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# Fractional-Order $PI^{\lambda}D^{\mu}$ Control and Optimization for Vehicle Active Steering

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***Abstract* - Active steering is a newly developed technology for passenger cars to enhance vehicle stability and handling performance. A fractional-order $PI^{\lambda}D^{\mu}$ control strategy is applied on active steering vehicle based on a multi-body vehicle dynamic model in this paper and the active control is expressed as a sum of $PI^{\lambda}$ and $PD^{\mu}$ control. The multi-body vehicle dynamic model using ADAMS can accurately predict the dynamic performance of the vehicle. A new hybrid steering scheme including both active front steering (applying an additional front steering angle besides the driver input) and rear steering is presented to control both yaw velocity and sideslip angle. The parameters of the controller were optimized by a genetic algorithm (GA) and it can adjust both sideslip angle and yaw velocity through the co-simulation between the ADAMS multi-body vehicle dynamic model and Matlab. The co-simulation scenario takes place at high speed with a single-sine steering angle input to validate the effectiveness of active steering system. Simulation result shows that active steering vehicle with fractional-order $PI^{\lambda}D^{\mu}$ control logic strategy can enhance vehicle stability and handling greatly comparing with classical PID controller and traditional front wheel steering vehicle.**

***Index Terms - Fractional-order $PI^{\lambda}D^{\mu}$ , active steering , multi-body, optimization.***

## I. Introduction

Active steering is a possible approach to enhance driving safety under critical situations. For active steering system, an additional front steering wheel angle controlled by Electronic Controller Unit (ECU) is combined to the driver input steering wheel angle while the permanent mechanical connection between steering wheel and road wheels remains, which is the major difference with steer-by-wire system. This new technology has been applied on the 2003 BMW-5 passenger sedan [1-3]. The steering system presented in this paper includes both active front wheel steering and rear wheel steering, which maintain the advantages (1) easy maneuverability at low speed; (2) improved handling and stability at high speed and (3) quick response to driver's input.

There is a wealth of literature that focuses on active steering research. As early as 1969, Kasselmann and Keranen [4] developed an active steering system based on feedback from a yaw rate sensor. In 1996, Ackermann [5] combined active steering with yaw rate feedback to robustly decouple yaw and lateral motions, which is effective in canceling out yaw generated when braking on a split friction surface. In Hiraoka's research [6-8], an estimated value for sideslip angle was used for active front steering control, and computer simulations demonstrated good estimates of sideslip angle and good performance of active front steering. He also proposed an active front steering law for lateral acceleration control at a center of percussion. The active steering controller designed by Huh and Kim [9] eliminated the difference in steering response between driving on slippery road and dry road based on feedback of lateral tire force. Segawa et al. [10] applied lateral acceleration and yaw rate feedback on a steer-by-wire vehicle and indicated that active steering maintains greater driving stability than differential brake control. Different from steer-by-wire, an additional steer angle was added by an actively controlled steering system introduced by Akita [11] using planetary steering box. Oraby and El-Demerdash [12] pointed out that significant improvements were achieved for the vehicle handling characteristics using active front steering control in comparison with four wheel steering and conventional two wheel steering. Among these research, nobody has considered both active front wheel steering and rear wheel steering system.

The active steering system is complex and requires accurate parameters. PID controller is one of the most popular industrial controllers due to its simple control structure, easiness of design, and inexpensive cost. More than 90% of all control loops are PID [13]. Based on classical PID controller, Podlubny proposed a generalization of the PID controller, namely the fractional-order $PI^{\lambda}D^{\mu}$ controller, involving an integrator of order $\lambda$ and a differentiator of order $\mu$ [14]. Due to the two extra degrees of freedom to better adjust the dynamical properties of a fractional-order control system, the fractional-order $PI^{\lambda}D^{\mu}$ was suit to design controller for dynamic system, and less sensitive to the changes of the parameters of a controlled system. The fractional-order $PI^{\lambda}D^{\mu}$ controller can also achieve better control performance than the classical PID controller [14-16].

