# A Fuzzy Control Strategy and Optimization for Four Wheel Steering System

Jie Zhang[a], Yunqing Zhang[a], Liping Chen[a], Jingzhou Yang[b]

[a]Center for Computer-Aided Design, Huazhong University of Science & Technology

Wuhan, Hubei 430074, P.R. China, jiehust@gmail.com
[b]Center for Computer-Aided Design, University of Iowa, Iowa City, IA 52246, USA
jyang@engineering.uiowa.edu

***Abstract*—This paper presents a fuzzy logic control strategy on four-wheel steering(4WS) vehicle based on a multi-body vehicle dynamic model. The multi-body vehicle dynamic model based on ADAMS can accurately predict the dynamics performance of the vehicle. Fuzzy logic is applied to track the yaw velocity of the two degrees of freedom ideal model through the co-simulation of ADAMS and Matlab Fuzzy control unit with the optimized membership function. The fuzzy control parameters are optimized and analyzed by a combined optimization algorithm (Genetic Algorithm (GA) and Nonlinear Programming Quadratic Line search (NLPQL) method) combined with response surface model (RSM). Single lane change test is chosen to validate the fuzzy control logic strategy. Simulation result shows that four-wheel steering vehicle with the fuzzy control logic strategy can improve vehicle handling stability greatly comparing with traditional front wheel steering.**

***Index Terms*—Four-wheel steering, fuzzy control, multi-body, genetic algorithm**

## I. INTRODUCTION

RECENT research and developments suggest that four-wheel steering systems can effectively improve transient response of vehicles in corning. Many actual and theoretical 4WS controllers are such that in high speed, steering angle of rear and front wheels are in the same direction to improve stability of vehicle and satisfy passenger relaxation. But in low speed, especially in parking geometry, the steering angle of front and rear wheels are chosen in the opposite direction to improve maneuverability of vehicle.

During the last 20 years, many different control methods have been applied on 4WS system. Nalecz[10-12] used a proportional controller in his nonlinear 3DOF model, while Furukawa[13] used proportional and compensator controllers in his control strategy. Whitehead[14] investigated a controller by setting the change rate of sideslip angle and sideslip angle to zero, which ultimately resulted in an uncoupled system for the sideslip angle. Nagai[8, 15] used a full state compensator with the use of error of state variables. To be extended, Song[7] introduced a dual steering schemes as a useful method for controlling both yaw rate and lateral velocity state variables. H-MLv[3] studied multi-objective H∞ optimization control based on yaw velocity feedback; Paul I. Ro and Hoyong Kim[5] applied sliding mode control in 4WS vehicle control indicating that handling performance in high speed was improved. Among them, only Will[2, 6] and Szosland[4] used fuzzy logic method to investigate the performance on controlling the wheel angle. In this paper, we choose fuzzy logic method to control the sideslip angle and yaw velocity according a 2DOF ideal model.

Fuzzy logic control is proved to be an efficient way to implement engineering heuristics into a control solution. The main advantages of using fuzzy logic is to reduce the very detailed models of controller. But fuzzy logic controller is usually built based on the operator's knowledge, resulting in that controllers designed by different experts may be various. To solve this problem, optimization strategy is introduced in this paper and fuzzy logic controller is designed form the view of optimization, which ensure the controller optimal. The membership function of the controller is optimized by a integrated algorithm (Genetic Algorithm (GA) and Nonlinear Programming Quadratic Line search (NLPQL) method) combined with RSM to maintain the controller accurate and stable.

This paper presents a multi-body vehicle dynamic model and a dynamic control strategy by comparing the multi-body model with the ideal model. The control strategy considers both yaw velocity $\gamma$ and sideslip angle $\beta$ objectives. The method is optimized and analyzed by a combined optimization algorithm (Genetic Algorithm (GA) and Nonlinear Programming Quadratic Line search (NLPQL) method) combined with RSM. Under extreme motion state, the simulation results indicate that the proposed method can gain dynamic stability and improve the accuracy of fuzzy control.

The paper consists of seven parts. The second section presents the multi-body vehicle dynamics model and 2DOF ideal reference model. In the third part, we integrated the dynamics model and controller for 4WS co-simulation using fuzzy logic control method. Fuzzy control strategy is the next part. In the fifth part, we optimize the membership function of fuzzy controller by a combined algorithm—GA and NLPQL. Finally, the simulation result and conclusion are presented.

