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Characterizing the Uncertainty Propagation from the Wind Conditions to the Optimal Farm Performance

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The planning of a wind farm, which minimizes the lifecycle project costs and maximizes the reliability of the expected power generation, presents significant challenges to today's wind energy industry. An optimal wind farm planning strategy that simultaneously (i) accounts for the key engineering design factors, and (ii) addresses the major sources of uncertainty in a wind farm, can offer a powerful solution to these daunting challenges. In this paper, we develop a new methodology to characterize the long term uncertainties, particularly those introduced by the ill-predictability of the annual variation in wind conditions (wind speed and direction, and air density). The annual variation in wind conditions is represented using a non-parametric wind distribution model. The uncertainty in the predicted annual wind distribution is then characterized using a set of lognormal distributions. The uncertainties in the estimated (i) farm power generation and (ii) Cost of energy (COE) are represented as functions of the variances of these lognormal distributions. Subsequently, we minimize the uncertainty in the COE through wind farm optimization. To this end, we apply the Unrestricted Wind Farm Layout Optimization (UWFLO) framework, which provides a comprehensive platform for wind farm design. This methodology for *robust wind farm optimization* is applied to design a 25MW wind farm in N. Dakota. We found that layout optimization is appreciably sensitive to the uncertainties in wind conditions.

Keywords: Cost of Energy, Farm layout, Particle Swarm Optimization, Turbine, Uncertainty, Wind distribution

I. Introduction

Renewable energy resources, particularly wind energy, have become a primary focus in Government policies, in academic research and in the power industry. The practical viability of energy production is generally governed by such factors as (i) the potential for *large scale energy production*, (ii) the predictability of the power to be supplied to the grid, and (iii) the expected *return on investment*. The various uncertainties in wind energy hinders the reliable determination of these viability factors. The 2009 worldwide nameplate capacity of wind powered generators was only approximately 2% of the worldwide electricity consumption.¹ For wind to play a major role in the future energy market, we need steady improvement in the wind power

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generation technology; such advancement can be realized in part through appropriate quantification of the various uncertainties, and explicit consideration of their influences in the optimal design of wind farms.

The engineering design of wind farms generally includes (but is not limited to) critical decision-making, regarding

1. the number of wind turbines to be installed,
2. the layout of the turbines in the wind farm, and
3. the selection of the types of wind turbines (defined by rated power, rated speed, rotor diameter and hub height).

The objectives of *optimal wind farm planning* are to (i) minimize the Cost of Energy (COE), expressed in \$/KW-h, and (ii) maximize the net power generation. Successful accomplishment of these objectives demands a robust and flexible wind farm optimization model that appropriately accounts for the critical factors.

The estimated power generation from a wind farm is a guiding factor in the planning of a wind energy project. The power extracted from turbines in a wind farm is a variable quantity that is a function of a series of parameters; several of these parameters are highly uncertain. It is important to recognize that uncertainty anywhere in the system will potentially lead to uncertainty everywhere - thereby eventually impacting the overall power output of the wind farm. Careful modeling and characterization of these uncertainties, together with their propagation into the overall system, will allow for the credible quantification of the overall wind farm power output.

A. Uncertainties in a Wind Farm

The uncertainties affecting the performance of a wind farm are primarily of two types:

1. Short term uncertainties: Uncertainties introduced by boundary layer turbulence and by other flow variations that occur in a small time scale (of the order of minutes)
2. Long term uncertainties: Uncertainties introduced by the long term variations in wind conditions, and by other environmental, operational and financial factors.

In this paper, the long term uncertainties are broadly classified into the categories shown in Fig. 1. In this paper, we specifically address the categories that are highlighted by the red-dashed rectangle in Fig. 1. The incoming wind condition (combination of speed, direction and air density) is a major factor that regulates the power generated by a wind farm. These conditions vary significantly over the course of a year. In the literature, the long term variation in wind conditions have been represented by several parametric^{2,3} and non-parametric distribution models.⁴ However, the predicted wind distributions themselves have been observed to be appreciably uncertain, which is evident from their year-to-year variations (as illustrated in Section III.

