$, the optimal consumption-wealth ratios with *forced* realizations are substantially lower than in the no-tax case and are monotonically decreasing with age. For example, with no embedded capital gain or loss on the portfolio (i.e., $p^* = 1.0$), the optimal consumption-wealth

¹¹ In the absence of a deferral option, the relationship between the optimal consumption-wealth ratio and the magnitude of the embedded capital gain (or loss) can be written as follows:

$$c_t = k_t \{1 - [s_t(1 - p_{t-1}^*)] \tau_g\},$$

where k_t is a coefficient that varies across age (t) and the term in square brackets is the total embedded capital gain (or loss) per dollar of pretax wealth. Intuitively k_t is the optimal consumption per dollar of *after-tax* wealth and the term in curly brackets measures the investor's *after-tax* wealth per dollar of *pretax* wealth. The values of k_t in the above expression can be derived numerically.

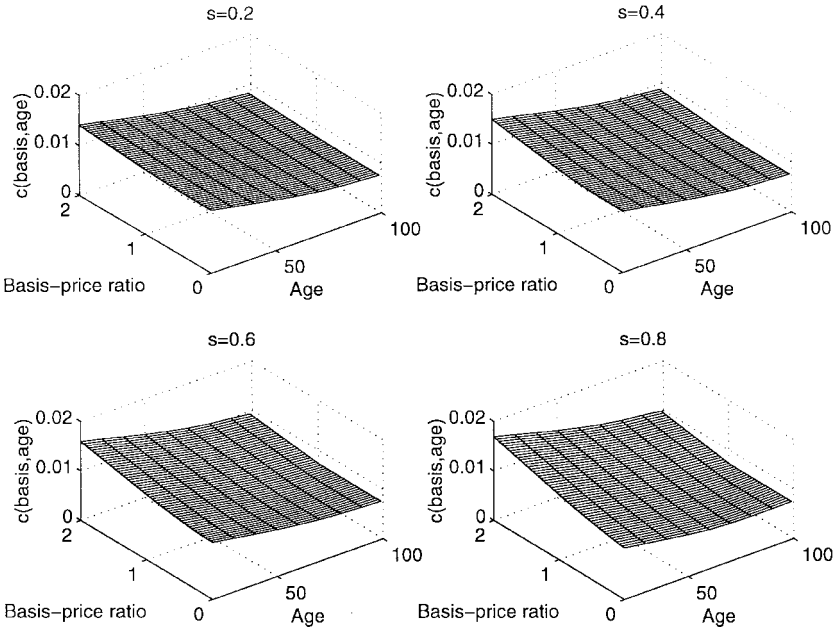


Figure 1
Optimal consumption policy

Optimal consumption-wealth ratios with the option to time the realization of capital gains and losses. The dividend and capital gains tax rates are assumed to be 36%. s is the initial stock holding as a fraction of beginning-of-period wealth.

ratio for ages 20 and 99 are 1.17% and 0.78%, respectively. These optimal consumption-wealth ratios indicate that the introduction of taxes has a stronger negative “wealth effect” on the consumption of the elderly. This is due to the fact that taxes have a disproportionate effect on the value of the investor’s bequest, which the elderly weight more heavily in making their consumption decisions.¹²

The optimal consumption-wealth ratio for the case in which investors have the option to realize or defer capital gains and losses each year is shown in Figure 1. Each of the four panels corresponds to a different initial endowment of equity, s . The optimal consumption-wealth ratios shown in Figure 1 are higher than those that emerged in the *forced* realization case. This is due to the fact that the deferral option, which increases the after-tax return on equity, essentially makes the investor wealthier on an after-tax basis and thus leads him to consume a larger fraction of his pretax wealth. This “wealth effect”

¹² Recall that the bequest is invested in an annuity contract that pays the after-tax interest rate in perpetuity ($H = \infty$). Thus an increase in the tax rate on ordinary income reduces the beneficiary’s consumption by a factor of $(1 - \tau_d)$. Since the beneficiary’s consumption is relatively more important in the utility functions of the elderly (due to their higher mortality rates), they respond to the increase in taxes by scaling back their own consumption by relatively more than do the young.

on consumption is more pronounced for the gains region ($p^* < 1$), where the deferral option is most valuable. Note also that the optimal consumption-wealth ratios in Figure 1 are now *convex* in the basis-price ratio. This reflects the convexity of the tax-timing option in the basis-price ratio (the exercise price of the tax-timing option). Consistent with standard option pricing theory, the sensitivity of the value of the tax-timing option (and the optimal consumption-wealth ratio) to the basis-price ratio is higher in the loss region (in-the-money option) than in the gains region (out-of-the-money option). Because young investors have longer horizons over which to benefit from the deferral option, their consumption-wealth ratios increase by more than do those for the elderly. Finally, because of the reset provision at death, the optimal consumption-wealth ratio is less sensitive to the basis-price ratio in the gains region for elderly investors when they have the option to defer their capital gains.

2.2 Optimal investment and liquidation policies

Having discussed the optimal consumption policy, we now consider the optimal investment and liquidation policies in the presence of taxes and short-sale restrictions. The no-tax case ($\tau_d = \tau_g = 0$) and the case with taxes ($\tau_d = \tau_g = 36\%$) and *forced* realizations of capital gains and losses each period will serve as benchmarks. Throughout the article, we report the optimal holding of equity as a proportion of total portfolio value, $f_i/(f_i + b_i)$. In the absence of taxes, the optimal proportion of the investor’s portfolio invested in equity is 25.2% for our base-case parameters. The corresponding optimal equity proportion for the case with taxes and *forced* realizations is 36.8%. While the introduction of taxes reduces the investor’s welfare, the higher equity holdings reflect the more favorable risk-return trade-off for equity on an after-tax basis.¹³ For both benchmarks, the optimal equity proportion is independent of the investor’s embedded capital gain (or loss) and age. Moreover, since there are no transaction costs, the investor rebalances his portfolio to the optimal equity proportion every period.

Figure 2 shows the optimal proportion of the portfolio invested in the risky stock for the case with taxes ($\tau_d = \tau_g = 36\%$), but where investors have the option to defer capital gains. The figure is drawn for an assumed initial stock holding (as a proportion of beginning-of-period wealth) of $s = 0.2$

¹³ When all sources of investment income are taxed at the same rate each period, the effect is to scale the returns on all assets by a factor of $(1 - \tau_d)$. This simple tax adjustment causes the ratio of the after-tax equity risk premium to the variance of after-tax equity returns to increase relative to the no-tax case, thereby causing the optimal equity holdings to increase above the no-tax optimum of 25.2%. To see the intuition underlying this result, consider the optimal holding of equity in a continuous-time model with all investment income taxed each period at the rate of τ_d . A simple tax adjustment to the Merton (1971) solution yields

$$\theta^* = \frac{(\mu - r)(1 - \tau_d)}{\gamma\sigma^2(1 - \tau_d)^2} = \frac{\mu - r}{\gamma\sigma^2(1 - \tau_d)},$$

where μ is the expected pretax return on the stock and all other variables have been defined previously. Note that the optimal holding of equity is increasing in the tax rate τ_d for all $0 \leq \tau_d < 1$.