## II. VEHICLE DYNAMICS MODEL AND CONTROL OBJECTIVES FOR 4WS

### *A. Linear vehicle model for 4WS*

Because 4WS system synthetically takes both transversal and yaw dynamic problem into account, we consider the multi-body dynamic model as a whole vehicle model and a 2DOF model (show in Fig. 1) as an ideal reference model. The vehicle is a four-wheel steering system.

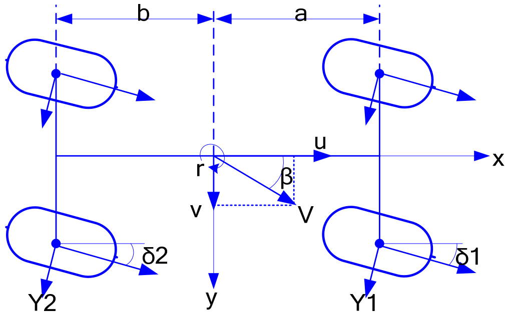

Fig. 1 A 2DOF ideal vehicle model

The dynamic equations for the ideal 2DOF reference model are formulated as:

$$\begin{cases} mv(\dot{\beta}+\gamma) = 2Y_1 + 2Y_2 \\ I\dot{\gamma} = 2aY_1 - 2bY_2 \end{cases} \tag{1}$$

Lateral force on wheels can be expressed as:

$$\begin{cases} Y_1 = -k_1\beta_1 = -k_1(\beta + \frac{a}{v}\gamma - \delta_1) \\ Y_2 = -k_2\beta_1 = -k_2(\beta - \frac{b}{v}\gamma - \delta_2) \end{cases} \tag{2}$$

where $m$ denotes the mass of vehicle, *v* is vehicle forward velocity, $\gamma$ represents the yaw velocity, $\beta$ is side-slip angle, $K_1$ and $K_2$ represent the front and rear wheels cornering stiffness, respectively. $\delta_1$ and $\delta_2$ represent front and rear wheel steering angle, respectively. Y1 and Y2 represent the lateral force of front and rear wheel. I is moment of inertia, *a* and *b* is length of front and rear axle to CG..

Then, Considering the vehicle velocity to be time invariant, the dynamic equations can be described in state space form:

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX + DU \end{cases} \tag{3}$$

Where $A = \begin{pmatrix} -\frac{k_1+k_2}{mv} & \frac{bk_2-ak_1}{mv^2}-1 \\ \frac{bk_2-ak_1}{Iv} & -\frac{a^2k_1+b^2k_2}{Iv} \end{pmatrix}$, $B = \begin{pmatrix} \frac{k_1}{mv} & \frac{k_2}{mv} \\ \frac{ak_1}{I} & -\frac{bk_2}{I} \end{pmatrix}$,

$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $\dot{X} = (\dot{\beta}\ \dot{\gamma})^T$, $U = (\delta_1\ \ \delta_2)^T$.

The parameters of the ideal 2DOF vehicle are shown in Table 1.

Table 1 Ideal vehicle model parameters

| | | | |
|---|---|---|---|
| *m( kg)* | 1400 | *I (Kgm^2)* | 1993 |
| *a(m)* | 1.063 | *b(m)* | 1.485 |
| *$K_1$ (kN/rad)* | 52.48 | *$K_2$( kN/rad)* | 88.416 |

### B. *Multi-body vehicle dynamic model for 4WS simulation*

Multi-body dynamic simulation is used to successfully simulate a wide variety of vehicles and predict the safety, mobility, stability, and operating loads of the complete system. The theoretical basis of multi-body vehicle dynamic model for 4WS simulation is multi-body dynamics, and the kinetic equation built with Lagrange Multiplier Method is presented as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)^T - \left(\frac{\partial T}{\partial q}\right)^T + \Phi_q^T \rho + \theta_q^T \mu = Q \tag{4}$$

Integrity constraint equations: $\Phi(q,t) = 0$

Nonholonomic constraint equations: $\theta(q,\dot{q},t) = 0$

Where T represents kinetic energy of the system, q is generalized coordinate vector, Q denotes generalized force vector, $\rho$ and $\mu$ represent the vector of Lagrange multipliers corresponding the Integrity constraints and Nonholonomic constrains.