This paper presents a multi-body vehicle dynamic model and a dynamic control strategy by comparing the multi-body model with the ideal model. The control strategy considers both yaw velocity and sideslip angle objectives. A fractional-order $PI^{\lambda}D^{\mu}$ active steering controller is designed to track the ideal reference model. The active control is expressed as a sum of $PI^{\lambda}$ and a $PD^{\mu}$ control, which choose yaw velocity discrepancy and sideslip angle discrepancy as input variable, respectively. The parameters were tuned by GA. The simulation results show that fractional-order $PI^{\lambda}D^{\mu}$ controller

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is very effective in handling and stability control, and achieve better control performance than classical PID controller.

The paper consists of seven sections. The second section presents the principle of active front wheel steering and rear wheel steering system, then multi-body vehicle dynamic model is built considering the two control objectives for active steering. In the third part, we integrate the dynamic model and controller for active steering co-simulation using fractional-order $PI^{\lambda}D^{\mu}$ control method. Fractional-order $PI^{\lambda}D^{\mu}$ control strategy is the next part. In the fifth part, the parameters of fractional-order $PI^{\lambda}D^{\mu}$ controller are optimized using Genetic Algorithm. Finally, the simulation result and conclusion are presented.

## II. VEHICLE DYNAMIC MODEL AND CONTROL OBJECTIVES FOR ACTIVE STEERING

This section first introduces a 2-DOF ideal vehicle model for active steering simulation, then presents a multi-body vehicle dynamic model for simulation of the fractional-order $PI^{\lambda}D^{\mu}$ control scheme.

### *A. Linear vehicle model*

Because the steering system synthetically takes both the transversal and yaw dynamic problems into account, we consider the multi-body dynamic model as a whole-vehicle model and the 2-DOF model (Fig. 1) as an ideal reference model. This 2-DOF model represents the driver's desired vehicle performance and driving tracking. If the driver's desired vehicle performance and driving tracking are expressed as linear functions, the vehicle will be much easier and safer to drive. In Fig. 1, the vehicle has a front-wheel steering system.

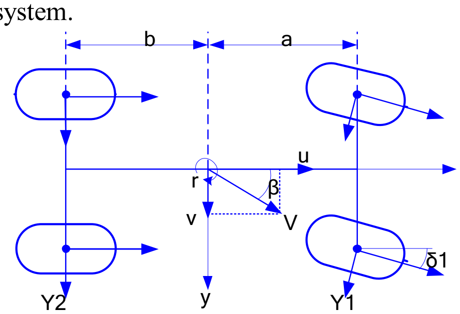


Fig. 1 A 2DOF ideal vehicle model

The dynamic equations for the ideal 2-DOF reference model are formulated as

$$\begin{cases} mv(\dot{\beta}+\gamma) = 2Y_1 + 2Y_2 \\ I\dot{\gamma} = 2aY_1 - 2bY_2 \end{cases} \quad (1)$$

Lateral force on wheels can be expressed as

$$\begin{cases} Y_1 = -k_1\beta_1 = -k_1(\beta + \frac{a}{v}\gamma - \delta_1) \\ Y_2 = -k_2\beta_1 = -k_2(\beta - \frac{b}{v}\gamma) \end{cases} \quad (2)$$

where $m$ denotes the mass of the vehicle; v is the vehicle forward velocity; $\gamma$ represents the yaw velocity; $\beta$ is the sideslip angle; $K_1$ and $K_2$ represent the front- and rear-wheel cornering stiffness, respectively; $\delta_1$ represents the front steering angle. Y1 and Y2 represent the lateral force of front- and rear-wheel, respectively; I is the moment of inertia; and a and b are the length from the front and rear axles, respectively, to the center of gravity of the vehicle.