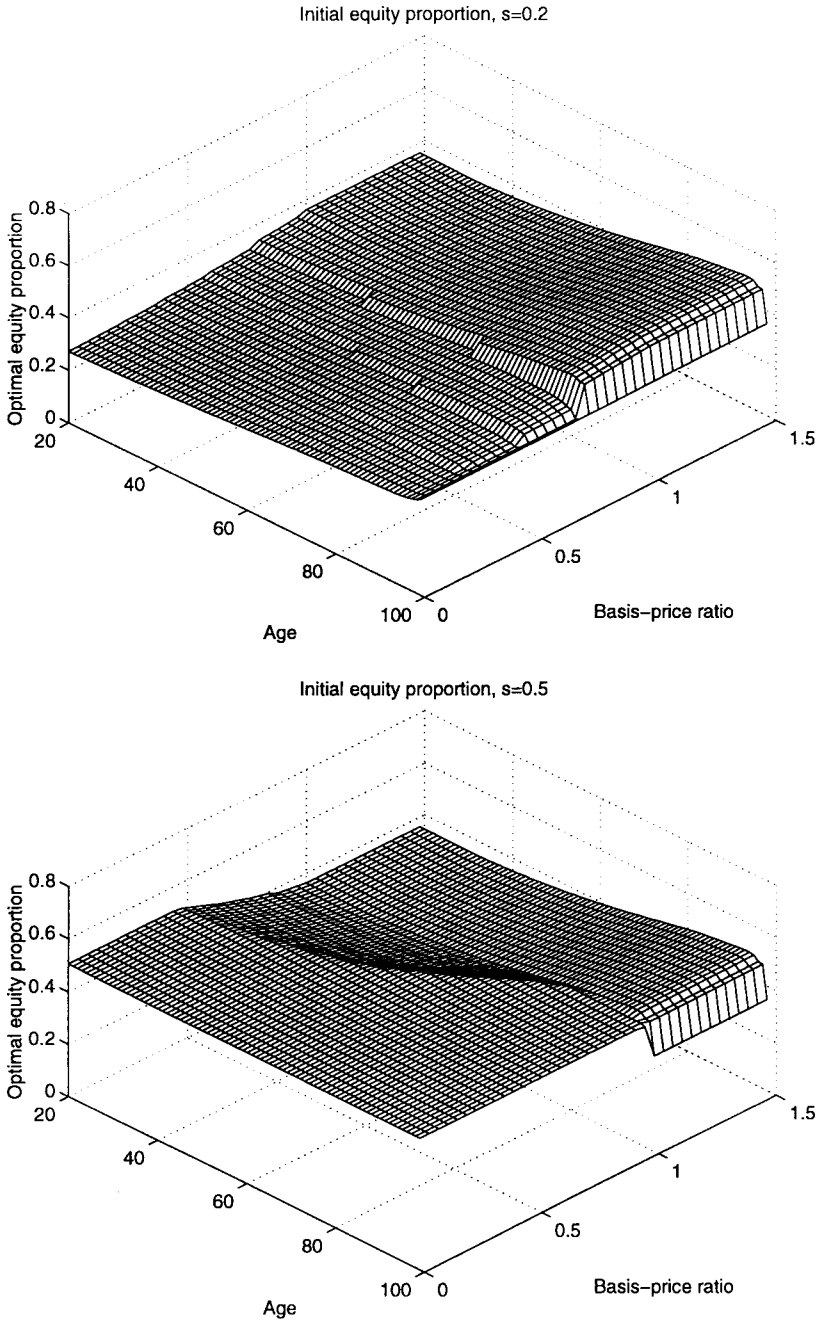


Figure 2
Optimal stock holding

Optimal stock holding with the option to time the realization of capital gains and losses. The dividend and capital gains tax rates are assumed to be 36%. The initial stock holding as a fraction of beginning-of-period wealth is set at $s = 0.2$ for the top panel and $s = 0.5$ for the bottom panel.

and $s = 0.5$. In contrast to the no-tax and *forced* realization cases, both the investor's basis-price ratio and age affect the optimal stock holding in the presence of the deferral option. In the upper panel ($s = 0.2$) the investor is initially underweighted in equity, while in the lower panel ($s = 0.5$) the investor is initially overweighted in equity.

In both panels of Figure 2, investors optimally sell all shares of stock with an embedded capital loss to benefit from the tax rebate before rebalancing their portfolios to the desired asset allocation. Because of the complete liquidation of shares with embedded capital losses, investors are unconstrained as to how they reallocate their portfolios. Thus the optimal holding of equity in the loss region ($p^* \geq 1.0$) after rebalancing depends only on the investor's age and is independent of his initial endowment of equity, s , and basis-price ratio, p^* . We refer to these optimal equity holdings as the *unconstrained optimum*. Note that the unconstrained optimal holding of equity is increasing with age until it reaches a maximum a few years prior to the terminal date. The unconstrained optimal holding of equity for young investors (through about age 60) is similar to the optimal equity holding in the *forced* realization case (36.8%) despite the fact that the deferral option makes equity more valuable. This is because young investors realize substantial capital gains for diversification reasons (see the discussion below) and, to the extent that they do defer capital gains, they run the risk of being overexposed to equity at later ages. To mitigate this risk, investors limit the extent to which they adjust their equity holdings at young ages. The unconstrained optimal equity holding increases for elderly investors because they not only benefit from the ability to realize losses, but in the event of a capital gain they also benefit from the reset provision at death. The age at which the combined benefit of the tax-timing option and the reset provision of the tax code are maximized occurs at a late age. This is also the age at which the optimal holding of equity is at its maximum.¹⁴ For the base-case parameters, the unconstrained optimal equity holding reaches its peak of 51.2% at age 94 in Figure 2, substantially higher than the optimal holding of equity in the *forced* realization case.

In the gains region ($p^* < 1.0$) in Figure 2, the investor's optimal stock holding depends upon his initial stock holding, basis-price ratio, and age. Investors increase their holding of equity in the gains region if they are initially underweighted in the stock ($s = 0.2$). Note, however, that the addition

¹⁴ The fact that the optimal holding of equity declines in the last few years before the terminal date is an artifact of the way we model the timing of events within a period. We assume that death occurs before the realization decision can be implemented and therefore all embedded capital gains *and losses* are untaxed at the time of death. Since the investor does not benefit from the ability to realize capital losses in the last period of his life, he optimally reduces his exposure to equity at very late ages when his mortality rate is the highest. If we had allowed investors to implement their optimal realization decisions before death occurs, then the optimal holding of equity would have continued to increase as the investor ages. In this case, the optimal holding of equity remains relatively unchanged through about age 80 due to the lower mortality rates for these investors, but then increases dramatically from about 50% at age 80 to nearly 78% at age 99. Similar behavior also arises in a model in which investors are allowed to trade more frequently than once per year (e.g., quarterly).

to the equity holdings increases as the magnitude of the investor's embedded capital gain falls. This is due to the fact that a higher embedded capital gain on existing shares reduces the value of the tax-timing option on any new shares purchased because the tax basis is a weighted average of the purchase prices of both old and new shares.¹⁵ In the lower panel of Figure 2 ($s = 0.5$), all investors retain their initial endowment of equity, and do not add to their holdings of equity, for embedded capital gains in excess of about 100% ($p^* \leq 0.5$). While some of these investors would like to reduce their exposure to equity for diversification reasons, the tax cost of doing so is prohibitive. For somewhat smaller gains ($0.5 < p^* < 1.0$), young and middle-aged investors scale back their holdings of stock. For these investors, the benefit of diversification outweighs the tax cost of rebalancing their portfolios. The smaller the embedded capital gain (p^* closer to 1.0), the smaller is the tax cost of rebalancing the portfolio and therefore the closer these investors get to their unconstrained optimum. In general, the amount of rebalancing the investor does will depend upon the size of the embedded capital gain, the investor's age, and the extent to which the investor's portfolio deviates from the unconstrained optimum.

In contrast to the standard financial advice that investors should reduce their exposure to equity as they age, the above discussion provides a tax-based rationale as to why investors may wish to increase their holdings of equity as they get older.¹⁶ Equity is more valuable to the elderly because it allows them to benefit from the realization of capital losses if they occur, but avoid paying the tax on capital gains by deferring them until death. The forgiveness of capital gains taxes at death is also why elderly investors are more reluctant to sell equity with an embedded capital gain in order to rebalance their portfolios. At very late ages, however, the option to realize losses decreases in value because the higher mortality rate increases the likelihood that death will occur before any losses can be realized (see note 14). At this point, investors should retain their holdings of equity with large embedded

¹⁵ The discrete jumps (steps) in the optimal stock holding that are present in the gains region in the upper panel ($s = 0.2$) of Figure 2 reflect the discrete binomial price process we assume for the stock. The discrete price process leads to discrete jumps in the value of the tax-timing option as we vary the basis-price ratio. With a continuous price process, the discrete jumps in the optimal stock holding will disappear. Moreover, if newly acquired shares were assigned a tax basis equal to their own purchase price, the effect of the existing tax basis on the optimal stock holding would be minimal. The effect of the existing tax basis on the investor's optimal stock holding would not disappear entirely, however, because it still affects the investor's after-tax wealth.