The multi-body vehicle dynamic model was built in ADAMS/CAR environment. This model includes seven subsystems—front suspension model, rear suspension model, brake system, powertrain system, steering system, tire and bodywork model, as shown in Fig. 2. During the building of the multi-body model, we have considered the joint constraints and the force element such as springs, dampers, bushing and so on. We have also considered the nonlinearity of tire and the flexibility of certain parts, which accurately reflects the practical vehicle system.

Fig. 2 Assemble vehicle model

### C. *Tire model*

In most time, 4WS has high nonlinearity and we should adopt nonlinear tire model. Thus, it introduces Pacejka's Magic Formula Model[9] which has high precision for longitudinal force of wheel and side-force and also has better confidence level out of range of limit value. It can be expressed in the following form.

$$\begin{cases} Y1 = y + S_v \\ y = G\sin(F\arctan(Ex - H(Ex - \arctan(Ex)))) \\ x = X1 + S_h \end{cases} \tag{5}$$

Where Y1(x) represents lateral-force, opposite rotary moment or longitudinal force, X1 is sideslip angle ($\beta$) or wheel slip ratio S. The coefficients E, F, G, H are determined by vehicle velocity and drive situation, and $S_v, S_h$ denotes the horizontal and vertical drift.

### *D. Control objectives for 4WS*

At the present time, there are two main control objectives in the study on 4WS. The sideslip angle control strategy reduces the lateral motion and transportation of vehicle, while it improves handling maneuverability and reduces the delay of response of the vehicle. A large number of researchers have used this strategy to control the 4WS vehicle, such as Nikzad[17], Nalecz[10-12], Furukawa[13], Cho[16], Whitehead[14] and Xia[18]. The yaw velocity control strategy minimizes the rotational motion of vehicle and leads the vehicle to lateral side tracking the desired trajectory. Researchers like Nikzad[17], Song[7] and Furukawa[13] used this strategy in their papers. Also, some researchers are performed under reference models, such as Nikzad[17], Song[7], Nagai[8, 15] and Palkovics[19].

In this paper, a fuzzy logic controller is used to track the yaw velocity of the 2DOF ideal model, which is detailed in the later section.

## III. INTEGRATED THE DYNAMICS MODEL AND CONTROLLER FOR 4WS CO-SIMULATION

The simulation system is established by the combination of ADAMS and MATLAB. The structure of co-simulation is showed in Fig. 3. The control objectives have been mentioned in the former part and we intend to make sideslip angle closest to zero and yaw velocity to track the 2DOF model. The ECU sends control instructions to the rear wheel steering system based on yaw velocity error to adjust the rear wheel angle. In the co-simulation, fuzzy logic method is applied on the control strategy.

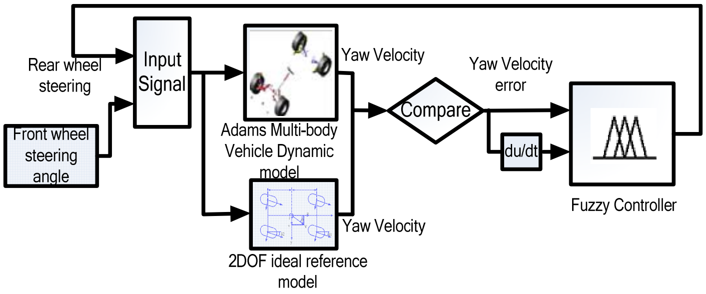

Fig. 3 Structure graph of Fuzzy control

## IV. FUZZY CONTROL STRATEGY DESIGN

### *A. Fuzzy controller design*

The fuzzy controller present in this paper chooses yaw velocity error (E) and yaw velocity error change rate (EC) as input variables, which can reflect the dynamic characteristics of output variable strictly. The input and output channels are presented in Fig. 4.