Atmospheric conditions also present various uncertainties, e.g. the occurrence of freezing rain, snow and ice accretion.^{5,6} The seasonal variation of these factors is likely to be correlated with the variation of wind conditions, thereby indirectly affecting the planning of optimal wind farm designs. However, the investigation of the uncertainties in these secondary environmental conditions is not within the scope of this paper.

B. Wind Farm Optimization

Wind farms generally consist of multiple wind turbines located in a particular arrangement over a substantial stretch of land (onshore) or water body (offshore). It has been shown that the total power extracted by a wind farm is significantly less than the simple product of the power extracted by a standalone turbine and the number of identical turbines (N) in the farm.⁷ This deficiency can be attributed to the loss in the availability of energy due to wake effects - i.e. the shading effect of a wind turbine on other wind turbines downstream from it.⁸ Energy deficit due to mutual shading effects is determined using wake models that give a measure of both the growth of the wake, and the velocity deficit in the wake with distance downstream from the wind turbine. The Park wake model, originally developed by N. O. Jensen⁹ and later by Katic et al.,¹⁰ is one of the most popular analytical wake models used in wind farm modeling. The modified Park wake model and the Eddy Viscosity wake model are other standard wake models. The reduction in the wind

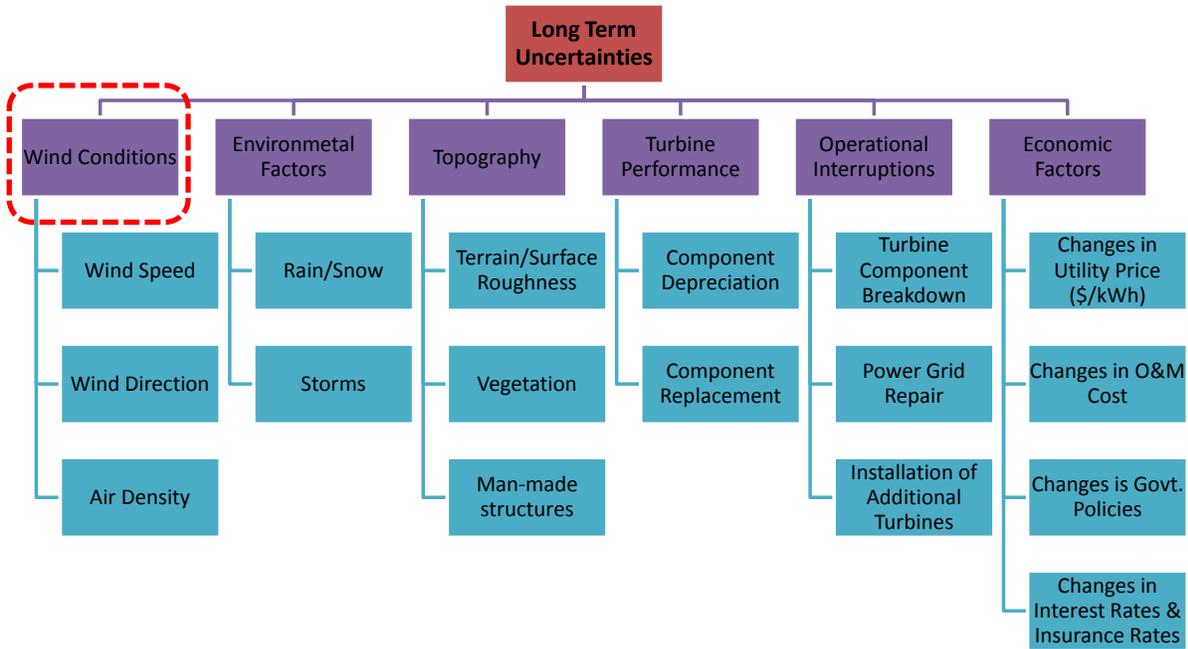


Figure 1. Classification of the long term uncertainties in wind energy

farm efficiency (loss in the effective energy available), due to this mutual shading, depends primarily on the geometric arrangement of wind turbines in a farm.

