Fear of the Unknown: 
Familiarity and Economic Decisions

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Abstract

Evidence indicates that individuals prefer proximate, familiar choices and dislike changes from status quo positions. We offer an integrated explanation for these phenomena based upon fear of change and of the unfamiliar. In our model, individuals who face model uncertainty focus on adverse scenarios in evaluating defections from familiar choice options. We derive excessive inertia in individual choices, consistent with the endowment effect, home and local biases, high hurdle rates and escalation bias in existing investments, limited investor diversification, and attention effects wherein news, on average, increases demand for a stock. Further, we demonstrate that with familiarity-biased investors participating in equilibrium, the standard CAPM may no longer hold using the aggregate stock holding as the market portfolio, but a modified CAPM holds with respect to the stock holding of investors not subject to familiarity bias.
The first time a Fox saw a Lion he was so terrified that he almost died of fright. When he saw him again, he was still afraid, but hid his fear. But when he met him the third time, he was so brave he began to talk to him as though they were old friends.

_Familiarity breeds contempt._

— Aesop’s Fables, “The Fox and the Lion.”

1 Introduction

People fear change and the unknown. This is supported by evidence from markets and the experimental laboratory, as discussed in Section 2 of this paper. Briefly, there is evidence that individuals favor gambles and investments that they are more familiar with and that are geographically and linguistically proximate (familiarity, local, or home bias); that investors are reluctant to trade away from their current ownership positions (the endowment effect) and are biased in favor of choice options made salient as default choices (_status quo_ bias); and hold strongly to past investment choices (escalation bias, sunk cost effects, inertia).

We offer an integrated explanation for a range of observed behaviors based on two phenomena. One is the tendency for individuals to use familiar or salient default choice alternatives as benchmarks for comparison. We refer to such a salient choice option as the “status quo”. The other is the tendency to evaluate skeptically those alternatives that deviate from the status quo. We argue that when individuals contemplate alternative, non-status quo choices in the face of uncertainty about how the world works, they fear change and the unfamiliar.

We model fear of the unfamiliar as arising from egocentrically pessimistic guesses about how the world works. For any given choice alternative, different possible models of the world imply different possible probability distributions over payoff outcomes. Thus, the individual faces a layering of gambles: first a gamble over which model describes the world, and second, for any given model, over the realized outcome. Ellsberg (1961) and later experiments establish that individuals are averse to the layering of gambles. Although the individual’s
action has no effect on the state of nature, the individual assesses probabilities of states of nature differently depending on what action choice he is contemplating.\(^1\)

Specifically, in our approach the decisionmaker acts as if he thinks that any choice that deviates from the status quo is likely to be countered by a structure of the world that minimizes his welfare. In other words, we model an inclination of individuals who are faced with model uncertainty to focus on the worst-case (or at least, bad-case) scenarios. This conditional pessimism causes what we call \textit{familiarity bias}.\(^2\)

There is a natural connection between fear of the unknown and model uncertainty. In our approach, aversion to model uncertainty is conditional. Individuals do \textit{not} penalize the status quo choice option for the model uncertainty associated with its outcomes. Pessimistic beliefs are primed only by contemplation of an action that deviates from the status quo choice (a familiar, endowed, or default choice option). This linkage between contemplated action and pessimism captures fear of change or of the unfamiliar—\textit{familiarity bias}.

We consider a preference relation that reflects aspects of the models of Bewley (2002) and Gilboa and Schmeidler (1989),\(^3\) but that emphasizes fear of the unfamiliar as reflected in a reluctance to deviate from a specified status quo action. In Bewley (2002), a consumption bundle dominates another if and only if its expected utility is higher than the expected utility of the other for all distributions in a probability set. In our setting, the status quo choice option is privileged. An individual selects a strategy over the status quo only if the strategy provides higher expected utility over a sufficiently large probability mass of possible models of the world.

We use this framework to examine how lack of familiarity can induce anomalies relat-

\(^1\)Our approach does not require that individuals have a declarative belief that the universe responds inimically to their choices. Rather, our premise is that emotions of fear and suspicion are involuntarily incited by the prospect of an unfamiliar course of action. Speculatively, familiarity preference may have arisen because of limited information processing power and the challenges of exchange with potentially exploitive partners.

\(^2\)Individuals’ choices in our model depend upon salient benchmarks, but in a fashion different from prospect theory (Kahneman and Tversky (1979)). In our approach decisionmakers fear deviations from a salient \textit{choice alternative}. In contrast, under prospect theory, individuals are averse to deviations from a benchmark \textit{payoff} level. Further, in our setting it is pessimistic beliefs rather than the shape of the utility function that induce investor’s conservatism.

ing to the unwillingness to trade or to shift investment policy: the endowment effect, the diversification puzzle, local investment biases, the home bias puzzle, the proximity puzzle in cross-listings, reluctance to engage in new investment, escalation bias, and attention effects in securities trading. (These effects are discussed in more detail in Section 2.) We also examine the implications of familiarity bias for equilibrium stock prices and returns.

In a trading context, a natural status quo is the investor’s current consumption or investment portfolio. Endowment effects arise because an investor evaluates purchases under a probability distribution that is adverse to buying, i.e., one in which the expected utility from the good or security is low. Similarly, an investor evaluates a possible sale under a distribution that is adverse to selling. We find that, given an endowed portfolio, there is an interval of prices within which the investor does not trade. Thus, familiarity bias acts like a shadow transaction cost that is proportional to the degree of uncertainty. Under constant absolute risk aversion utility and a family of normal payoff distributions parameterized by an uncertain mean, the gap between the willingness to pay and to accept is proportional to the product of the degree of uncertainty and risk. Thus, our approach offers implications for the magnitude as well as the direction of the effect of familiarity bias.

In a capital budgeting context, the willingness to pay can be interpreted as the amount the manager values the payoff distribution from a new investment project. A lower willingness to pay implies a higher implicit discount rate, and therefore, an excessively high hurdle rate for new investment. For an ongoing project, the willingness to accept can be interpreted as the level of the liquidation value that would make the manager just willing to terminate the project. A willingness to accept that is too high indicates that the implicit hurdle rate for continuing the project is too low. Thus, familiarity preference is consistent both with managers using excessively high hurdle rates in investment choices, and with managers being subject to escalation bias (reluctance to terminate) in existing investments.

If investors are endowed with portfolios that include only a subset of available goods or securities, then this pessimism about trades provides a quantifiable explanation for various puzzles of non-participation in securities markets and investors’ limited diversification across stocks and asset classes. Special cases include the home bias puzzle, the preference of individuals to invest in company stock, and attachment to investing “styles” (such as preferences among firm size and industries, value versus growth).
In calibration analyses we find that with a reasonable degree of uncertainty about the mean stock returns, our model can explain why many investors hold poorly diversified portfolios. Familiarity bias substantially reduces the perceived certainty equivalent diversification gains from holding foreign in addition to domestic equity. Modest levels of model uncertainty induce a home bias comparable to the observed magnitude.

Our approach is also consistent with attention effects, wherein stocks that receive greater publicity or have greater news arrival tend to be purchased more heavily (even if the news is, on average, neutral). Stocks whose names are mentioned in the news or by others become more familiar. In our approach this implies that investors will feel greater comfort with such stocks, and will assess them more favorably—the stocks have good “buzz.” This implies increased purchases by otherwise-non-participating investors.

Our paper builds upon previous research relating ambiguity aversion and model uncertainty to underdiversification and the home bias puzzle. Dow and Werlang (1992) examine a setting with a risk-free asset and a risky asset when investors are averse to model uncertainty and demonstrate that when their uncertainty aversion is high, investors will, regardless of initial endowment, not hold the risky asset. Uppal and Wang (2003) and Epstein and Miao (2003) suggest that home bias results from differences in uncertainty across different agents about different assets. Domestic investors are more uncertain about foreign assets than about domestic assets, causing them to favor domestic assets. In these papers, home bias is endowment-independent, so that even if an investor is endowed with a diversified portfolio that includes foreign stocks, the individual will trade to a home-biased portfolio.

Our paper differs from previous studies in that we consider agents who are influenced by their initial endowments. In our setting, pessimistic beliefs are triggered by trading—the move from the initial endowment to another position—not just by the final position being contemplated. Different from Uppal and Wang (2003) and Epstein and Miao (2003), investors in our model are equally uncertain about the returns of different assets. Home bias results because the pessimism induced by a given degree of uncertainty is greater in an unfamiliar asset or portfolio than in a familiar one. Thus, in our approach, home bias is endowment-dependent. When familiarity bias and uncertainty are high, investors do not trade. Individuals who are endowed with domestic assets retain home-biased portfolios, and

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if they are endowed with diversified portfolios, they retain these as well.

Furthermore, in previous models, individuals generically participate in all asset markets. In our analysis, owing to pessimism about contemplated trades, individuals may choose complete non-participation in certain markets.\footnote{In an equilibrium model with heterogeneous uncertainty averse agents, Cao, Wang and Zhang (2005) show that investors with low uncertainty participate in the risky asset market, while investors with high uncertainty optimally choose to stay sidelined. Their result is also endowment-independent.} Thus, our model is consistent not just with poorly chosen portfolio weights, but with investors holding zero quantities of certain securities and asset classes. For example, the model is consistent not just with home bias, but with a null position in a foreign stock market.

Finally, our paper differs from much of the related literature in considering not just portfolio choices, but equilibrium price determination. For example, we show that in equilibrium, stock markets in which investors have strong familiarity bias are overpriced relative to stock markets in which investors have weaker familiarity bias. We also show that when some investors are subject to the familiarity bias, CAPM may not hold (the market portfolio may not be on the mean-variance efficient frontier), but a version of the CAPM holds with respect to the aggregate stock portfolio of the rational investors.

The remainder of the paper is organized as follows. Section 2 reviews evidence relating to human attitudes toward the familiar and toward deviations from salient benchmark choice alternatives. Section 3 describes the preferences and beliefs in a model with familiarity bias. Section 4 analyzes the endowment effect and no trade condition in the case of one stock. Section 5 extends the analysis to multiple stocks to perform calibration analysis of home bias. Section 6 explores the under-diversification puzzle. Section 7 examines the equilibrium asset pricing implications of familiarity bias. Section 8 concludes. All proofs are in the appendix.

2 Motivating Evidence

We begin by summarizing the evidence relating to human attitudes toward the familiar and toward deviations from salient benchmark choice alternatives. Starting with Zajonc (1968), psychologists have documented a strong and robust mere exposure effect: individuals tend
to like stimuli that are more familiar [e.g., Bornstein and Dagostino (1992), Moreland and Beach (1992)]. Advertisers try to take advantage of this by repeatedly exposing consumers to the name of a brand. People also prefer familiarity and similarity in choice of friends and mates [e.g., Aronson, Wilson and Akert (2006), Berscheid and Reis (1998)]. Of course, greater understanding of or experience with a familiar phenomenon may, ceteris paribus, resolve uncertainty, reducing risk. However, the preference for familiarity goes beyond this. For example, individuals prefer to bet in a decision domain within which they feel expert than on another gamble with an identical distribution of payoff outcomes [Heath and Tversky (1991)].

Individuals tend to dislike risks that derive from active choices more than risks that result from remaining passive. Psychologists have referred to this as the omission bias [Ritov and Baron (1990), Josephs et al. (1996)]. For example, individuals are reluctant to take seemingly risky actions such as getting vaccinated, often preferring to bear the much bigger risks associated with remaining passive.

It has been well documented that people often demand a higher price to give up an object than they would be willing to pay to acquire it [e.g., Knetsch and Sinden (1984), Kahneman, Knetsch and Thaler (1991)]. For example, Kahneman, Knetsch and Thaler (1991) show that students who were randomly chosen to receive mugs demand higher prices to give them up than the prices students who did not have mugs were willing to pay to obtain them. An agent values an object more highly simply because it is his object. This difference between the willingness to pay and the willingness to accept is called the endowment effect [Thaler (1980)].

