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Exploring Key Factors Influencing Optimal Farm Design Using Mixed-Discrete Particle Swarm Optimization

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The planning of a wind farm, which minimizes the project costs and maximizes the power generation capacity, presents significant challenges to today's wind energy industry. An optimal wind farm planning strategy that accounts for the key factors (that can be designed) influencing the net power generation offers a powerful solution to these daunting challenges. This paper explores the influences of (i) the number of turbines, (ii) the farm size, and (iii) the use of a combination of turbines with differing rotor diameters, on the optimal power generated by a wind farm. We use a recently developed method of arranging turbines in a wind farm (the Unrestricted Wind Farm Layout Optimization (UWFLO)) to maximize the farm efficiency. Response surface based cost models are used to estimate the cost of the wind farm as a function of the the turbine rotor diameters and number of turbines. Optimization is performed using a Particle Swarm Optimization (PSO) algorithm. A robust mixed-discrete version of the PSO algorithm is implemented to appropriately account for the discrete choice of feasible rotor diameters. The use of an optimal combination of turbines with differing rotor diameters was observed to significantly improve the net power generation. Exploration of the influences of (i) the number of turbines, and (ii) the farm size, on the *cost per KW of power produced*, provided interesting observations.

Keywords: energy, mixed-discrete optimization, Particle Swarm, turbine, wind farm

I. Introduction

In recent years, growing climate change concerns and ill-predictability of fossil fuel prices have increased the focus on sustainable energy resources, such as wind and solar energy. The horizontal axis wind turbine is the most popular wind turbine, which has been in existence since the 13th century.[1] The practical viability of energy production is generally governed by such factors as (i) the potential for *large scale energy production*, (ii) consistency in the power supplied to the grid, (iii) and the expected *return on investment*. These factors have been restraining the exploitation of the full potential of wind energy. The 2009 worldwide nameplate capacity of wind powered generators was only approximately 2% of the worldwide electricity consumption.[2]

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For wind to play a major role in the future energy market, we need steady improvement in the wind power generation technology, which can be realized in part through optimal planning of wind farms.

Wind farm planning generally includes (but is not limited to) critical decision-making, regarding

1. the number of wind turbines to be installed,
2. the layout of the turbines in the wind farm, and
3. the selection of the types of wind turbines (defined by rated power, rated speed, rotor diameter and hub height).

The objectives of *optimal wind farm planning* are to (i) minimize the Cost of Energy (COE), expressed in $/KW-h, and (ii) maximize the net power generation. Successful accomplishment of these objectives demands a robust and flexible wind farm optimization model that appropriately accounts for the critical factors.

A. Wind Farm Optimization

Wind energy sources generally appear in the form of wind farms that consist of multiple wind turbines located in a particular arrangement over a substantial stretch of land (onshore), or water body (offshore). It has been shown that the total power extracted by a wind farm is significantly less than the simple product of the power extracted by a standalone turbine and the number of identical turbines (N) in the farm.[3] Comparison of (i) the product of the power curve of a standalone turbine and N, and (ii) the power curve of the whole wind farm (Park Power Curve (PPC)) reveals this phenomena. Sorensen et al.[3] shows that this discrepancy is approximately 12.4% of the former (loss in farm efficiency) in the case of an offshore wind farm in Denmark.

This deficiency can be attributed to the loss in the availability of energy due to wake effects - i.e. the shading effect of a wind turbine on other wind turbines downstream from it.[4] Energy deficit due to mutual shading effects is determined using wake models that give a measure of both the growth of the wake, and the velocity deficit in the wake with distance downstream from the wind turbine. The Park wake model, originally developed by N. O. Jensen[5] and later by Katic et al.,[6] is one of the most popular analytical wake models used in wind farm modeling. The modified Park wake model and the Eddy Viscosity wake model are other standard wake models. The reduction in the wind farm efficiency (loss in the effective energy available), due to this mutual shading, depends primarily on the geometric arrangement of wind turbines in a farm.

Two popular class of approaches have been reported in wind farm layout modeling: (i) models that assume an array like (row-column) farm layout[3,7] , and (ii) models that divide the wind farm into a discrete grid in order to search for the optimum grid locations of turbines.[4,8–10] A majority of these approaches are not readily applicable to the broad commercial scenario that requires simultaneous (i) optimization of the layout and selection of turbines, and (ii) consideration of the appropriate number of turbines and farm size. The original Unrestricted Wind Farm Layout Optimization (UWFLO) framework, introduced by Chowdhury et al.,[11] avoids key limiting assumptions (presented by other methods) regarding the layout pattern and the selection of turbines.

II. Development of the Unrestricted Wind Farm Layout Optimization (UWFLO) Model

In the UWFLO model, the growth of the wake behind a turbine is determined using the wake growth model proposed by Frandsen et al.[12] The corresponding energy deficit behind a turbine is determined using the velocity deficit model presented by Katic et al.;[6] this velocity deficit model has been widely adopted in wind farm modeling ([9,10,13]). In a wind farm, the velocity of the wind approaching a turbine can be affected by the wake of multiple turbines upstream from it. Crespo et al.[14] provides a review of different methods that account for the merging of wakes (wake superposition), in evaluating the wake velocity deficits. UWFLO implements the wake superposition model developed by Katic et al.[6] The possibility of a turbine being 'partially' in the wake of another turbine (upwind) has also been taken into account, in the UWFLO power generation model. The wind farm model developed in UWFLO has been successfully validated against recently published experimental data.[15]

The net power generated by the wind farm, which is to be maximized, is evaluated by the sum of the power generated by the individual turbines. The farm dimensions and the minimum distance required between any two turbines are treated as system constraints during optimization. In the commercial scenario, other factors, such as the terrain, the load bearing capacity of the soil, and the road layout in a wind farm,[10] might further restrict the arrangement of turbines. However, these practical constraints have not yet been considered in the current framework. A modified Particle Swarm Optimization algorithm[16] is applied to optimize the farm layout with the objective of maximizing the total power generation.