¹⁶ Focusing exclusively on the ability of investors to defer capital gains taxes, Balcer and Judd (1987) use a model with deterministic equity returns and no bequest motive or reset provision at death to argue that young (long-horizon) investors hold all-equity portfolios because they benefit the most from the deferral option, while elderly (short-horizon) investors hold all-bond portfolios. With uncertainty, however, all investors will hold diversified portfolios of stocks and bonds. Moreover, in our framework, the tax-timing option coupled with the forgiveness of capital gains taxes at death makes equity relatively more attractive to the elderly. Thus the optimal holding of equity is increasing with age in our model.

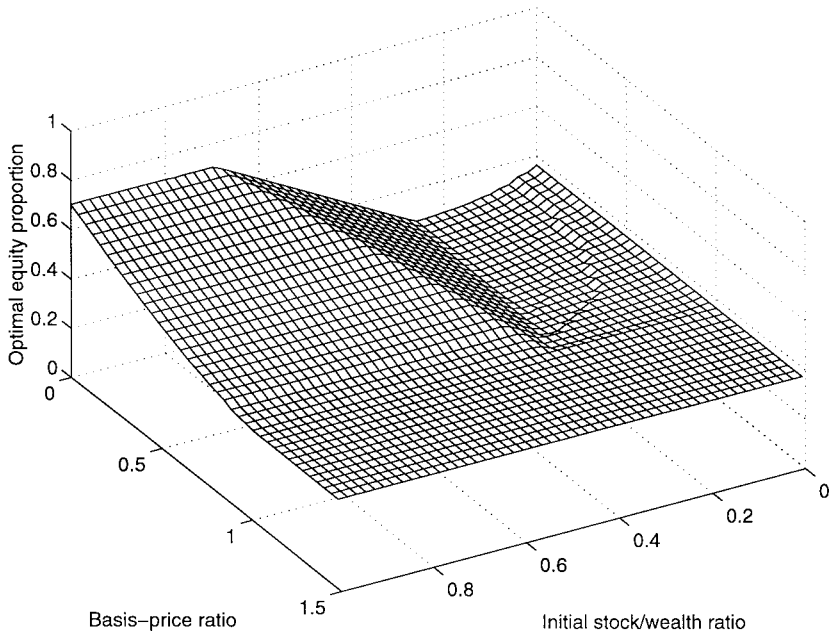


Figure 3
Optimal stock holding at age 20

Optimal stock holding as a function of the basis-price ratio and initial stock holdings at age 20. The dividend and capital gains tax rates are assumed to be 36%.

capital gains to benefit from the reset provision at death, and reduce their equity exposure only if the tax cost is relatively small.¹⁷

Figure 3 provides an overall picture of the optimal investment and liquidation policies for investors at age 20. Because investors at age 20 have very low mortality rates, these policies are a close approximation to the optimal policies in the infinite-horizon case. With an embedded capital loss, investors optimally liquidate their initial stock holdings and immediately rebalance to their unconstrained optimal holding of equity (36.3%). This is illustrated by the flat portion of the figure for the region in which $p^* \geq 1.0$.

In the gains region in Figure 3 ($p^* < 1.0$), investors with initial equity holdings above the unconstrained optimum face a trade-off between the diversification benefits and tax costs of rebalancing their portfolios. The smaller the gain, the closer the investor moves toward the unconstrained optimal equity holding, as illustrated by the front side of the pyramid in the figure. At each point in this region, the diversification benefits and tax costs of rebalancing are equated at the margin. For investors whose initial equity holdings far exceed the unconstrained optimum, the diversification benefits

¹⁷ Poterba (2001) documents that elderly investors with substantial appreciation in their portfolios tend to postpone the distribution of a larger proportion of their assets until after death compared to their contemporaries of similar wealth who possess smaller embedded capital gains. This is evidence of how the value of the reset provision varies across elderly investors with different embedded capital gains.

of rebalancing are substantial and can lead to the realization of large embedded capital gains. For example, Figure 3 indicates that a 20-year-old investor with an all-equity portfolio ($s = 1.0$) and an embedded capital gain of 100% ($p^* = .5$) will optimally sell about one-half of his equity holdings to rebalance his portfolio. Thus it is not unreasonable that undiversified investors would occasionally realize substantial capital gains to rebalance their portfolios. For sufficiently high embedded capital gains, however, the tax cost of rebalancing outweighs the diversification benefits and the investor optimally retains his initial equity holding. The right side of the pyramid in the figure illustrates this behavior.

Investors with embedded capital gains who are initially underweighted in equity (below the unconstrained optimum) face a different trade-off. For these investors, adding equity to their portfolios moves them closer to their unconstrained optimum, but the value of the tax-timing option on new shares is negatively affected by the embedded capital gain on existing shares. This accounts for the decline in the optimal stock holdings as the size of the investor's embedded gain increases in this region.

2.3 Welfare analysis

The previous section examined the effect of taxes on the optimal investment and liquidation policies. In this section we investigate the welfare cost of ignoring the optimal policies by following some alternative policies that do not take full advantage of the tax-timing option. The two alternative policies that we consider are (1) realizing all capital gains and losses each period and (2) never realizing capital gains or losses except to finance consumption (buy and hold). The first alternative takes advantage of the ability to take losses, but ignores the option to defer capital gains. The second alternative takes advantage of the ability to defer capital gains, but ignores the option to realize losses. We solve numerically for the optimal consumption and portfolio rules for each of these alternative realization policies using the base-case parameters. The resulting value functions are compared to the value function from our model over the entire state space. For each point in the state space, we calculate the percentage increase in wealth that would be needed to bring the value function for the alternative policy up to the level of the value function for our model. The results are illustrated in Figure 4 for an initial equity proportion of $s = 0.5$, which is only a slight deviation from the unconstrained optimal equity holdings.

The top panel of Figure 4 shows the welfare cost calculations for the first alternative of realizing gains and losses every period. Note that the welfare costs are increasing in the size of the embedded capital gain and decreasing in the age of the investor. This makes intuitive sense because, under the optimal policy, investors defer most large embedded capital gains and young investors, with longer horizons, have the most to lose from deviating from the optimal policy. The welfare cost is nonzero in the loss region ($p^* > 1$),