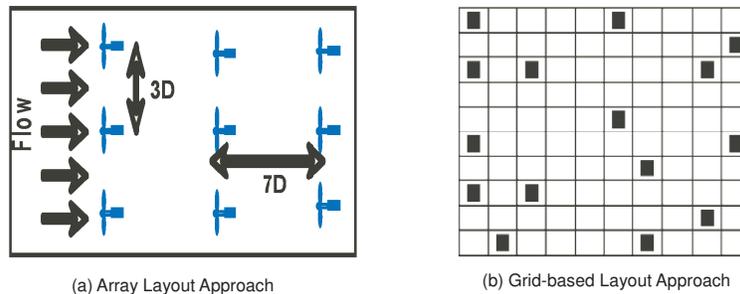


Figure 2. Existing approaches in wind farm optimization: (a) Array layout approach, and (b) Grid-based approach

To address this energy deficiency, two popular class of approaches have been reported in wind farm layout modeling: (i) models that assume an array like (row-column) farm layout^{7,11}, and (ii) models that divide the wind farm into a discrete grid in order to search for the optimum grid locations of turbines.^{8,12-14} Figure 2 illustrates these two class of approaches. These approaches are often not readily applicable to the broad commercial scenario that requires synergistic consideration of the arrangement and the selection of turbines, as shown by Chowdhury et al.¹⁵ A limited set of choices of wind turbines (turbine types) is available in the commercial market. The selection of appropriate turbines-types yields a mixed-discrete optimization problem. Commercial wind farms often comprise a large number of turbines (up to a hundred or more); for a wind farm with N turbines, the optimization problem is characterized by at least $2N$ design variables. Together with the likely multimodal nature of the power generation model,¹⁵ this characteristic leads to a challenging optimization problem. Robust optimization methodologies are necessary to address this challenge of designing large scale commercial wind farms.

The Unrestricted Wind Farm Layout Optimization (UWFLO) methodology, introduced by Chowdhury et

al.,¹⁵ avoids limiting assumptions presented by other methods, regarding the layout pattern and the selection of turbines. In the UWFLO method, the turbine location coordinates are treated as continuous variables, which provides a helpful flexibility in the arrangement of the turbines; in addition, the type of turbine to be installed is allowed to be selected from commercially available types. A response surface based wind farm cost model¹⁶ is implemented to evaluate the *cost per KW of power produced*. This paper advances the current UWFLO framework by addressing the uncertainties in the wind conditions within the scope of wind farm design. This advancement is accomplished through

- (1) the classification and the characterization of these uncertainties,
- (2) the stochastic modeling of the propagation of uncertainty into the annual energy production, and
- (3) the development of a wind farm design methodology to minimize the uncertainty in the COE.

The advanced UWFLO method thus presents a framework to design wind farms that are reliably high performing. To the best of the authors' knowledge, such a robust *optimal wind farm design* strategy is unique in the literature.

The UWFLO methodology is summarized in Section II. The characterization of the uncertainties in wind conditions and subsequent propagation of these uncertainties into the farm performance are presented in Section III. Reformulation of the wind farm optimization method to explicitly consider the modeled uncertainties is presented in Section IV. In the same section, we also present the application of the new robust optimization method to design a 25MW wind farm in N. Dakota.