Considering the vehicle velocity to be time invariant, the dynamic equations can be described in state-space form:

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX + DU \end{cases} \quad (3)$$

Where $A=\begin{pmatrix} -\frac{k_1+k_2}{mv} & \frac{bk_2-ak_1}{mv^2}-1 \\ \frac{bk_2-ak_1}{Iv} & -\frac{a^2k_1+b^2k_2}{Iv} \end{pmatrix}$, $B=\begin{pmatrix} \frac{k_1}{mv} & \frac{k_2}{mv} \\ \frac{ak_1}{I} & -\frac{bk_2}{I} \end{pmatrix}$,

$C=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $D=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $X=(\beta\ \gamma)^T$ and U is the front steering angle.

The parameters are given in Table I.

Table I Ideal vehicle model parameters

| | | | |
|---|---|---|---|
| m( kg) | 1400 | I (Kgm^2) | 1993 |
| a(m) | 1.063 | b(m) | 1.485 |
| $K_1$ (kN/rad) | 52.48 | $K_2$( kN/rad) | 88.416 |

### *B. Active front steering system overview*

Fig. 2 shows the principle of the active front steering system. The driver controls the vehicle via the hand steering wheel (the steering wheel angle is denoted by $\delta_s$) and the actuator provides an additional steering wheel angle $\delta_a$ according to the signal from ECU. Both angles result in a pinion angle down at the steering track.

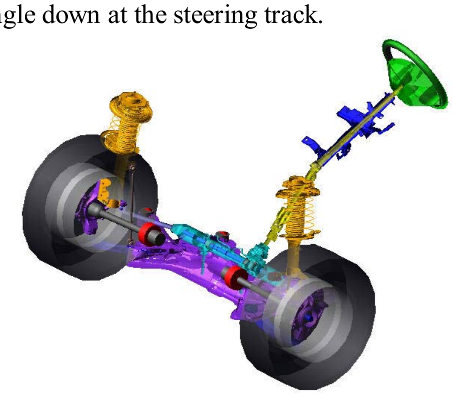

Fig. 2 Principle of the active front steering system

### *C. Active rear steering system overview*

Fig. 3 shows the principle of the active rear steering system. According to the signal from ECU, the rear wheel electric motor actuates an steering angle resulting in a pinion angle down at the steering track. At a high speed, rear wheel steering and front wheel steering are in the same direction to improve stability of vehicle and satisfy passenger relaxation; but at a low speed, especially in parking geometry, the steering angles of rear and front wheels are in opposite direction to improve the maneuverability of vehicle.

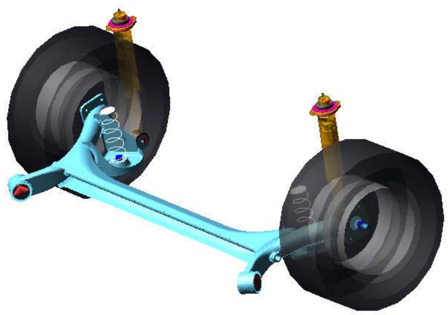

Fig. 3 Principle of the active rear steering system

### D. *Multi-body vehicle dynamic model for active steering*

Multi-body dynamic simulation is used to successfully simulate a wide variety of vehicles and predict the safety, mobility, stability, and operating loads of the complete system. The theoretical basis of multi-body vehicle dynamic model for active steering simulation is multi-body dynamics, and the kinetic equation built with Lagrange multiplier method is presented as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)^T - \left(\frac{\partial T}{\partial q}\right)^T + \Phi_q^T \kappa + \theta_q^T \sigma = Q \quad (4)$$

Integrity constraint equations: $\Phi(q,t)=0$

Nonholonomic constraint equations: $\theta(q,\dot{q},t)=0$

where $T$ represents kinetic energy of the system, $q$ is generalized coordinate vector, $Q$ denotes generalized force vector, $\kappa$ and $\sigma$ represent the vector of Lagrange multipliers corresponding the Integrity constraints and Nonholonomic constrains.

