

# Modeling the Uncertainty in Farm Performance Introduced by the Ill-predictability of the Wind Resource

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A wind farm planning strategy that simultaneously accounts for the key engineering design factors, and addresses the major sources of uncertainty in a wind farm, can offer a powerful impetus to the development of wind energy. The distribution of wind conditions, including wind speed, wind direction, and air density, vary significantly from year to year. The resulting ill-predictability of the annual distribution of wind conditions introduces significant uncertainties in the estimated resource potential as well as in the predicted performance of the wind farm. In this paper, a new methodology is developed (i) to characterize the uncertainties in the annual distribution of wind conditions, and (ii) to model the propagation of uncertainty into the local Wind Power Density (WPD) and the farm performance: Annual Energy Production (AEP) and Cost of Energy (COE). Both parametric and non-parametric uncertainty models are formulated, which can be leveraged in conjunction with a wide variety of stochastic wind distribution models. The AEP and the COE are evaluated using advanced analytical models, adopted from the Unrestricted Wind Farm Layout Optimization (UWFLO) framework. The year-to-year variations in the wind distribution and the quantified uncertainties are illustrated using two case studies: (i) an onshore wind site at Baker, ND, and (ii) an offshore wind site near Boston, MA. Appreciable uncertainties are observed in the estimated yearly WPDs over the ten year period - approximately 11% for the onshore site, and 30% for the offshore site. Likewise, an appreciable uncertainty of 4% is observed in the performance of an optimized wind farm layout at the onshore site.

**Keywords:** Annual Energy Production, Cost of Energy, Farm layout, Turbine, Uncertainty, Wind distribution

## I. Introduction

### A. An Overview of Wind Farm Planning

Renewable energy resources, particularly wind energy, have become a primary focus in Government policies, in academic research and in the power industry. The practical viability of energy production is generally governed by such factors as (i) the potential for *large scale energy production*, (ii) the predictability of the power to be supplied to the grid, and (iii) the expected *return on investment*. The various uncertainties in wind energy hinders the reliable determination of these viability factors. The end of 2010 worldwide

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nameplate capacity of wind powered generators was approximately 2.5% of the worldwide electricity consumption;<sup>1</sup> and the annual growth rate of wind power in 2010 was the lowest since the year 2004.<sup>1</sup> For wind to play a more prominent role in the future energy market, we need steady improvement in the wind power generation technology; such advancement can be realized in part through appropriate quantification of the various uncertainties, and explicit consideration of their influences in the optimal design of wind farms.

The primary objectives of *robust and optimal wind farm planning* should include (i) the optimal selection of a site based on the quality of the local wind resource (ii) the maximization of the annual energy production and/or minimization of the Cost of Energy (COE) of the farm, and (iii) the maximization of the reliability of the predicted energy production. One of the major activities in site selection is to determine the wind resource potential at candidate sites, a popular measure of which is the estimated local Wind Power Density (WPD). Other site selection criteria include (but not limited to) the local topography, the vegetation, the distance to major grid connections, the ease of land acquisition, and the accessibility for turbine transport and maintenance.

Post site selection, the second objective can be accomplished through effective planning of the following engineering factors:

1. the rated capacity of the wind farm,
2. the farm land configuration (including land area, land shape and N-S-E-W orientation),
3. the layout of the wind farm, and
4. the type(s) of wind turbines to be installed in the wind farm.

The power extracted from turbines in a wind farm is a variable quantity that is a function of a series of parameters; several of these parameters are highly uncertain. A careful characterization of these uncertainties, and the modeling of their propagation into the overall system should provide a credible quantification of the wind farm energy production. The subsequent mitigation of these uncertainties in the optimal wind farm design stage would accomplish the third objective. At the same time, regarding site selection, it is generally considered that a desirable wind resource is one that has **stable high speeds**.<sup>2</sup> In this context, there are two long term stability aspects to be considered:<sup>3</sup> (i) *the annual variation of wind speed* (over the year), which can be estimated through wind distribution modeling; and (ii) *the variation of the annual wind distribution from year to year*, which can be characterized using the uncertainty models presented in this paper.

## B. Wind Farm Performance

The Annual Energy Production (AEP) capacity and the Cost of Energy (COE) can be considered two primary measures of the wind farm performance. Considering the variation of wind speed and direction, the expected AEP is estimated by the aggregate of the farm power generation over the annual wind distribution. For any given wind condition, the total power extracted by a wind farm is significantly less than the simple product of the power extracted by a standalone turbine and the number of such turbines in the farm.<sup>4</sup> This deficiency can be attributed to the loss in the availability of energy due to wake effects, which is the shading effect of a wind turbine on other wind turbines downstream from it.<sup>5</sup> Energy deficit due to mutual shading effects can be determined using wake models that give a measure of the growth of the wake and the velocity deficit in the wake with distance downstream from the wind turbine. Standard analytical wake models include the Park wake model,<sup>6,7</sup> the modified Park wake model and the Eddy Viscosity wake model. The reduction in the wind farm efficiency caused by wake effects depends primarily on the geometric arrangement of wind turbines in the farm (farm layout) and on the expected distribution of wind conditions at the concerned site.

To address this energy deficiency, several wind farm layout modeling approaches have been reported in the literature: (i) models that assume an array like (row-column) farm layout,<sup>4,8</sup> (ii) models that divide the wind farm into a discrete grid in order to search for the optimum grid locations of turbines,<sup>5,9-11</sup> and (iii) models that consider the location-coordinates of each turbine to be independent and continuous variables.<sup>12</sup> The Unrestricted Wind Farm Layout Optimization (UWFLO) method<sup>12,13</sup> avoids limiting assumptions presented by other methods regarding the layout pattern and the selection of turbines. In this paper, we adopt the power generation model from the UWFLO method<sup>13</sup> to evaluate the AEP.

Numerous techniques have been developed to evaluate the cost of onshore and offshore wind farms in the last twenty years. Notable examples include: Short-cut model,<sup>14</sup> cost analysis model for the Greek market,<sup>15</sup>

OWECOP-Prob cost model,<sup>16</sup> JEDI-wind cost model<sup>17</sup> and the Opti-OWECS cost model.<sup>18</sup> In this paper, we implement the *Response Surface-based Wind Farm Cost* (RS-WFC) model developed by Chowdhury et al.;<sup>13</sup> this model is founded on the principles proposed by Zhang et al.<sup>19</sup> Such a cost model has two major advantages - (i) the cost is represented by a continuous analytical function that can be easily used as a criterion function in optimization irrespective of the search strategy; and (ii) the estimated cost function is highly adaptive to the local cost data provided for training the model.

### C. Uncertainties in a Wind Farm

The physical uncertainties affecting the performance of a wind farm are primarily of two types:

1. Short term uncertainties: Uncertainties introduced by boundary layer turbulence and by other flow variations that occur in a small time scale (of the order of minutes)
2. Long term uncertainties: Uncertainties introduced by the long term variations in wind conditions, and by other environmental, operational and financial factors.

The physical sources of long term uncertainties can be further broadly classified into the categories shown in Fig. 1. In this paper, the category that is highlighted by the red-dashed rectangle in Fig. 1 is specifically