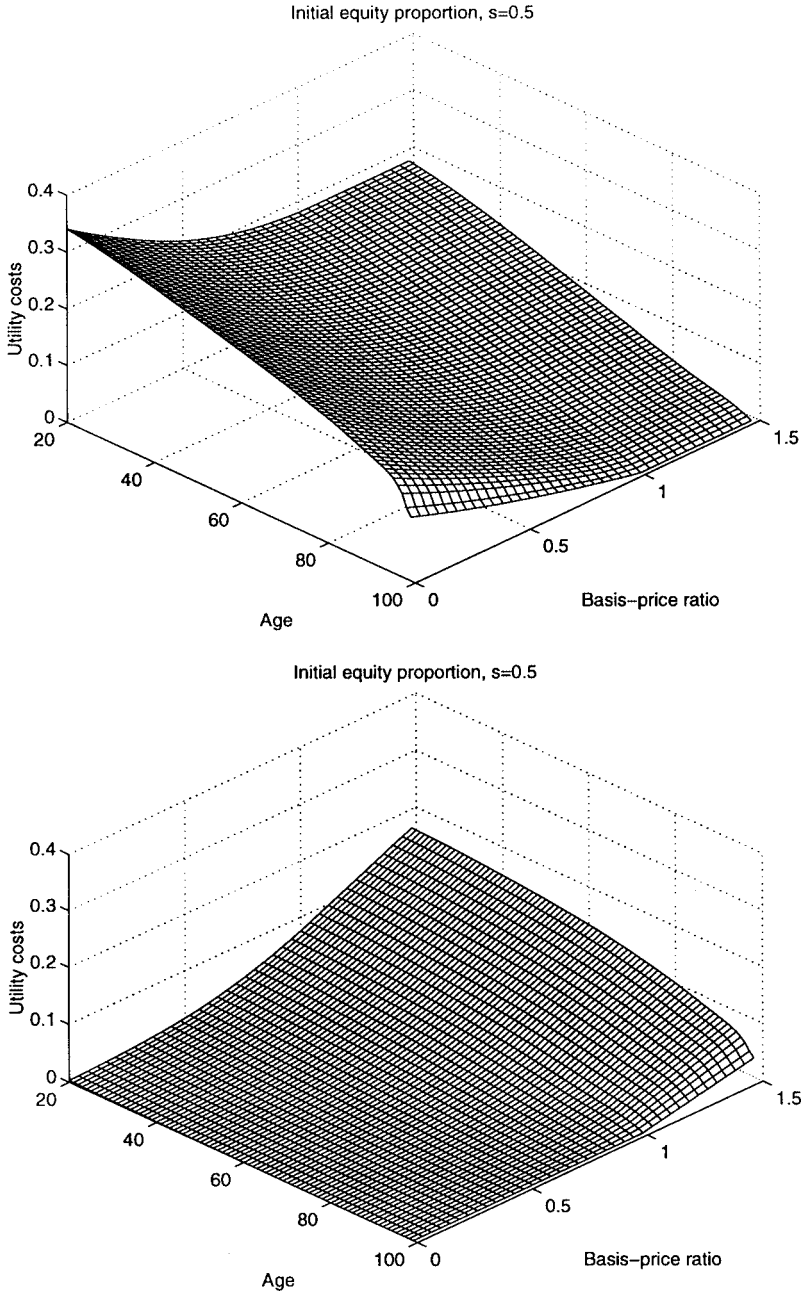


Figure 4

Welfare costs of alternative investment policies

Welfare costs measured as the extra wealth needed to compensate an investor who follows an alternative suboptimal investment policy as compared to the optimal policy. The top panel shows the welfare cost per unit of wealth for the policy of realizing both gains and losses every period. The bottom panel shows the welfare cost per unit of wealth for a buy-and-hold strategy. The initial stock holding as a fraction of beginning-of-period wealth is set at $s = 0.5$ for both panels.

even though it is optimal to realize all capital losses, because the alternative policy is still suboptimal for the future. The welfare cost of following the alternative policy of realizing capital gains and losses every period is between 15% and 30% for young investors.¹⁸ The bottom panel of Figure 4 shows the welfare cost calculations for the second alternative of deferring capital gains and losses every period (buy and hold). As expected, the welfare costs are increasing in the size of the embedded capital loss and decreasing in the investor's age. Note that the welfare costs are relatively small (near zero) for large embedded capital gains. This reflects the fact that with a large embedded capital gain, it is highly unlikely that the investor will incur a capital loss in the future. Thus, for large embedded capital gains, the buy-and-hold policy and the optimal policy are effectively the same. This is not true, however, for small embedded capital gains, where there is a higher probability of a future capital loss.

The above discussion begs the question as to whether the buy-and-hold policy, augmented to take advantage of capital losses, is substantially different from the optimal realization policy derived in our model. The augmented buy-and-hold policy advocates realizing all capital losses and deferring all capital gains. The main difference, of course, is that the optimal policy in our model advises young investors to sell some equity with embedded capital gains for diversification reasons when they are substantially overweighted in equity. Elderly investors are advised to avoid realizing capital gains to benefit from the reset provision at death. The welfare cost calculations for the augmented buy and hold policy (not shown) indicate that it provides nearly identical benefits as the optimal policy in our model. Not surprisingly, the only points in the state space at which the welfare costs are noticeably different are for young investors with small embedded capital gains. But even in this case the incremental wealth needed to fully compensate the investor is less than 5% when $s = 0.5$. The welfare costs of the augmented buy-and-hold policy will increase as the volatility of the stock, risk aversion of investors, or the initial overweighting of equity increases.

2.4 Comparative static analysis

We now provide some comparative static results by varying the capital gains tax rate, the volatility of stock returns, and the number of periods the investor wishes to provide consumption support to his beneficiary. Because the most interesting changes are reflected in the investor's portfolio composition, our discussion will be focused on the optimal investment policies. Figure 5 illustrates the optimal equity holdings for a tax rate on capital gains and losses

¹⁸ If the reset provision is eliminated, so that all capital gains and losses are fully taxed at death, the welfare analysis will change slightly. For young investors, there will be almost no effect on the welfare cost calculations, since young investors have low mortality rates and are not influenced much by the treatment of capital gains and losses at death. Elderly investors, on the other hand, will find realizing capital gains every period less costly since these capital gains, if deferred, will be taxed at death. As a consequence, the welfare costs are reduced, but not eliminated, in the gains region for elderly investors when the reset provision is eliminated.

of 20% (top panel) and for a 31.05% standard deviation of nominal equity returns (bottom panel). Both figures are constructed assuming an initial equity holding of $s = 0.5$ and by holding all other parameters fixed at their base-case values. The lower panel of Figure 2 provides the appropriate comparison to determine the marginal effects of these changes.

A cut in the capital gains tax rate from 36% (lower panel of Figure 2) to 20% (upper panel of Figure 5) has two major effects on the optimal equity holdings. First, because the tax cost of rebalancing is reduced when the capital gains tax rate is cut, investors are willing to realize higher capital gains in order to rebalance their portfolios. This is particularly true for young investors who benefit the most from rebalancing. Second, in the loss region, the optimal holding of equity is less sensitive to age when the capital gains tax rate is reduced. This is to be expected, since a cut in the capital gains tax rate reduces the value of the option to realize losses and defer gains, especially for elderly investors who benefit the most from the reset provision at death. The optimal holding of equity for young investors is only slightly higher with a lower capital gains tax rate because the lower tax cost of realizing gains to rebalance the portfolio is nearly offset by the lower tax rebate on capital losses.

An increase in the volatility of stock returns from 20.7% (lower panel of Figure 2) to 31.05% (lower panel of Figure 5) has a dramatic effect on the optimal equity holdings. Because equity is riskier, the optimal holding of equity is reduced for all investors, except for those elderly investors with large embedded capital gains who continue to hold their initial equity positions to avoid paying the capital gains tax. With an increase in volatility, investors are willing to bear the tax cost of realizing larger capital gains in order to maintain a well-diversified portfolio. This is evident in the lower panel of Figure 5, where young and middle-aged investors reduce their exposures to equity even when the tax basis is zero. Although the decline in the optimal holding of equity is quite dramatic, it is muted to some extent by the fact that a higher level of volatility increases the value of the tax-timing option on equity. Similar results to those for an increase in volatility can be obtained with an increase in the risk aversion of investors.

As a final comparative static exercise, we examine the importance of the investor's bequest function for the optimal consumption, portfolio allocation, and liquidation decisions. Consider an investor whose bequest is invested in a 5-year ($H = 5$) annuity that provides consumption for his beneficiary. To equate his own marginal utility of consumption with that of his beneficiary, the investor's optimal consumption-wealth ratio will increase dramatically as he ages. The investor's optimal consumption rate is affected very little by the bequest function at young ages because of the low mortality risk. Assuming an initial basis-price ratio of $p^* = 1.0$, the optimal consumption-wealth ratio is 2.7% at age 20, 3.7% at age 50, and 7.8% at age 80. By the time the investor reaches age 99, his optimal consumption-wealth ratio is in excess