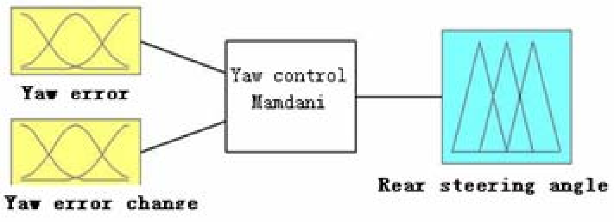

Fig. 4 2-input and 1-output fuzzy controller

### *B. The choice of fuzzy controller parameters*

Input and output variables can be divided into 5 levels. They are defined as negative big (NB), negative median (NM), median zero (ZE), positive median (PM) and positive big (PB). In order to improve defuzzification speed, Trimf is chosen as membership function and the weight is equal to one. Figs. 5-7 show the membership functions of input and output variables, the range of yaw velocity error and yaw velocity error change rate are [-6 6] and [-10 10], respectively. The range of output variable is normalized within 0 to 1.

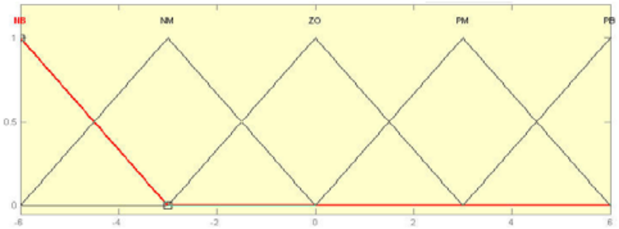

Fig. 5 membership function of yaw velocity error

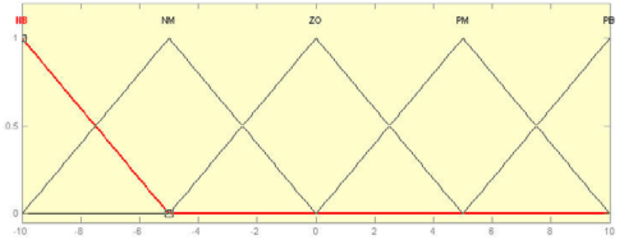

Fig. 6 membership function of yaw velocity error change rate

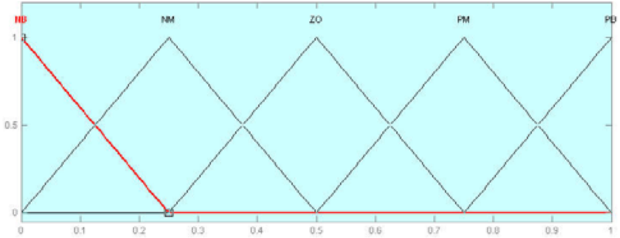

Fig. 7 membership function of rear steering angle

### *C. Establishing fuzzy controlling rules*

We adopt average gravity center method for defuzzification. When error and error change rate are both negative big, the error will have an increasing trend and it requires the output is positive big to track the setting value. Thus, the rear steering wheel angle can be controlled intelligently according to the fuzzy control rules established based on control strategy showed in Table 2. The response surface of rear steering angle is showed in Figs. 8.

Table 2 Fuzzy control rulers

| U | | E | | | | |
|---|---|---|---|---|---|---|
| | | PB | PM | ZE | NM | NB |
| EC | PB | NB | NB | NB | NS | ZE |
| | PM | NB | NB | NM | ZE | PS |
| | ZE | NB | NM | ZE | PM | PB |
| | NM | NS | ZE | PM | PB | PB |
| | NB | ZE | PS | PB | PB | PB |

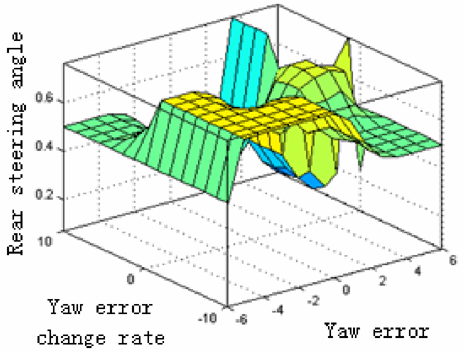

Fig. 8 Response surface of Yaw velocity

## V. OPTIMIZATION OF THE FUZZY CONTROLLER

For the reason that fuzzy control rules are determined by expert's experience, it is difficult to optimal fuzzy control rules. However, we can consider to build fuzzy controller from the view of optimization. Fuzzy control rules determined by the control strategy is reliable, and the objective is to optimize membership functions. Assuming that membership function of each variable is centrosymmetric, we can identify each membership function with only two parameters—position and width. Thus, we need seven parameters to identify a variable's universe and we have total 3 variables, so we need 21 factors to determine the membership of the whole fuzzy control. This number is so large that we use design of experiment (DOE) analysis to select the main factors, then optimize these factors. The flow chart of the optimization is shown in Fig. 9.

we choose single lane change test that to study the function of dynamical parameters and handling stability.