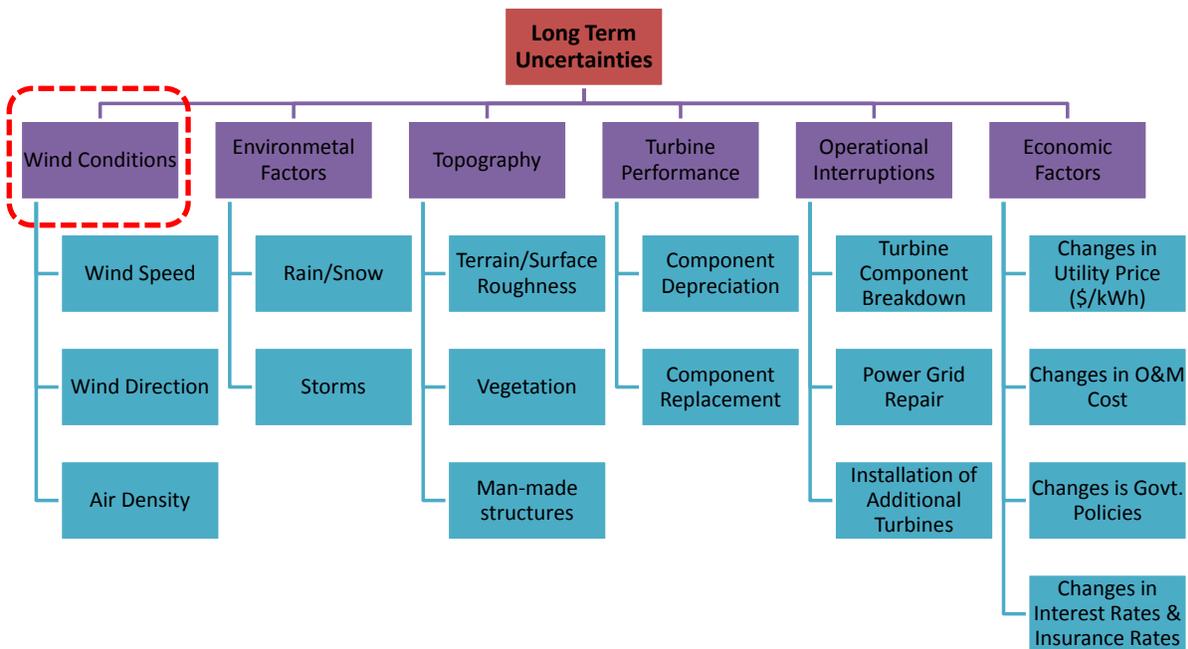


Figure 1. Classification of the long term uncertainties in wind energy

addressed. The incoming wind condition, comprising wind speed, wind direction and air density, is a major factor that regulates the power generated by a wind farm. Owing to seasonal effects, these conditions vary significantly over the course of a year. The overall trend of the annual distribution of wind conditions is however expected to remain fairly similar over years. In the literature, the long term variation in wind conditions have been represented by several parametric<sup>20,21</sup> and non-parametric distribution models.<sup>22</sup> **The predicted wind distributions themselves have been observed to be appreciably uncertain as is evident from their year-to-year variations, which is later illustrated in Section B.**

From an optimization standpoint, a reliable wind farm design should be minimally sensitive to the uncertainties in the predicted wind distribution at the concerned site. For a given wind distribution, the AEP strongly depends on the design factors 2 to 4: farm-land configuration, farm layout, and turbine selection. Chowdhury et al.<sup>23</sup> reported that robust optimization of the farm design factors 3 and 4 should seek to reduce the sensitivities of the AEP and the COE to the more uncertain wind conditions. In order

to accomplish the reliability objective, we need to quantify the uncertainty in the farm performance that is introduced by the ill-predictability of the local wind distribution. The quantification of this uncertainty is the primary development in this paper, and is accomplished through

1. a careful characterization of the uncertainties in the predicted wind distribution, and
2. the modeling of the propagation of these uncertainties into the AEP and the COE.

Both parametric and non-parametric uncertainty models are developed in this paper, each with uniquely helpful attributes; together, they provide a generalized methodology for uncertainty quantification that is uniquely helpful for robust optimization of wind farm configurations. These uncertainty models address the variations in all the three major attributes of an incoming wind condition: wind speed, wind direction and air density. However, for easier illustration purposes, only wind speed and wind direction are considered in the case studies provided in this paper.

Environmental conditions also present various secondary uncertainties, e.g. the occurrence of freezing rain, snow and ice accretion.<sup>24,25</sup> The seasonal variation of these factors is likely to be correlated with the variation of wind conditions, thereby indirectly affecting the planning of optimal wind farm designs. However, the investigation of the uncertainties in these secondary environmental conditions is not within the scope of this paper.

Section II provides the details of the wind sites explored in this paper (case studies) and the local distribution of wind at these sites; the illustration of the year-to-year variations in the predicted annual wind distribution is presented in the same section. The resource potential and the farm performance models adopted from the UWFLO method are summarized in Section III. The development of new methods to characterize the uncertainties in wind conditions and to model the propagation of these uncertainties into the farm performance is presented in Section IV. The application of the new uncertainty quantification methods to the case studies developed in this paper is presented in Section V.

## II. Case Study - Uncertainty in the Wind Distribution

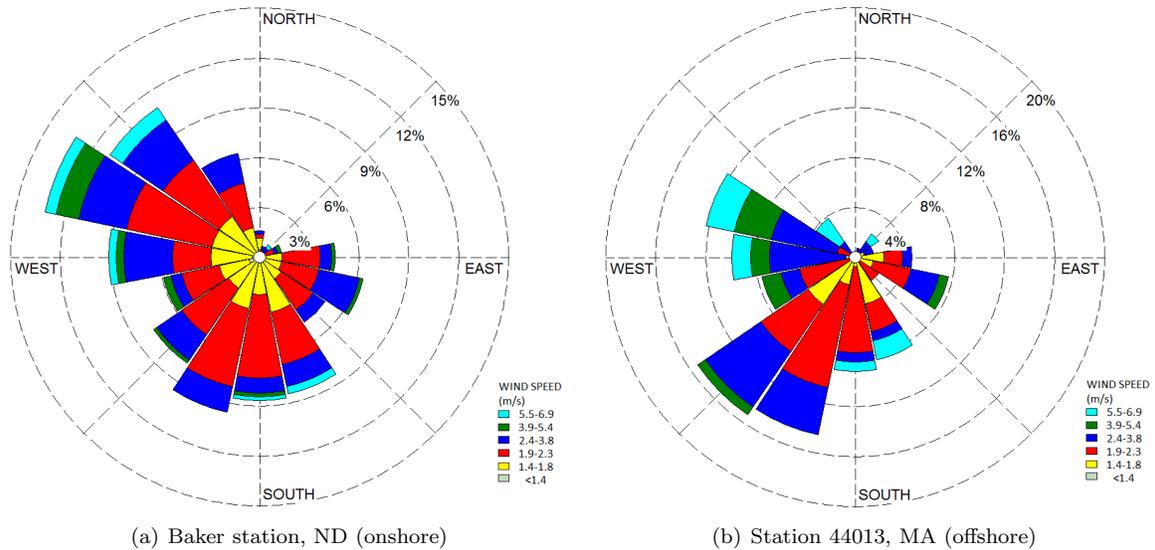
### A. Estimation of the Local Wind Distribution

To explore the uncertainties in the predicted wind distribution, two case studies are developed and analyzed in this paper: (1) an onshore wind farm at the Baker station in N. Dakota, and (2) an offshore wind farm at the Station 44013-BOSTON, 16 NM East of Boston, off the coast of Massachusetts. The wind data for the onshore wind farm is obtained from the North Dakota Agricultural Weather Network (NDAWN),<sup>26</sup> and that for the offshore wind farm is obtained from the National Data Buoy Center (NDBC).<sup>27</sup> In both cases, we consider the daily averaged recorded data over the span of ten years from January 2000 to December 2009. Pertinent details of the Baker station and Station 44013 near Boston are provided in Table 1. The recorded annual distributions of wind speed and direction at the two stations are illustrated by Windrose diagrams in Figs. 2(a) and 2(b). In the Windrose diagrams, each of the sixteen sectors represent the respective probability of wind blowing from that direction.

**Table 1. Details of the two wind stations**

Parameter	Onshore station <sup>26</sup>	Offshore station <sup>27</sup>
Location	Baker, ND	Station 44013, MA
Latitude	48.167° N	42.346° N
Longitude	99.648° W	70.651° W
Elevation	512m	sea level
Measurement height	3m above site elevation	5m above site elevation
Water depth	NA	61m

One of the most widely-used model for characterizing the annual variation in wind speed is the 2-parameter Weibull distribution.<sup>20,28</sup> Other models used to characterize this variation include the 1-parameter Rayleigh distribution, the 3-parameter generalized Gamma distribution, the 2-parameter Lognormal distribution, the 3-parameter Beta distribution, the bimodal Weibull model, the 2-parameter inverse Gaussian



**Figure 2. Wind rose diagrams: Recorded wind data from year 2000 to 2009**

distribution, the singly truncated normal Weibull mixture distribution, and the maximum entropy probability density function.<sup>20,21</sup> The majority of these wind distribution models make limiting assumptions regarding the correlativity and the modality of the variations in wind speed and wind direction.