The UWFLO framework allows for the use of *a combination of turbines with differing rotor diameters* in a wind farm. Chowdhury et al.[11] illustrated that an optimal combination of such non-identical turbines can significantly increase the power generation without any unfavorable effects on the cost; the rotor diameter of each turbine was treated as an additional continuous design variable in that case. However, in practice, only a finite choice of rotor diameters (for wind turbines) are commercially available. Hence, the treatment of rotor diameters as discrete design variables (constrained to a set of discrete values) will allow for further realistic observations. In this paper, we implement a recently developed mixed-discrete version of the PSO algorithm[17] to account for the selection of optimal turbine rotor diameters. The proposed mixed-discrete PSO uses continuous optimization as its primary search technique, in conjunction with a deterministic approximation in the discrete variable domain at each iteration. The resulting algorithm is expected to be computationally less expensive than existing binary non-gradient based optimization algorithms.[18–20]

Factors, such as (i) the number of turbines in a farm, and (ii) the farm size, are also expected to play important roles in deciding the optimum farm layout. In this paper, case studies are performed (using the UWFLO technique) to investigate the nature of influence of these factors. Two representative cost models (polynomial response surfaces) are formulated to express the cost of a wind farm as functions of the turbine rotor diameters and the number of turbines in a farm. However, we could not analyze the relationship between the farm cost and the farm size owing to a lack of data regarding the size of existing commercial wind farms. The authors would like to point out that this paper does not intend to develop an extensive cost analysis of commercial wind farms; the actual cost depends on other important factors as well (that are not considered in this paper).

A. Modeling the Layout and the Net Power Generation

The power generated by a wind farm is an intricate function of the configuration and location of the individual wind turbines. The flow pattern inside a wind farm is complex, primarily due to the wake effects and the highly turbulent flow. Hence, the velocity of the wind approaching a turbine and the corresponding power generated are determined separately for each turbine. The incoming wind profile is given by[15]

$$\frac{U}{U_\infty} = b_1\left(\frac{z}{b_2}\right)^{0.15} \tag{1}$$

where z is the vertical distance from the ground, and b_1 and b_2 are constants that depend on the terrain, and the atmospheric conditions. However, in this model we assume a uniform incoming flow equivalent to the velocity (in Eq. 1) integrated and averaged over the rotor area (U_0).

The layout modeling process is laconically represented using a influence matrix (M), defined as

$$M_{ij} = \begin{cases} +1 & \text{if Turbine}-i \text{ influences Turbine}-j \\ -1 & \text{if Turbine}-j \text{ influences Turbine}-i \\ \ \ 0 & \text{if there is no mutual influence} \end{cases} \tag{2}$$

where Turbine-j is in the influence of the wake created by Turbine-i if and only if

$$\begin{array}{l} \Delta x_{ij} < \ 0 \quad \text{and} \quad |\Delta y_{ij}| - \frac{D_j}{2} < \frac{D_{wake,ij}}{2} \ , \quad \text{where} \\ \Delta x_{ij} = x_i - x_j, \quad \Delta y_{ij} = y_i - y_j \end{array} \tag{3}$$

In Eq. 3, D_j is the rotor diameter of Turbine-j; $D_{wake,ij}$ is the diameter of the wake produced by Turbine-i, and approaching Turbine-j; x and y represent the co-ordinate axes "along" and "perpendicular to" the streamwise direction, respectively.

Detailed formulation of the total power generated (P_{farm}) by the wind farm can be found in the paper by Chowdhury et al.[11] Having determined the net power generation, the farm efficiency[3] is given by

$$\eta_{farm} = \frac{P_{farm}}{NP_0} \tag{4}$$

where P_0 is the power that a standalone turbine would generate, for the given uniform incoming wind speed. However, for ease of comparison, we redefine the farm efficiency (maximized in this paper) as

$$\eta_{farm} = \frac{P_{farm}}{NP_rated} \tag{5}$$

where P_rated is the rated power of the wind turbines used.

B. UWFLO Cost Model

Numerous techniques have been developed to evaluate the cost (installation, operation and maintenance) of both onshore and offshore wind farms in the last twenty years; namely, the Short-cut model,[21] cost analysis model for the Greek market,[22] OWECOP-Prob cost model,[23] JEDI-wind cost model[24] and the Opti-OWECS cost model.[25] Among these cost models, only[21] and[22] present analytical expressions of the cost as a function of the critical contributing factors. Most of these models do not explicitly consider the effect of the rotor diameter of the wind turbines in the farm. Generally, the rated power of the wind turbines is considered, which however, does not account for the effect of turbine dimensions on the nature of the flow inside the wind farm. A recent response surface based cost model developed by Zhang et al.[26] showed that the cost of energy (COE) of a wind farm depends substantially on the turbine rotor diameters.

Two single variable (1D) response surfaces have been developed, which express the cost of a wind farm as functions of the turbine rotor diameter and the number of turbines in the farm. The cost functions, in this paper, represent *cost per KW of installed capacity* (total rated capacity of the wind farm). These functions are evaluated using data for different 1.5 MW turbines installed in wind farms in the state of New York; the data is provided by the Wind and Hydropower Technologies program (US Department of Energy).[24] The *rotor diameter based cost* function is given by

$$Cost_D(D) = 143.85 - 0.32447D - 1.4841 \times 10^{-3}D^2 \tag{6}$$

where D is the diameter of the wind turbines in the farm. The cost function in Eq. 6 was evaluated with a relative error of 0.2%. Sufficient cost information (for commercial wind farms) is not available for wind farms with non-identical wind turbines (turbines with differing rotor diameters). Therefore, the cost of a wind farm with non-identical wind turbines is approximated by

$$Cost_D = \frac{1}{N}\sum_{i=1}^{N} Cost(D_i) \tag{7}$$

The *number of turbines based cost* function is given by

$$Cost_N(N) = 133.3938 - 0.1501N - 7.9 \times 10^{-4}N^2 \tag{8}$$

The cost function in Eq. 8 was evaluated with a relative error of 0.21%. In this case, the *effective cost per KW of power produced* ($Cost_{N,eff}$) can be expressed as

$$Cost_{N,eff} = \frac{Cost_N \times P_0 \times N}{P_{farm}} \implies Cost_{N,eff} = Cost_N \times \eta_{farm} \tag{9}$$