A manager’s reluctance to invest or to terminate investment is much like the endowment effect. Investment is the exchange of cash for a project, and termination is the opposite exchange. Previous studies document that firms commonly use hurdle rates that exceed the cost of capital, thereby discouraging new projects [e.g., Poterba and Summers (1995), Graham and Harvey (2001)]. Several researchers have also argued that managers are reluctant to terminate ongoing projects. Psychologists have described a tendency for individuals to escalate commitment to a previously selected strategy [Staw (1976), Staw and Ross (1987)]. This is referred to as escalation bias, or inertia. A related phenomenon is the sunk-cost effect [Arkes and Blumer (1985)], wherein an initial investment in a project creates reluctance to
terminate it.

The status quo bias shares some of the flavor of the endowment effect. An individual who is subject to the status quo bias prefers either the current state or some choice alternative that has been made salient as the default option that will apply should no alternative be selected explicitly [Samuelson and Zeckhauser (1988), Fox and Tversky (1995)]. For example, in a set of experiments on portfolio choices following a hypothetical inheritance, Samuelson and Zeckhauser (1988) find that an option becomes significantly more popular when it is designated as the status quo while others are designated as alternatives. As another illustration of status quo bias and inertia, Madrian and Shea (2001) and Ameriks and Zeldes (2004) find that people stick for long periods of time to the default offered by their firm and make no change in composition of their retirement portfolios.

There is a great deal of evidence suggesting that individuals prefer familiar investment choices. U.S. individual investors experience lower return performance for their local stock picks than for their remote stock trades, suggesting that individuals trade local stocks despite a lack of information about these stocks [Seasholes and Zhu (2005)]. U.S. investment managers invest disproportionately in locally headquartered firms, though this may be due to superior information rather than a bias toward the familiar [Coval and Moskowitz (1999)]. Both institutional and individual investors in Finland tend to hold the shares of firms that have nearby headquarters and communicate in investors’ native tongue [Grinblatt and Keloharju (2001)]. Customers of a given U.S. Regional Bell Operating Company (RBOC) tend to hold more of its shares and invest more money in it than in other RBOCs [Huberman (2001)]. Swedish investors tend to concentrate holdings in stocks to which the investor is geographically or professionally close or that he has held for a long period [Massa and Simonov (2006)]. Experimentally, Ackert et al. (2005) find that investors prefer to invest in stocks that have more familiar names. They also provide evidence that participants have a greater perceived familiarity with local and domestic securities and, in turn, invest more in such securities.

When the firm’s own stock is available for investment in its pension fund, many employees invest a significant fraction of their discretionary contributions in it. For example, Mitchell

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6 Guiso, Sapienza, and Zingales (2007) argue that low levels of trust in the stock market explain individuals’ choice not to invest in the stock market.
and Utkus (2002) and Meulbroek (2002) find that the percentage of assets in company stock in defined contribution plans is around 29 percent. In a sample of S&P 500 firms, Benartzi (2001) finds that about one third of the assets in retirement plans are invested in company stock, and of the discretionary contributions, about a quarter are invested in company stock.

In international financial markets, investors tend to hold domestic assets instead of diversifying across countries, a puzzle known as home bias (e.g., French and Poterba (1991), Uppal (1992), Tesar and Werner (1995)). Although various explanations such as transaction costs, differential taxes, political risk, exchange rate risk, asymmetric information, purchasing power parity and non-tradable assets have been offered, none has been shown to explain the magnitude of observed home bias (see Lewis (1999), Stulz (1999)). A related phenomenon is the familiarity bias in international cross-listings: firms tend to cross list their stocks in countries where investors are more familiar with the firms to be listed (e.g., Pagano et al (2002), Sarkissian and Schill (2004)).

Both individuals and portfolio managers have more pessimistic expectations about foreign stocks than about domestic stocks (Shiller, Konya and Tsutsui (1996), Strong and Xu (2003), and Kilka and Weber (2000)). This is consistent with our beliefs-based approach to the home bias puzzle. The preference for the familiar goes above and beyond motivations based upon lower true risk or higher returns.

3 The Model

To highlight the intuition of the model, we consider a two-date setting in which investment decisions are made at date 0, and consumption takes place at date 1. We first consider a portfolio choice decision for an investor who is subject to familiarity bias in an economy with one stock and a risk-free asset. We later extend the model to multiple stocks.

We consider a preference relation that reflects aspects of the preferences described by Bewley (2002) and Gilboa-Schmeidler (1989), but which emphasizes fear of the unfamiliar as reflected in a reluctance to deviate from a specified status quo action. The unique subjective probability distribution used in standard expected utility calculation is replaced by a set of probability distributions. When there is uncertainty, an individual is assumed to stick to
the status quo action (and resulting wealth distribution) unless it is dominated by another action in the sense that the alternative is preferred for all beliefs in the relevant set.

Investors’ uncertainty about the distribution of asset payoffs presents them with two-layered gambles. Based on their prior experience or the results of econometric analysis, investors have in mind a family of distributions from which they believe the true distribution is drawn. We call this set of distributions over payoff outcomes \( P \). The set \( P \) captures the degree of model uncertainty perceived by investors. A larger set \( P \) corresponds to a higher degree of model uncertainty. To take a concrete example, investors may believe that asset payoffs are distributed normally with given variance, but do not know the value of the mean. In this example, each possible mean and variance corresponds to one distribution in \( P \). The range of possible means provides a measure of uncertainty.

Each individual has a twice differentiable and concave utility function \( U(W) \) defined over the end-of-period wealth, \( W \). Let \( W(x) \) denote the wealth random variable for an investor following a given strategy \( x \). The following definition describes a preference relation that captures fear of change and unfamiliar choices.

**Definition 1 Status Quo Deviation Aversion**

Let \( x \) be a feasible strategy and \( s \) be the status quo strategy. Then \( x \) is strictly preferred over \( s \) if and only if \( x \) gives higher expected utility than \( s \) under any probability distribution \( Q \) in \( P \).

\[
x \succ s \iff \min_{Q \in P} \{E[Q[U(W(x))]] - E[Q[U(W(s))]]\} > 0.
\]

Status Quo Deviation Aversion (SQDA) gives a privileged position to the status quo strategy. A strategy is preferred to the status quo strategy only if it provides higher expected utilities under all probability models in \( P \).

Status quo deviation aversion is an incomplete preference relation, as it does not specify how to compare two non-status-quo alternatives. The following definition gives one way to complete the preference ordering:

**Definition 2 Strong Status Quo Deviation Aversion**
Let $x$ and $y$ be any two strategies and $s$ be the status quo strategy. Then

$$x \succ y \text{ iff } \min_{Q \in \mathcal{P}} \{E^Q[U(W(x))] - E^Q[U(W(s))]\} > \min_{Q \in \mathcal{P}} \{E^Q[U(W(y))] - E^Q[U(W(s))]\}.$$

Under strong status quo deviation aversion (SSQDA), each strategy is compared to other strategies by evaluating its utility gains over the status quo strategy under the most adverse scenario for the utility of that strategy relative to the utility provided by the status quo. Thus, under SSQDA the status quo choice is privileged. This reflects fear of change and uncertainty. It is easy to show that SSQDA implies SQDA. Our results on the endowment effect in Section 4 require only the milder SQDA. The analysis in the later sections apply SSQDA.

Status Quo Deviation Aversion, both in its basic form and its strong form, assigns a privileged role to a single status quo alternative. This familiar option is chosen unless there exists an alternative that is preferred for all possible beliefs within the set $\mathcal{P}$. Thus, a familiar choice option acts as an anchor from which deviations are pessimistically considered. When there is uncertainty, deviations from more familiar choices will be scrutinized with skepticism and suspicion. This results in a tendency to prefer more familiar choices, or choices that seem to preserve the status quo.

SSQDA implies that when there are choices that dominate the status quo option, the investor chooses among them according to a procedure similar to that described by Gilboa and Schmeidler (1989), i.e., the investor evaluates each strategy under the scenario that is most adverse to that strategy. Thus, if the status quo action is dominated by an alternative strategy $x$, then strategy $x$ is evaluated according to the minimum gains in expected utility, and the alternative strategy with the highest minimum gains in expected utility is selected.

4 The Endowment Effect

4.1 The Basic Model

In this section, we analyze the endowment effect induced by preference for the familiar. We consider the case of acquiring more shares of a stock whose random payoff is denoted $r$. The risk-free rate of interest is set to zero. We assume that the individual perceives making
no trade as the default or status quo choice option. Let $W_0$ denote the initial wealth in the risk-free bond, $e$ denote the endowment in the stock, $c$ denote the dollar amount the individual pays for the additional shares of the stock, and $d$ denote the dollar amount the individual receives for giving up the additional shares of the stock under measure $Q$. For small additional shares in the stock $\Delta e$ we let $\Delta C_P$ denote the greatest amount an investor would be willing to give up in exchange for the additional quantity of the asset

\[
\Delta C_P \equiv \sup_{e} \{ c | \min_{Q \in P} E^{Q} [U(W_0 + (e + \Delta e)r - c)] - E^{Q} [U(W_0 + er)] > 0 \}. \tag{1}
\]

Similarly, we let $\Delta C_A$ denote the least amount of cash required to induce an individual to give up a small amount of the stock,

\[
\Delta C_A \equiv \inf_{e} \{ c | \min_{Q \in P} E^{Q} [U(W_0 + (e - \Delta e)r + c)] - E^{Q} [U(W_0 + er)] > 0 \}. \tag{2}
\]

Assuming that the limits of $\Delta C_A/\Delta e$ and $\Delta C_P/\Delta e$ exist, we define the willingness to accept (WTA) and willingness to pay (WTP) as

\[
WTA = \lim_{\Delta e \to 0} \frac{\Delta C_A}{\Delta e}, \tag{3}
\]

\[
WTP = \lim_{\Delta e \to 0} \frac{\Delta C_P}{\Delta e}. \tag{4}
\]

Because $U$ is concave, the first order condition for (1) and (2) implies that:

\[
\min_{Q \in P} E^{Q} \left[ \left( r - \frac{\Delta C_P}{\Delta e} \right) U'(W_0 + er) \right] = 0,
\]

\[
\min_{Q \in P} E^{Q} \left[ \left( \frac{\Delta C_A}{\Delta e} - r \right) U'(W_0 + er) \right] = 0.
\]

Letting $\Delta e \to 0$, we get

\[
\min_{Q \in P} E^{Q} [(r - WTP)U'(W_0 + er)] = 0, \tag{5}
\]

\[
\min_{Q \in P} E^{Q} [(WTA - r)U'(W_0 + er)] = 0. \tag{6}
\]
Equation (6) is equivalent to
\[
\max_{Q \in \mathcal{P}} \mathbb{E}[(r - WTA)U'(W_0 + er)] = 0. \tag{7}
\]

Equations (5) and (7) indicate that there is a kink around the endowed stock position. When determining his willingness to pay, an individual considers the scenario most adverse to buying the stock. On the other hand, when determining his willingness to accept, he contemplates the best case scenario for holding on to the stock.

We must have \(WTP \leq WTA\). To show this, suppose that \(WTP > WTA\). Then
\[
0 = \min_{Q \in \mathcal{P}} \mathbb{E}[(r - WTP)U'] = \max_{Q \in \mathcal{P}} \mathbb{E}[(WTP - r)U'] > \min_{Q \in \mathcal{P}} \mathbb{E}[(WTA - r)U'] = 0,
\]
a contradiction.

**Proposition 1** Under Status Quo Deviation Aversion, there is an endowment effect, i.e., \(WTA\) is always greater than or equal to \(WTP\).

To derive closed-form solutions for \(WTA\) and \(WTP\) (as well as for portfolio choice and equilibrium in later sections), from now on we assume that investors have CARA utility with risk aversion \(\gamma\), and that the stock payoff is normally distributed. Further, investors have precise knowledge of the variance of stock payoff but do not know the mean. This is motivated by the fact that it is much easier to obtain accurate estimates of the variances and covariances than of the expected values (e.g., Merton (1992)). Thus, in our model, fear of the unfamiliar derives from aversion to model uncertainty about the mean payoffs of unfamiliar choice alternatives. Investors will consider a set of probability distributions with different means when making their investment decisions.