In this paper, the multi-body vehicle dynamic model was built in the ADAMS/CAR environment. This model includes seven subsystems: the front suspension system, the rear suspension system, the brake system, the powertrain system, the steering system, the tire and the bodywork system, as shown in Fig. 4. During the building of the multi-body model, we have considered the joint constraints and the force element such as springs, dampers, bushing and so on. We have also considered the nonlinearity of tire and the flexibility of certain parts, which accurately reflects the practical vehicle system.

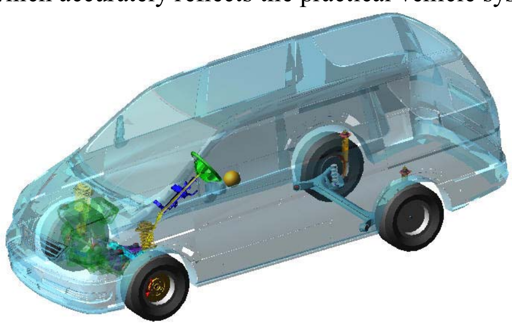

Fig. 4 Multi-body vehicle dynamic model

### E. *Tire model*

Most of the time, the active steering system has high nonlinearity and we should adopt nonlinear tire model. Thus, Pacejka's Magic Formula Model (MFM) [17], which has high precision for longitudinal wheel force and side-force, is introduced. This is a standard model that a lot of people already understand and it can be expressed in the following form.

$$\begin{cases} Y1 = y + S_v \\ y = G\sin(F\arctan(Ex - H(Ex - \arctan(Ex)))) \\ x = X1 + S_h \end{cases} \quad (5)$$

where *Y1(x)* represents the lateral-force, the opposite rotary moment or the longitudinal force, *X1* is the sideslip angle ($\beta$) or the wheel slip ratio (*S*). The coefficients E, F, G, H are determined by vehicle velocity and drive situation, and $S_v, S_h$ denotes the horizontal and vertical drift.

### E. *Control objectives for active steering*

There are two main control objectives in the study of steering system control. The sideslip angle control strategy reduces the lateral motion and transportation of vehicle, while it improves handling maneuverability and reduces the delay of response of the vehicle; the yaw velocity control strategy minimizes the rotational motion of vehicle and leads the vehicle to lateral side tracking the desired trajectory. We intent to minimize sideslip angle close to zero and to make the yaw velocity track the reference model, which will be implemented by introducing an additional front steering wheel angle according to feedback of yaw velocity and adjusting the rear steering wheel angle based on the signal of sideslip angle.

## III. INTEGRATED THE DYNAMIC MODEL AND CONTROLLER FOR ACTIVE STEERING CO-SIMULATION

The simulation system is established by the combination of ADAMS and MATLAB/SIMULINK. The structure of co-simulation is showed in Fig. 5. The two control objectives (sideslip angle and yaw velocity) have been mentioned in the former part, and we intend to minimize the sideslip angle closest to zero and to make the real yaw velocity following the ideal reference model. The ECU sends control instructions to the front steering system and rear steering system based on the yaw velocity and sideslip angle signal of bodywork to generate the additional front steering wheel angle and to adjust the rear wheel steering angle. During the co-simulation, fractional-order $PI^{\lambda}D^{\mu}$ method is applied on the controlling of sideslip angle and yaw velocity.

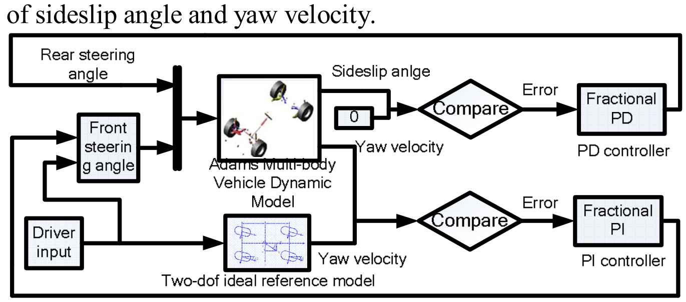


Fig. 5 Fractional-order $PI^{\lambda}D^{\mu}$ control flow chart

## IV. FRACTIONAL-ORDER $PI^{\lambda}D^{\mu}$ CONTROL STRATEGY DESIGN

### A. *Fractional-order $PI^{\lambda}D^{\mu}$ overview*

Fractional-order $PI^{\lambda}D^{\mu}$ control is the generalization and development of the integer-order PID control. It is described as follows [14, 15]:

$$u(t) = k_p e(t) + k_i D_t^{-\lambda} e(t) + k_d D_t^{\mu} e(t) \quad (6)$$

where $e(t)$ is the error, $\lambda$ is the fractional integral order, $\mu$ is the fractional derivative order. When $\lambda = 1$, and $\mu = 1$, classical integer order PID controller is obtained. $\lambda = 1$ and $\mu = 0$ define a PI controller; $\lambda = 0$ and $\mu = 1$ give a PD controller. $\lambda = 0$ and $\mu = 0$ give a gain.

Compared with traditional PID controller, fractional-order $PI^{\lambda}D^{\mu}$ controller can be assigned any real values to $\lambda$ and $\mu$. Fractional-order $PI^{\lambda}D^{\mu}$ controller can choose parameters in a plane, and have limitless combinations. Due to the two extra degrees of freedom to better adjust the dynamical properties of a fractional-order control system, it is more flexible to design a fractional-order $PI^{\lambda}D^{\mu}$ controller. Also, it maintains more superior performance compared with classical PID controller[16].

*B. Controller Design*

In this part, we establish an active steering fractional-order $PI^{\lambda}D^{\mu}$ controller which is expressed as a sum of $PI^{\lambda}$ and a $PD^{\mu}$ control.

To keep the real yaw velocity follow the ideal reference model, A $PI^{\lambda}$ controller with yaw velocity discrepancy as input variable is designed to generate additional front steering wheel angle.

The additional front steering wheel angle $\delta_a$ is described as follows:

$$\delta_a = K_{pYawFra} e_{\gamma}(t) + K_{iYawFra} D_t^{-\lambda} e_{\gamma}(t) \quad (7)$$

where $e_{\gamma}(t)$ is the yaw velocity discrepancy between the multi-body vehicle dynamic model and the 2-DOF ideal reference model. Parameters like $K_{pYawFra}$, $K_{iYawFra}$, and fractional integral order $\lambda$ are tuned by GA optimization algorithm.

To decrease sideslip angle of the body, sideslip angle discrepancy was chosen as the input variable to generate rear steering wheel angle. A $PD^{\mu}$ controller is designed to keep the body sideslip angle at the set value. Ideally we want to achieve zero sideslip angle. So the value was set to zero.

The rear steering wheel angle $\delta_r$ is described as follows:

$$\delta_r = K_{pSideFra} e_{\beta}(t) + K_{dSideFra} D_t^{\mu} e_{\beta}(t) \quad (8)$$

where $e_{\beta}(t)$ is the discrepancy between the real sideslip angle and the set value. The parameters like $K_{pSideFra}$, $K_{dSideFra}$ and fractional derivative order $\mu$ are also obtained by GA optimization algorithm.

## V. Optimization of $PI^{\lambda}D^{\mu}$ Controller

The key objective of active steering is to enhance handling and stability performance. $PI^{\lambda}D^{\mu}$ controller is adopted to control sideslip angle and yaw velocity by producing additional front steering wheel angle and rear steering wheel angle. For the reason that ride comfort is conflict with handling quality and that rollover may occur under critical situations, it is difficult to design the parameters for fractional-order $PI^{\lambda}D^{\mu}$ controller. However, we can consider to build the $PI^{\lambda}D^{\mu}$ controller from the view of optimization. In this paper, Genetic Algorithm is selected to design the parameters of fractional-order $PI^{\lambda}D^{\mu}$ controller.

The objective function selected to be optimized is: (1) sideslip angle discrepancy; (2) yaw velocity discrepancy; (3) the time lag between steering wheel angle and yaw velocity; and (4) the time lag between yaw velocity and lateral acceleration. It can be expressed as follows.