The cost of the wind farm is a complex function that depends on several other economic and environmental factors, as well. The objectives of the 1D quadratic cost models in Eqs. 7 and 9 is to, respectively, explore (i) the benefits of using non-identical turbines and (ii) the influence of the number of turbines on the net economic utility of a wind farm.

III. Mixed-Discrete Particle Swarm Optimization

PSO is one of the most well known stochastic optimization algorithms, initially coined by an Electrical Engineer (Russel Eberhart) and a Social Psychologist (James Kennedy) in 1995.[16] Later, several improved variations of the algorithm have appeared in the literature and been used in popular commercial optimization packages. The PSO algorithm used in this paper has been derived from the unconstrained version presented by Colaco et al.[27] The basic steps of the algorithm are summarized as

$$\begin{aligned} x_i^{t+1} &= x_i^t + v_i^{t+1} \\ v_i^{t+1} &= \alpha v_i^t + \beta_l r_1 \left(p_i - x_i^t\right) + \beta_g r_2 \left(p_g - x_i^t\right) \end{aligned} \tag{10}$$

where, x_i^t is i^{rmth} member of the population (swarm) at the t^{rmth} iteration; r_1 and r_2 are random numbers between 0 and 1; p_i is the best candidate solution found for the i^{rmth} member; p_g is the best candidate solution for the entire population; α, β_l and β_g are user defined constants in the range [0, 1].

In this algorithm, candidate solutions (particles) are compared using the principle of constrained non-dominance, introduced by Deb et al.[20] and later adopted by.[28] In this technique, solution-i is said to dominate solution-j if,

- solution-i is feasible and solution-j is infeasible or,
- both solutions are infeasible and solution-i has a smaller constraint violation than solution-j or,
- both solutions are feasible and solution-i weakly dominates solution-j.

If none of the above conditions apply (possible only in the case of a multi-objective problem), then both of the solutions are considered non-dominated with respect to each other.

A. Vertex Approximation Approach: Addressing Discrete Variables

One of the earliest discrete binary version of the Particle Swarm Optimization (PSO) algorithm was developed by Kennedy and Eberhart.[19] In this algorithm, a candidate solution is located at a discrete point, using a sigmoid function based probability distribution. Various other versions of discrete PSO algorithm have also been reported in the literature.[29–31]

In the vertex approximation approach, the design space for a mixed-discrete problem is divided into a continuous domain and a discrete domain (corresponding to the continuous variables and the discrete variables, respectively). In the discrete domain, feasible candidate solutions should be located only at discrete locations. Hence, within a typically continuous optimization process, the location of a candidate solution (at each iteration) can be defined by the local hypercube, H_d, expressed as

$$\begin{aligned} & H_d = \left\{ \left(\tilde{x}_1^L, \tilde{x}_1^U\right), \ \left(\tilde{x}_2^L, \tilde{x}_2^U\right), \ \ldots, \ \left(\tilde{x}_m^L, \tilde{x}_m^U\right) \right\}, \qquad \text{where} \\ & \tilde{x}_i^L \leq \tilde{x}_i \leq \tilde{x}_i^U, \quad \forall\ i = 1, 2, \ldots, m \end{aligned} \tag{11}$$

In Eq. 11, m is the number of discrete design variables, and $\tilde{x}_i$'s denote the current location of the candidate solution in the discrete domain. The parameters $\tilde{x}_i^L$ and $\tilde{x}_i^U$ represent two consecutive possible values of the i^{th} discrete variable that bound the local hypercube. The total number of vertices in the hypercube is equal to 2^m.

The values $\tilde{x}_i^L$ and $\tilde{x}_i^U$ can be readily obtained from the discrete set that need to be specified apriori for each discrete design variable. The vertex approximation technique(s) relocates the candidate solution (in the discrete variable domain) to one of the vertices of its local hypercube H_d. The values of the continuous design variables remain unchanged. The identification of the approximating vertex (for each candidate solution, at each iteration) is performed using the Nearest Vertex Approach (NVA) and the Shortest Normal Approach (SNA), separately. The NVA and SNA are illustrated in Fig. 1. Detailed description of these techniques can be found in the paper by Chowdhury et al.[17]

The concept of the vertex approximation approaches can be readily implemented in PSO. The discrete component of the velocity vector, $\tilde{v}_i^{t+1}$, which connects the parent solution (x_i^t) and the child solution (x_i^{t+1}) (in the discrete domain), serves as the *connecting vector*. Accordingly, the NVA and the SNA would approximate the child solution to the "NVA vertex" and the "SNA vertex", respectively, as shown in Fig. 1. This implementation of the NVA/SNA provides a modified version of PSO that can appositely deal with mixed-discrete optimization problems.