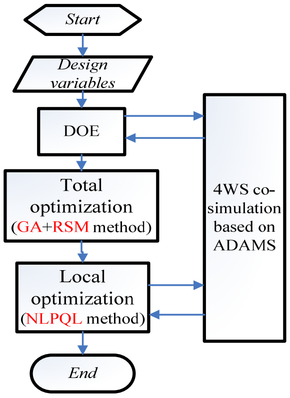

Fig. 9 Co-optimization flow chart of GA and RSM+NLPQL

We adopt Latin Hypercube method with uniform space sampling and random combination. Fig. 10 shows the normalized Pareto graph of all factors. We choose the first 10 factors which have great contribution to the result and optimize these factors. The vertical axis represents fuzzy control variables, *A* represents the center of Trimf, *B* represents width of Trimf, and numerical value after the alphabet represent 5 levels of variables.

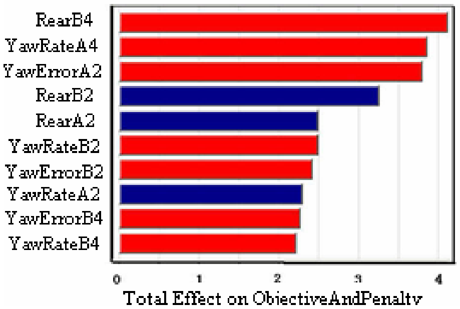

Fig. 10 Pareto graph analyzed by DOE

There are four objectives f or the optimization, and they are: (1)sideslip angle; (2)yaw velocity; (3) the time lag between steering wheel angle and yaw velocity; and (4)the time lag between yaw velocity and lateral acceleration. It can be formulated as follows.

*To find*: Design variables

*to minimize*

$$F_t = \sqrt{\Psi}$$

$$\text{where } \Psi = \lambda_1\left(\frac{\bar{\gamma}}{B_{\bar{\gamma}}}\right)^2 + \lambda_2\left(\frac{\bar{\beta}}{B_{\bar{\beta}}}\right)^2 + \lambda_3\left(\frac{\gamma_{\max}}{B_{\gamma_{\max}}}\right)^2 + \lambda_4\left(\frac{\gamma_{\min}}{B_{\gamma_{\min}}}\right)^2$$

$$+ \lambda_5\left(\frac{\beta_{\max}}{B_{\beta_{\max}}}\right)^2 + \lambda_6\left(\frac{\beta_{\min}}{B_{\beta_{\min}}}\right)^2 + \lambda_7\left(\frac{T_\gamma}{B_{T_\gamma}}\right)^2 + \lambda_8\left(\frac{T_a}{B_{T_a}}\right)^2$$

*subject to*:

$$0 \le \mathrm{Re}\,arB4 \le 0.5, 0 \le YawRateA4 \le 10$$
$$-6 \le YawErrorA2 \le 0, 0 \le \mathrm{Re}\,arB2 \le 0.5$$
$$0 \le \mathrm{Re}\,arA2 \le 0.5, 0 \le YawRateB2 \le 10$$
$$0 \le YawErrorB2 \le 6, -10 \le YawRateA2 \le 0$$
$$0 \le YawErrorB4 \le 6, 0 \le YawRateB4 \le 10$$

Where $B_{\bar{\gamma}}$, $B_{\bar{\beta}}$, $B_{\gamma_{\max}}$, $B_{\gamma_{\min}}$, $B_{\beta_{\max}}$, $B_{\beta_{\min}}$, $B_{T_r}$, $B_{T_a}$ are the maximum values of $\bar{\gamma}$, $\bar{\beta}$, $\gamma_{\max}$, $\gamma_{\min}$, $\beta_{\max}$, $\beta_{\min}$, $T_\gamma$, $T_a$ respectively. $\lambda_1 \ldots \lambda_8$ are the corresponding weights. $T_\gamma$ is the time lag between steering wheel angle and yaw velocity. $T_a$ is the time lag between yaw velocity and lateral acceleration. $\bar{\beta}$ is the mean values of sideslip angle. $\beta_{\max}$ and $\beta_{\min}$ are the maximum and minimum values of sideslip angle, respectively. $\overline{\gamma}$ is the mean value of yaw velocity. $\gamma_{\max}$ and $\gamma_{\min}$ are the maximum and minimum values of the yaw velocities.