In more recent publications, a limited number of methods have been proposed to capture the joint annual variation of wind speed and wind direction.<sup>29–32</sup> However, these methods do not explicitly account for the variation in air density; Zhang et al.<sup>22</sup> has shown that the air density at a site in N. Dakota can vary up to 30% over the year. In this paper, we use the the Multivariate and Multimodal Wind Distribution (MMWD) model developed by Zhang et al.<sup>22</sup> for both univariate and multivariate wind distributions. The MMWD model can capture the joint annual variation of wind speed, wind direction and air density, and also allows representation of multimodally distributed data. Multivariate kernel density estimation (KDE) method<sup>33</sup> is used to develop the MMWD model. KDE is a non-parametric method of estimating the probability density function of random variables. For the onshore and the offshore sites explored in this paper, the MMWD model provides the most accurate representation of the recorded wind data.<sup>22</sup>

In addition to the case studies 1 and 2, further analysis is performed to determine the uncertainties in the AEP and the COE for an optimized wind farm comprising 25 turbines at the studied onshore site. The optimized wind farm design is adopted from a recent paper by Chowdhury et al.<sup>13</sup> This wind farm design was obtained through the application of layout optimization using the UWFLO method, for a specified turbine type (GE 1.5MW xle) and a specified rectangular wind farm land (fixed area). The objective of the layout optimization was to maximize the AEP for the estimated wind distribution at that location. The AEP of the optimized wind farm was found to be 4.4% higher than that of a reference wind farm having a  $5 \times 5$  array layout.<sup>13</sup>

## B. Illustrating the Ill-predictability of the Wind Distribution

For the sake of lucid illustration of the year-to-year variations in the wind distribution, we consider the univariate distribution of wind speed. Figures 3(a) and 3(b) show the wind speed distributions for the years 2000 to 2009 at the onshore and the offshore sites, respectively. Figures 4(a) and 4(b) show the estimated Wind Power Densities (WPD) for the same period at the onshore and the offshore sites, respectively. The determination of the WPD from the estimated wind distribution is detailed in Section D.

Figure 3(a) and 3(b) show that the wind speed distributions at the onshore and the offshore sites have been shifting from year to year. Similarly, the WPD figures illustrate that the available energy at the onshore and offshore wind sites vary significantly from year to year. Most importantly, it is observed from these figures that, the distribution of wind speed in a particular year could be significantly deviated from the overall ten-year distribution. Expectedly, over the ten-year period the available wind energy at the offshore site is more stable than that at the onshore site, except for the year 2001. Overall, it is evident from these

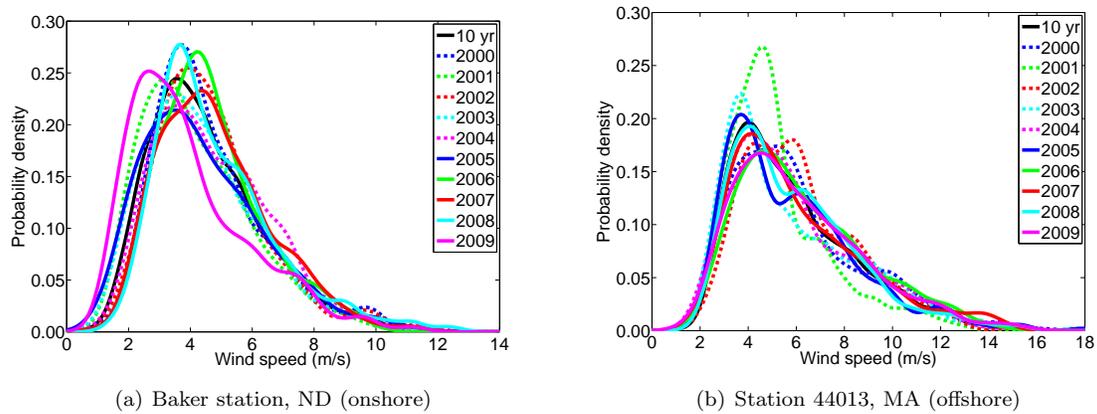


Figure 3. Annual and ten-year distributions of wind speed (years: 2000-2009) estimated by the MMWD model

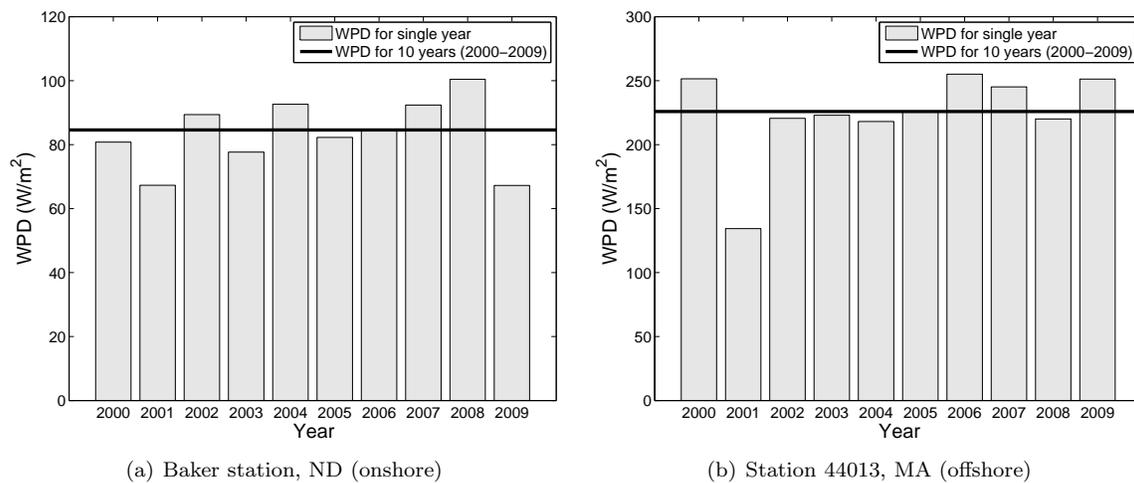
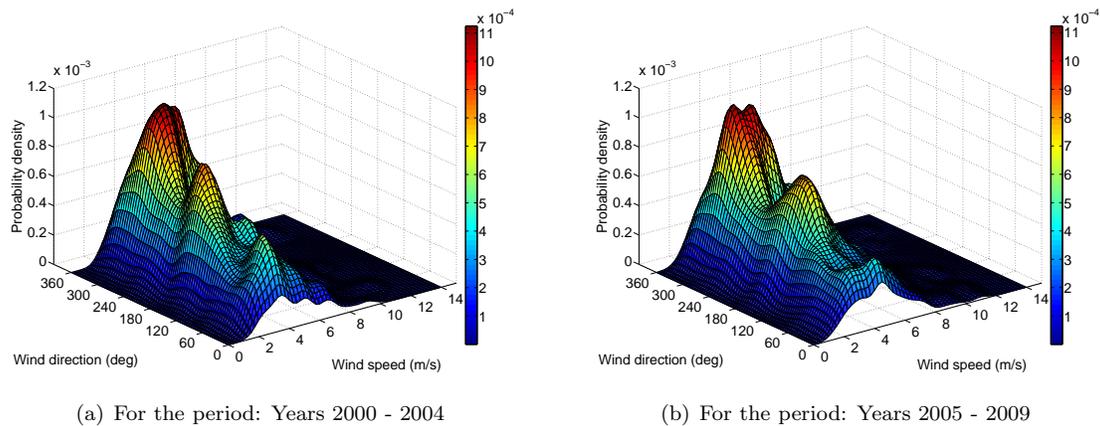


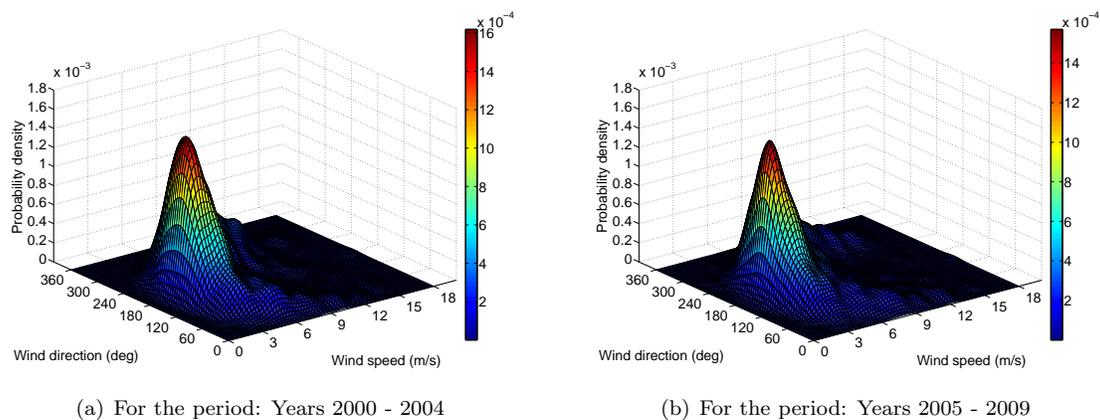
Figure 4. Annual and ten-year WPD (years: 2000-2009) determined by the MMWD model<sup>22</sup>

illustrations that - *the wind distribution estimated from data recorded for a single year cannot be considered **reliably** representative of the long term wind distribution at the concerned site.* Commercial wind farms are designed for a lifetime of 15-25 years, and hence require a more reliable prediction of their expected performance over the lifetime.