Following the notation above, we use \(\mathcal{P}\) to denote the set of probability distributions over the stock payoff. When specifying the set \(\mathcal{P}\), we consider a reference distribution and form the set around this reference distribution based on the log likelihood ratio. Specifically, let \(P\) be a reference probability distribution obtained, say, from econometric analysis. We define the set \(\mathcal{P}\) as the collection of all probability distributions \(Q\) satisfying \(\mathbb{E}^{Q}[-\ln(dQ/dP)] < \beta\) for a preselected positive value \(\beta\). Intuitively, since \(\mathbb{E}^{Q}[-\ln(dQ/dP)]\) is the log likelihood
ratio under $Q$, $\mathcal{P}$ can be viewed as a confidence region around $P$, and $\beta$ can be viewed as the critical value for, say, 95% confidence. For a fixed level of confidence, the size of $\mathcal{P}$ captures an investor’s uncertainty about the reference probability distribution $P$. If the investor has more information about $P$, he would be able to estimate the true probability law more precisely. Then for the same level of confidence, $\beta$ will be small. Conversely, if the investor has little information about $P$, then $\beta$ will be large. In this sense, $\beta$ can be viewed as a measure of uncertainty.

If $\mathcal{P}$ is chosen to be the set of normal distributions with a common known variance, Kogan and Wang (2002) show that the confidence region can always be described by a set of quadratic inequalities. In our case, it takes the form of $\mu + v$ where $v$ measures the adjustment made to the estimated mean $\mu$ under probability measure $Q$ and satisfies

$$Tv^2\sigma^{-2} \leq \beta^2,$$

where $\beta$ is a parameter that captures the investor’s uncertainty about the mean of the payoff of the stock, and $T$ is the number of periods for which data on the stock are available. We therefore define the set $\mathcal{P}$ as the collection of all normal distributions with mean $\mu + v$ and variance $\sigma^2$ such that $v$ satisfies (8). The higher is $\beta$, the wider is the range for the expectation of $r$. We refer to $\beta$ as the level of uncertainty of the investor about the mean of $r$. Condition (8) also implies that the deviation from the sample mean is less than the standard error of the sample mean times a constant, $\beta$, the degree of uncertainty, i.e., $|v| \leq \beta\sigma/\sqrt{T}$.

Consider an investor with endowment $e$ of the stock. Under the assumption of CARA utility and the normal distributions for stock payoff, the amount that the investor is willing to pay for $\Delta e$ additional units of the stock is the gain in certainty equivalence associated with the change of stock holding $e$ to $e + \Delta e$ under the worst case scenario for holding stock, i.e.,

$$\Delta C_P = \min_v \left\{ \left[ W_0 + (e + \Delta e)(\mu + v) - \frac{\gamma\sigma^2}{2}(e + \Delta e)^2 \right] - \left[ W_0 + e(\mu + v) - \frac{\gamma\sigma^2}{2}e^2 \right] \right\}.$$

Under the assumption of normal distributions for the set $\mathcal{P}$, the worst distribution for holding additional shares of stock is a normal distribution with mean payoff $(\mu - \beta\sigma/\sqrt{T})$ (the mean stock payoff adjusted downward by $-\beta\sigma/\sqrt{T}$). Substituting $v = -\beta\sigma/\sqrt{T}$ into the above
equation and combining terms, we arrive at the following expression for the amount that an investor is willing to pay for $\Delta e$ additional units of stock:

$$\Delta C_P = \Delta e (\mu - \beta \sigma / \sqrt{T}) - \frac{\gamma \sigma^2}{2} [(e + \Delta e)^2 - e^2].$$

Letting $\Delta e$ approach zero, we obtain the marginal willingness to pay:

$$WTP = \mu - \beta \sigma / \sqrt{T} - \gamma e \sigma^2.$$

Similarly, we derive the amount that the investor requires to give up $\Delta e$ units of the stock as the gain in certainty equivalence associated with the change of stock holding $e$ to $e - \Delta e$ under the worst case scenario for selling stock. Since the worst case for selling stocks is a normal distribution with a mean payoff $(\mu + \beta \sigma / \sqrt{T})$ (the mean stock payoff adjusted upward by $\beta \sigma / \sqrt{T}$), we arrive at the following expression for the amount that an investor requires to give up $\Delta e$ units of stock

$$\Delta C_A = \Delta e (\mu + \beta \sigma / \sqrt{T}) - \frac{\gamma \sigma^2}{2} [(e - \Delta e)^2 - e^2].$$

The marginal willingness to accept is given by

$$WTA = \mu + \beta \sigma / \sqrt{T} - \gamma e \sigma^2.$$

The difference between WTP and WTA is

$$WTA - WTP = 2 \beta \sigma / \sqrt{T}. \quad (9)$$

This gap is proportional to the product of the degree of uncertainty, measured by $\beta$, and the degree of risk, measured by $\sigma$. When the degree of model uncertainty $\beta = 0$, or when the amount of available data $T$ goes to infinity, the gap approaches zero. The gap occurs because when an investor purchases a share of stock, he considers the scenario that is most adverse to buying, and when he sells a share of stock, he considers the scenario that is most adverse to selling. The disparity in WTA and WTP comes from the difference in perceived outcome distribution.
4.2 Aversion to Bad Cases Instead of Worst Cases

In the basic approach, investors who exhibit familiarity bias focus on the worst case scenarios associated with contemplated deviations from status quo choices. In this section, we show that similar results can be obtained under a less extreme assumption: that investors focus on bad cases instead of worst cases.

In order to define “bad cases” specifically, we consider an investor who is uncertain about which model of the world is valid. Let $s$ be the status quo strategy and $x$ be an alternative strategy that the investor is contemplating. We rank the probability distributions $Q$ in $P$ by $E^Q[U(W(x))] - E^Q[U(W(s))]$, the increments each provides under strategy $x$ over the expected utility under the status quo action. An individual who is maximally pessimistic about an action would use the probability distribution in $P$ with the lowest expected utility relative to that provided by the status quo. These beliefs generate SQDA (and SSQDA), as in Section 3.

More generally, however, the individual may pessimistically select a probability distribution $Q$ at the $1 - \delta$ quantile of this ranking ($\delta > 0.5$). The corresponding $E^Q[U(W(x))] - E^Q[U(W(s))]$ is defined as the quantile utility gain $\text{QUG}^\delta(x; s)$. The quantile utility gain describes the gain for a given degree of pessimism $\delta$.

Formally, we define $\text{QUG}^\delta(x; s) = L(Q^\delta, x; s)$, where $Q^\delta$ is the set of payoff distributions corresponding to the $\delta$ quantile of the distributions in $P$ ranked according to the increments each provides under strategy $x$ over the expected utility under the status quo action. For any subset $Q \subset P$ of probability distributions, $L(Q, x; s)$ is the lower bound of expected utility gains associated with strategy $x$ among the set $Q$, defined as

$$L(Q, x; s) = \inf_{Q \in Q} \{E^Q[U(W(x))] - E^Q[U(W(s))]\}.$$ 

We define status quo deviation aversion based on QUG as follows:

**Definition 3 Status Quo Deviation Aversion Based on QUG** Let $x$ be a feasible strategy and $s$ be the status quo strategy, and let $\delta$ be a given quantile level. Then $x$ is strictly preferred to $s$ if and only if the quantile utility gain for strategy $x$ at quantile level $\delta$
is positive:

\[ x \succ s \text{ if and only if } \text{QUG}^{\delta}(x; s) > 0. \]

Under Status Quo Deviation Aversion Based on QUG, a strategy \( x \) is preferred to the status quo choice if it provides higher expected utility for \( \delta \) quantile of the probability distributions. This is a milder condition than SQDA, which requires that strategy \( x \) provides higher expected utility under all possible distributions in \( \mathcal{P} \). For example, if \( \delta = 0.95 \), then the investor considers the five percent lower tail of expected utility gains that can be generated from distributions in \( \mathcal{P} \). The individual prefers the status quo unless the alternative action generates higher expected utility than the status quo under a large set of probability distributions in \( \mathcal{P} \)—a set that has probability mass of 0.95.

We can define the strong status quo deviation aversion based on the QUG in a fashion analogous to SSQDA in Section 3.

**Definition 4 Strong Status Quo Deviation Aversion Based on QUG** Let \( x \) and \( y \) be two strategies and \( s \) be the status quo strategy. Then

\[ x \succ y \text{ if and only if } \text{QUG}^{\delta}(x; s) > \text{QUG}^{\delta}(y; s) \text{ for a given quantile } \delta. \]

Intuitively, a investor would weakly prefer choice \( x \) to choice \( y \) if, for most of the probability distributions in the pre-selected set \( \mathcal{P} \) (0.5 < \( \delta \) ≤ 1), the expected utility of choice \( x \) is at least as high as that of choice \( y \).

The worst case scenario SQDA and SSQDA preferences of the preceding subsection are the special cases of the above definitions in which \( \delta = 1 \). Similar results of autarky on the part of individuals (the endowment effect) still obtain under the more moderate familiarity bias described by the QUG approach. Using similar definitions as in (1)-(4),\(^7\) we have:

**Proposition 2 Under Status Quo Deviation Aversion Based on QUG, there is an endowment effect, i.e., WTA is always greater than or equal to WTP.**

\(^7\)For example, we can define the willingness to pay for \( \Delta e \) units of stock as the set of \( c \) such that,

\[ \Delta C_P \equiv \sup_c \{ c | \text{QUG}^{\delta}[(e + \Delta e)r - c; e] \geq 0 \}. \]
In the special case of normally distributed stock payoffs and CARA utility, we can derive the WTA and WTP in closed form as follows. The maximum amount that the investor is willing to pay for $\Delta e$ additional shares of the stock is given by the gain in certainty equivalence for stock holding $e + \Delta e$ over stock holding $e$ under the QUG at the quantile level $\delta$, i.e.,

$$\Delta C_p = \min_{v^\delta} \left\{ \left[ W_0 + (e + \Delta e)(\mu + v^\delta) - \frac{\gamma \sigma^2}{2} (e + \Delta e)^2 \right] - \left[ W_0 + e(\mu + v^\delta) - \frac{\gamma \sigma^2}{2} e^2 \right] \right\},$$

where the minimum is taken over the mean adjustments corresponding to probability distributions in $Q^\delta$.

Let $F$ be the cumulative density function (CDF) for the adjustments ($v$) to the estimated mean payoff corresponding to probability distributions in $P$. The worst case mean stock payoff corresponding to the $\delta$ quantile under the ordered probability distribution based on QUG is $\mu + F^{-1}(1 - \delta)$, where $F^{-1}(\cdot)$ is the inverse function of $F$. Thus $F^{-1}(1 - \delta)$ is the mean stock payoff adjustment that the investor considers when he exchanges $\Delta e$ units of stock for $\Delta C$ units of cash under the QUG at quantile $\delta$. Hence,

$$\Delta C_p = \Delta e \left[ \mu + F^{-1}(1 - \delta) \right] - \frac{\gamma \sigma^2}{2} [(e + \Delta e)^2 - e^2].$$

Letting $\Delta e$ approach 0, we obtain the marginal willingness to pay under the QUG at quantile level $\delta$,

$$WT P = \mu + F^{-1}(1 - \delta) - \gamma e \sigma^2.$$

Similarly, we can find the amount that the investor requires to give up $\Delta e$ shares of stock as the certainty equivalent gain for the trade from stock holding $e$ to $(e - \Delta e)$ under the QUG at quantile level $\delta$,

$$\Delta C_A = \min_{v^\delta} \left\{ \left[ W_0 + (e - \Delta e)(\mu + v^\delta) - \frac{\gamma \sigma^2}{2} (e - \Delta e)^2 \right] - \left[ W_0 + e(\mu + v^\delta) - \frac{\gamma \sigma^2}{2} e^2 \right] \right\}.$$

The case corresponding to the trade from stock holding $e$ to $e - \Delta e$ is the opposite of that in which the investor acquires additional shares. The adjusted mean stock payoff is thus $\mu + F^{-1}(\delta)$. Substituting the adjusted mean stock payoff into above yields

$$\Delta C_A = \Delta e [\mu + F^{-1}(\delta)] - \frac{\gamma \sigma^2}{2} [(e - \Delta e)^2 - e^2].$$
Letting $\Delta e$ approach 0, we obtain the marginal willingness to accept under the QUG at quantile level $\delta$,

$$WTA = \mu + F^{-1}(\delta) - \gamma e \sigma^2. \tag{10}$$

The gap between WTP and WTA is $F^{-1}(\delta) - F^{-1}(1 - \delta)$, which is positive when the distribution of $F$ is strictly monotone and $\delta > 0.5$. When $\delta = 1$, the analysis reduces to the Status Quo Deviation Aversion case, as shown in equation (9). When uncertainty decreases to zero, the gap between WTA and WTP also shrinks to zero. In particular, when the distribution of $F$ is uniform, the gap between WTP and WTA is $(4\delta - 2)\beta\sigma/\sqrt{T}$. The gap depends on the degree of familiarity bias, characterized by $\delta$. As $\delta$ increases between 0.5 and 1, the gap between WTP and WTA also increases. Intuitively, if the investor chooses the alternative strategy $x$ over the status quo strategy $s$ only under the condition that the alternative is preferable to the status quo under a wider set of probability distributions, he must have valued his endowment highly relative to the alternative. This leads to a large gap between WTP and WTA.