To use a genetic algorithm, the initial point should be in feasible region first. Then, we stochastically choose some code array from feasible region as the first code array of the beginning of evolution and compute the objective function of each solution. In addition, some codes are selected randomly as code sample of pre-production and produce the next generation code array through crossover and variance. Repeat the process above until obtaining the optimal solution of the last generation, the solution is the final solution of genetic algorithm. This algorithm can prevent to trap into local optimization in the process of searching optimal point.

Programming Quadratic Line search is called numerical optimization and mathematical programming established by quadratic objective function and linear constrained function. It can search the optimal solution in the continuous design space with single-peak.

The combination of two optimization methods can prevent to trap in local optimization and improve optimization speed using RSM approximation model. Fig. 11-13 show the membership function after optimization.

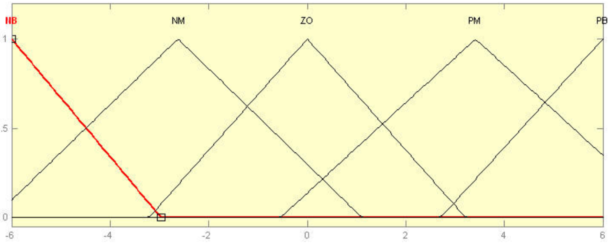

Fig. 11 Membership function of yaw velocity error after optimization

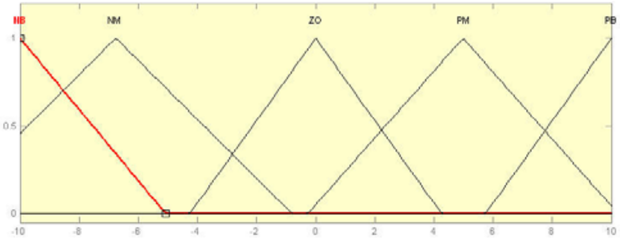

Fig. 12 Membership function of yaw velocity error change rate after optimization

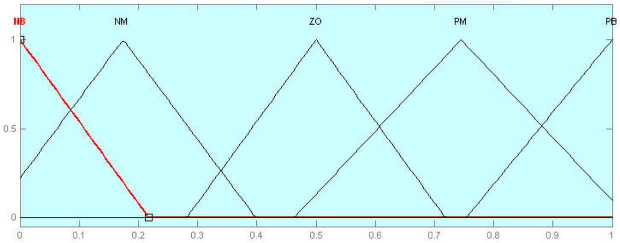

Fig. 13 Membership function of rear steering angle after optimization

## VI. SIMULATION RESULTS AND DISCUSSION

Based on the above formulation, we illustrate the procedure and results using single lane change experiment. The vehicle velocity is 100km/h, and the steering wheel angle is 20 degree. Figs. 14-17 show the simulation result of 4WS vehicle before optimization, after optimization and Front Wheel Steering with the same condition. The results show that with fuzzy control yaw velocity of 4WS vehicle can track the ideal reference model accurately while it is also reduces comparing with that of front wheel steering. In addition, the sideslip angle of 4WS vehicle can decrease a lot comparing with the Front wheel steering vehicle, and the performance of fuzzy controller after optimization is better than that before optimization. What is more, the lag phase between steering wheel angle and yaw velocity and that between yaw velocity and lateral acceleration reduces after optimization (shown in Table 3).