To further explain the “the reliability of representation issue”, we present illustrations of bivariate wind distributions for consecutive five-year periods: 2000 - 2004 and 2005 - 2009. Considering the year-to-year variations in the univariate wind speed distribution, the shifting is expected to be more pronounced in the case of the joint distribution of wind speed and direction. Figures 5(a) and 5(b) show the bivariate distributions of wind speed and direction at the onshore site for the periods 2000 to 2004 and 2005 to 2009, respectively. Figures 6(a) and 6(b) show the bivariate distributions of wind speed and direction at the offshore site for the periods 2000 to 2004 and 2005 to 2009, respectively. In the bivariate distribution plots, the wind direction axis in degrees represent “the direction from which wind is flowing, measured clockwise with respect to the North direction”. The same convention is used for all the graphical and the numerical illustrations of wind direction in the remainder of the paper. Interestingly, it is observed from Figs. 5(a) and 5(b) that, for



**Figure 5. Five-year distribution of wind speed and direction at the onshore site (Baker Station, ND), estimated by the MMWD model**



**Figure 6. Five-year distribution of wind speed and direction at the offshore site (Station 44013, MA), estimated by the MMWD model**

the onshore site the shifting between the consecutive five-year distributions is somewhat smaller than the year-to-year variations. The shifting in the consecutive five-year distributions in the case of the offshore site (Figs. 6(a) and 6(b)) is even smaller than that in the onshore site. These observations indicate that *if the length of time of the recorded data (past number of years) is (i) reasonably long and (ii) close to the succeeding length of time for which the prediction is being made, the estimated wind distribution is likely to be more representative.* If either of the above criterion is not fulfilled, the predicted wind distribution may

not be *reliably representative*.

Owing to economic and timeline constraints in commercial wind farm development, wind measurements are generally recorded at potential sites for a period of a year or so. Measure-Correlate-Predict (MCP) methods are often used to correlate this one-year on-site data to the concurring data at nearby meteorological station(s), and predict the approximate long term wind distribution at the site.<sup>3,28</sup> Nevertheless, in a practical scenario, the accuracy of long term predictions obtained using MCP methods is still subject to (i) the availability of a nearby meteorological station, (ii) the uncertainty associated with a specific correlation methodology,<sup>3</sup> and (iii) the likely dependence of this correlation on physical features such as the topography, the distance between the monitoring stations, and the type of the local climate regime.<sup>34</sup> As a result, given the unavoidable practical constraints, the overall reliability of the predicted long term wind distribution remains highly sensitive to the one-year distribution of the recorded on-site data.

The uncertainty models proposed in this paper can therefore prove to be uniquely helpful in quantifying the uncertainty in the predictions made, and provide more credibility to the wind resource assessment and the farm performance estimation (during optimization). Where MCP methods are used, these uncertainty models could be applied to the long term data recorded at the meteorological stations, and the estimated uncertainty can be used to determine the expected uncertainty in the onsite wind conditions. In that case, an investigation of how the uncertainties explored in this paper interact with the *uncertainties inherent in the MCP correlation methodology* is also necessary. This investigation is a potential topic for future research.

### III. Modeling the Resource Potential and the Farm Performance Measures

#### A. Wind Farm Power Generation Model

The power generated by a wind farm is an intricate function of the configuration and the location of the individual wind turbines. The flow pattern inside a wind farm is complex, primarily due to the wake effects and the highly turbulent flow. Hence, the velocity of the wind approaching a turbine and the corresponding power generated are determined separately for each turbine. Chowdhury et al.<sup>12,35</sup> implemented a rotor averaged uniform wind velocity derived from the incoming wind profile developed by Cal et al.<sup>36</sup> This wind profile can be expressed as,

$$\frac{U}{U_\infty} = b_1 \left( \frac{z}{b_2} \right)^{0.15} \quad (1)$$

where  $U$  and  $U_\infty$  represent the wind speed at a height  $z$ , and the free stream wind speed, respectively;  $b_1$  and  $b_2$  are constants that depend on the terrain and the atmospheric conditions. In the case of atmospheric boundary layer, a similarity study can be performed to describe the vertical profiles of turbulence statistics, when fully developed conditions are reached.<sup>37</sup> Assuming neutral conditions (negligible thermal effects) in the surface layer, which is for heights less than 100m, the mean velocity is commonly represented by the log profile.<sup>37</sup> For a known measured wind speed  $U_m$  at a height  $z_m$ , the log profile can be expressed as

$$\frac{U}{U_m} = \frac{\ln \frac{z}{z_0}}{\ln \frac{z_m}{z_0}} \quad (2)$$

where  $z_0$  is the average roughness length in the farm region, which depends on the terrain and the type of vegetation; and  $U$  is the wind speed at a height  $z$ . A uniform incoming flow that is equivalent to “*the logarithmic velocity profile (in Eq. 2) integrated and averaged over the rotor area*” is used in this model.

Detailed description of the power generation model used in this paper can be found in previous publications by Chowdhury et al.<sup>12,35</sup> The critical aspects of the power generation model for a given wind speed and wind direction are briefly summarized in this section. The wind turbines are assigned fixed  $XY$  coordinates, from which transformed coordinates can be determined for any given wind direction - the transformed positive  $X$ -axis ( $x$ ) is always aligned along the wind direction. Subsequently, the scope of mutual wake influence between turbines is concisely represented using an influence matrix  $M$ . This matrix is defined as

$$M_{ij} = \begin{cases} +1 & \text{if Turbine-}i \text{ influences Turbine-}j \\ -1 & \text{if Turbine-}j \text{ influences Turbine-}i \\ 0 & \text{if there is no mutual influence} \end{cases} \quad (3)$$

$\forall i, j = 1, 2, \dots, N; \quad i \neq j$

where  $N$  represents the number of turbines in the wind farm. Turbines with differing hub-heights are allowed in this power generation model with a simple modification of the original UWFLO model. Turbine- $j$  is in the influence of the wake created by Turbine- $i$  if and only if

$$\begin{aligned} \Delta x_{ij} < 0 \quad \text{and} \quad \sqrt{(\Delta y_{ij})^2 + (\Delta H_{ij})^2} - \frac{D_j}{2} < \frac{D_{wake,ij}}{2}, \quad \text{where} \\ \Delta x_{ij} = x_i - x_j, \quad \Delta y_{ij} = y_i - y_j, \quad \Delta H_{ij} = H_i - H_j \\ \forall i, j = 1, 2, \dots, N; \quad i \neq j \end{aligned} \quad (4)$$

In Eq. 4,  $D_j$  and  $H_j$  are, respectively, the rotor-diameter and the hub-height of Turbine- $j$ ;  $H_i$  is the hub-height turbine- $i$ ;  $D_{wake,ij}$  represents the diameter of the wake that is produced by Turbine- $i$ , and approaching Turbine- $j$ ; and  $x_i$  and  $x_j$  respectively represent the coordinates of turbine- $i$  and turbine- $j$  measured “along” the streamwise direction, and  $y_i$  and  $y_j$  respectively represent the coordinates of turbine- $i$  and turbine- $j$  measured “perpendicular to” the streamwise direction. The growth of the wake behind a turbine is determined using the wake growth model proposed by Frandsen et al.<sup>38</sup> The turbine-wake influence criterion given by Eq. 4 accounts for the possibility of a turbine being *partially* in the wake of another turbine located upwind.