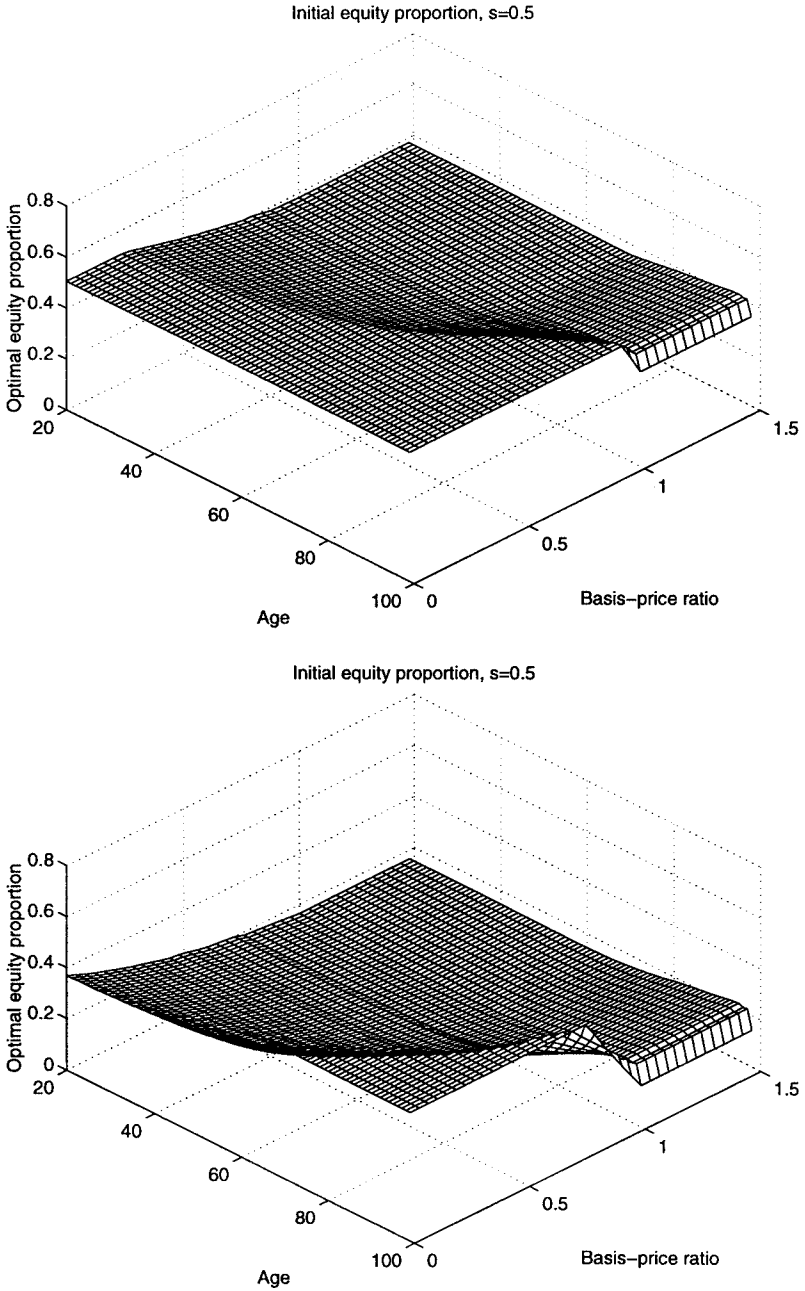


Figure 5
Comparative static analysis

The top panel shows the optimal stock holding as a function of the basis-price ratio and initial stock holdings when the tax rate on capital gains and losses is lowered to 20%, while the dividend tax rate remains at 36%. The bottom panel shows the optimal stock holding when the stock return volatility is increased from 20.7% to 31.05%, while the dividend and capital gains tax rates are assumed to be 36%. The initial stock holding as a fraction of beginning-of-period wealth is set at $s = 0.5$.

of 17%, which is close to the beneficiary's initial draw rate on the bequest. The investor's optimal consumption-wealth ratio is higher (lower) at each of these ages as the investor's basis-price ratio increases (decreases).

Figure 6 shows the optimal holding of equity for this case ($H = 5$). To provide a better view, the axes for age and the basis-price ratio have been switched compared to the earlier figures. The upper panel is for an initial stock holding of $s = 0.2$ and the lower panel is for an initial stock holding of $s = 0.5$. The optimal equity proportions shown in Figure 6 are heavily influenced by the investor's higher consumption rates in this case. To finance their higher consumption levels, investors are forced to liquidate a larger portion of their portfolio holdings each period. When investors are underweighted in equity (upper panel) they sell bonds to finance their higher consumption levels and, in addition, may add equity to their portfolios. When investors are overweighted in equity (lower panel) and have large embedded capital gains, there is a choice as to how much of each security to sell. For elderly investors, the reset provision at death induces them to sell only bonds to finance their higher consumption rates, thereby causing their optimal equity holdings to increase to nearly 60% in the lower panel of Figure 6. For young and middle-aged investors, diversification considerations induce them to sell both bonds and stocks to finance consumption and to realize larger gains than those shown in Figure 2. Because the realization of large embedded capital gains makes equity less valuable, the unconstrained optimal holding of equity (i.e., the region where $p^* > 1$) is lower for middle-aged investors than that shown in Figure 2.

2.5 Simulation analysis

Our discussion so far has been focused on the optimal decision rules over the state space at different ages. Given the investor's optimal consumption and investment policies defined on the state space, we can obtain time-series profiles of consumption and portfolio choices using simulation. First, we simulate realizations of the capital gain return using the binomial process. We then use the optimal decision rules from the state-space analysis to calculate the investor's consumption and portfolio allocations. The basis-price ratio, stock holdings, and age are updated in each period of the simulation to provide time-series profiles of the optimal decisions and state variables for the investor.

Table 1 summarizes the results of 5,000 simulation trials. The basis-price ratio (panel A), consumption-wealth ratio (panel B), and equity proportion (panel C) are shown for two alternative bequest functions: one in which the bequest provides a perpetual annuity ($H = \infty$) and the other in which the bequest provides a 5-year annuity ($H = 5$). We present the means and the quartiles of the distributions for each of these variables. Although the distributions are obviously related to each other, it should be noted that the quartiles for each distribution are not from the same simulation trial. Our

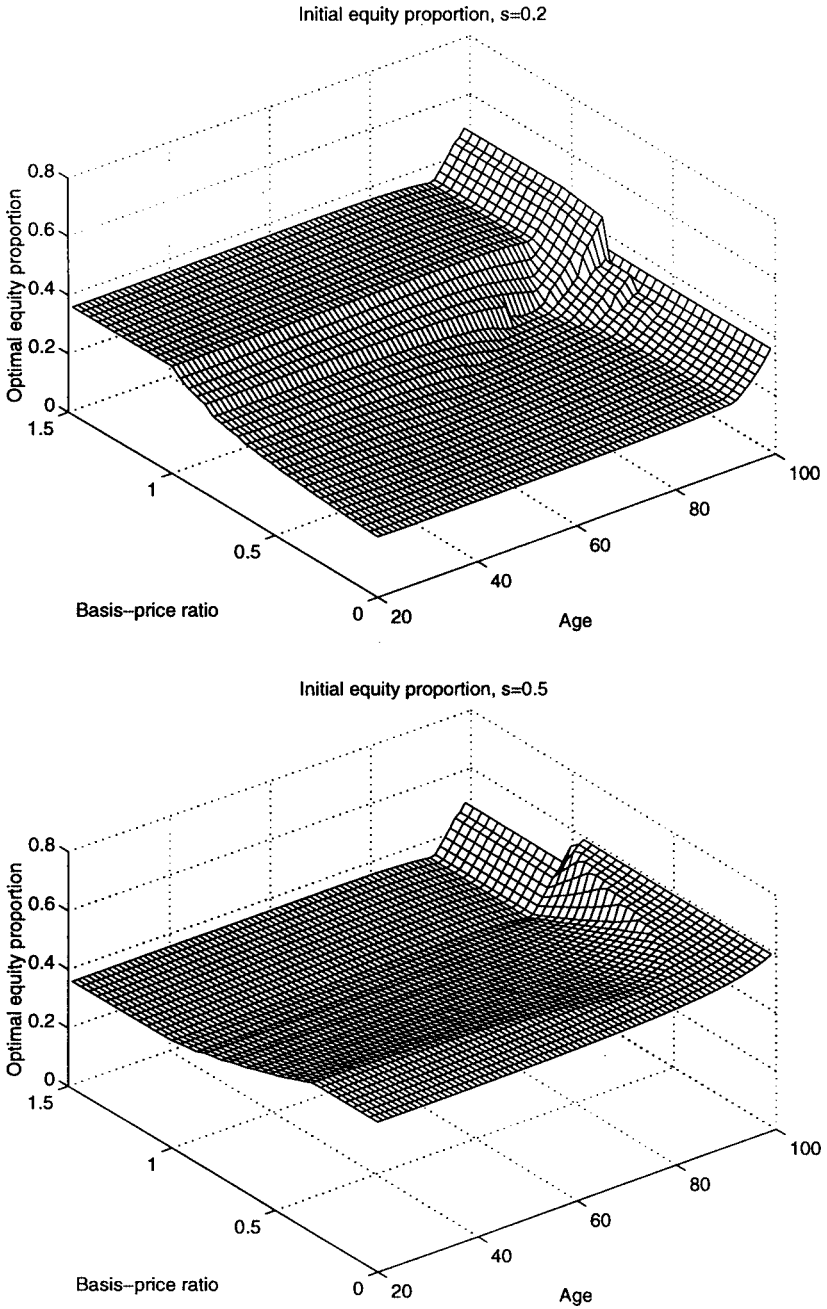


Figure 6

Optimal stock holding with a weaker bequest motive

Optimal stock holding for an alternative bequest function in which the investor provides consumption support to his beneficiary for five periods. The dividend and capital gains tax rates are assumed to be 36%. The initial stock holding as a fraction of beginning-of-period wealth is set at $s = 0.2$ for the top panel and $s = 0.5$ for the bottom panel.