Table 3 Lag phase comparing

| Lag phase | Before optimization | After optimization |
|---|---|---|
| Between steering wheel angle and yaw velocity | 0.07s | 0.065s |
| Between yaw velocity and lateral acceleration | 0.155s | 0.0514s |

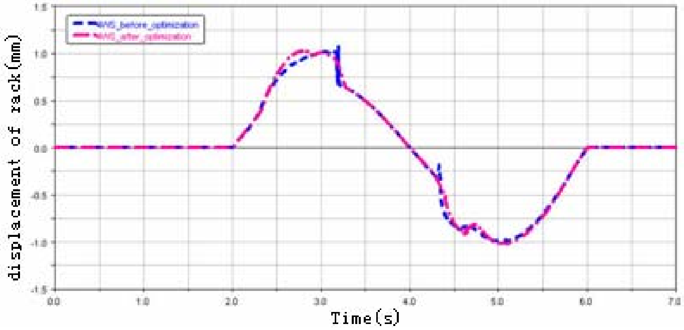

Fig. 14 Comparison of rear steering angle
(Dash and Blue: before optimization; Dotdash and Magenta: after optimzation)

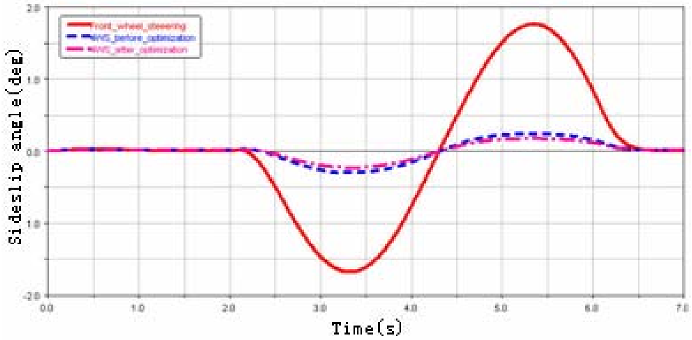

Fig. 15 Comparison of sideslip angle
(Solid and Red: front wheel steering; Dash and blue: four wheel steering before optimization; Dotdash and Magenta: four wheel steering after optimization)

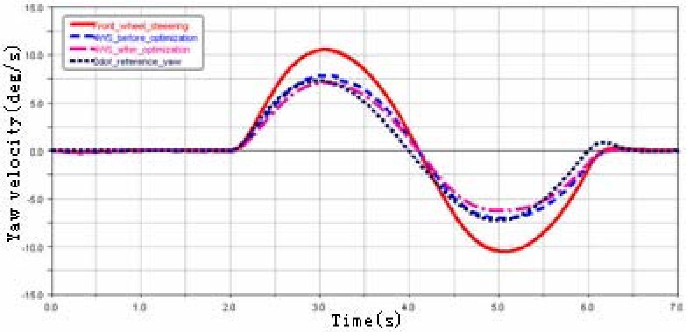

Fig. 16 Comparison of yaw velocity
(Solid and Red: front wheel steering; Dash and blue: four wheel steering before optimization; Dotdash and Magenta: four wheel steering after optimization; Dot and Black: 2DOF reference model)

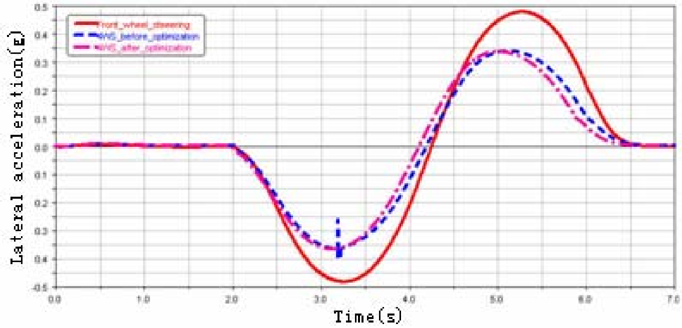

Fig. 17 Comparison of lateral acceleration
(Solid and Red: front wheel steering; Dash and blue: four wheel steering before optimization; Dotdash and Magenta: four wheel steering after optimization)

## VII. CONCLUSIONS

The results show that 4WS system can enhance the vehicle stability and handling. We also see that fuzzy logic maintain good character in the control of sideslip angle and yaw velocity. For fuzzy logical control and high nonlinear model like multi-body model, this paper adopts a combined optimization method of Genetic Algorithm global search, RSM approximation model and Nonlinear Programming Quadratic Line search. The results show that using the optimization method of co-simulation of multi-body dynamical model and fuzzy control model combined with the experimental method for the 4WS system can generate desired solutions. The results also show that fuzzy control becomes more accurate after optimization, and the 4WS can effectively provide more stable responses.

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