4.3 The No-Trade Condition under Familiarity Bias

We now consider the portfolio choice with one risky asset (stock) and one risk-free asset using the basic model described in Section 4.1. We continue to assume CARA utility and normally distributed stock payoff with uncertainty about the mean. For a range of prices that depend on the investor’s endowment, neither buying nor selling is desired, as the prospect of buying makes the good seem less attractive, and the prospect of selling makes it seem more attractive. We provide conditions under which no trade is perceived to be optimal for an investor who is subject to Status Quo Deviation Aversion.

**Proposition 3** When

$$\left| \frac{\mu - P}{\gamma \sigma^2} - e \right| \leq \frac{\beta}{\gamma \sigma \sqrt{T}}, \tag{10}$$

the investor will not deviate from his status quo position.

Proposition 3 implies that for every initial endowment, $e$, there is a price interval $[\mu - \gamma \sigma^2 e - \beta \sigma / \sqrt{T}, \mu - \gamma \sigma^2 e + \beta \sigma / \sqrt{T}]$ such that within the interval the investor does
not trade. The effect of familiarity bias on an investor’s portfolio choice resembles that of transaction cost. In the case of a single stock with 50% sample standard deviation based on 100 observations, a risk aversion $\gamma = 1$, and an uncertainty parameter $\beta = 1$, the effect of familiarity bias on the investor’s portfolio choice is similar to a setting in which there is a 5% proportional transaction cost without familiarity bias.

5 Calibration Analysis of the Home Bias Puzzle

A well known puzzle in international finance is that investors in aggregate tend to hold mostly the assets of the country they reside in, rather than diversifying internationally. Here, we calibrate familiarity bias under Status Quo Deviation Aversion to determine how much uncertainty is needed for investors to hold mostly local assets.

Let $\mu$ be the vector of mean returns to the domestic and world stock market, and $\Sigma$ be the covariance matrix of the returns: 8 say, estimated from historical data of $T$ observations. Familiarity biased investors make adjustments $v$ to perceived mean stock returns given by the following elliptical set 9

$$v^\top \Sigma^{-1} v \leq \beta^2 / T,$$

where $\beta$ represents the investor’s degree of uncertainty on the expected stock payoff (common for both countries). These adjustments form the probability set $\mathcal{P}$, which the investors utilize to evaluate their investment strategies.

Let $e \equiv (e_d, 1 - e_d)^\top$ denote the investor’s current equity portfolio, where $e_d$ is the weight in the domestic stock. The contemplated new portfolio is $(e_d + \Delta D, 1 - e_d - \Delta D)^\top$, where $\Delta D$ denotes change in investor’s weight in the domestic equity market. The investor’s initial wealth is set to be one. Given the CARA utility considered here, this normalization does not affect his portfolio choice.

Under the SSQDA preference, the perceived certainty-equivalent gains of moving from

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8 In this section, we work with return distributions in order to facilitate the calibration analysis that makes use of stock returns data. Elsewhere in the paper, $\mu$ and $\Sigma$ are the mean and variance of stock payoffs rather than returns.

9 Under the normal distribution assumption, this implies that the ratio of the log-likelihood function of the joint stock returns under the distributions in $\mathcal{P}$ relative to that under the reference probability lies above some threshold level. In Section 7, we consider another specification for the adjustments to perceived expected returns when there are multiple stocks.
endowment portfolio \((e_d, 1-e_d)\top\) to a portfolio \((e_d + \Delta D, 1-e_d - \Delta D)\top\) is

\[
G(\Delta D, e) \equiv \min_v \{-e^{-\gamma[(\Delta Du + e)\top(\mu + v) - \frac{\gamma}{2}(\Delta Du + e)\top\Sigma(\Delta Du + e)]} + e^{-\gamma(\mu + v) + \frac{\gamma^2}{2}e\top\Sigma e}\},
\]

where \(\gamma\) is the risk aversion coefficient and \(u \equiv (1, -1)\top\). The certainty equivalent gain can be expressed as

\[
G(\Delta D, e) \approx \gamma C(\Delta D, e),
\]

\[
C(\Delta D, e) = \min_v \{(\Delta Du)\top(\mu + v) - \frac{\gamma}{2}[\Delta Du\top\Sigma(\Delta Du + e) - e\top\Sigma e]\}
\]

\[
= \Delta D[u\top \mu - \text{sign}(\Delta D)v_m] - \frac{\gamma}{2}[(\Delta D^2 u\top \Sigma u + 2\Delta Du\top \Sigma e],
\]

where \(v\) satisfies (11) and \(v_m \equiv -\min_{Q \in P} u\top v\). It is straightforward to show that

\[
v_m = \beta \sqrt{u\top \Sigma u / T}.
\]

Given initial endowment \(e\), the optimal trade \(\Delta D\) maximizes the certainty equivalent gain \(C(\Delta D, e)\). The unconstrained first order condition is:

\[
u\top \mu - \text{sign}(\Delta D)v_m - \gamma \Delta Du\top \Sigma u - \gamma u\top \Sigma e = 0.
\]

There are two scenarios: (1) No trading is perceived to be optimal, i.e., \(\Delta D = 0\); (2) Trading is perceived to be optimal and satisfies the first order condition above, which implies

\[
\Delta D = \frac{u\top \mu - \text{sign}(\Delta D)v_m - \gamma u\top \Sigma e}{\gamma u\top \Sigma u}.
\]

The no trade scenario occurs if and only if

\[-v_m < u\top \mu - \gamma u\top \Sigma e < v_m.\]

Otherwise, net trade in domestic equity \(\Delta D\) is positive when \(u\top \mu - \gamma u\top \Sigma e > v_m\), and is negative when \(u\top \mu - \gamma u\top \Sigma e < -v_m\).

The following proposition summarizes the optimal trading strategy under the familiarity bias.
Proposition 4 The net trade in domestic equity perceived to maximize the certainty equivalent gain is given by

$$
\Delta D = \begin{cases} 
\frac{\mu_d - \mu_w - v_m - \gamma u^\top \Sigma e}{\gamma u^\top \Sigma u}, & \text{if } \mu_d - \mu_w - \gamma u^\top \Sigma e > v_m \\
0, & \text{if } |\mu_d - \mu_w - \gamma u^\top \Sigma e| \leq v_m \\
\frac{\mu_d - \mu_w + v_m - \gamma u^\top \Sigma e}{\gamma u^\top \Sigma u}, & \text{if } \mu_d - \mu_w - \gamma u^\top \Sigma e < -v_m,
\end{cases}
$$

where $u \equiv (1, -1)^\top$ and $v_m \equiv -\min_{Q \in P} u'v = \beta \sqrt{u^\top \Sigma u/T}$.

We calibrate the model to the data for four countries, including Germany, Japan, United Kingdom, and the United States. Table 1 shows the summary statistics of annual stock market returns for the four countries and the world market portfolio, based on data from 1975-2006. To facilitate comparison, we use value-weighted dollar returns for all four countries and the world market portfolio. Investor initial endowment is assumed to be 100 percent in domestic stock market ($e_d = 1$). This offers the highest level of certainty equivalent gains for diversifying into the world equity market. It therefore creates the most challenging situation for home bias. The risk aversion is set at $\gamma = 2$.

For each country we calculate the optimal combination of the domestic portfolio and the world equity portfolio for the familiarity biased investor with different levels of model uncertainty. We assume no short sales because it is costly to sell short in international markets. We also quantify the perceived gains of moving from the endowment to the more diversified optimal combination of the domestic and world equity portfolios.

The portfolio chosen by investors in each country reflects the fear of the high model uncertainty (though low risk) associated with defecting from the endowment in order to invest more globally. Figure 1 plots the domestic equity proportion perceived as optimal in investor’s total portfolio as a function of the model uncertainty for the four countries. At low levels of model uncertainty, the perceived optimal portfolios for investors in all four countries fall below their respective initial domestic endowments, suggesting that it is beneficial for these investors to shift from entirely domestic equity to the world market portfolio.

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10 We thank Kenneth French for making available on his webpage http://mba.tuck.dartmouth.edu/pages/faculty/ken.french the data used in our analysis.

11 It turns out that this constraint is binding only for Japan when the uncertainty is low. Based on the sample estimates, the Japanese stock market is dominated in the mean-variance space by the world market. Thus, the unconstrained rational optimal portfolio has a short position in the Japanese stock market.
On the other hand, with sufficient high levels of familiarity bias and model uncertainty about stock returns, Figure 1 shows that investors in all four countries perceive their endowment (which is 100 percent domestic equity) as optimal. This is consistent with empirically observed home bias. There is also cross-sectional variation in the amount of uncertainty needed to induce investors to hold on to their endowed (100 percent domestic equity) portfolio. In Germany, model uncertainty needs to be above two. In Japan, the uncertainty has to be even higher (almost three) before investors find it unattractive to add some world stock market exposure to their portfolio. In contrast, UK and U.S. investors stop diversifying into world stock market at much lower levels of uncertainty (about one-half).

When there is no familiarity bias, the perceived certainty equivalent gain in moving from the initial all domestic equity position to the rational optimal combination of the domestic portfolio and the world equity portfolio is about 6.2 percent for Germany, 10.7 percent for Japan, 0.6 percent for the UK, and 0.5 percent for the United States. Figure 2 shows that the perceived certainty equivalent gain decreases to zero as the model uncertainty increases. The decline is faster for the UK and U.S., mirroring the finding that UK and U.S. investors stop diversifying into the world stock market at low levels of uncertainty.

The investor’s aversion to risk interacts with model uncertainty in determining the effect on the perceived certainty equivalent gain and the investor’s portfolio decision. The gains from diversification are higher when investors are more risk averse. As an investor becomes more risk averse, the level of uncertainty required to induce investors to remain at their initial endowments increases. When investors contemplate deviating from their initial endowment positions by increasing their exposure to the world market portfolio to achieve a more balanced portfolio, the level of uncertainty at which the perceived certainty equivalent gain reaches zero rises. This reflects the tradeoff between the benefit of risk reduction and the fear of the model uncertainty associated with deviating from the initial endowment.

Our results indicate that for plausible parameter values, individuals may reach an autarkic outcome. Assuming that domestic investors are endowed with domestic stock, this implies a home bias. In the model of Dow and Werlang (1992), owing to ambiguity aversion, individuals may choose to hold only a risk-free asset and not hold an uncertain risky asset. However, this finding is endowment independent, whereas in our approach it is the fact that individuals start out with portfolios tilted toward domestic assets (and in the extreme, hold
only domestic assets) that leads to a home bias. Furthermore, in Dow and Werlang, the zero
holding result occurs because one asset is much more uncertain than the other. In contrast,
in our analysis, the domestic and foreign assets can be comparably uncertain. Nevertheless,
the uncertainty of the foreign asset (which is not part of the domestic investor’s endowment)
makes the investor pessimistic about trading to acquire it.