*To find*: Design variables

*to minimize:*

$$F_t = \sqrt{\Psi}$$

Where

$$\Psi = \rho_1\left(\frac{\bar{e}_{\gamma}}{B_{\bar{e}_{\gamma}}}\right)^2 + \rho_2\left(\frac{\bar{e}_{\beta}}{B_{\bar{e}_{\beta}}}\right)^2 + \rho_3\left(\frac{e_{\gamma\max}}{B_{e_{\gamma\max}}}\right)^2 + \rho_4\left(\frac{e_{\gamma\min}}{B_{e_{\gamma\min}}}\right)^2 + \rho_5\left(\frac{e_{\beta\max}}{B_{e_{\beta\max}}}\right)^2 + \rho_6\left(\frac{e_{\beta\min}}{B_{e_{\beta\min}}}\right)^2 + \rho_7\left(\frac{T_{\gamma}}{B_{T_{\gamma}}}\right)^2 + \rho_8\left(\frac{T_a}{B_{T_a}}\right)^2$$

*subject to*:

$$3\sqrt{\frac{\sum_{i=1}^{N}(a_z)^2}{N}} + 2\max(a_z) \le 1 m/s^2$$

$$a_y \le 7 m/s^2$$

$$\phi \le 3 \deg$$

where $B_{\bar{e}_{\gamma}}$, $B_{\bar{e}_{\beta}}$, $B_{e_{\gamma\max}}$, $B_{e_{\gamma\min}}$, $B_{e_{\beta\max}}$, $B_{e_{\beta\min}}$, $B_{T_r}$, and $B_{T_a}$ are the maximum values of $\bar{e}_{\gamma}$, $\bar{e}_{\beta}$, $e_{\gamma\max}$, $e_{\gamma\min}$, $e_{\beta\max}$, $e_{\beta\min}$, $T_{\gamma}$, and $T_a$, respectively. $\rho_1 \ldots \rho_8$ are the corresponding weights; $T_{\gamma}$ is the time lag between the steering wheel angle and the yaw velocity; $T_a$ is the time lag between the yaw velocity and the lateral acceleration. $\bar{e}_{\beta}$ is the mean value of difference between the real sideslip angle and set value; $e_{\beta\max}$ and $e_{\beta\min}$ are the maximum and minimum values, respectively, of the difference between the real sideslip angle and set value; $\bar{e}_{\gamma}$ is the mean value of difference between the real and ideal yaw velocity; $e_{\gamma\max}$ and $e_{\gamma\min}$ are the maximum and minimum values, respectively, of the difference between the real and ideal yaw velocities; $a_z$ is the body acceleration; $a_y$ is the lateral acceleration; and $\phi$ represents the roll angle of bodywork.

To use a genetic algorithm, the initial point should be in feasible region first. Then, we stochastically choose some code array from the feasible region as the first code array of the beginning of the evolution and compute the objective function of each solution. In addition, some codes are selected randomly as a pre-production code sample and produce the next generation code array through *crossover* and *variance*. The process above is repeated until the optimal solution of the last generation is obtained, and the results are the final solution of the genetic algorithm. This algorithm can prevent to trap into local optimization in the process of searching optimal point.

## IV. SIMULATION RESULTS AND DISCUSSION

Active steering is a driver-vehicle-environment closed-loop system, and we choose a lane-change test that is a typical experiment for studying the function of dynamical parameters and handling stability in the driver-vehicle-environment system. The vehicle velocity is 100km/h, and the driver input steering wheel angle is 20 degrees.

The active steering controllers used in this paper are fractional-order $PI^{\lambda}D^{\mu}$ controller and classical PID controller. The fractional-order $PI^{\lambda}D^{\mu}$ controller is described in detail in former section and the design for classical PID controller is similar to fractional-order $PI^{\lambda}D^{\mu}$ controller. To obtain a classical PID controller, we only need to assign $\lambda$ and $\mu$ the value one, respectively.