II. Wind Farm Optimization Framework

In this paper we adopt the Unrestricted Wind Farm Layout Optimization (UWFLO) method¹⁷ to simultaneously optimize the farm layout and turbine selection. In the UWFLO model, the growth of the wake behind a turbine is determined using the wake growth model proposed by Frandsen et al.¹⁸ The corresponding energy deficit behind a turbine is determined using the velocity deficit model presented by Katic et al.;¹⁰ this velocity deficit model is widely adopted in wind farm modeling.^{13, 14, 19} In a wind farm, the velocity of the wind approaching a turbine can be affected by the wake of multiple turbines upstream from it. Crespo et al.²⁰ provide a review of different methods that account for the merging of wakes (wake superposition), in calculating the wake velocity deficits. UWFLO implements the wake superposition model developed by Katic et al.¹⁰ In the UWFLO power generation model, we also account for the possibility of a turbine being 'partially' in the wake of another turbine located upwind. The wind farm model developed in UWFLO has been successfully validated by Chowdhury et al.^{15, 21} against published experimental data.²²

The net power generated by the wind farm, for a given wind speed and direction, is evaluated by the sum of the power generated by the individual turbines. The cost of the farm (in \$/KW installed) is estimated using a radial basis function based cost model. The farm dimensions and the minimum distance required between any two turbines are treated as system constraints during optimization. In commercial wind farm planning, other factors such as (i) the grid connection, (ii) the terrain, (iii) the load bearing capacity of the soil, and (iv) the road layout in a farm,¹⁴ might further govern the optimal arrangement of turbines. These factors are, however, not within the scope of this paper. A mixed-discrete Particle Swarm Optimization¹⁷ is applied to minimize the Cost of Energy (COE) of the farm. In the subsequent subsections, we discuss the key components of this wind farm optimization methodology.

A. Modeling the Net Power Generation

The power generated by a wind farm is an intricate function of the power characteristics and location of the individual wind turbines. The flow pattern inside a wind farm is complex, primarily due to the wake effects and the highly turbulent flow. Hence, the velocity of the wind approaching a turbine and the corresponding power generated are determined separately for each turbine. Assuming neutral conditions (negligible thermal effects), the mean velocity in the surface layer (for heights less than 100m) is commonly represented by the log profile.²³ For a known recorded wind speed U_m at a height z_m , the log profile can be expressed as

$$\frac{U}{U_m} = \frac{\ln \frac{z}{z_0}}{\ln \frac{z_m}{z_0}} \quad (1)$$

where z_0 is the average roughness length (terrain dependent) in the farm region, and U is the wind speed at a height z . In this paper, we use a uniform incoming flow equivalent to “the logarithmic velocity profile (in Eq. 1), integrated and averaged over the rotor area”.

The layout modeling process in the original UWFLO method, for a wind farm comprised of N turbines, is concisely represented using an influence matrix (M). This matrix is defined as

$$M_{ij} = \begin{cases} +1 & \text{if Turbine-}i \text{ influences Turbine-}j \\ -1 & \text{if Turbine-}j \text{ influences Turbine-}i \\ 0 & \text{if there is no mutual influence} \end{cases} \quad (2)$$

$$\forall i, j = 1, 2, \dots, N; \quad i \neq j$$

Turbine- j is in the influence of the wake created by Turbine- i if and only if

$$\Delta x_{ij} < 0 \quad \text{and} \quad \sqrt{(\Delta y_{ij})^2 + (\Delta H_{ij})^2} - \frac{D_j}{2} < \frac{D_{wake,ij}}{2}, \quad \text{where} \quad (3)$$

$$\Delta x_{ij} = x_i - x_j, \quad \Delta y_{ij} = y_i - y_j, \quad \Delta H_{ij} = H_i - H_j$$

$$\forall i, j = 1, 2, \dots, N; \quad i \neq j$$

In Eq. 3, D_j and H_j are, respectively, the rotor-diameter and the hub-height of Turbine- j ; $D_{wake,ij}$ represents the diameter of the wake produced by Turbine- i , and approaching Turbine- j ; x_i and y_i represent the coordinates of turbine- i measured “along” and “perpendicular to” the streamwise direction, respectively.