For the given wind direction, the turbines are ranked in the increasing order of their streamwise location. The approaching wind speed for each turbine is then determined in the order of their rank. The wake velocity deficits behind each turbine are determined using the wake model based on the 1D stream-tube concept.<sup>38</sup> Considering the possibility of an influence of multiple upstream turbines, a standard wake superposition principle<sup>7</sup> is used to determine the effective speed ( $U_j$ ) of the wind approaching turbine- $j$ . The power generated by turbine- $j$  ( $P_j$ ), is generally given by

$$P_j = k_g k_b C_p \left( \frac{1}{2} \rho \pi \frac{D_j^2}{4} U_j^3 \right) \quad (5)$$

where  $C_p$ ,  $k_b$ , and  $k_g$  are the power coefficient, the mechanical efficiency, and the electrical efficiency of the turbine, respectively; and  $\rho$  represents the density of air. Within the model, the power generated by each turbine is determined using approximated power curves. A normalized power curve is developed through polynomial regression, using the power curve data for a representative GE turbine.<sup>39</sup> For any other particular commercial turbine, the normalized power curve is scaled using the corresponding information on the *cut-in*, the *cut-out*, and the *rated* wind speeds and the rated power, reported online by commercial turbine manufacturers. This generalized power characteristics estimation strategy has been used for ready implementation purposes; if power response data is available for a particular wind turbine, a unique power curve specific to that turbine should ideally be determined. The net power generated by the farm,  $P_{farm}$ , is given by

$$P_{farm} = \sum_{j=1}^N P_j \quad (6)$$

where  $P_j$  represents the power generated by Turbine- $j$ . Accordingly, the farm efficiency<sup>4</sup> can be expressed as

$$\eta_{farm} = \frac{P_{farm}}{\sum_{j=1}^N P_{0j}} \quad (7)$$

where  $P_{0j}$  is the power generated by Turbine- $j$  when operating as a standalone entity for the given uniform incoming wind speed. The farm efficiency measure,  $\eta_{farm}$ , reflects the loss of power owing to the wake effects.

It is helpful to note that the accuracy of the analytical power generation model is significantly sensitive to that of the analytical wake models used. A discussion of the inherent assumptions in the analytical wake model and that of the other assumptions made in the power generation model can be found in the paper by Chowdhury et al.<sup>12</sup> Accurate determination of the wake growth, the wake velocity deficit, and the wake superposition is challenging, and is a key topic of ongoing research in the wind energy community. As more advanced wake models become available, they can be readily incorporated into the global optimization framework presented in this paper.

## B. Annual Energy Production (AEP) Model

The prediction of the annual energy production from a wind farm should account for the correlated long term variations in wind speed, wind direction, and air density, which are together termed as “*wind condition*”. To this end, the annual distribution of wind conditions is first represented using a suitable probability density function, as discussed in Section A. Subsequently, the AEP is determined by integrating the power generation function over the estimated annual wind distribution.

The AEP of a wind farm in kWh,  $E_{farm}$ , at a particular location can therefore be expressed as

$$E_{farm} = 8760 \int_{\rho_{min}}^{\rho^{max}} \int_{0^\circ}^{360^\circ} \int_0^{U^{max}} P_{farm}(U, \theta, \rho) p(U, \theta, \rho) dU d\theta d\rho \quad (8)$$

where,  $U^{max}$  is the maximum possible wind speed at that location, and  $\rho_{min}$  and  $\rho^{max}$  represent the minimum and the maximum air densities at the concerned location, respectively; and  $P_{farm}(U, \theta, \rho)$  represents the power generated by the farm for a wind speed  $U$ , a wind direction  $\theta$ , and an air density  $\rho$ . In Eq. 8,  $p(U, \theta, \rho)$  represents probability of occurrence of wind conditions defined by speed  $U$ , direction  $\theta$ , and air density  $\rho$ . The power generated by the entire wind farm is a complex function of the incoming wind attributes, the arrangement of turbines, and the turbine features. Hence, a numerical integration approach<sup>40</sup> is suitable for estimating the annual energy production as given by Eq. 8.

To this end, the Monte Carlo integration method is implemented using the Sobol’s quasirandom sequence generator.<sup>41</sup> This class of integration methods is easy to apply for multidimensional integrals, and is likely to provide greater accuracy for the same number of sample evaluations when compared to the repeated integrations using one-dimensional Quadrature Rule methods. The approximated total annual energy produced by the wind farm is expressed as

$$E_{farm} = 8760 \sum_{i=1}^{n_p} P_{farm}(U^i, \theta^i, \rho^i) p(U^i, \theta^i, \rho^i) \Delta U \Delta \theta \Delta \rho, \quad \text{where} \quad (9)$$

$$\Delta U \Delta \theta \Delta \rho = (U_{max} \times 360^\circ \times (\rho_{max} - \rho_{min})) / n_p$$

and where  $n_p$  is the number of sample points used; the parameters  $U^i$ ,  $\theta^i$ , and  $\rho^i$ , respectively, represent the wind speed, the wind direction, and the air density of the incoming wind for the  $i^{\text{th}}$  sample point. Hence, the annual energy is readily determined by the summation of the estimated power generation ( $P_{farm}$ ) over the set of randomly distributed  $n_p$  wind conditions. In the remainder of the paper, the *predicted yearly probability of the  $i^{\text{th}}$  sample wind condition*,  $p(U^i, \theta^i, \rho^i)$ , is abbreviated as  $p_i$ .

## C. Wind Farm Cost Model

In the context of optimizing the farm layout and the turbine selection, the estimated annual cost of the farm can be represented as a function of *the number of turbines in the farm and the turbine rated powers*.<sup>13</sup> The annual farm cost is expressed in *dollars per kW installed* (\$/kW). Radial Basis Functions are used to develop the Response Surface-based Wind Farm Cost (RS-WFC) model. In creating the response surface, the annual farm cost is expressed in *dollars per kW installed* (\$/kW).

For a wind farm comprising  $N$  turbines, each with rated power  $P_r$ , the RS-WFC model  $Cost(P_r, N)$  can be represented as

$$Cost(P_r, N) = \sum_{i=1}^{n_p} \sigma_i \sqrt{(P_r - P_r^i)^2 + (N - N^i)^2 + c^2} \quad (10)$$

where  $P_r^i$  and  $N^i$  respectively denote the turbine rated power and the number of turbines in a farm corresponding to the  $i^{\text{th}}$  training data. The value of the prescribed constant  $c$  is specified to be 0.9 in this paper. The unknown coefficients ( $\sigma_i$ ) are evaluated using the pseudo-inverse technique. In the case of a wind farm comprising multiple types of turbines, the cost function should be modified. To this end, the total annual cost in dollars (\$) can be represented using a more generic expression, as given by

$$Cost_{farm} = \sum_{k=1}^{n_t} Cost(P_r^k, N^k) \times P_r^k \times N^k \quad (11)$$

where  $n_t$  denotes the number of different turbines types used in the wind farm; the parameter  $N^k$  represents the number of turbines of type- $k$  in the farm, which have a rated power  $P^k$ . In this case, the total number of turbines ( $N$ ) in the farm is equal to  $\sum_{k=1}^{n_t} N^k$ . Subsequently, the COE in \$/kWh can be estimated as

$$COE = \frac{Cost_{farm}}{E_{farm}} \quad (12)$$

where  $P_{farm}$  is the net power generated by the farm (as given by Eq. 6), expressed in kW. The above cost functions are estimated using data provided by the Wind and Hydropower Technologies program (US Department of Energy).<sup>17</sup>

#### D. Wind Power Density Estimation

*Wind Power Density* (WPD) is a standard measure of the wind resource potential of a candidate site. WPD can be defined as the power available from wind per unit area (perpendicular to the wind direction), averaged over an year at a particular site, and is expressed in *watts per square meter*. Mathematically, WPD can be defined as

$$WPD = \int_{\rho_{min}}^{\rho_{max}} \int_0^{U_{max}} \frac{1}{2} \rho U^3 p(U, \rho) dU d\rho \quad (13)$$

The terms and parameters in Eq. 13 are same as those defined for Eq. 8. If a parametric wind distribution model is used, the probability density function (pdf),  $p(U, \theta, \rho)$ , is a well defined analytical function; in that case, a standard analytical integration may be applicable. However, if a non-parametric wind distribution model is used, numerical integration techniques should be leveraged.