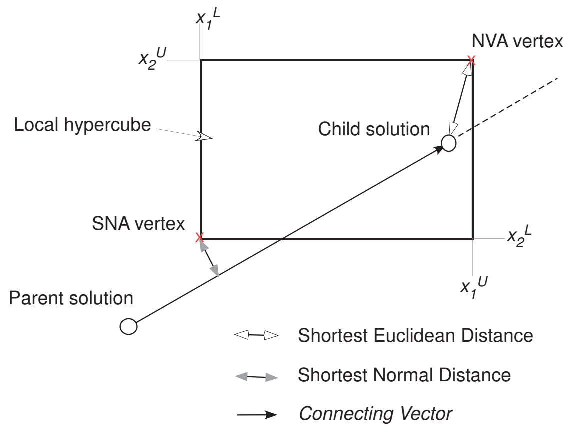


Figure 1. Illustration of the NVA and the SNA approximation

IV. Application of UWFLO Framework

The UWFLO technique has been applied to two different cases (1 and 2) in order to explore the scope of wind farm optimization for:

Case 1 Wind farm with non-identical turbines (differing rotor diameters).

Case 2 Wind farm with identical turbines that can adapt to wind conditions similar to commercial turbines.

Case 2 is further applied to investigate the influences of the number of turbines and the farm size, on the optimal layout of the wind farm. Both the cases assume a unidirectional wind of constant speed (constant wind velocity), since specific information regarding the distribution of wind speed and direction was not available for the experiment.[15] However, in the commercial scenario, the speed and the direction of wind change with time. In the literature,[1] the long term wind speed variation is often represented using a Weibull distribution, which is controlled by the shape parameter K and the scale parameter C. The values of these parameters have not been reported in the case of the experimental setup.[15] Nevertheless, this assumption does not restrict the application of the UWFLO framework; if the Weibull distribution (of the wind velocity) is available for a particular experimental/commercial wind farm, a numerical integration procedure can be readily applied to approximate the power generated over the entire range of wind speed and direction (as in[32]).

The wind farms simulated in this paper have similar attributes to the experimental wind farm,[15] which is summarized in Tables 1 and 2.

Table 1. Wind Farm Attributes

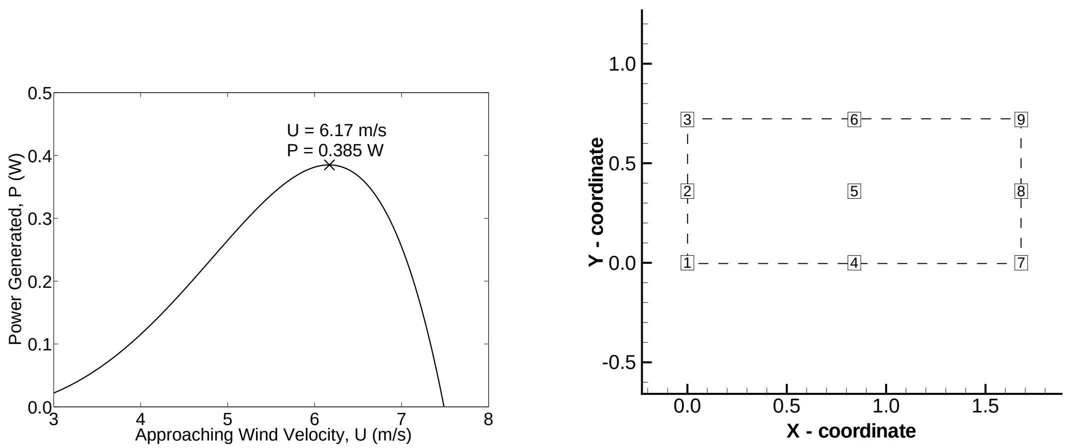
Attribute	Value
Length	1.68 m
Breadth	0.72 m
Turbine Hub Height (H)	0.12 m
Turbine Rotor Diameter (D)	0.12 m
Downwind Separation	$7 \times D$ m
Crosswind Separation	$3 \times D$ m
Average Surface Roughness	0.001 m

The power characteristics used in Case 1 are the same as reported by Chowdhury et al.[11] for the experimental model turbines; the power curve is illustrated Fig. 2(a). The experimental farm layout[15] is illustrated in Fig. 2(b).

It is noteworthy that the optimum wind farm layout is not necessarily unique. There can be different optimal arrangements of turbines with nearly equal amount of total power output. This results in a multimodal

Table 2. Wind Characterisitcs

Parameter	Value
Rotor Averaged Wind Speed (U_0)	7.09 m/s
Mean Velocity profile	Refer Eq. 1
Wind Direction	$0°$ with positive X-axis
Density of Air	1.2 kg/m^3



(a) Power curve for the model wind turbines

(b) Layout of the farm

Figure 2. Wind tunnel experiment on a scaled down wind farm

optimization problem. To compensate for the performance fluctuations induced by the random generators in PSO (used in creating the initial population and other swarm operators), the algorithm is run five times for each case.

A. UWFLO Case 1

The objective of this study (Case 1) is to explore the effect of using *a combination of turbines with differing rotor diameters* on the total power generation from the wind farm. This exploration demands simultaneous optimization of the location and the rotor diameter of each turbine placed in the wind farm. A *rotor diameter based cost* constraint (g_3) ensures that any feasible wind farm design requires a net investment equal to or less than that required for a wind farm with identical wind turbines (as in the experimental setup[15]). Using the data from major turbine manufacturers (available online), the mean rotor diameter for currently installed commercial wind turbines (worldwide) was determined to be 75m. These commercial-scale design dimensions are scaled down to the dimensions of the experiment,[15] where the mean rotor diameter is 0.12m. Subsequently, the set of feasible rotor diameters at the experimental scale was developed and provided as input (feasible discrete set) for the optimization.