In Uppal and Wang (2003) and Epstein and Miao (2003), domestic investors hold less
of foreign assets because, to them, foreign assets are highly uncertain. However, in contrast
with our model, in their settings investors always trade and always hold non-zero amounts
of every foreign asset.\textsuperscript{12} Also, in our model a crucial source of home bias is that investors
are endowed disproportionately in domestic stocks. In contrast, in Uppal and Wang and
in Epstein and Miao, endowment does not affect the final allocation. In their settings, if a
domestic investor were endowed with foreign stock (perhaps received as an inheritance), he
would still sell it to trade to a position heavy in domestic stock.

6 Underdiversification

Blume and Friend (1975) find that investors hold highly undiversified portfolios. Using
more recent data from a major discount brokerage firm, Barber and Odean (2000) find that
investors, on average, hold 4.3 stocks at this brokerage firm, with the median being only 2.6
stocks. This phenomenon is in sharp contrast to the recommendation of traditional portfolio
theory, and especially puzzling prior to the rise of mutual funds in recent decades. We
illustrate here that when deviating from the status quo choice triggers investor aversion to
model uncertainty, investors may remain at poorly diversified endowment positions and do
not perceive further diversification to be beneficial.

Consider the case of \( N \) stocks with identically distributed returns. Assume that asset
returns are jointly normally distributed, with sample variance \( \sigma^2 \) and correlation \( \rho \) estimated
from historical data of \( T \) observations. The amount of uncertainty \( \beta \) is assumed to be the
same for each stock. We define a portfolio \( p_e \) as \textit{undominated} if a familiarity-biased investor
who starts with \( p_e \) as his status quo prefers to hold \( p_e \). Thus, a portfolio \( p_e \) is undominated

\textsuperscript{12}In these models if uncertainty is large, a close-to-zero-holding outcome can occur.
if, for any arbitrary portfolio $p$, 

$$\min_{q \in P} [CE(p) - CE(p_e)] \leq 0.$$ 

Given the symmetry of the model, all risk-averse investors would hold equally weighted portfolios. For any integer $K$, let $e_K$ denote an equally weighted portfolio of $K$ stocks. We now characterize the minimum number of stocks $K$ so that $e_K$ is undominated. For this purpose, we examine the conditions under which a familiarity-biased investor endowed with $e_K$ would not want to combine $e_K$ with any $e_{M-K}$, $K < M \leq N$, where $e_{M-K}$ denotes the equally weighted portfolio of $M - K$ of the $N - K$ stocks not contained in $e_K$.

Let $v_K$ and $v_{M-K}$ be the adjustments to the mean returns of portfolios $e_K$ and $e_{M-K}$, respectively. As in Section 5, familiarity-biased investors contemplating a trade make adjustments $v = (v_K, v_{M-K})^\top$ to perceived mean portfolio returns satisfying 

$$v^\top \Sigma_M^{-1} v \leq \beta^2 / T,$$

where $\Sigma_M$ is the variance-covariance matrix of returns of $e_K$ and $e_{M-K}$. It is straightforward to show that 

$$\Sigma_M = \begin{pmatrix} \frac{1+(K-1)\rho}{K} & \frac{\rho}{1+\rho} \\ \frac{\rho}{1+\rho} & \frac{1+(M-K-1)\rho}{M-K} \end{pmatrix} \sigma^2.$$ 

Applying Proposition 4, familiarity-biased investors would hold onto the endowment portfolio $e_K$ if 

$$v_M - \gamma u^\top \Sigma_M (1,0)^\top \geq 0,$$ 

where $u = [1 \ -1]^\top$, and 

$$v_M \equiv -\min_{v^\top \Sigma_M^{-1} v \leq \beta^2 / T} u^\top v = \beta \sqrt{u^\top \Sigma_M u / T} = \beta \sigma \sqrt{\frac{M(1-\rho)}{K(M-K)T}}.$$ 

With some algebra, (12) reduces to 

$$K \geq \frac{\gamma^2 \sigma^2 (1-\rho)M}{M\beta^2 / T + \gamma^2 \sigma^2 (1-\rho)}.$$ 

Thus, given the uncertainty about mean stock returns described by $\beta$, a familiarity-biased investor who holds the portfolio $e_K$ with $K$ stocks will not want to diversify further, as long
as (13) holds for all $M$ such that $K < M \leq N$. Since the right-hand side of (13) increases with $M$, the minimum number of stocks $K$ so that a familiarity-biased investor endowed with $e_K$ will not diversify further is

$$K^* = 1 + \text{Int} \left[ \frac{\gamma^2 \sigma^2 (1 - \rho) N}{N \beta^2 / T + \gamma^2 \sigma^2 (1 - \rho)} \right],$$

where $\text{Int}[x]$ represents the largest integer below $x$.

When $\beta = 0$, the investor acts as a rational Bayesian and invests in all $N$ stocks. However, when $\beta > 0$, a familiarity-biased investor holds on to portfolios with a much fewer number of stocks. For example, take $N = 500$, $\rho = 0.5$, $\gamma = 1$, $\sigma = 0.5$, $T = 100$. With the model uncertainty associated with deviation from the status quo at $\beta = 2$, an equally weighted portfolio with only four stocks ($K^* = 4$) is undominated. Thus, our model can generate empirically observed underdiversification with reasonable parameter values.\(^\text{13}\)

Figure 3 plots the minimum number of stocks needed to construct an undominated portfolio against the model uncertainty, for different levels of risk aversion. It illustrates the tradeoff between the benefit of risk reduction through diversification and the fear of the model uncertainty associated with deviating from the initial endowment. As model uncertainty increases, the minimum number of stocks needed to construct an undominated portfolio decreases monotonically, reflecting the investor’s desire to reduce the overall uncertainty in his portfolio. Furthermore, as the investor’s risk aversion increases, the investor increasingly desires to hold a well diversified portfolio. Holding fixed the amount of model uncertainty, the number of stocks in an investor’s undominated portfolio is uniformly larger for higher values of risk aversion.

Our finding that limited diversification can derive from a fear of unfamiliar choice options suggests that mutual funds (and especially index funds) provide a social benefit for a reason different from standard explanations. A standard argument is that mutual funds reduce the transaction cost needed for investors to diversify. However, for a long-term buy-and-hold investor, it is not really all that costly to form a reasonably diversified portfolio on an individual account. In our model, investors stop adding stocks to their portfolios because a large diversification gain is needed to offset the aversion to buying an unfamiliar stock.\(^\text{13}\)

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\(^\text{13}\)When there are $T = 100$ data points, the amount of uncertainty corresponding to $\beta = 2$ implies that the investors adjust the sample mean stock return up and down by, at most, one-fifth of the standard deviation.
A mutual fund can address this issue in two ways. First, the individual needs to add just a single new asset to his portfolio, the mutual fund. Second, by focusing on marketing to investors, mutual funds can make their product more familiar to investors. In other words, where corporations specialize in making profits, mutual funds can specialize in being invested in. Our approach suggests that there is a socially valuable complementarity between being good at marketing that assuages investor fears about stocks, and providing a diversified portfolio of securities in which individuals can invest.\footnote{In a study that examines the role of advertising in the mutual fund industry, Cronqvist (2006) finds that funds took advantage of investor familiarity in their advertisements (e.g., Absolut Strategi Fund associated itself with the Vodka brand Absolut). Fund advertising is shown to affect investors’ choices, although it provides little information (e.g., about fund fees or manager ability). In particular, advertising induces more home bias.}

7 Capital Market Equilibrium with Familiarity Bias

We have analyzed the strategies perceived to be optimal by investors who have familiarity bias. We now turn to the question of how familiarity affects trading and prices as part of an endogenously determined market equilibrium. There are domestic and foreign stock markets.\footnote{Although we consider international stock markets, our results also apply to a cross-section of stocks in one market, e.g., to settings where investors have preferred habitats or styles.} We allow for the possibility that a subset of investors is immune to familiarity bias, and acts as a rational Bayesian. The population size of each country is normalized to one, and the proportion of rational investors in each country is denoted $m$. Thus, there are four groups of investors: domestic and foreign rational investors, and domestic and foreign investors who are subject to familiarity bias. All investors have CARA utility function with constant absolute risk aversion coefficient $\gamma$. We use subscript “$d$” to denote home country and subscript “$f$” to denote foreign country.

The payoffs $V$ of the stocks in the two countries are assumed to be multi-variate normally distributed with mean vector $\mu = (\mu_d, \mu_f)^\top$. The variance-covariance matrix of the payoffs, $\Sigma$, has diagonal elements of $\sigma_d^2$ and $\sigma_f^2$. The correlation of stock payoffs is $\rho$. $\Sigma$ is known to all investors. For the rational Bayesian investors, the expected payoff is $\mu$. For investors with familiarity bias, the expected payoff is denoted by a set $\mu + v$, where the adjustments...
to perceived mean payoffs \((v)\) are uniformly distributed on a rectangular set given below

\[
v \in [-\alpha \sigma_d, \alpha \sigma_d] \times [-\alpha \sigma_f, \alpha \sigma_f],
\]

(14)

where for the ease of notation we denote \(\alpha = \beta / \sqrt{T}\), so in the rest of the paper \(\alpha\) represents investor’s degree of uncertainty on the expected stock payoffs after analyzing historical data of sample size \(T\).

The per capita supplies of the domestic and foreign stocks are denoted \(x_d\) and \(x_f\), respectively. We assume that the entire supply of domestic stocks is initially endowed among domestic investors evenly, while the entire supply of foreign stocks is endowed among foreign investors evenly. Besides the risky stocks, there is a risk-free asset with rate of return normalized to 0.

Consider an investor’s optimal portfolio choice corresponding to a given price vector \(P = (P_d, P_f)^\top\). Let \(W_0\) denote his initial wealth in the risk-free asset, \(e \equiv (e_1, e_2)^\top\) denote his initial share endowment in the stock markets, and \(\Delta D \equiv (\Delta D_1, \Delta D_2)^\top\) denote deviation of his share holdings from the initial endowment. To find the equilibrium prices, we need to solve the optimal portfolio holdings of domestic Bayesian, domestic familiarity-biased, foreign Bayesian, and foreign familiarity-biased investors, denoted respectively by \(D_{dr}\), \(D_{db}\), \(D_{fr}\), and \(D_{fb}\).

The rational Bayesian investors maximize \(E[e^{-\gamma W_1}]\), where \(W_1\) is the wealth next period,

\[
W_1 = W_0 + (e + \Delta D)^\top \hat{V} - (\Delta D)^\top P.
\]

It follows that

\[
D_{dr} = D_{fr} = \frac{1}{\gamma} \Sigma^{-1}(\mu - P).
\]

(15)

When the investor has uncertainty about the mean stock payoffs and is subject to familiarity bias, his optimal trade \(\Delta D^*\) can be computed in two steps. First, for each proposed demand deviation \(\Delta D\), evaluate the certainty equivalent gains \(G(\Delta D, e)\) of deviating from endowment \(e\) by \(\Delta D\). Second, choose \(\Delta D^*\) to maximize \(G(\Delta D, e)\) for a given endowment \(e\).

The equilibrium stock prices can be found by aggregating the demand of the rational Bayesian and familiarity-biased investors from the two countries and clearing the market.
The equilibrium prices and investors’ equilibrium portfolio holdings are given in the following proposition. The proofs are in the Appendix.