The power generated by turbine- j (P_j), for an incoming wind speed U_j , is given by

$$P_j = k_g k_b C_p \left(\frac{1}{2} \rho \pi \frac{D_j^2}{4} U_j^3 \right) \quad (4)$$

where C_p , k_b , and k_g are the power coefficient, the mechanical efficiency, and the electrical efficiency of the turbine, respectively; and ρ represents the density of air. The power generated by each turbine is determined using approximated power curves. The power curves are represented by normalized polynomial functions that are determined (function fitting) using the *cut-in*, the *cut-out*, and the *rated* wind speeds. The normalized curves can be scaled according to the rated power of the concerned turbine-type.

The net power generated by the farm, P_{farm} , is given by

$$P_{farm} = \sum_{j=1}^N P_j \quad (5)$$

where P_j represents the power generated by Turbine- j . Accordingly, the farm efficiency⁷ can be expressed as

$$\eta_{farm} = \frac{P_{farm}}{N P_{0j}} \quad (6)$$

where P_{0j} is the power that Turbine- j would generate if operating as a standalone entity, for the given uniform incoming wind speed.

B. Wind Farm Cost Model

The UWFLO method uses a *response surface-based wind farm cost* (RS-WFC) model that is founded on the principles presented by Zhang et al.²⁴ The estimated annual cost of the farm is represented as a function of the number and the rated power of turbines in the farm. The annual farm cost is expressed in *dollars per kW installed* ($\$/kW$).

Radial Basis Functions (RBFs) are used to develop the cost model. The RS-WFC model can be represented as

$$Cost(P_r, N) = \sum_{i=1}^{n_p} \sigma_i \sqrt{(P_r - P_r^i)^2 + (N - N^i)^2 + c^2} \quad (7)$$

where P_r denotes rated power of turbines. The unknown coefficients (σ_i) are evaluated using the pseudoinverse technique. The total annual cost of the farm in dollars is estimated as

$$Cost_{farm} = \sum_{k=1}^{n_t} Cost(P_r^k, N^k) \times P_r^k \times N^k \quad (8)$$

where n_t denotes the number of different turbines types used in the wind farm; the parameters P_r^k and N^k represent the rated power and the number of turbines of type- k . In this case, the total number of turbines (N) in the farm is equal to $\sum_{k=1}^{n_t} N^k$. Subsequently, the COE (in \$/kWh) can be estimated as

$$COE = \frac{Cost_{farm}}{8760 \times P_{farm}} \quad (9)$$

where P_{farm} is the net power generated by the farm (as given by Eq. 5), expressed in KW. The above cost functions are estimated using data provided by the Wind and Hydropower Technologies program (US Department of Energy).²⁵

C. Annual Variation in the Power Generated by the Wind Farm

The power generated by an individual turbine is a cubic function of the approaching wind speed (as seen from Eq. 4). At the same time, for a given farm layout, the wind direction is a major factor that regulates the overall flow pattern and wake losses inside the wind farm. The determination (or prediction) of the annual power generation from a wind farm should thereby account for the long term variation in wind speed and direction. To this end, we can apply the following two-step procedure:

1. Estimate the annual distribution of the wind speed and direction (as a probability distribution function).
2. Integrate the power generation model over the entire annual distribution of the wind speed and direction.

One of the most widely-used model for characterizing the wind speed is the 2-parameter Weibull distribution.^{2,26} Other models used to characterize wind speed include the 1-parameter Rayleigh distribution, the 3-parameter generalized Gamma distribution, the 2-parameter Lognormal distribution, the 3-parameter Beta distribution, the bimodal Weibull model, the 2-parameter inverse Gaussian distribution, the singly truncated normal Weibull mixture distribution, and the maximum entropy probability density function.^{2,3} The majority of these wind distribution models make limiting assumptions regarding the correlativity and the modality of the variations in wind speed and wind direction. In this paper, we use the newly developed Multivariate and Multimodal Wind Distributions (MMWD) model⁴ that avoids such limiting assumptions. This model is developed using multivariate kernel density estimation (KDE)²⁷ that is also known as the Parzen-Rosenblatt window method.^{28,29} KDE is a non-parametric method of estimating the probability density function of random variables.