During optimization, the rotor diameter of each turbine is treated as a discrete design variable. Hence there are a total of $2N$ continuous design variables and N discrete design variables in Case 1. In order to maximize the power generated by the wind farm, the optimization problem is formulated as

$$\begin{aligned}
&\text{Max} \quad f(V) = \eta_{farm} \\
&\text{subject to} \\
&\qquad g_1(V) \leq 0 \\
&\qquad g_2(V) \leq 0 \\
&\qquad g_3(V) \leq 0 \\
&\qquad V = \{X_1, X_2, ..., X_N, Y_1, Y_2, ..., Y_N, D_1, D_2, ..., D_N\} \\
&\qquad 0 \leq X_i \leq X_{farm} \\
&\qquad 0 \leq Y_i \leq Y_{farm} \\
&\qquad D_{min} \leq D_i \leq D_{max}
\end{aligned} \tag{12}$$

where η is given by Eq. 5. The rated power of the scaled down model turbines (with $D = 0.12$m) was reported to be 0.385 W.[11]

The inequality constraint g_1 in Eq. 12 represents the minimum clearance required between any two turbines, and is given by

$$\begin{aligned}
g_1(V) &= \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \max\left(\left(D_i + D_j + \Delta_{\min} - d_{ij}\right), 0\right) \\
d_{ij} &= \sqrt{\Delta x_{ij}{}^2 + \Delta y_{ij}{}^2}
\end{aligned} \tag{13}$$

In Eq. 13, Δ_{min} is the minimum clearance required between the outer edge of the rotors of the two turbines. The value of Δ_{min} is set at zero, to allow maximum flexibility in turbine spacing. In practice, a higher value of Δ_{min} might be necessary. The parameters X_{farm} and Y_{farm} in Eq. 12 represent the extent of the rectangular wind farm in X and Y directions, respectively. To ensure the placement of the wind turbines within the fixed size wind farm, the X_i and Y_i bounds are reformulated into an inequality constraint, $g_2(V) \leq 0$, where g_2 is expressed by

$$g_2(V) = \frac{1}{2N}\left(\frac{1}{X_{farm}}\sum_{i=1}^{N}\max\left(-X_i, X_i - X_{farm}, 0\right) + \frac{1}{Y_{farm}}\sum_{i=1}^{N}\max\left(-Y_i, Y_i - Y_{farm}, 0\right)\right) \tag{14}$$

The *rotor diameter based cost* of a wind farm is implemented as the third constraint, which can be expressed as

$$g_3 = Cost_D \tag{15}$$

B. UWFLO Case 1 Results

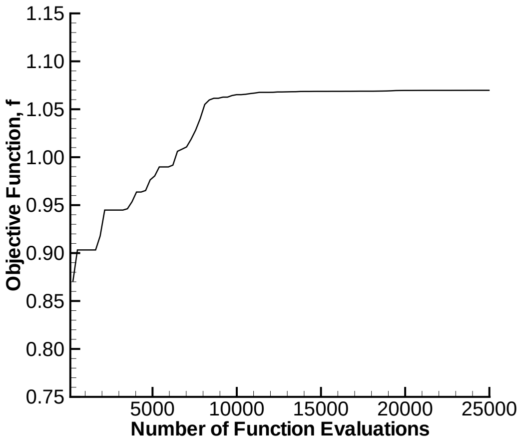
Optimization is performed using the PSO algorithm, which is initiated with a population of random wind farm layouts. The user defined constants involved in PSO are summarized in Table 3. The outcomes of the

Table 3. User-defined constants in PSO

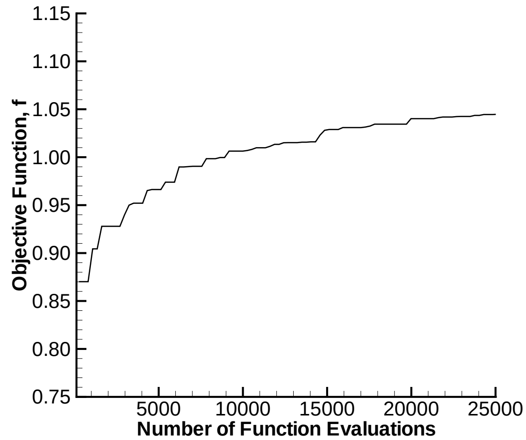
Constant	Value
α	0.5
β_g	1.4
β_l	1.4
Population size	$10 \times N_{var}$
Allowed number of function calls	25000

one of the representative optimizations for Case 1, each using the NVA and SNA are illustrated in next two subsections.

Figures 3(a) and 3(b) show that the Nearest Vertex Approach provides faster convergence than the Shortest Normal Approach. The power generated by the farm increases by approximately 21% (average for



(a) Using NVA

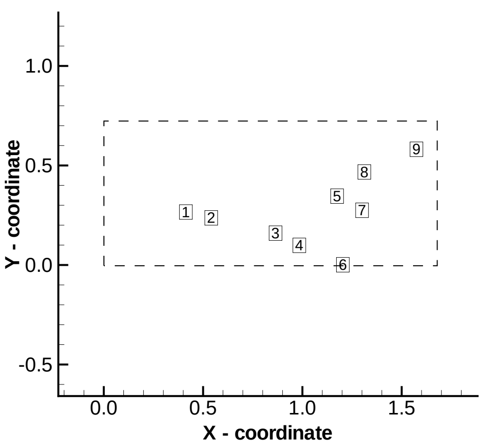


(b) Using SNA

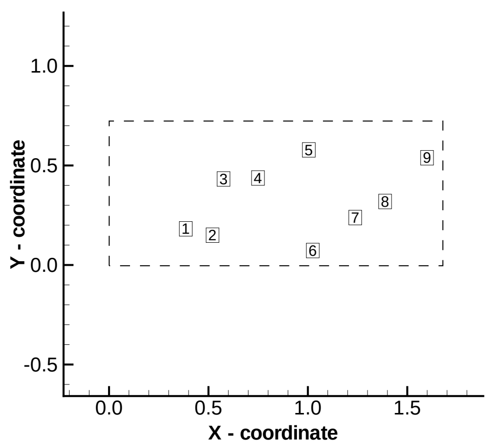
Figure 3. Convergence history of PSO (Case 1)

the two approaches) during optimization, which is approximately 44% higher than that generated by the original farm layout in the experiment (Fig. 2(b)).