**Proposition 5** (1) When $\alpha < \min\{(1 - \rho)\gamma\sigma_dx_d/2, (1 - \rho)\gamma\sigma_fx_f/2\}$, both rational Bayesian investors and investors who are subject to familiarity bias trade in both domestic and foreign markets. The equilibrium stock returns are

$$
\left( \begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array} \right) = \left( \begin{array}{c} x_d \\ x_f \end{array} \right) \frac{\gamma}{2} \sum.
$$

(16)

The rational Bayesian investors’ equilibrium holdings are $D_{dr} = D_{fr} = (x_d/2, x_f/2)\top$. The familiarity-biased investors’ equilibrium holdings are

$$
D_{db} = \left( \begin{array}{c} \frac{1}{2}x_d + \frac{\alpha}{(1-\rho)\gamma\sigma_d} \\ \frac{1}{2}x_f - \frac{\alpha}{(1-\rho)\gamma\sigma_f} \end{array} \right),
$$

(17)

and

$$
D_{fb} = \left( \begin{array}{c} \frac{1}{2}x_d - \frac{\alpha}{(1-\rho)\gamma\sigma_d} \\ \frac{1}{2}x_f + \frac{\alpha}{(1-\rho)\gamma\sigma_f} \end{array} \right).
$$

(18)

(2) When $\alpha \geq \max\{(1 - \rho)\gamma\sigma_dx_d/2, (1 - \rho)\gamma\sigma_fx_f/2\}$, rational Bayesian investors trade internationally, whereas investors subject to familiarity bias remain at their endowment positions. The equilibrium stock returns are also given by (16). The rational Bayesian investors’ equilibrium holdings are $D_{dr} = D_{fr} = (x_d/2, x_f/2)\top$. The familiarity-biased investors’ equilibrium holdings are $D_{db} = (x_d, 0)\top$, and $D_{fb} = (0, x_f)\top$.

(3) When $\alpha$ is between $(1 - \rho)\gamma\sigma_dx_d/2$ and $(1 - \rho)\gamma\sigma_fx_f/2$, rational Bayesian investors trade internationally, while familiarity-biased investors based in the country with a higher uncertainty threshold trade in their home market, and familiarity-biased investors based in the country with a low uncertainty threshold remain at their endowment positions. Furthermore, if the domestic country has a higher uncertainty threshold, the equilibrium stock returns are

$$
\left( \begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array} \right) = \sum \left( \begin{array}{c} \frac{1 - m}{1+m} \gamma x_d - \frac{(1-m)\alpha}{(1+m)(1-\rho^2)\sigma_d} \\ \frac{1}{2}\frac{(1-m)\alpha}{\gamma x_f} \end{array} \right).
$$

(19)
The rational Bayesian investors’ equilibrium holdings are

\[ D_{dr} = D_{fr} = \left( \frac{1}{1+m}x_d - \frac{(1-m)\alpha}{(1+m)(1-\rho^2)\gamma \sigma_d} \right). \]  

(20)

The familiarity-biased investors’ equilibrium holdings are

\[ D_{db} = \left( \frac{1}{1+m}x_d + \frac{2m\alpha}{(1+m)(1-\rho^2)\gamma \sigma_d} \right), \]  

(21)

and \( D_{fb} = (0, x_f)^\top \). The case in which the foreign country has a higher uncertainty threshold is symmetric.

Case (2) here is the equilibrium analog of the no-trade results in Proposition 3 and Proposition 4. Even when a familiarity-biased investor trades away from his endowment, he does not move all the way to the rational optimal position. The equilibrium holdings of a familiarity biased investor differ more from those of a rational Bayesian when the uncertainty is higher, and when the correlation between domestic and foreign stocks is higher.

The equilibrium stock returns in cases (1) and (2) of Proposition 5 coincide with the case when everyone is a rational Bayesian. To gain more intuition about Case (3), consider \( \rho = 0 \) for the moment. Let \( i = d \) or \( f \) such that \( \sigma_i x_i = \max(\sigma_d x_d, \sigma_f x_f) \), and \( j \) is such that \( \sigma_j x_j = \min(\sigma_d x_d, \sigma_f x_f) \). For the country \( j \) with a low uncertainty threshold, the equilibrium stock return is given by

\[ \mu_j - P_j = \frac{1}{2} \gamma \sigma_j^2 x_j. \]

For the country \( i \) with a higher uncertainty threshold, the equilibrium stock return is given by

\[ \mu_i - P_i = \left( \frac{1}{1+m} \right) \gamma \sigma_i^2 x_i - \left( \frac{1-m}{1+m} \right) \alpha \sigma_i. \]

Thus, the stock return in the country with a high uncertainty threshold is most affected by familiarity bias. For example, suppose that domestic uncertainty threshold is higher than foreign uncertainty threshold. When the degree of uncertainty is between foreign and domestic uncertainty thresholds, domestic familiarity-biased investors will sell shares, while
foreign familiarity-biased investors will remain at their endowment positions. Given the proportion of familiarity biased investors in home and foreign countries, the domestic stock return is lower and decreasing in the degree of model uncertainty. Intuitively, domestic familiarity-biased investors will not sell their shares unless selling is beneficial even after adjusting the expected stock return upward to account for the model uncertainty. The uncertainty risk thus increases equilibrium price for domestic stock and reduces equilibrium return.

The proposition above assumes symmetric familiarity bias in the two countries. More generally, the relative prices of different stock markets will be influenced by the proportion of familiarity biased investors and investors’ risk tolerance in different markets. When familiarity bias is asymmetric, in equilibrium stock markets in which investors have strong familiarity bias will be overpriced relative to stock markets in which investors have weaker familiarity bias.

The next proposition concerns the validity of international CAPM when some investors are subject to familiarity bias. In the cases (1) and (2) of Proposition 5, equilibrium stock returns are the same as in the case when everyone is rational. It is not surprising that the CAPM holds in these cases. But in case (3), uncertainty and familiarity bias affect stock returns, and the traditional CAPM fails. Interestingly, in this case, a modified CAPM holds with respect to the rational Bayesian investors’ aggregate stock portfolio rather than the world market.

Proposition 6 (1) When \( \alpha < \min\{(1 - \rho)\gamma \sigma_d x_d/2, (1 - \rho)\gamma \sigma_f x_f/2\} \), or when \( \alpha \geq \max\{(1 - \rho)\gamma \sigma_d x_d/2, (1 - \rho)\gamma \sigma_f x_f/2\} \),

\[
E[r_i] = \beta_i E[r_M],
\]  
(22)

where \( r_i \) and \( r_M \) are the return of country \( i \)’s stock market \( (i = d \text{ or } f) \) and the value-weighted world stock market \( M \), \( \beta_i \) is the beta of stock \( i \)’s return with respect to the world market return. (2) When \( \alpha \) is between \( (1 - \rho)\gamma \sigma_d x_d/2 \) and \( (1 - \rho)\gamma \sigma_f x_f/2 \),

\[
E[r_i] = \frac{\beta_i}{k_i} E[r_M],
\]  
(23)

where \( k_i \neq 1 \) are constants that depend on model parameters (mean and variance of stock payoffs, investor’s risk aversion and uncertainty parameters, fraction of familiarity-biased
investors, per capital supply of risky stocks). (3) When \( \alpha \) is between \((1 - \rho)\gamma \sigma_d x_d/2\) and \((1 - \rho)\gamma \sigma_f x_f/2\),

\[
E[r_i] = \beta'_i E[r_{M'}],
\]

where \( M' \) is the rational Bayesian investors’ aggregate stock portfolio, and \( \beta'_i \) is the beta of stock \( i \)’s return \((i = d \ or \ f)\) with respect to \( M' \).

We conclude this section by examining the amount of home bias that results in equilibrium with familiarity-biased investors. The measure of home bias for domestic investors is the ratio of their domestic holdings in the total risky portfolio relative to that of the weight of domestic holdings in the world market portfolio:

\[
H_d \equiv \frac{y_d P_d}{y_d P_d + y_f P_f} - \frac{x_d P_d}{x_d P_d + x_f P_f} = \frac{(y_d x_f - y_f x_d) P_d P_f}{(y_d P_d + y_f P_f)(x_d P_d + x_f P_f)},
\]

where \( y_d \) is the total holdings of domestic stock by domestic investors, and \( y_f \) is the total holdings of foreign stock by domestic investors. Substituting the equilibrium prices analyzed in Proposition 5 into the optimal shareholdings given by (15) for the rational Bayesians and (28) for the familiarity-biased investors, we obtain the following expressions for the total holdings of domestic stock by domestic investors, and the total holdings of foreign stock by domestic investors:

(1) when \( \alpha < \min\{(1 - \rho)\gamma \sigma_d x_d/2, (1 - \rho)\gamma \sigma_f x_f/2\} \),

\[
y_d = \frac{x_d}{2} + \frac{\alpha(1 - m)}{(1 - \rho)\gamma \sigma_d},
\]

\[
y_f = \frac{x_f}{2} - \frac{\alpha(1 - m)}{(1 - \rho)\gamma \sigma_f}.
\]

(2) when \( \alpha \geq \max\{(1 - \rho)\gamma \sigma_d x_d/2, (1 - \rho)\gamma \sigma_f x_f/2\} \),

\[
y_d = \left(1 - \frac{m}{2}\right) x_d,
\]

\[
y_f = \left(\frac{m}{2}\right) x_f.
\]
(3) when \((1 - \rho)\gamma \sigma dx_f / 2 < \alpha < (1 - \rho)\gamma \sigma dx_d / 2\),

\[
y_d = \left(\frac{1}{1 + m}\right)x_d + \frac{m(1 - m)}{1 + m} \frac{\alpha}{(1 - \rho^2)\gamma \sigma d},
\]
\[
y_f = \left(\frac{m}{2}\right)x_f.
\]

(4) when \((1 - \rho)\gamma \sigma dx_d / 2 < \alpha < (1 - \rho)\gamma \sigma dx_f / 2\),

\[
y_d = \left(1 - \frac{m}{2}\right)x_d,
\]
\[
y_f = \left(\frac{m}{1 + m}\right)x_f - \frac{m(1 - m)}{1 + m} \frac{\alpha}{(1 - \rho^2)\gamma \sigma f}.
\]

If all domestic investors are rational \((m = 1)\), then \(y_d / x_d = y_f / x_f = 1/2\), there is no home bias, and the home bias measure \((H_d)\) takes a value of 0. If there is a positive fraction of investors subject to familiarity bias \((m < 1)\), then in all of the cases above, \(y_d / x_d \geq 1/2\) and \(y_f / x_f \leq 1/2\). Thus, \(y_dx_f - y_fx_d > 0\), and the home bias measure \(H_d\) in (25) is positive. The extent of home bias depends upon the degree of model uncertainty perceived by these investors. When the model uncertainty is beyond the threshold of trading, familiarity-biased investors choose to hold their endowment and do not trade. In this case, home bias reaches the highest level and no longer varies with the degree of model uncertainty.

In Figure 4, we plot the home bias ratio as a function of the model uncertainty for Germany, Japan, the United Kingdom, and the United States. In all four panels, the home bias ratio rapidly increases in the model uncertainty as long as the participation condition for the uncertainty averse investors is satisfied. At very high levels of model uncertainty, familiarity-biased investors choose not to trade. In this case, only Bayesian investors trade the risky assets, and the home bias ratio stays at the peak level.

8 Conclusion

Experimental and capital market evidence indicates that individuals favor geographically and linguistically proximate and more familiar investments; are biased in favor of sticking
to current consumption/investment positions or strategies and in favor of choice alternatives made salient as default options; and are averse even to small gambles when presented as increments relative to a salient reference point. More generally, individuals are more reluctant to take action that imposes risk than to bear risk associated with remaining passive; tend to like stimuli they have been exposed to more, tend to like people they are located close to, and are prone to malice toward outsiders.

These effects have on the whole been discussed separately, as reflected by a variety of labels: familiarity, local or home bias; the endowment effect; status quo bias; escalation bias, sunk cost effects, inertia; omission bias; the mere exposure effect; xenophobia; proximity bias in international cross listings, and propinquity effects. We offer an integrated explanation for these effects based upon fear of change and of the unfamiliar.

Building upon previous research, we model an inclination of individuals who are faced with model uncertainty to focus on worst-case scenarios in relation to contemplated choices. However, in contrast with most past literature, in our approach, pessimism about the uncertainty associated with a choice is triggered by the deviation of the individual’s contemplated decision from some familiar default or status quo choice alternative. Thus, in our setting the aversion to model uncertainty is not general, but rather is triggered by change or lack of familiarity.

In our setting, individuals evaluate a purchase of a security or product under a probability distribution that is adverse to buying, and a sale under a distribution that is adverse to selling. The endowment effect arises endogenously in our setting. The gap between willingness to pay and willingness to accept increases with both the degree of uncertainty and risk. Similar reasoning leads to a general tendency to adhere to status quo options, and to excessive inertia in individual choices. We show that, for each investor, and for every endowment, there exists a set of prices at which an investor will not trade. Thus, unfamiliarity-induced aversion to model uncertainty acts like a shadow transaction cost.