The total annual energy produced by a wind farm (in kWh), E_{farm} at a particular location can be expressed as³⁰

$$E_{farm} = 8760 \int_{\rho_{min}}^{\rho_{max}} \int_{0^\circ}^{360^\circ} \int_0^{U^{max}} P_{farm}(U, \theta, \rho) p(U, \theta, \rho) dU d\theta d\rho \quad (10)$$

where, U^{max} is the maximum possible wind speed at that location, and $P_{farm}(U, \theta, \rho)$ represents the power generated by the farm for a wind speed U , a wind direction θ , and an air density ρ . In Eq. 10, $p(U, \theta, \rho)$ represents probability of occurrence of wind conditions defined by speed U , direction θ , and air density ρ . A numerical integration approach³¹ is suitable for estimating the annual energy production as given by Eq. 10. Numerical integration is performed using the Monte Carlo method that is implemented through the Sobol's quasirandom sequence generator.³² The approximated total annual energy produced by the wind farm is expressed as

$$E_{farm} = 8760 \sum_{i=1}^{N_p} P_{farm}(U^i, \theta^i, \rho^i) p(U^i, \theta^i, \rho^i) \Delta U \Delta \theta \Delta \rho, \quad \text{where} \quad (11)$$

$$\Delta U \Delta \theta \Delta \rho = (U_{max} \times 360^\circ \times (\rho_{max} - \rho_{min})) / N_p$$

and where N_p is the number of sample points used; the parameters U^i , θ^i , and ρ^i , respectively, represent the wind speed, the wind direction, and the air density of the incoming wind for the i^{th} sample point. Hence, the annual energy is readily determined by the summation of the estimated power generation (P_{farm}) over a set of randomly distributed N_p wind conditions.

III. Modeling the Propagation of Uncertainties in a Wind Farm

A. Case Study - Illustrating the Uncertainties in Wind Conditions

The expected wind conditions at a candidate wind site is one of the guiding factors (i) in farm siting (based on the predicted wind resource potential) and (ii) for the subsequent design of the wind farm. We believe that an exploration of the estimated wind distribution for a real life wind site is helpful to understand (and appreciate) the nature of the uncertainties in wind conditions. For this purpose, we present the estimated distribution of wind speed and wind direction at a site in North Dakota. The wind data used in this paper is obtained from the North Dakota Agricultural Weather Network (NDAWN).³³ We use the daily averaged data for wind speed and direction, measured at the Baker station between the years 2000 and 2009. Pertinent details of the Baker station are provided in Table 1. The annual distribution of wind speed and wind direction is determined using the Multivariate and Multimodal Wind Distributions (MMWD) model.⁴ Details of the MMWD method can be found in the paper by Zhang et al.⁴ The estimated bivariate distribution is illustrated by a Windrose diagram in Fig. 3; in this diagram, each of the sixteen sectors represent the respective probabilities of wind blowing from that direction.

Table 1. Details of the NDAWN station at Baker, ND³³

Parameter	Value
Location	Baker, ND
Period of record	01/01/2000 to 12/31/2009
Latitude	48.167°
Longitude	-99.648°
Elevation	512m
Measurement height	3m