The resulting rotor diameters, in this case, can be higher than the rotor diameters of the model turbines used in the experiment ($D = 0.12$m). Consequently, the maximum possible power generation, from a single turbine, is not restricted by the power curve shown in Fig. 2(a). Thereby, the objective function can reach values higher than unity as shown in Figs. 3(a) and 3(b). The optimized farm layouts obtained by the NVA and the SNA are shown in Figs. 4(a) and 4(b), respectively. Figures 5(a) and 5(b) illustrate the rotor diameters of each turbine in the optimized wind farms obtained by the two approaches.



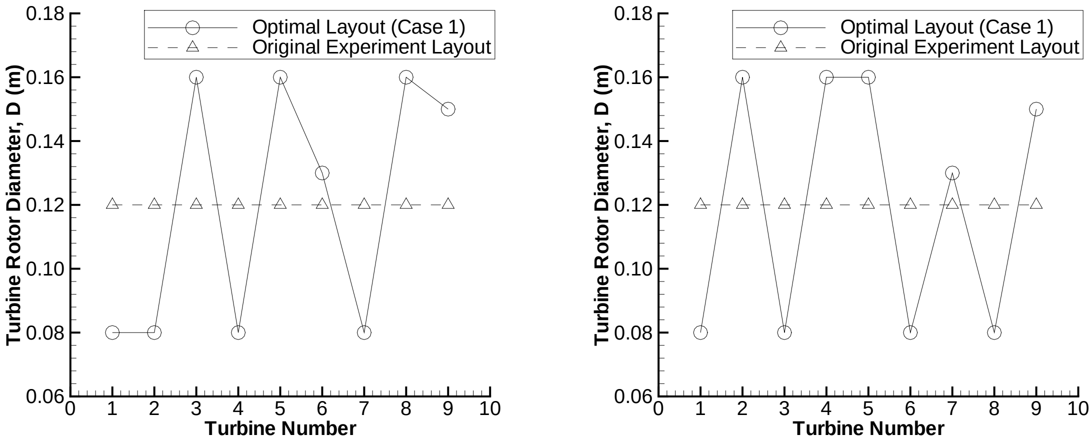
(a) Using NVA



(b) Using SNA

Figure 4. Optimized wind farm layout (Case 1)

The incoming wind speed (which is 7.0896 m/s) is greater than the rated wind speed. As a result, owing to the nature of the power curve (that has a maxima at 6.17m/s, as shown in Fig.2(a)), we observe a deliberate positioning of certain turbines in the wakes of other turbines located upstream (Figs. 4(a) and 4(b)); this arrangement of turbine allows more number of turbines to operate at wind speed closer to the maximum (of 6.17m/s). Figures 5(a) and 5(b) show that the optimized wind farms present a combination of turbines with significantly different rotor diameters. However, further investigation is necessary to determine



(a) Using NVA

(b) Using SNA

Figure 5. Rotor diameter of each turbine (D) for Case 1

if there exists a correlation between the rotor diameters of the turbines and their relative location in the wind farm. The primary observation, from the results of Case 1, is the remarkable increase in the total power generation, accomplished using turbines with differing rotor diameters.

Wind turbines with differing rotor diameters might have different hub heights and different performance characteristics. In this case study, such data (for the experimental scale model turbines) was not available and hence the same performance curves (as in Fig. 2(a)) and hub height were used for all the turbines. In the case of a commercial wind farm, other design and economic factors might affect the cost of using differing rotor diameters. Owing to the above reasons, further research (regarding use of differing rotor diameters) needs to be undertaken that accounts for:

1. the appropriate hub height, corresponding to different rotor diameters,
2. the use of appropriate performance characteristics specific to each turbine, and
3. the application of a comprehensive cost model.

C. UWFLO Case 2

Case 2 is designed to illustrate a more appropriate characterization of the commercial wind farm scenario. Hence, modified power characteristics of the turbines are used in this case - the power generated by a turbine is assumed to remain constant and equal to the rated power (of 0.385 W), if the approaching wind speed is above the rated speed (of 6.17 m/s). This change is introduced to better explain the effect of turbine wakes on the optimal layout of the wind farm. The incoming wind velocity is specified to be 6.2 m/s. This is because, higher incoming wind velocities allowed turbines to readily operate at approaching wind speeds above the rated speed. Other user-defined constants and parameters in this case are the same as in Case 1. On account of the above changes, application of UWFLO to Case 2 is expected to produce a farm layout that minimizes the mutual shading effect of wind turbines. In Case 2, only turbines with rotor diameter of 0.12m are considered.

Two parametric studies are also performed in Case 2. These studies investigate the effects of (i) the number of turbines in a farm, and (ii) the size of the farm, on the maximum power generation. Parametric Study-I (Section 1) applies UWFLO on five different wind farms, with number of turbines equal to 6, 9, 12, 15 and 18, respectively. These wind farm have the same fixed dimensions, as in the experimental setup,[15] which is $14D \times 6D$. The five different wind farms, analyzed in Parametric Study-2 (Section 2), are listed in Table 4. All the farms in Study-2 are assumed to be rectangular in shape, with a length to breadth ratio $(\frac{X_{farm}}{Y_{farm}})$ of 7/3.

Table 4. **Parametric Study-II: Wind farms**

Farm	Farm dimensions ($X_{farm} \times Y_{farm}$)	Farm area (m^2)	Number of turbines
1	$7D \times 3D$	0.3024	18
2	$14D \times 6D$	1.2096	18
3	$21D \times 9D$	2.7216	18
4	$28D \times 12D$	4.8384	18
5	$35D \times 15D$	7.5600	18

D. UWFLO Case 2 Results

PSO is run with the same user defined constants as specified in Table 3, except that the allowed number of function evaluations is 15,000 in this case. The outcomes of the one of the representative runs for Case 2 are illustrated in this paper. The farm layout obtained through optimization is shown in Fig. 6.