For the case studies in this paper, numerical integration has been used to evaluate the WPD from both parametric and non-parametric wind distributions. Similar to the numerical estimation of the AEP, for a set of  $n_p$  Monte Carlo sample wind conditions, the approximated WPD is given by

$$WPD = \sum_{i=1}^{n_p} \frac{1}{2} \rho^i U^i{}^3 p(U^i, \rho^i) \Delta U \Delta \rho, \quad \text{where} \quad (14)$$

$$\Delta U \Delta \rho = (U_{max} \times (\rho_{max} - \rho_{min})) / n_p$$

For the case studies in this paper, the variation of air density has not been considered. It is also helpful to note that WPD is not a comprehensive measure of the wind resource potential, since it does not account for the variation of wind direction. Owing to the wake effects, the actual power that can be extracted by a wind farm is significantly deviated from the WPD, or in other words, does not necessarily scale with the WPD. This deviation strongly depends on the distribution of the wind direction and the farm layout. Further discussion on the limitation of using WPD as a measure of the resource potential can be found in the paper by Zhang et al.<sup>42</sup>

## IV. Characterizing the Uncertainties in Wind Conditions

For a given farm layout, the AEP is a function of the distribution of wind speed, wind direction, and air density (as shown in Eqs. 8 and 14). Majority of the wind distribution models are *parametric* probability distribution functions, as stated in Section B. A handful of promising non-parametric wind distribution models have also been recently developed.<sup>22</sup>

The ill-predictability of the annual wind distribution introduces uncertainties into the predicted probabilities of the sample wind conditions that are used to evaluate the annual energy production (as in Eq. 14). These uncertainties then propagate into the AEP and the subsequently estimated COE. In this paper, two methods are developed to characterize the uncertainties in wind conditions and model the propagation of these uncertainties into the farm performance: (i) the *Parametric Wind Uncertainty (PWU) model*, and (ii) the *Non-Parametric Wind Uncertainty (NPWU) model*. Multiple year data of the wind conditions are required by these two methods.

### A. Parametric Wind Uncertainty (PWU) model

The uncertainty in *the frequency of wind approaching at a particular speed  $U$ , from a particular direction  $\theta$ , and with an air density  $\rho$*  can be represented in terms of the uncertainties in the parameters of the wind

distribution. In this case, the distribution parameters are themselves considered to be stochastic over years. The variance of a continuous stochastic parameter provides a standard measure of the uncertainty in that parameter. As illustrated in Section B, the annual distribution of wind conditions vary significantly from year to year. The parametric wind distribution model represented by  $p(U, \theta, \rho)$ , can therefore be considered a nonlinear function of uncertain distribution parameters. Based on standard uncertainty propagation principles,<sup>43</sup> for a  $m_p$ -parameter wind distribution model, the corresponding uncertainty can be expressed as

$$\Sigma_p = J \Sigma_q J^T, \quad \text{where}$$

$$J = \begin{bmatrix} \frac{\partial p_1}{\partial q_1} & \cdots & \frac{\partial p_1}{\partial q_{m_p}} \\ \vdots & \vdots & \vdots \\ \frac{\partial p_{n_p}}{\partial q_1} & \cdots & \frac{\partial p_{n_p}}{\partial q_{m_p}} \end{bmatrix} \quad (15)$$

In Eq. 15,  $\Sigma_p$  represents the uncertainty in the *predicted yearly probabilities of the sample wind conditions*;  $q_k$  represents the  $k^{\text{th}}$  parameter of the wind distribution model; the  $n_p \times m_p$  covariance matrix for the wind distribution parameters,  $\Sigma_q$ , represents the uncertainty in these parameters; the matrix  $J$  represents the Jacobian of the probabilities with  $J_{ik} = \frac{\partial p_i}{\partial q_k}$ ; and  $n_p$  is the number of random sample points used to implement the wind distribution in the estimation of the AEP (Eq. 14).

The covariance matrix  $\Sigma_q$  can be determined by fitting a suitable probability distribution to the set wind distribution parameters estimated for each year over a  $n$ -year period. The multivariate normal distribution has been used for this purpose in this paper. The Jacobian matrix  $J$  depends on the distribution model that is used to represent the yearly variation of wind conditions. A generic row of the Jacobian matrix for three popular wind distribution models are provided in Table 2.

**Table 2. Jacobian of the parameters for popular univariate wind distribution models**

Distribution	Distribution pdf	Jacobian
Rayleigh	$p(u) = \frac{u}{\sigma^2} \exp[-x^2 2\sigma^2]$	$J_i = \left[ \frac{p_i}{\sigma} \left( \frac{u^2}{\sigma^2} - 2 \right) \right]$
Lognormal	$p(u) = \frac{1}{u\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln u - \mu)^2}{2\sigma^2}\right]$	$J_i = \left[ p_i \frac{\ln u_i - \mu}{\sigma^2}, \frac{p_i}{\sigma} \left( \frac{(\ln u - \mu)^2}{\sigma^2} - 1 \right) \right]$
Weibull	$p(u) = \frac{k}{\lambda} \left( \frac{u}{\lambda} \right)^{k-1} \exp\left[-\left(\frac{u}{\lambda}\right)^k\right]$	$J_i = \left[ p_i \left( \frac{1}{k} + \ln\left(\frac{u}{\lambda}\right) - \left(\frac{u}{\lambda}\right) \ln\left(\frac{u}{\lambda}\right) \right), \frac{kp_i}{\lambda} \left( \left(\frac{u}{\lambda}\right)^k - 1 \right) \right]$

Equation 14 shows that, the annual energy production of a wind farm can be represented as a linear combination of the *predicted yearly probabilities of wind approaching with different sample combinations of wind speed, wind direction, and air density*. Hence, the uncertainty propagating into the annual energy production,  $\sigma_{E_{farm}}$ , can be modeled as

$$\sigma_{E_{farm}}^2 = C \Sigma_p C^T, \quad \text{where} \quad (16)$$

$$C = [C_1 \ C_2 \ \cdots \ C_{n_p}]; \quad C_i = 8760 \times \Delta U \Delta \theta \Delta \rho \times P_{farm}(U^i, \theta^i, \rho^i)$$

and where  $P_{farm}(U^i, \theta^i, \rho^i)$  represents the power generated by the farm for a wind coming at speed  $U^i$ , from the direction  $\theta^i$ , and with air density  $\rho^i$ ; and the term  $\Delta U \Delta \theta \Delta \rho$  is representative of the 3D interval size used for the wind condition sampling. The COE of a farm is inversely proportional to the annual energy production as given by Eq. 12. According to standard uncertainty propagation principles,<sup>44</sup> the uncertainty in the COE,  $\sigma_{COE}^2$ , can therefore be represented by,

$$\sigma_{COE} = COE \frac{\sigma_{E_{farm}}}{E_{farm}} \quad (17)$$

It is helpful to note that the uncertainty in the wind conditions and the corresponding uncertainties in (i) the AEP and (ii) the COE can also be represented in terms of confidence intervals. Such confidence intervals can be generally determined using Maximum Likelihood Estimation (MLE) techniques.

## B. Non-Parametric Wind Uncertainty (NPWU) model

The nonparametric model provides a more generalized characterization of the uncertainties in wind conditions - not dependent on the type of wind distribution model used to predict the annual variation in wind conditions. In this generalized model, we conceive *the predicted yearly frequency of wind approaching with a particular speed, direction, and air density* to be itself a stochastic parameter; the predicted yearly frequency of a given wind condition can therefore be directly expressed by a probability distribution. Such a distribution readily provides the covariance matrix,  $\Sigma^P$ , that quantifies the uncertainty in the *predicted yearly sample wind probabilities*. The derivation of  $\Sigma^P$  from the uncertainties in the wind distribution parameters  $\Sigma^q$ , as given by Eq. 15, is thereby bypassed.

Figure 7 illustrates the frequency for five samples conditions. The wind probabilities for the individual years and for the ten-year period are represented by the circle symbols and the dashed line, respectively. The Y-axis in this figure represents the logarithm of the wind probabilities. The yearly wind probabilities shown in the figure have been normalized through division by the corresponding ten-year wind probability value for the ease of illustration. The distribution of the yearly frequency of each sample wind condition is illustrated using representative Gaussian kernels in Figure 7. These Gaussian kernels are labeled as *Distribution of the Probability of Sample Wind Conditions (DPSWC)* in the figure. Interestingly, it is observed Figure 7 that,

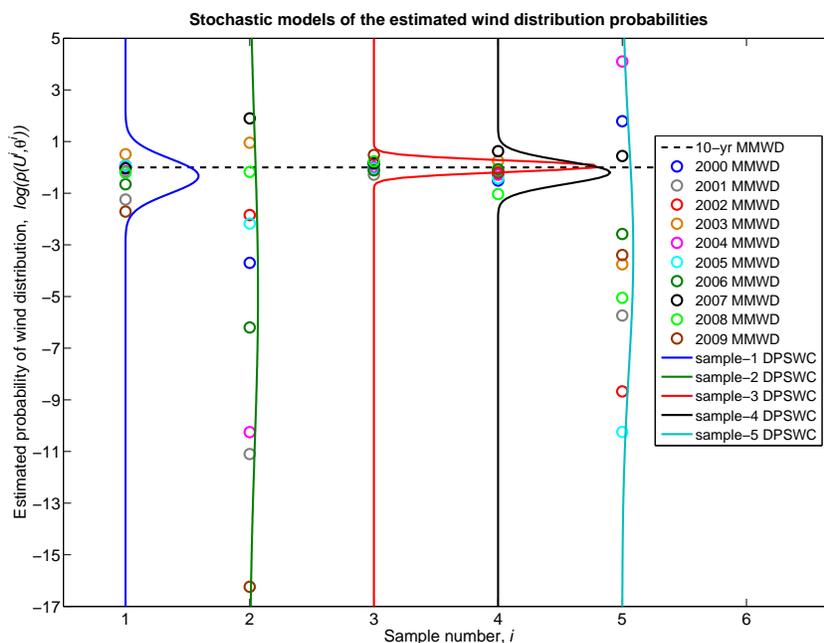


Figure 7. Sample wind probability measures (over 10 years) and their stochastic variations

the order of the *predicted sample wind probabilities* oscillate over the ten-year period.