Table 1
The distribution of the simulated basis-price ratio, consumption-wealth ratio, and the equity proportion

Age	$H = \infty$				$H = 5$			
	Mean	25%	50%	75%	Mean	25%	50%	75%
Panel A: Basis-price ratio								
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	.5540	.3590	.5310	.7110	.5539	.3590	.5310	.7110
40	.3665	.1730	.3100	.4870	.3655	.1730	.3100	.4830
50	.2569	.0962	.1770	.3440	.2537	.0962	.1730	.3440
60	.1827	.0487	.1070	.2330	.1763	.0487	.1070	.2220
70	.1407	.0274	.0667	.1670	.1291	.0272	.0656	.1650
80	.1118	.0167	.0410	.1190	.0947	.0167	.0404	.1070
90	.0890	.0089	.0273	.0811	.0686	.0089	.0248	.0728
99	.0766	.0055	.0176	.0630	.0513	.0055	.0167	.0495
Panel B: Consumption-wealth ratio								
20	.0131	.0131	.0131	.0131	.0272	.0272	.0272	.0272
30	.0116	.0113	.0116	.0119	.0276	.0268	.0277	.0284
40	.0105	.0101	.0105	.0109	.0297	.0286	.0296	.0306
50	.0096	.0092	.0096	.0100	.0333	.0321	.0331	.0343
60	.0089	.0085	.0089	.0093	.0393	.0380	.0390	.0402
70	.0083	.0080	.0084	.0087	.0495	.0479	.0493	.0506
80	.0079	.0077	.0080	.0082	.0673	.0652	.0672	.0685
90	.0078	.0076	.0078	.0080	.0943	.0916	.0937	.0963
99	.0077	.0076	.0077	.0077	.1650	.1640	.1640	.1660
Panel C: Equity proportion								
20	.3630	.3630	.3630	.3630	.3690	.3690	.3690	.3690
30	.4672	.4010	.4580	.5280	.4413	.3980	.4350	.4870
40	.5187	.4300	.5220	.6070	.4763	.4260	.4870	.5310
50	.5552	.4570	.5630	.6530	.4964	.4540	.5100	.5480
60	.5882	.4810	.5970	.7000	.5093	.4790	.5250	.5560
70	.6137	.4950	.6200	.7300	.5174	.4960	.5330	.5560
80	.6387	.5180	.6470	.7640	.5365	.5220	.5480	.5640
90	.6597	.5370	.6750	.7900	.7251	.6890	.7410	.7780
99	.6724	.5510	.6900	.8050	1.1973	1.2000	1.2000	1.2000

The percentages in the table refer to percentiles of the distribution. The results are based on 5,000 simulation trials, starting at age 20. The simulated capital gain return paths are identical for the two bequest functions examined.

simulation begins for an investor at age 20 with an initial basis-price ratio of $p_{-1}^* = 1$. While the investor has some limited opportunities to realize losses at early ages, the basis-price ratios in Table 1 indicate that he quickly becomes “locked in” to a capital gain. By age 30, the investor’s basis-price ratio has fallen below $p^* = .711$ in 75% of the trials. This “lock in” effect becomes more severe as the investor ages.¹⁹

The optimal consumption-wealth ratios in Table 1 show a sharp contrast between the two bequest functions. When $H = \infty$, the investor consumes

¹⁹ Related to the “lock in” effect, the portfolio turnover rates also drop precipitously as the investor ages. At age 21, the portfolio turnover rate is 50.4% on average, reflecting the investor’s early opportunities to realize losses for tax reasons. At age 30, the average turnover rate drops to less than 8% and by age 65 it drops to less than 1%.

about 1.3% of his wealth at age 20 and gradually reduces his consumption as he ages. As mentioned previously, this is due to the fact that the investor has a relatively strong bequest motive. However, when $H = 5$, the investor's consumption level is much higher and increases rapidly as he ages, approaching 16.5% on average at age 99. Intuitively, under the alternative bequest arrangement, the beneficiary's per period consumption level is much higher and his marginal utility much lower. Thus, to equate his own marginal utility to that of his beneficiary, the investor must increase his consumption significantly as he ages.

Panel C of Table 1 shows that the investor's stock holdings increase as he ages under both bequest arrangements. This reflects the fact that elderly investors refrain from selling equity with large embedded capital gains in order to benefit from the reset provision at death. While the optimal portfolio in the case of a perpetual annuity ($H = \infty$) remains less than 100% equity, the optimal portfolio under the alternative bequest arrangement ($H = 5$) approaches 120% equity at age 99. This is closely related to the investor's optimal consumption discussed previously. To finance his optimal consumption plan under the alternative bequest arrangement, the investor must liquidate a portion of his portfolio each period as he ages. Because the investor is "locked in" to a large capital gain, he refrains from selling equity to finance his consumption. The investor first liquidates his holdings of bonds to finance consumption, and in the last few years actually borrows to consume. As a result, the optimal equity proportion exceeds 100% at late ages.²⁰

3. Alternative Model Specifications

In this section we examine the optimal investment and liquidation policies under three alternative model specifications. The alternatives we consider are (1) mandatory capital gains realization at death (i.e., no reset provision), (2) asymmetric long-term and short-term tax rates, and (3) the introduction of nonfinancial income.

3.1 Mandatory capital gains realization at death

We first discuss the optimal investment and liquidation policies with mandatory capital gain and loss realization at death. Investors retain the option to defer capital gains prior to death. In this case, the investor's assets are liquidated at death, taxes are paid on any embedded capital gain or loss, and the

²⁰ The results reported in Table 1 are based on the restriction that the optimal equity holding as a fraction of beginning-of-period wealth, f_t , is not allowed to exceed 1.0. This restriction is imposed on the simulation because our state-space analysis derived the optimal decision rules only for initial stock holdings, s_t , between 0 and 1. Using the budget constraint in Equation (14), this restriction on f_t limits the investor's total outstanding borrowing at date t , $-b_t$, to the sum of the investor's contemporaneous consumption, c_t , and realized capital gains tax liability, $\tau_g \delta_t$. As a consequence, the maximum equity holding as a fraction of total portfolio value is $f_t / (f_t + b_t) = 1 / (1 - c_t - \tau_g \delta_t)$. Without the restriction on f_t , the investor's borrowing and optimal equity proportion at late ages would exceed those reported in Table 1.

after-tax proceeds are invested in a perpetual annuity ($H = \infty$) to finance the beneficiary's consumption. Capital gains and losses are again calculated relative to a tax basis equal to the weighted average purchase price. The mandatory realization of capital gains and losses at death and the average tax basis calculation are both consistent with the current tax code in Canada.

Figure 7 shows the optimal stock holdings as a function of age and the basis-price ratio. Similar to earlier results, investors optimally realize all losses to benefit from the tax rebate and immediately rebalance their portfolios to the unconstrained optimum. Since young investors have very low mortality rates, it should not be surprising that their optimal holdings of equity are relatively insensitive to the treatment of embedded capital gains at death. This is the case in both the gain and loss regions of the two panels ($s = 0.2$ and $s = 0.5$) in Figure 7. Elderly investors, however, are seriously negatively affected by the elimination of the reset provision at death and respond by altering their optimal portfolio allocation and liquidation policies. With mandatory realization of capital gains at death, the benefit of deferring capital gains is limited to the time value of the capital gains tax, which is relatively small for elderly investors who have high mortality rates. As a consequence, elderly investors optimally realize substantially higher capital gains and hold less equity than in the case with the reset provision at death. This can be seen by comparing Figure 7 to Figure 2. Since capital gains taxes cannot be avoided through deferral, elderly investors become more concerned about maintaining an optimally diversified portfolio. This makes elderly investors behave more like the young, with similar benefits from holding equity. Consequently the optimal holding of equity is similar across ages in Figure 7.