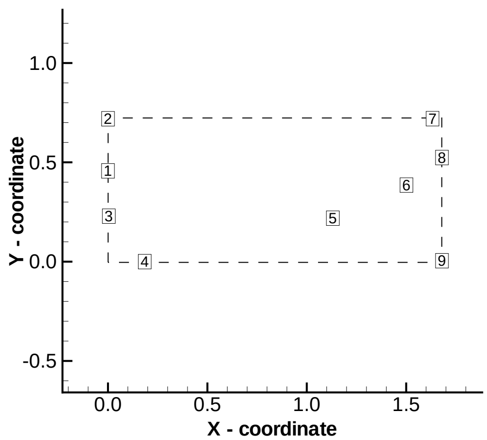


Figure 6. Optimized wind farm layout (Case 3)

It can be observed from the optimized layout shown in Fig. 6 that the nine wind turbines have been divided (by location) into two broad regions - (i) one, located at the front end of the wind farm with respect to the incoming wind, and (ii) the other, located at the rear end of the farm. Such a layout minimizes the mutual shading effects, since the wake velocity deficit decreases with distance downstream from a turbine. However, if variation of wind speed and direction are considered, the resulting optimized layout is likely to be different.

1. Parametric Study-I Results

In the case of a fixed size wind farm, the mutual shading effect of turbines are expected to get more pronounced with increasing number of operational turbines. The influence of this crowding effect on the farm efficiency, and thereby on the *cost per KW of power produced*, is investigated in this study. The *cost per KW of power produced* is evaluated using Eq. 9 in Section B. The convergence histories, of the application of UWFLO to the five wind farms, are shown in Fig. 7. The total power generated, the farm efficiency and the *cost per KW of power produced*, for each farm, are illustrated in Figs.8(a), 8(b) and 9, respectively. These critical output parameters (for Parametric Study-I) are also summarized in Table 5. The farm number prefix (II- or I-) in column 1 of this table indicates which parametric study the wind farm corresponds to.

Higher number of turbines present a higher dimensional (in design variable domain) problem, which demands more function evaluations during optimization; this phenomena is shown in Fig. 7. Figure 8(a) illustrates that the crowding effect (mutual shading of turbines) undermines the marginal increment in

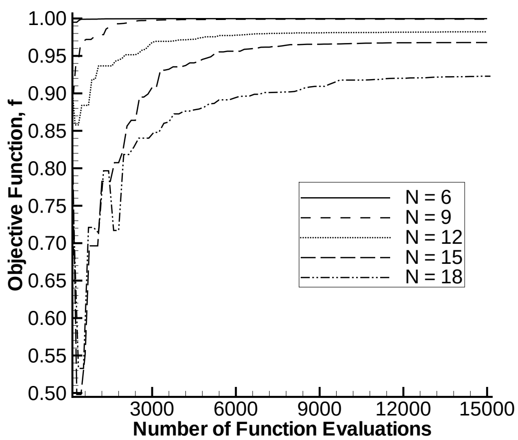
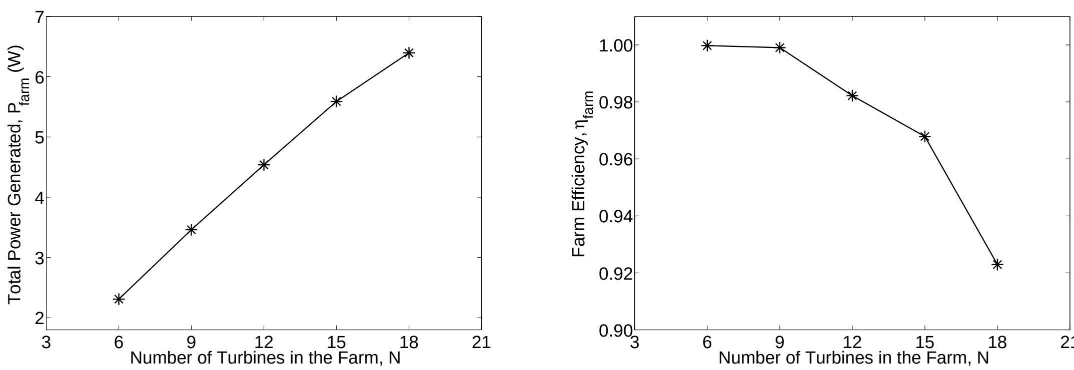


Figure 7. Convergence history of PSO (Case 2, Study-I)



(a) Power generated

(b) Farm efficiency

Figure 8. Critical output parameters for the wind farms with different number of turbines (Case 2, Study-I)

power with increasing number of turbines. The reduction in the farm efficiency (as shown in Fig. 8(b)), with increasing number of turbines, further elucidates this phenomena.

The most interesting observation is furnished by the variation of the *Cost per KW of power produced* with increasing number of turbines. This variation is illustrated by Fig. 9, which indicates the existence of a minimum (for the cost at $N = 9$) with respect to number of turbines used. Hence, it is crucial to determine the optimal number of wind turbines in a farm, which ensures the lowest cost per KW of power production. Such information can be extremely beneficial in evaluating the genuine potential and profitability of a wind energy project. The planning of large scale wind farms involving hundreds of turbines can significantly benefit from such an analysis. Nevertheless, further investigation (regarding the influence of the number of turbines) is necessary in order to account for distributions of wind speed and direction that exist in the commercial scenario.