The model also offers an explanation for limited diversification of investors across stocks and asset classes; special cases include the home bias puzzle and the preference of individuals to invest in company stock. In calibrations, we find that the observed magnitude of home bias can be consistent with a reasonable level of model uncertainty. We calculate the minimum number of stocks in a portfolio such that defection-induced fear of uncertainty deters
individuals from diversifying further. We find that for plausible parameter values, investors may settle for very undiversified portfolio with just a few stocks.

In an international securities market equilibrium, investors with familiarity bias may not buy foreign stocks. More generally, the relative prices of different stock markets will be influenced by the extent of familiarity bias and the risk tolerances of investors in the different countries. We show that with familiarity-biased investors participating in the equity market, in equilibrium the standard CAPM may no longer hold using aggregate stock holdings as the market portfolio. Interestingly, however, a modified CAPM holds when we use rational Bayesian investors’ stock holdings as the market portfolio. Our result points to a new direction for the failure of empirical tests of the standard capital asset pricing model. This is, the market portfolio used in empirical tests may include shares held by investors who are subject to familiarity bias. A valid empirical test of the CAPM should be based on the stock portfolios held by rational Bayesian investors who are not subject to the familiarity bias.

Our approach is consistent with the evidence that stocks that receive greater publicity or have greater news arrival tend to be purchased more heavily (even if the news is, on average, neutral). Stocks whose names are prominently mentioned in the media or by other individuals become more familiar. In our approach, investors therefore perceive the uncertainty as smaller in highly-publicized firms. Less familiarity bias with respect to a stock that is not currently part of the portfolio will therefore encourage many individuals to switch to holding a positive quantity. Increased publicity about a stock expands breadth of ownership, increases net demand for the stock, and thereby induces a positive stock price reaction. Thus, our approach is consistent with the fact that firms sometimes make non-substantive advertisements prominently emphasizing the name of their firm, apparently directed to potential stockholders. Stocks that receive heavy publicity should be purchased disproportionately by otherwise-non-participating investors.

We have argued that the emotions of fear and suspicion are directed to the unfamiliar and toward potential change, and that this phenomenon explains several biases in individual psychology as well as economic and financial decisions. One issue we have not addressed is the effect of these feelings on decisions made in response to the arrival of new information. Such news will occasionally stimulate new uncertainty about the economic environment, thereby making individuals reluctant to trade. For example, it seems likely that extreme economic
news could raise doubts among investors about whether their beliefs about how the world is structured are correct. In such circumstances of heightened uncertainty, familiarity bias effects could become especially strong, leading to reduction in trade.\textsuperscript{16} Fear of the unfamiliar deserves further study as a possible explanation for the dynamics of market participation, liquidity, and prices.

\textsuperscript{16}See Routledge and Zin (2003) on how ambiguity aversion can lead to fluctuations in liquidity, such as the extreme illiquidity and “flight to quality” that occurred in international bond markets during the Russian debt crisis of August 1998.
References


APPENDIX

Proof of Proposition 3: Let $\Delta D$ denote the investor’s net trade. Under the assumption of CARA utility and normality of payoffs, comparison of utility functions reduced to the perceived certainty equivalent comparison between two choices. Let $G(\Delta D, e)$ denote the perceived certainty equivalent gains of a trade $\Delta D$ relative to endowment $e$ in the most adverse scenario. The certainty equivalent gains $G$ can be written as:

$$G(\Delta D, e) = \min_{|v| \leq \frac{\beta \sigma}{\sqrt{T}}} \Delta D(\mu - v - P) - \frac{\gamma \sigma^2}{2}[(\Delta D + e)^2 - e^2]$$

$$= \Delta D \left[ \mu - \frac{\text{sign}(\Delta D) \beta \sigma}{\sqrt{T}} - P \right] - \frac{\gamma \sigma^2}{2}[(\Delta D + e)^2 - e^2]$$

$$= -\frac{\gamma \sigma^2}{2} \left\{ \left[ \Delta D - \left( \mu - \frac{\text{sign}(\Delta D) \beta \sigma}{\sqrt{T}} - P \right) \right]^2 - \left[ \mu - \frac{\text{sign}(\Delta D) \beta \sigma}{\sqrt{T}} - P \right] \right\},$$

where $\text{sign}(\cdot)$ denotes the sign function. For no trade to be perceived as an optimal choice, the status quo position cannot be dominated by any other strategy. So we must have $G(\Delta D, e) \leq 0$, for all $\Delta D$. Given condition (10), when $\Delta D > 0$, we have

$$\left| \Delta D - \left( \mu - \frac{\text{sign}(\Delta D) \beta \sigma}{\sqrt{T}} - P \right) \right| > \left| \Delta D - \left( \mu - \frac{\beta \sigma}{\sqrt{T}} - P \right) \right|.$$ 

Similarly, when $\Delta D < 0$, we have

$$\left| \Delta D - \left( \mu - \frac{\text{sign}(\Delta D) \beta \sigma}{\sqrt{T}} - P \right) \right| > \left| \Delta D - \left( \mu + \frac{\beta \sigma}{\sqrt{T}} - P \right) \right|.$$ 

Thus $G(\Delta D, e) \leq 0$ for all $\Delta D$ and the investor will not deviate from the status quo position.

Q.E.D.

Proof of Proposition 5:
Under SSQDA preferences, the perceived certainty equivalent gains of moving from endowment portfolio $e$ to a portfolio $e + \Delta D$ is

$$G(\Delta D, e) \approx \gamma C(\Delta D, e),$$

where $v$ is the adjustments to perceived mean returns as in (14), and

$$C(\Delta D, e) \equiv \min_v \{\Delta D^\top (\mu + v - P) - \left(\frac{\gamma}{2}\right) [\Delta D^\top \Sigma \Delta D + 2\Delta D^\top \Sigma e]\}.\tag{26}$$

The familiarity-biased investor evaluates any deviation in the worse case scenario among the possible probability distributions. For $i = 1$ (corresponding to domestic stock) or $i = 2$ (corresponding to foreign stock), if $\Delta D_i > 0$ (buy more shares), the worse case scenario mean adjustment is $-\alpha \sigma_i$; if $\Delta D_i < 0$ (sell some shares), the worse case scenario mean adjustment is $\alpha \sigma_i$. Thus,

$$C(\Delta D, e) = \Delta D^\top [\mu - P - \text{sign}(\Delta D)v_m] + \left(\frac{\gamma}{2}\right) [\Delta D^\top \Sigma \Delta D + 2\Delta D^\top \Sigma e],\tag{26}$$

where $\text{sign}(\Delta D)$ is a vector that gives the sign of each component of the vector $\Delta D$, and $v_m$ is a vector defined as $v_m \equiv \alpha (\sigma_d, \sigma_f)^\top$.

The optimal trade $\Delta D_b^*$ for a familiarity biased investor corresponding to a given endowment $e$ maximizes $G(\Delta D, e)$. If it is nonzero, then it necessarily satisfies the first order condition derived from (26),

$$\mu - P - \text{sign}(\Delta D_b^*)v_m - \gamma \Sigma \Delta D_b^* - \gamma \Sigma e = 0,\tag{27}$$

which implies that familiarity biased investor’s optimal holding is

$$\Delta D_b^* + e = \left(\frac{1}{\gamma}\right) \Sigma^{-1} [\mu - P - \text{sign}(\Delta D_b^*)v_m].\tag{28}$$

This applies to both domestic and foreign familiarity biased investors, with $e = (x_d, 0)$ and $e = (0, x_f)$ respectively.

There are several possibilities for the familiarity biased investors’ demand in equilibrium. In the first case when the amount of uncertainty is sufficiently low in both countries, we will
show that familiarity biased investors would sell some of their own country’s stock and buy some of the other country’s stock. In the second case when the amount of uncertainty is sufficiently high in both country, the familiarity biased investors would keep their endowment. We also consider a third case where the amount of uncertainty is too high in only one country. 

**Case (1):** when uncertainty is low for both domestic and foreign stock markets, so that familiarity biased investors in both countries sell some of their own country’s stock and buy some of the other country’s stock. Then by (28), the optimal demand of domestic familiarity biased investor is

$$D_{db} = \left( \frac{1}{\gamma} \right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d + \alpha \sigma_d \\ \mu_f - P_f - \alpha \sigma_f \end{pmatrix},$$

(29)

and the optimal demand by the foreign familiarity biased investor is

$$D_{fb} = \left( \frac{1}{\gamma} \right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d - \alpha \sigma_d \\ \mu_f - P_f + \alpha \sigma_f \end{pmatrix}.$$ 

(30)

Aggregating the rational investors’ demand in (15) and familiarity biased investors’ demand in (29) and (30), the market clearing condition is

$$\left( \frac{2m}{\gamma} \right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} + \left( \frac{1-m}{\gamma} \right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d + \alpha \sigma_d \\ \mu_f - P_f - \alpha \sigma_f \end{pmatrix} + \left( \frac{1-m}{\gamma} \right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d - \alpha \sigma_d \\ \mu_f - P_f + \alpha \sigma_f \end{pmatrix} = \begin{pmatrix} x_d \\ x_f \end{pmatrix}. $$

This simplifies to

$$\left( \frac{2}{\gamma} \right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \begin{pmatrix} x_d \\ x_f \end{pmatrix},$$

which implies that the equilibrium stock prices in the first case satisfy

$$\begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \left( \frac{\gamma}{2} \right) \Sigma \begin{pmatrix} x_d \\ x_f \end{pmatrix},$$

just as claimed in Case (1) of Proposition 5 (see equation (16)). The equilibrium stock prices in Case 1 coincides with the equilibrium stock prices when all investors are rational. Corresponding to the this price vector, the familiarity biased investors’ demand are given by (17) and (18).
We need to check that familiarity biased investors in both countries sell some of their own country’s stock and buy some of the other country’s stock. For this to obtain the model parameters must satisfy:

\[
\frac{1}{2}x_d + \frac{\alpha}{(1 - \rho)\gamma\sigma_d} < x_d,
\]

\[
\frac{1}{2}x_d - \frac{\alpha}{(1 - \rho)\gamma\sigma_d} > 0,
\]

\[
\frac{1}{2}x_f - \frac{\alpha}{(1 - \rho)\gamma\sigma_f} > 0,
\]

\[
\frac{1}{2}x_f + \frac{\alpha}{(1 - \rho)\gamma\sigma_f} < x_f.
\]

The necessary and sufficient condition for the above to hold is

\[
\alpha < \min \left\{ \left( \frac{1 - \rho}{2} \right) \gamma\sigma_d x_d, \left( \frac{1 - \rho}{2} \right) \gamma\sigma_f x_f \right\}.
\]

**Case (2):** \( \alpha > \max \left\{ \left( \frac{1 - \rho}{2} \right) \gamma\sigma_d x_d, \left( \frac{1 - \rho}{2} \right) \gamma\sigma_f x_f \right\} \). In this case, both domestic and foreign familiarity biased investors choose to stay at the endowment because the perceived amount of uncertainty is too high. The market clearing condition is

\[
\left( \frac{2m}{\gamma} \right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} + (1 - m) \begin{pmatrix} x_d \\ 0 \end{pmatrix} + (1 - m) \begin{pmatrix} 0 \\ x_f \end{pmatrix} = \begin{pmatrix} x_d \\ x_f \end{pmatrix},
\]

which implies that the equilibrium stock prices satisfy (16).