3.2 Asymmetric taxation

Under the current U.S. tax code, long-term capital gains and losses are taxed at a lower rate than short-term capital gains and losses. We now investigate the implications of this asymmetry in tax rates for the investor's optimal portfolio allocation. Under asymmetric taxation, Constantinides (1984) has shown that it can be optimal to sell (and immediately repurchase) stock with an embedded long-term capital gain in order to reset the tax basis to the current market price and restart the option to realize future capital losses short term.²¹ In his model, the optimality of realizing long-term capital gains each year depends upon the mean and standard deviation of stock returns, the risk-free interest rate, the size of transaction costs, and the magnitudes of the short-term and long-term tax rates. Assuming a short-term tax rate of $\tau_s = 36\%$ and a long-term tax rate of $\tau_l = 20\%$, the condition derived by Constantinides (1984) for optimally realizing long-term capital gains each

²¹ In a model with multiple trading dates within the year, Dammon and Spatt (1996) have shown that it also can be optimal to defer the realization of short-term capital losses prior to the end of the short-term holding period, even in the absence of transaction costs. This is due to the fact that the value of the restarting option is a concave function of the time remaining in the short-term region.

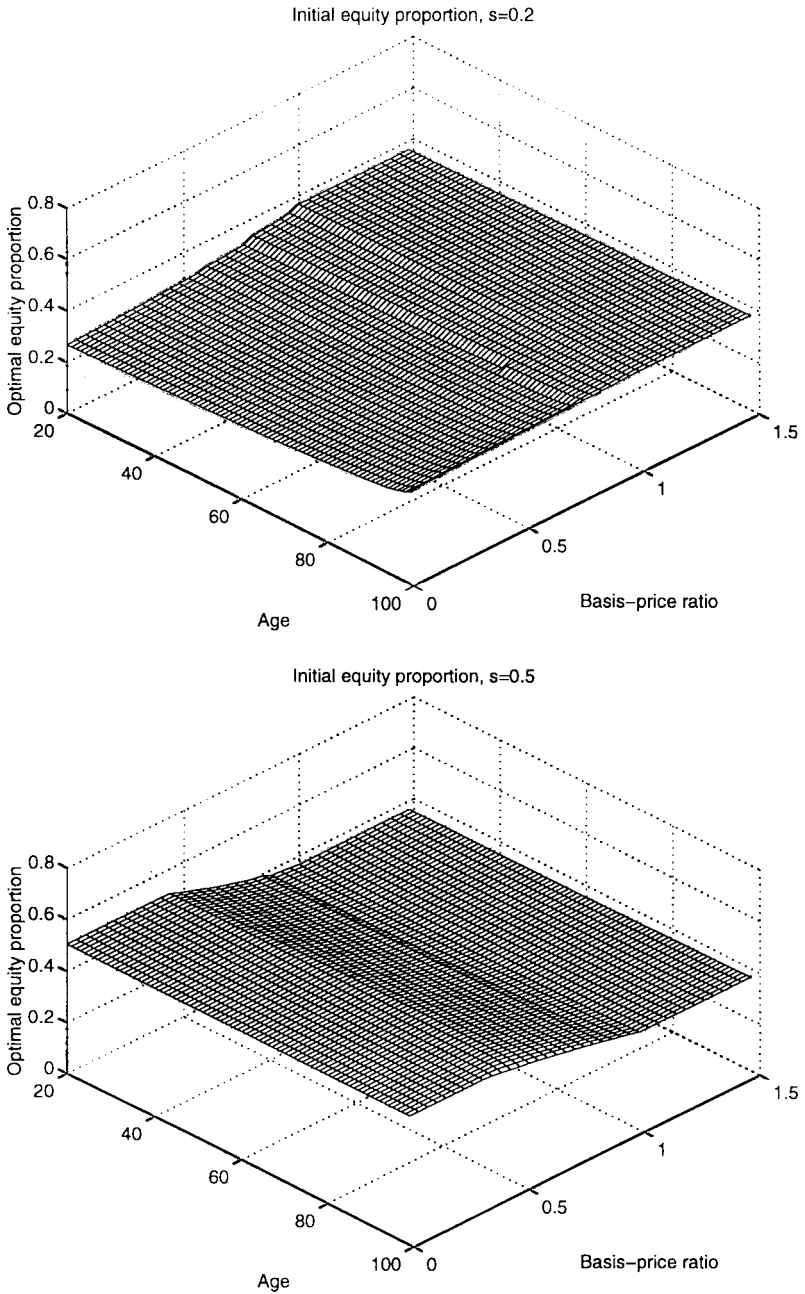


Figure 7

Optimal stock holding with mandatory realization at death

Optimal stock holding for a tax code with mandatory realization of capital gains at death. The dividend and capital gains tax rates are assumed to be 36%. The initial stock holding as a fraction of beginning-of-period wealth is set at $s = 0.2$ for the top panel and $s = 0.5$ for the bottom panel.

year is satisfied for our base-case parameter values. Therefore, in what follows, we assume that all capital gains are realized long term each year, while all capital losses are realized short term.²² As a consequence of realizing all gains and losses each year, the optimal holding of equity will depend upon the investor's age, but will be independent of the investor's initial equity holdings and tax basis.

The optimal holding of equity under asymmetric taxation (not shown) increases relative to the unconstrained optimum in the symmetric taxation case (see Figure 2) at all but very late ages. The increase in equity holdings is most pronounced for young investors, who benefit the most from a reduction in the tax rate on capital gains because it reduces the cost of realizing capital gains to rebalance their portfolios. Younger investors also have a longer horizon over which the asymmetry between the long-term and short-term tax rates can be exploited. The optimal holding of equity with asymmetric taxation is slightly more than 60% for investors younger than age 60. Recall that in the symmetric tax case the unconstrained optimum was less than 40% for these same investors. Since elderly investors do not benefit as much from the realization of long-term capital gains, it is not surprising that the optimal holding of equity declines sharply after age 80, falling from about 57.5% at age 80 to 36.8% at age 99.

3.3 Nonfinancial income

The models examined thus far have assumed that the investor's sole source of income is from financial assets. We now want to investigate how nonfinancial income interacts with capital gains taxes in determining the optimal investment policy.²³ Ideally we would like to model nonfinancial income with its own stochastic process that is less than perfectly correlated with equity returns. However, this would require us to introduce an additional state variable, which greatly complicates the numerical analysis. Therefore, to model nonfinancial income without increasing the dimensionality of the state space, we assume that an investor's pretax nonfinancial income in period t is a constant fraction, y , of his contemporaneous beginning-of-period wealth, W_t .²⁴

²² As in Constantinides (1984), we assume a 1-year short-term holding period and allow the investor to accelerate or defer the sale of stock by 1 day to achieve short-term or long-term tax treatment. There is, however, an important difference between our model and Constantinides (1984). Constantinides (1984) assumes an infinite horizon with no mortality risk for the investor, while our model assumes that mortality risk is increasing with age. With increasing mortality risk, the benefit of realizing long-term capital gains is declining with age because the probability of realizing the tax benefits on any subsequent capital losses is reduced. Consequently the optimal realization policy will be age dependent with increasing mortality rates. We do not attempt to solve for the age-dependent optimal realization policy for the case of asymmetric taxation, but simply assume that all capital gains and losses are realized each year for all ages. Obviously this policy is a better approximation to the optimal policy for young investors (with low mortality rates) than it is for elderly investors (with high mortality rates).

²³ Several studies show that nonfinancial income has important effects on investors' optimal portfolio holdings [e.g., Merton (1971), Jagannathan and Kocherlakota (1996), and Viceira (2001)]. However, these studies do not consider the interaction between nonfinancial income and capital gains taxes on optimal portfolio decisions.

²⁴ We also examined a model in which nonfinancial income is an age-dependent fraction, y_t , of the investor's contemporaneous beginning-of-period wealth, where y_t is assumed to take the following form:

Thus, with nonfinancial income, the gross growth rate of the investor's real wealth becomes

$$w_{t+1} = \frac{R_{t+1}}{1+i} (1 - \tau_g \delta_t - c_t) / [1 - y(1 - \tau_d)]. \quad (22)$$

While this formulation is not ideal, it does provide us with some useful insights about how nonfinancial income affects the investor's asset allocation decisions at different stages of the life cycle. In our numerical analysis, we assume that an investor receives $y = 15\%$ of his pretax wealth as nonfinancial income through age 65 and nothing thereafter.