Chowdhury et al.<sup>23</sup> used Lognormal distributions to fit the pdfs of *the yearly frequency of each sample wind condition*. In that approach, *the yearly frequencies of sample wind conditions* are considered to be random independent variables. The Lognormal distribution was found to be a reasonably appropriate choice to represent the stochastic nature of the logarithm of the *predicted sample wind probabilities*.<sup>23</sup> The correlation between the frequencies of the individual sample wind conditions are ignored in this approach.

The corresponding uncertainty in the annual energy production was represented by

$$\sigma_{E_{farm}}^2 = \sum_i^{n_p} C_i^2 (p_i \sigma_{\bar{p}_i})^2 \quad (18)$$

where  $n_p$  is the number of sample wind conditions;  $p_i$  is the predicted yearly frequency of the  $i^{\text{th}}$  sample wind condition; and  $\sigma_{\bar{p}_i}$  represents the uncertainty in the logarithm of the predicted yearly frequency of the  $i^{\text{th}}$  sample wind condition. The parameter  $\sigma_{\bar{p}_i}$  is given by the standard deviation of the Lognormal distribution. The cross-covariance terms in the  $\Sigma_p$  matrix are assumed to be zero in this approach. For any given wind distribution, the *predicted yearly frequencies of the sample wind conditions* are actually not independent of each other - the corresponding uncertainties are therefore expected to be correlated.

In this paper, we explore the feasibility of accounting for the correlations between the frequencies of the individual sample wind conditions in the NPWU model. A reasonably accurate determination of the AEP of a farm requires a relatively high number of sample wind conditions - e.g.  $n_p = 100$  for a bivariate scenario including wind speed and direction. At the same time, to account for the cross-covariance, a multivariate distribution is required instead of independent univariate distributions for each sample wind condition. Therefore, a distribution model that can be readily leveraged for such a high dimensional multivariate representation is suitable to determine the uncertainty in the *predicted yearly sample wind probabilities*. To this end, a multivariate normal distribution of the logarithm of the *predicted yearly wind probabilities* is used, as given by

$$p_{\bar{p}} = \frac{1}{(2\pi)^{k/2} |\Sigma_{\bar{p}}|^{1/2}} \exp \left[ -(\bar{p} - \mu_{\bar{p}}) (\Sigma_{\bar{p}})^{-1} (\bar{p} - \mu_{\bar{p}})^T \right] \quad \text{where} \quad (19)$$

$$\bar{p} = [\ln p_1 \quad \ln p_2 \quad \cdots \quad \ln p_{n_p}]$$

In Eq. 19,  $\mu_{\bar{p}}$  and  $\Sigma_{\bar{p}}$  are the mean vector and the covariance of the logarithm of the predicted yearly wind probabilities. The uncertainty in the *predicted yearly sample wind probabilities*,  $\Sigma_p$ , can then be expressed as

$$\Sigma_p = K \Sigma_{\bar{p}} K^T \quad (20)$$

where  $K$  is a diagonal matrix such that  $K_{ii} = p_i$ . The uncertainties in the AEP and in the COE can then be determined using Eqs. 16 and 17, respectively. Nevertheless, it is helpful to note that, the number of wind condition samples used ( $n_p$ ) is significantly higher than the number of years for which wind data is available; as a result, the estimation of the probability  $p_{\bar{p}}$  (from Eq. 19) requires fitting a high dimensional data with a significantly small number of data points. The accuracy of  $p_{\bar{p}}$  thus remains a challenging issue.

A comparison of the attributes of the parametric and the non-parametric wind uncertainty models is provided in Table 3. Positive attributes are marked by bold font in Table 3

**Table 3. Comparison of the attributes of the parametric and the non-parametric wind uncertainty models**

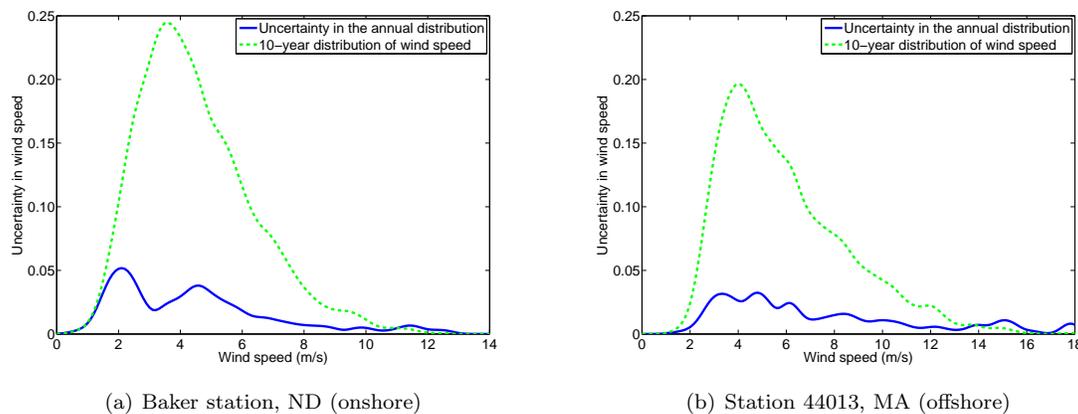
PWU model	NPWU model
Cannot be readily applied to non-parametric wind distribution models	<b>Can be readily applied to non-parametric wind distribution models</b>
Requires separate derivation of the Jacobian and determination of the covariance matrix for differing wind distribution models	<b>Derivation of the Jacobian is not required; provides a generalized uncertainty propagation model</b>
<b>Less sensitive to the selected set of Monte Carlo sample wind conditions</b>	More sensitive to the selected set of Monte Carlo sample wind conditions
<b>High dimensional distribution is not required</b>	Requires fitting a high-dimensional distribution to the frequencies of the sample wind conditions

## V. Application of the Uncertainty Models

### A. Case Studies 1 and 2: Results and Discussion

The Non-Parametric Wind Uncertainty models (NPWU) is applied to the ten-year univariate wind speed distributions estimated using the MMWD model. The Parametric Wind Uncertainty models (PWU) is applied to the ten-year univariate wind speed distribution estimated using the the Lognormal distribution. Zhang et al.<sup>22</sup> has shown that the Lognormal distribution and the MMWD compare well for the concerned onshore and offshore sites; the MMWD is however more accurate in both cases.<sup>22</sup> The Lognormal distribution comprises 2 parameters:  $\sigma$  and  $\mu$ ; the corresponding  $2 \times 2$  parameter-covariance matrix  $\Sigma_q$  is obtained by fitting a multivariate normal distribution to these parameters, estimated for each year from 2000 to 2009.

For the purpose of graphical illustration of the uncertainties, we present the square root of the variances: diagonal terms of the covariance matrix  $\Sigma_p$  from Eq. 20. In other words, the uncertainty in the yearly frequency of wind speed  $U^i$  and wind direction  $\theta^i$  is represented by  $\Sigma_p^{ii}$  in the following figures. Figures 8(a) and 8(b) show the uncertainties in the annual univariate wind speed distribution at the onshore and the offshore sites, respectively. The corresponding ten-year wind speed distributions are also presented in these figures. It is observed from Figs. 8(a) and 8(b) that the uncertainties in the predicted annual



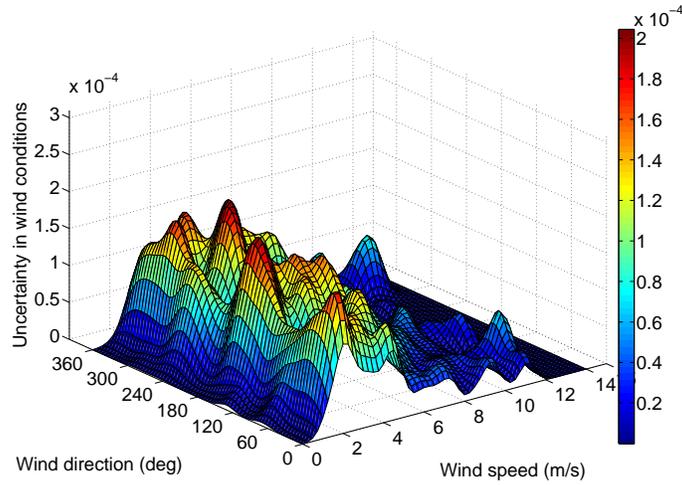
**Figure 8. Uncertainty in the distribution of wind speed (variance terms in the univariate NPWU model)**

wind distributions somewhat scale with the corresponding wind distributions. However, relative to the distribution, the uncertainties are higher for higher wind speeds: approximately, above 10 m/s for the onshore site, and above 13m/s for the offshore site. Most importantly, we observe that for a major portion of the wind distribution (onshore and offshore), the uncertainties are almost 20% of the corresponding ten-year wind frequencies.