The parameters for fractional-order $PI^{\lambda}D^{\mu}$ controller tuned by GA are given as follows:

$K_{pYawFra} = 0.323$, $K_{iYawFra} = 2.365$, $\lambda = 0.974$,

$K_{pSideFra} = 20.637$, $K_{dSideFra} = 0.116$, $\mu = 0.545$.

The parameters for classical controller tuned by GA are given as follows:

$K_{pYawPID} = 0.302$, $K_{iYawPID} = 0.547$, $K_{pSidePID} = 10.326$,

$K_{dSidePID} = 0.124$.

Where $K_{pYawPID}$ and $K_{iYawPID}$ are the parameters of the PI controller, $K_{pSidePID}$ and $K_{dSidePID}$ are the parameters of the PD controller.

Figs. 6-11 show the active steering simulation results with classical PID controller, with fractional-order $PI^{\lambda}D^{\mu}$ controller, and Front-wheel steering vehicle with the same condition. The results indicate that active steering vehicle can track the objective path closely. Fig. 11 shows that the yaw velocities of active steering system with both the classical PID controller and the fractional-order $PI^{\lambda}D^{\mu}$ controller can track the 2-DOF ideal reference model accurately, and they are also a little smaller comparing with front wheel steering vehicle. In addition, the sideslip angle of active steering vehicle can decrease a lot comparing with front wheel steering vehicle, and the sideslip angle of active steering system with the $PI^{\lambda}D^{\mu}$ controller is even smaller than that with the classical PID controller, which can enhance the vehicle safety greatly (shown in Fig 10). Fig. 12 shows that the lateral acceleration response of active steering system is more quickly than that of front wheel steering system, and the performance of the fractional-order $PI^{\lambda}D^{\mu}$ controller is better than the classical PID controller. Table 2 gives the numerical values of the lag phase between steering wheel angle and yaw velocity and that between yaw velocity and lateral acceleration.

Table II Lag phase comparing

| Lag phase | Front wheel steering | AFS with PID controller | AFS with $PI^{\lambda}D^{\mu}$ controller |
|---|---|---|---|
| Between $\delta_s$ and $\gamma$ | 0.105s | 0.01s | 0.005 |
| Between $\gamma$ and $a_y$ | 0.14s | 0.025s | 0.005 |