**Case (3):** The amount of uncertainty is too high in one country but not the other. Without loss of generality, assume the parameters are such that \( \left( \frac{1 - \rho}{2} \right) \gamma\sigma_f x_f < \alpha < \left( \frac{1 - \rho}{2} \right) \gamma\sigma_d x_d \). In this case, the domestic familiarity biased investor sells some of his endowment but he does not buy any shares of the foreign stock. The foreign familiarity biased investor stays at his endowed foreign stock shares and does not invest in the domestic stock. The market clearing
condition is
\[
\frac{2m}{\gamma} \Sigma^{-1} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} + \left( \frac{1-m}{(1-\rho^2)\gamma \sigma_d^2 \sigma_f^2} \right) \begin{pmatrix} (\mu_d - P_d + \alpha \sigma_d) \sigma_f^2 - \rho \sigma_d \sigma_f (\mu_f - P_f) \\ 0 \end{pmatrix} \\
+ (1-m) \begin{pmatrix} 0 \\ x_f \end{pmatrix} = \begin{pmatrix} x_d \\ x_f \end{pmatrix}.
\]

This is equivalent to the following system of linear equations for \( \mu_d - P_d \) and \( \mu_f - P_f \):
\[
\begin{pmatrix} \frac{1}{(1-\rho^2)\sigma_d^2 \sigma_f^2} & -\rho \sigma_d \sigma_f \\ -\rho \sigma_d \sigma_f & \frac{\sigma_f^2}{\sigma_d^2} \end{pmatrix} \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \begin{pmatrix} \frac{1}{1+m} \left( \gamma x_d - \frac{(1-m)\alpha}{1-\rho^2} \sigma_d \right) \\ \frac{1}{2} \gamma x_f \end{pmatrix}.
\]

But
\[
\Sigma^{-1} = \frac{1}{(1-\rho^2)\sigma_d^2 \sigma_f^2} \begin{pmatrix} \frac{\sigma_f^2}{\sigma_d^2} & -\rho \sigma_d \sigma_f \\ -\rho \sigma_d \sigma_f & \frac{\sigma_f^2}{\sigma_d^2} \end{pmatrix},
\]
so the equilibrium stock prices satisfy (19) as claimed in the case (3) of Proposition 5.

**Proof of Proposition 6:** The world stock market \( M \) consists of \( x_d \) shares of the domestic stock and \( x_f \) share of the foreign stock. Its payoff next period is normally distributed as
\[
\tilde{V}_M \sim N \left( x_d \mu_d + x_f \mu_f, (x_d x_f) \Sigma \left( \begin{array}{c} x_d \\ x_f \end{array} \right) \right).
\]

The value of the world stock market \( P_M \) is \( x_d P_d + x_f P_f \). Stock returns are
\[
r_d = \frac{V_d - P_d}{P_d}, \quad r_f = \frac{V_f - P_f}{P_f}.
\]

\[
r_M = \frac{V_M - P_M}{P_M} = \frac{x_d (V_d - P_d) + x_f (V_f - P_f)}{P_M}.
\]

It follows that
\[
E[r_M] = \left( \frac{1}{P_M} \right) (x_d x_f) \begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix},
\]
\[
\text{Var}(r_M) = \left( \frac{1}{P_M^2} \right) (x_d x_f) \Sigma \left( \begin{array}{c} x_d \\ x_f \end{array} \right),
\]
\[
\text{Cov}(r_i, r_M) = \left( \frac{1}{P_i P_M} \right) \left[ \Sigma \left( \begin{array}{c} x_d \\ x_f \end{array} \right) \right]_i,
\]

45
where $[-]_i$ denotes the $i$th component of a vector, $i = 1$ (respectively $i = 2$) corresponds to the domestic (foreign) stock. Thus, the beta of the domestic (respectively foreign) stock return with respect to the world market return $\beta_d$ (respectively $\beta_f$) is

$$
\beta_d = \left( \frac{P_M}{P_d} \right) \left[ \frac{\Sigma (x_d \ x_f)}{(x_d \ x_f) \Sigma (x_d \ x_f)} \right]_1,
$$

$$
\beta_f = \left( \frac{P_M}{P_f} \right) \left[ \frac{\Sigma (x_d \ x_f)}{(x_d \ x_f) \Sigma (x_d \ x_f)} \right]_2.
$$

It follows that

$$
\beta_i E[r_M] = \left( \frac{\beta_i}{P_M} \right) (x_d \ x_f) \left( \frac{\mu_d - P_d}{\mu_f - P_f} \right), \quad i = d \text{ or } f.
$$

(31)

CAPM holds if and only $E[r_i] = \beta_i E[r_M]$ in the equilibrium (the riskfree rate is zero in our economy).

For Case 1 and Case 2 of Proposition 5, equilibrium prices $P_d$ and $P_f$ satisfy (16). Substituting (16) into (31),

$$
\beta_d E[r_M] = \left( \frac{\gamma \beta_d}{P_M} \right) (x_d \ x_f) \left( \frac{\mu_d - P_d}{\mu_f - P_f} \right)
$$

$$
= \left( \frac{\gamma}{2P_d} \right) \left[ \Sigma (x_d \ x_f) \right]_1
$$

$$
= \frac{\mu_d - P_d}{P_d}
$$

$$
= E[r_d]
$$

Thus, CAPM holds for the domestic stock. Similarly, CAPM holds for the foreign stock in these cases as well.

For Case 3 of Proposition 5, suppose $(1+\rho^2) \frac{\gamma \sigma_f}{2} < \alpha < (1+\rho^2) \frac{\gamma \sigma_d}{2}$.\(^{17}\) Then the conclusion is the same (i.e., the CAPM does not hold) when $(1+\rho^2) \frac{\gamma \sigma_d}{2} < \alpha < (1+\rho^2) \frac{\gamma \sigma_f}{2}$.
equilibrium prices \( P_d \) and \( P_f \) satisfy (19). It follows that

\[
\beta_d \mathbb{E}[r_M] = \left( \frac{1}{P_d} \right) \frac{\left( \sum x_d \right) \left( \sum x_f \right)}{(x_d x_f) \Sigma \left( \begin{array}{c} x_d \\ x_f \end{array} \right)} \left( \frac{1}{1+m} \left( \frac{\gamma x_d - (1-m)\alpha}{(1-\rho^2)\sigma_d} \right) \right)
\]

\[
= k_1 \mathbb{E}[r_d],
\]

where

\[
k_1 = \frac{(x_d x_f) \Sigma \left( \begin{array}{c} x_d \\ x_f \end{array} \right) \left( \sum x_d \right) \left( \sum x_f \right)}{(x_d x_f) \Sigma \left( \begin{array}{c} x_d \\ x_f \end{array} \right) \Sigma \left( \begin{array}{c} x_d \\ x_f \end{array} \right)}.\]

Similarly, for the foreign stock market,

\[
\beta_f \mathbb{E}[r_M] = k_2 \mathbb{E}[r_f],
\]

where

\[
k_2 = \frac{(x_d x_f) \Sigma \left( \begin{array}{c} x_d \\ x_f \end{array} \right) \left( \sum x_d \right) \left( \sum x_f \right)}{(x_d x_f) \Sigma \left( \begin{array}{c} x_d \\ x_f \end{array} \right) \Sigma \left( \begin{array}{c} x_d \\ x_f \end{array} \right)}.\]

The constants \( k_1 \) and \( k_2 \) are not equal to one in general. Thus, the CAPM does not hold in Case 3 as considered in Proposition 5.

Finally, we show that a modified version of CAPM holds in Case (3). Suppose \( \frac{(1+\rho^2)}{2} \gamma \sigma_d x_d < \alpha < \left( \frac{1+\rho^2}{2} \right) \gamma \sigma_d x_d \). The rational Bayesian investors’ optimal holdings are given by

\[
\left( \frac{1}{\gamma} \right)^{-1} \left( \begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array} \right)
\]

Substituting the equilibrium stock returns given by (19), the rational Bayesian investors’ portfolio \( M' \) consist of \( n_1 x_d \) shares of the domestic stock and \( n_2 x_f \) shares of the foreign
stock, where
\[ n_1 = \frac{1}{1 + m} - \left( \frac{1 - m}{1 + m} \right) \frac{\alpha}{(1 - \rho^2)^2 \sigma_d x_d}, \quad n_2 = \frac{1}{2}. \]

Note that \( n_1 > n_2 \), and the difference increases with the amount of uncertainty \( \alpha \).

The expected return of the portfolio \( M' \) is:
\[ \mathbb{E}[r_{M}] = \left( \frac{1}{P_{M'}} \right) (n_1 x_d n_2 x_f) \left( \begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array} \right). \]

The beta of stock \( i \) with respect to the portfolio \( M' \) is \((i = 1 \text{ for the domestic stock}, i = 2 \text{ for the foreign stock})\)
\[ \beta_i = \left( \frac{P_{M'}}{P_i} \right) \left[ \Sigma \left( \begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right) \right]_i \left( \begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right) \Sigma \left( \begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right). \]

By the definition of \( n_1 \) and \( n_2 \), and the equilibrium return relation (19),
\[ \left( \begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array} \right) = \Sigma \left( \begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right). \]

Using equations (8) and (8), it follows that for the domestic stock,
\[ \beta_d \mathbb{E}[r_{M'}] = \left( \frac{1}{P_d} \right) \left[ \Sigma \left( \begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right) \right]_1 \left( \begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right) \left( \begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array} \right) \]
\[ = \left( \frac{1}{P_d} \right) \left[ \Sigma \left( \begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right) \right]_1 \]
\[ = \frac{\mu_d - P_d}{P_d} \]
\[ = \mathbb{E}[r_d]. \]

Thus, the CAPM holds for the domestic stock with respect to the modified market portfolio \( M' \). The case for the foreign stock is similar.
Table 1. Summary statistics of annual stock market returns for various countries.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.1356</td>
<td>0.2431</td>
<td>0.5679</td>
</tr>
<tr>
<td>Japan</td>
<td>0.1434</td>
<td>0.3017</td>
<td>0.8508</td>
</tr>
<tr>
<td>UK</td>
<td>0.1890</td>
<td>0.2504</td>
<td>0.6076</td>
</tr>
<tr>
<td>US</td>
<td>0.1478</td>
<td>0.1569</td>
<td>0.5004</td>
</tr>
<tr>
<td>World</td>
<td>0.1495</td>
<td>0.2084</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

The reported statistics are for the annual value-weighted dollar returns from January 1975 to December 2006. “Correlation” measures the sample correlation between the stock market return in each country and the return on the world market portfolio. The original datasets are obtained from Kenneth French’s website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french.
Figure 1: Optimal weight on domestic equity for familiarity biased investors

This figure plots the perceived optimal domestic equity proportion as a function of model uncertainty ($\beta$) for Germany, Japan, United Kingdom, and the United States. Investors are allowed to hold their domestic market portfolio and the world market portfolio. Their initial endowment is 100% in domestic equity. The adjustments $v$ for the mean stock returns for familiarity biased investors satisfies $v^\top \Sigma^{-1} v \leq \beta^2 / T$, where $\Sigma$ is the covariance matrix of return of a country with the world market using the annual return data from January 1975 to December 2006 ($T = 32$). The risk aversion coefficient is set to $\gamma = 2$ for all four panels.
This figure plots the certainty equivalent gains as a function of model uncertainty ($\beta$) for Germany, Japan, United Kingdom, and the United States when familiarity biased investors move from their 100% domestic equity initial endowment to the optimal combination of domestic and world market (see Figure 1). The adjustments $v$ for the mean stock returns for familiarity biased investors satisfies $v^\top \Sigma^{-1} v \leq \beta^2 / T$, where $\Sigma$ is the covariance matrix of return of a country with the world market using the annual return data from January 1975 to December 2006 ($T = 32$). The risk aversion coefficient is set to $\gamma = 2$ for all four panels.
The minimum number of stocks in an investor’s portfolio when defection from the endowment induces aversion to model uncertainty, for various risk aversion coefficients. The total number of available stocks is 500. Assume the investor uses $T = 100$ data points and estimates that the annual standard deviation of stock return is $\sigma = 0.3$, and the pairwise correlation is 0.5.
Home bias ratio as a function of model uncertainty ($\alpha$) for Germany, Japan, United Kingdom, and the United States in a general equilibrium model with familiarity bias and both Bayesian and uncertainty averse investors. The home bias ratio is defined as the ratio of each country’s domestic holdings in the total risky portfolio relative to that of the weighted domestic holdings in the world market portfolio. The proportion of Bayesian investors is set at 20 percent. The risk aversion is set at $\gamma = 2$. 

Figure 4: Home bias ratio