2. Parametric Study-II Results

In the case of a fixed number of turbines, an increase in the farm size is likely to reduce the mutual shading effect of turbines, thereby enhancing the farm efficiency. This phenomena might also depend on the shape of the wind farm. In this paper, only rectangular wind farms have been studied. The convergence histories, of the application of UWFLO to the five wind farms of different dimensions, are shown in Fig. 7. Figure 11 shows the variation of the farm efficiency (obtained through optimization) with increasing farm size. In this

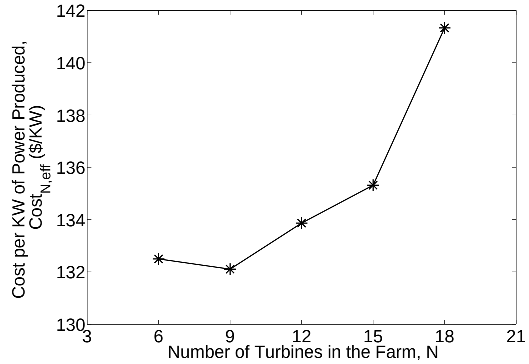


Figure 9. ***Cost per KW of power produced*** **by wind farms with different number of turbines (Case 2, Study-I)**

Table 5. Case 3, Parametric Study results

Farm	Farm dimensions ($X_{farm} \times Y_{farm}$)	Number of turbines	Farm efficiency after optimization
I-1	$14D \times 6D$	6	0.999
I-2	$14D \times 6D$	9	0.999
I-3	$14D \times 6D$	12	0.982
I-4	$14D \times 6D$	15	0.968
II-1	$7D \times 3D$	18	0.553
I-5/II-2	$14D \times 6D$	18	0.923
II-3	$21D \times 9D$	18	0.993
II-4	$28D \times 12D$	18	0.994
II-5	$35D \times 15D$	18	0.999

figure, the horizontal axis represents the area of the wind farm (S_{farm}) as a multiple of the area of the smallest farm studied, which is given by

$$S_{farm} = \frac{X_{farm} \times Y_{farm}}{7D \times 3D} \tag{16}$$

The critical output parameters (for Parametric Study-II) are also summarized in Table 5.

It is readily observed from Fig. 11 that the farm efficiency increases with increase in the farm size only to a certain level (which is $21D \times 9D$) in this study); beyond this level, there is no additional gain in the total power generated, by increasing the farm size. Considering that a bigger farm generally demands a higher cost of land and an increased cost of Operation and Maintenance (O&M), the determination of the optimum farm size (for a wind energy project) is crucial to wind farm planning. However, from a commercial perspective, we also need to consider factors such as the availability of land at a particular site and the local terrain. The latter factor will be explored in future research.

V. Conclusion

This paper presents a foundational wind farm optimization strategy. The original Unrestricted Wind Farm Layout Optimization (UWFLO) framework presents a layout optimization technique that does not make any limiting assumptions regarding the arrangement of turbines in a wind farm. The advanced UWFLO model, presented in this paper, allows simultaneous optimization of (i) the farm layout, and (ii) the selection of turbine rotor diameters (discrete in nature). The mixed-discrete Particle Swarm Optimization (PSO) algorithm, recently developed by the same authors, appositely deals with the combinatorial nature of the corresponding wind farm model. The advanced UWFLO framework is applied on a wind farm that is subjected to farm attributes and conditions similar to that in an experimental scale wind farm (recently

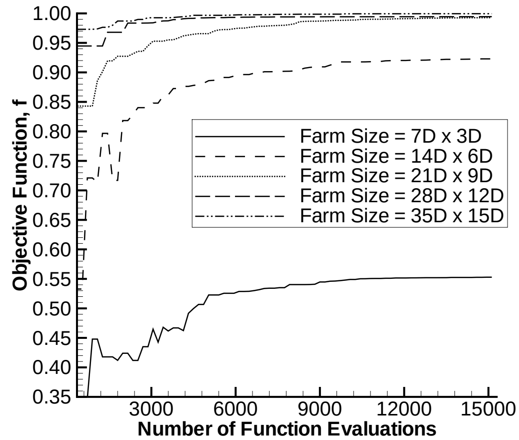


Figure 10. Convergence history of PSO (Case 2, Study-II)

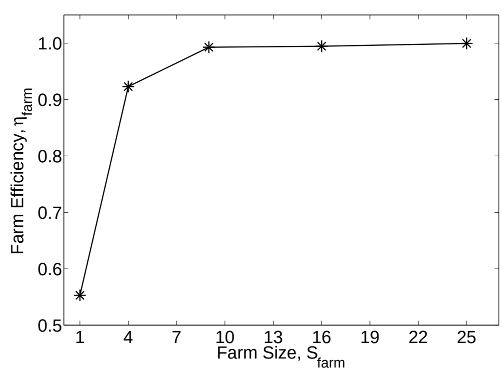


Figure 11. Farm efficiency of wind farms with different number of turbines (Case 2, Study-II)

reported in the literature). Subsequent case studies exhibited that, the UWFLO technique has the flexibility to ensure maximum improvement in the effective power available for the whole farm, for any given turbine power characteristics.

The use of turbines with differing rotor diameters produced a remarkable increase in the total power generated by the farm (43% compared to the experimental farm). A cost constraint was applied to ensure that the use of non-identical turbines does not escalate the cost of the wind farm. Hence the improvement in power generation accentuates the potential benefits of using of non-identical turbines in a wind farm. However, further research that considers the appropriate hub-height, pertinent performance characteristics and a comprehensive cost model, would provide a better insight in this direction. Additional parametric studies indicated that the selection of the optimal number of turbines, and the determination of the optimal farm size is essential to planning an efficient wind farm. We can summarize that the reliable determination of the requirements of a wind farm (number of turbines, type of turbines, and farm size) can be accomplished through robust modeling and optimization, as presented in this paper. Such an approach can favorably corroborate the prospects of wind becoming a major player in the future energy market.

Future developments in the UWFLO technique will account for the uncertainties in a wind farm, and other factors of commercial importance. Further application of the advanced UWFLO framework to a commercial scale wind farm would establish the true potential of this technique.

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