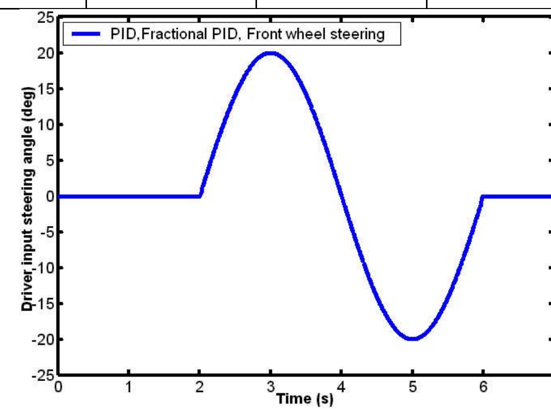

Fig. 6 Driver input steering angle

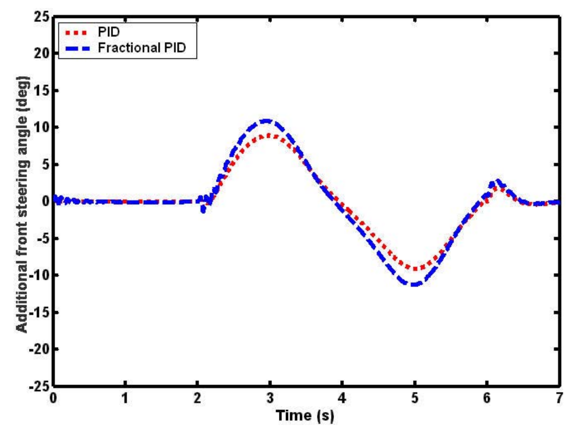

Fig. 7 Comparison of additional front steering angle

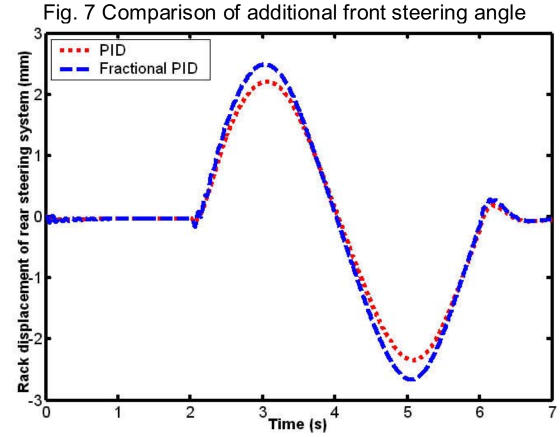

Fig. 8 Comparison of rack displacement of rear steering system

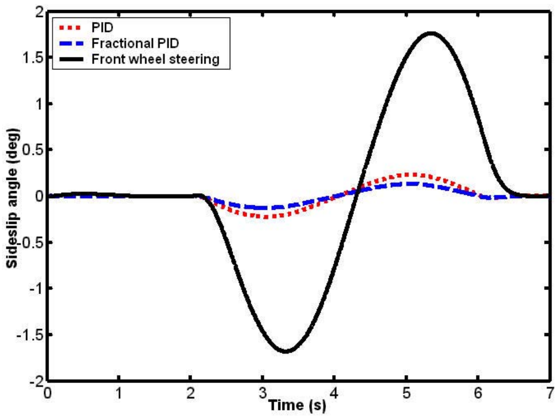

Fig. 9 Comparison of sideslip angle

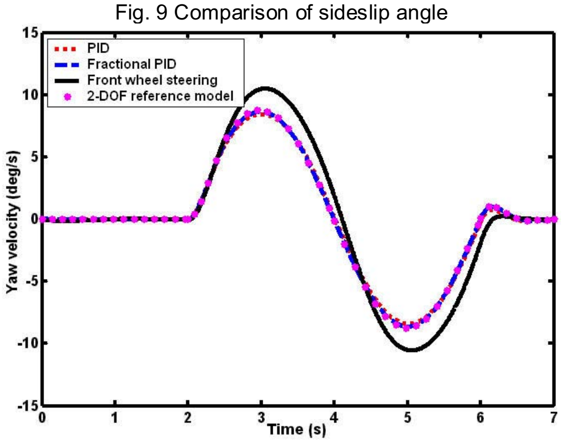

Fig. 10 Comparison of yaw velocity

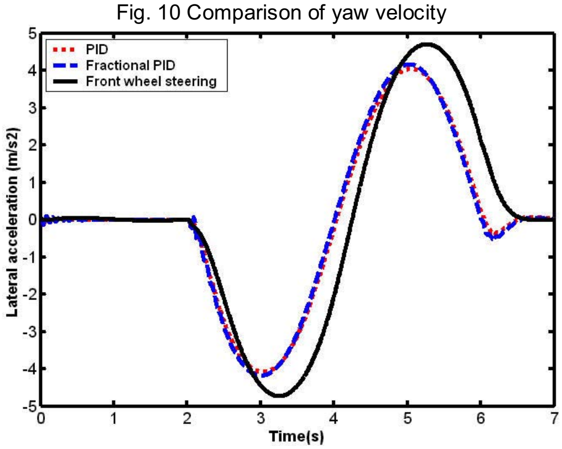

Fig. 11 Comparison of lateral acceleration

## IV. Conclusion

This paper utilizes a fractional-order $PI^{\lambda}D^{\mu}$ controller in active steering design to achieve good handling and stability performance. The parameters of the controller are tuned by GA optimization algorithm. The results show that with the additional front steering wheel angle and rear steering wheel angle, active steering system can improve the vehicle performance greatly. It is also proved that $PI^{\lambda}D^{\mu}$ controller is effective in steering control and achieve better control performance than classical PID controller.

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