Figure 8 shows the optimal stock holding for an investor with nonfinancial income and initial stock holdings of either $s = 0.5$ (upper panel) or $s = 0.8$ (lower panel). The optimal holdings of equity are unaffected by the introduction of nonfinancial income for investors older than age 65 because these investors do not receive nonfinancial income. For investors younger than age 65, the introduction of nonfinancial income has led to a dramatic increase in the optimal holding of equity. This is due to the fact that nonfinancial income increases the investor's total wealth level (financial plus nonfinancial wealth) and does so in a way that reduces the investor's overall risk exposure.²⁵ To reestablish the desired risk level, young investors shift more of their financial wealth into equity. This can be seen by comparing the optimal holdings of equity in Figure 8 (upper panel) to those in Figure 2 (lower panel).

In the upper panel of Figure 8 ($s = 0.5$), young investors are slightly *underweighted* in equity and add to their holdings of equity only if the embedded capital gain on existing shares is relatively small ($p^* > 0.8$). Once again, this reflects the reluctance of investors to add equity to their portfolios when existing shares have large embedded capital gains because of the averaging rule used for the tax basis. In the lower panel of Figure 8 ($s = 0.8$), all investors, young and old, are *overweighted* in equity. The way in which investors deal with this situation varies by age. For investors older than age 65, there is a clear trade-off between the size of the embedded capital gain and the amount of equity that is sold. The larger the embedded capital gain, and the older the investor, the less willing the investor is to rebalance his portfolio. As discussed previously, the increasing mortality rates and the tax forgiveness on capital gains at the time of death drive

$$y_t = \left(\frac{45-t}{45} \right)^2,$$

which takes the value of $y_0 = 1.0$ (at age 20) and $y_{45} = 0$ (at age 65). In this case, the unconstrained optimal holding of equity is 90.0% at age 20 and gradually declines to 41.5% at age 65. Beyond age 65, the optimal equity holdings are identical to those in Figure 8.

²⁵ Nonfinancial income is received with certainty and, although the amount is somewhat uncertain, is much less risky than the investor's financial wealth. Thus, holding the investor's portfolio composition fixed, the introduction of nonfinancial income reduces the overall risk of the investor's total wealth. The same intuition is also provided in Campbell et al. (1999).

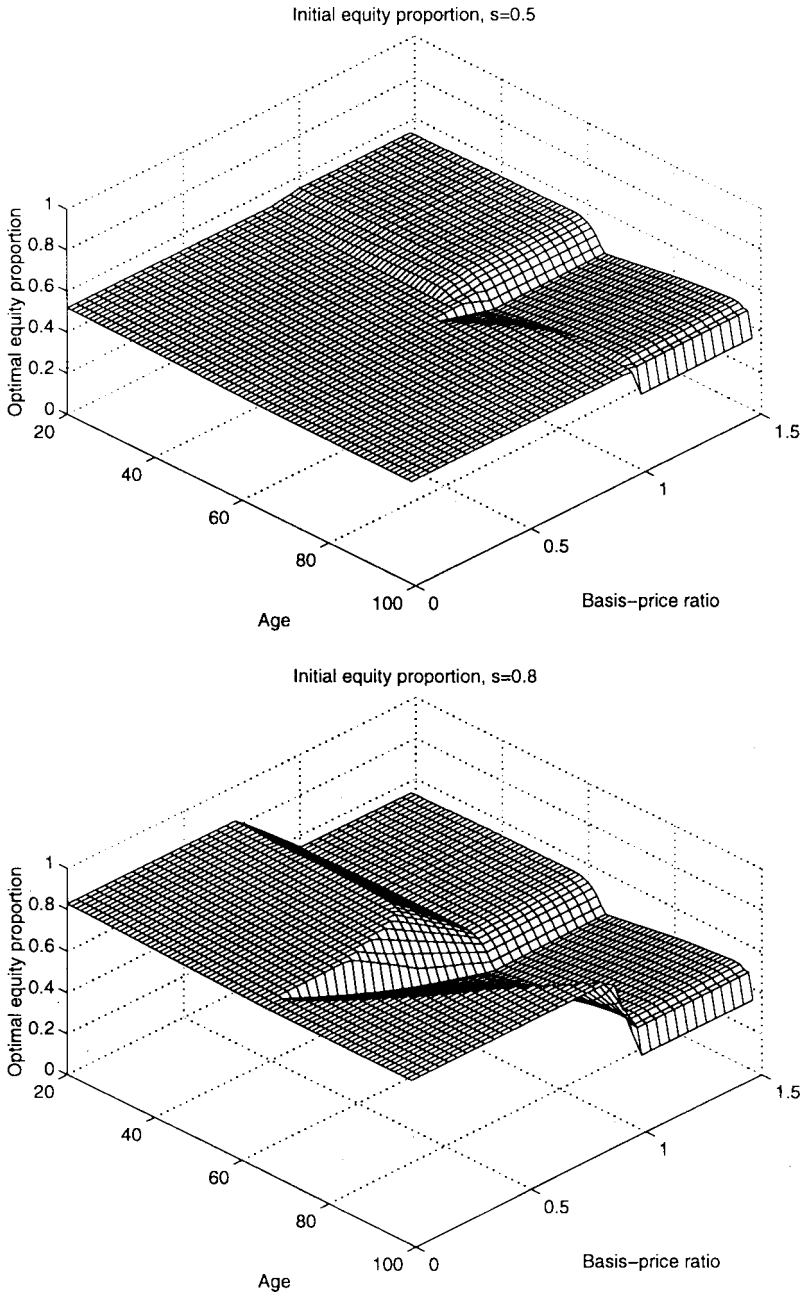


Figure 8
Optimal stock holding with nonfinancial income
Optimal stock holding in the presence of nonfinancial income. Nonfinancial income is assumed to be 15% of the investor's wealth each year through age 64. The dividend and capital gains tax rates are assumed to be 36%. The initial stock holding as a fraction of beginning-of-period wealth is set at $s = 0.5$ for the top panel and $s = 0.8$ for the bottom panel.

this behavior. For young investors (those younger than age 65), there is less willingness to realize embedded capital gains to rebalance their portfolios in the presence of nonfinancial income. This is because young investors can use their nonfinancial income to rebalance their portfolios over time without the need to sell stock with embedded capital gains. The ability to use nonfinancial income in this way declines with age and disappears altogether at age 65.

4. Concluding Remarks

In this article we examined the dynamic optimal consumption, investment, and liquidation decisions for a risk-averse investor in the presence of capital gains taxes and short-sale constraints. We derived a tractable intertemporal model by limiting the investment opportunities to a riskless bond and a single risky stock, and by treating the investor's tax basis in the risky asset as the weighted average purchase price. Our numerical results capture the trade-off between diversification and taxes over the investor's lifetime. In particular, we show how the investor's optimal portfolio holdings depend upon his age, existing portfolio composition, and the embedded capital gain (or loss) on his portfolio. The incentive to sell stock with an embedded capital gain in order to rebalance the portfolio is inversely related to the size of the gain and the age of the investor. We find that the forgiveness of capital gains taxes at death under the current U.S. tax code gives elderly investors a strong incentive to defer the realization of embedded capital gains and to increase their holdings of equity as they age. Our results provide additional new insights regarding the effect of taxes on investor's optimal consumption and portfolio behavior.

The simplifying assumptions we made were driven by the desire to maintain the tractability of our model. However, some of these simplifying assumptions have the effect of understating the value of tax timing. For example, with multiple risky assets, and separate tax bases for each asset purchase, the value of tax timing would increase substantially (see Dammon, Spatt, and Zhang [2000] for an analysis of the multiple asset problem). This would induce investors to hold more equity and would not discourage investors from adding equity to their portfolios when they were "locked in" to large gains on existing holdings of equity. Increasing the frequency of trading would also enhance the value of tax timing, especially for the elderly who have the shortest horizons and who benefit the most from the ability to realize losses more frequently. Finally, because transaction costs are assumed to be zero, we may have overstated the value of tax timing to some extent. With transaction costs, investors may realize fewer gains and losses than our model suggests. However, the relatively low portfolio turnover rates in our simulation analysis provide some assurance that transaction costs will not dramatically alter our qualitative results. One interesting extension of our

model is to allow for tax-deferred investing, which provides a framework for examining the joint determination of the optimal asset allocation in taxable and tax-deferred accounts. An analysis of this issue is provided in Dammon, Spatt, and Zhang (1999).

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