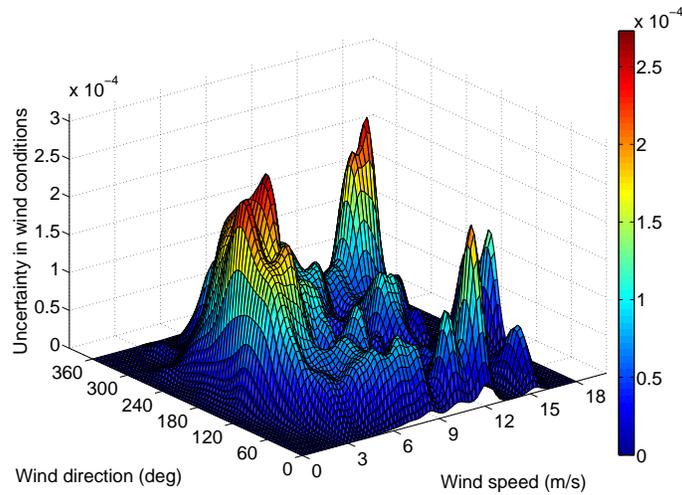
Figure 9(a) and 9(b) show the uncertainties in the bivariate annual distribution of wind speed and direction at the onshore and the offshore sites, respectively. In the case of the bivariate distributions of wind speed and direction (Figs. 9(a) and 9(b)), the scaling of the uncertainties with the corresponding distribution is less pronounced. In addition, for the offshore wind site, Fig. 9(b) shows that the higher wind speeds coming from the North direction are relatively more uncertain. The visualization of these uncertainties in the predicted yearly wind distributions (as shown in Figs. 9(a) and 9(b)) is particularly helpful in designing wind farms for more reliable performance. *A reliable wind farm design should seek to be minimally sensitive to the more uncertain wind conditions.*

In order to determine the WPD and quantify the uncertainty in the WPD, a set of 100 sample wind conditions are used. The estimated WPDs for the onshore and the offshore sites are summarized in Table 4. The uncertainties in the WPD, estimated using the PWU and NPWU models, are given in Table 5. The first two columns in Table 5 represent the standard deviations (std) of the estimated individual year WPDs obtained using the Lognormal distribution and the MMWD. These standard deviations help in validating the PWU and the NPWU models. In Tables 4 and 5, LND represents the Lognormal distribution.

It is seen from Table 4 that the WPDs estimated using the MMWD and the Lognormal distributions fairly agree with each other for the onshore and offshore wind sites. Interestingly, the standard deviations of the WPD estimated from the two different distributions also agree well for the onshore and the offshore stations.



(a) Baker station, ND (onshore)



(b) Station 44013, MA (offshore)

Figure 9. Uncertainty in the distribution of wind speed and direction (variance terms in the bivariate NPWU model)

Table 4. Estimated ten-year WPDs at the two wind stations

Site	Using LND ( $W/m^2$ )	Using MMWD ( $W/m^2$ )
Onshore	93.49	91.97
Offshore	212.83	219.72

Table 5. Uncertainty in the predicted yearly WPDs at the two stations

Site	Std. of WPD (LND)	Std. of WPD (MMWD)	Using PWU	Using NPWU without cross-covariance	Using NPWU
Onshore	11.25	11.90	11.48	3.52	13.54
Offshore	30.10	30.83	31.54	19.23	96.30

Table 5 shows that the uncertainty in the WPD, estimated using the PWU model in conjunction with the Lognormal distribution, is reasonably close to the standard deviations of the estimated yearly WPDs. It is also seen from Table 5 that

- when the cross-covariance terms are ignored, the NPWU model underestimates the uncertainty, and
- when the cross-covariance terms are considered, the NPWU model overestimates the uncertainty.

For the offshore site particularly, the overestimation is substantial. This observation indicates that further research is necessary to develop more accurate determination of the covariance matrix  $\Sigma_p$  for the NPWU model. Nevertheless, the variance terms in the NPWU  $\Sigma_p$  matrix can still be used as a measure of the uncertainty in the wind resource potential and the wind farm performance, when a non-parametric wind distribution model is applied.

## B. Uncertainty in the Optimized Wind Farm: Results and Discussion

The MMWD model was implemented to determine the AEP for the layout optimization performed by Chowdhury et al.<sup>13</sup> The determination of the AEP in that case included the joint variation of wind speed and wind direction over the period 2000-2009 at the onshore site in Baker, ND. We applied the NPWU model without the cross-covariance terms to estimate the uncertainty in the AEP and the COE for the optimized wind farm. The AEP, the COE, and the corresponding uncertainties in these farm performance measures are summarized in Table 6.

**Table 6. Uncertainties in the wind farm performance measures (onshore site)**

AEP (kWh)	Uncertainty in AEP (kWh)	COE (\$/kWh)	Uncertainty in COE (\$/kWh)
1.93e08	7.59e06	0.024	0.001

Table 6 shows that the relative uncertainties in the AEP and in the COE are approximately 4%. Considering that *the NPWU model without the cross-covariance terms* underestimate the uncertainties, the actual uncertainties in the AEP and the COE is likely to be higher than 4%. Such uncertainties would have an appreciable influence on the economic feasibility of a wind farm. Hence, an accurate estimation and due consideration of these uncertainties is of primary importance when conceiving a wind energy project.

## VI. Conclusion

Efficient planning of a wind energy project demands appropriate consideration of the factors that influence the farm performance, several of which are highly uncertain. The ill-predictability of wind conditions is one of the primary sources of such uncertainties. This paper presents a new methodology to (i) characterize the *uncertainties in the predicted yearly wind distribution*, and (ii) model the propagation of these uncertainties into the overall farm performance. Illustrations of the annual wind distributions over the ten year period 2000-2009 show that significant year to year variations exist at the studied onshore and offshore sites. Two uncertainty models are developed to capture these variations: a parametric model that can be implemented in conjunction with a wide variety of parametric wind distribution models; and a non-parametric model that can be implemented with both parametric and non-parametric uncertainty models. The parametric model is developed based on standard uncertainty propagation principles, and the pertinent expressions are provided for the 1-D Weibull, Lognormal and Rayleigh distributions. In the non-parametric model, the annual frequency of occurrence of a particular wind condition is itself treated as a stochastic parameter, and represented by a multivariate normal distribution. Although the non-parametric model is more universally applicable, its accuracy is limited by the requirement of *a high-dimensional multivariate stochastic modeling with a small data set* - further advancement of this model is necessary to avoid this limitation. Expectedly, the parametric model was found to provide a more accurate estimation of the uncertainty in the predicted WPD.

It is observed that the uncertainties in the predicted yearly wind frequencies are of the order of approximately 10% at the onshore and the offshore sites. For the offshore site, the uncertainties did not necessarily scale with the probability of the wind conditions, thereby indicating the existence of attractive trade-offs

between “higher farm performance” and “higher reliability of the farm performance”. The application of the non-parametric uncertainty model to an optimized wind farm design illustrated significant uncertainties in the AEP and the COE. Appropriate estimation and consideration of the long term wind uncertainties are therefore crucial for both wind resource assessment and wind farm layout optimization. Future research in this direction should also investigate the interaction of “the uncertainties occurring due to year-to-year variations” with “the uncertainties introduced by the measure-correlate-predict method used in commercial wind resource assessment”.

## VII. Acknowledgements

Support from the National Found from Awards CMMI-0533330, and CMII-0946765 is gratefully acknowledged.

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