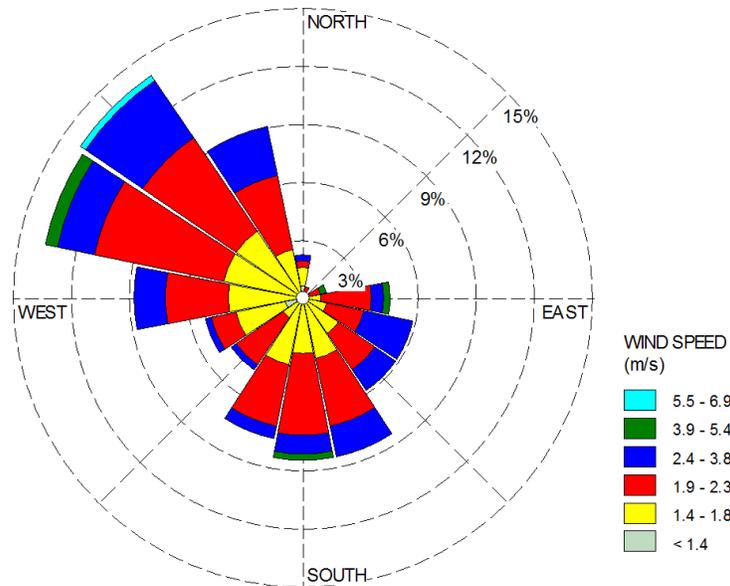


Figure 3. Windrose diagram for Baker station, ND (Years 2000-2009)

Figure 4(a) shows the univariate distribution of the wind speed for each year (from 2000 to 2009), as well as the overall distribution for 10 years. Figure 4(b) shows the estimated Wind Power Densities (WPD) for the 10 years. From figure 4(a) we observe that the wind speed distribution has been shifting (e.g. in terms of the approximate mean wind speed) over the ten years. More importantly, it is observed (from Figure 4(a)) that, the distribution of wind speed in a particular year could be significantly deviated from the overall 10-year distribution. This deviation leads to significant variations in the WPD from year to year, as seen

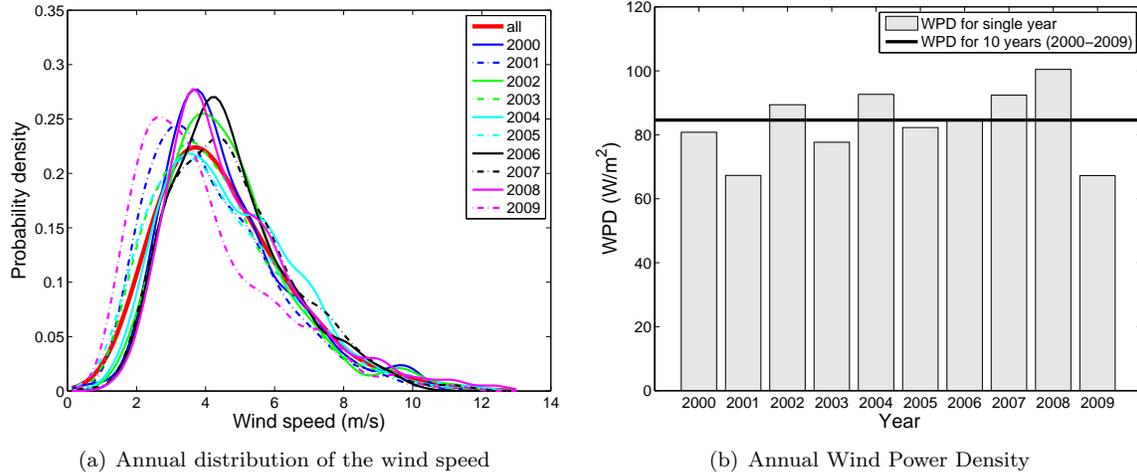


Figure 4. Year-to-year variation in the wind distribution estimated by the MMWD model⁴

from Fig. 4(b). Other important factors to consider are (i) climatic conditions at a location might evolve over years, and (ii) the general lifetime of wind turbines is 15-20 years, approximately. Therefore, in the authors' opinion, the uncertainty in the energy production owing to the ill-predictability of the wind resource is not adequately captured by an averaged distribution (developed using data from a certain number of past years).

In this paper, we develop a generalized methodology to (i) characterize the uncertainties in the predicted wind distribution and (ii) model the propagation of these uncertainties into the net energy production. This methodology is described in the next section.

B. Characterizing the Uncertainties in Wind Conditions

For a given farm layout, the net annual energy production is a function of the distribution of wind speed, wind direction, and air density (as shown in Eqs. 10 and 11). Majority of the wind distribution models are *parametric* probability distribution functions, e.g. the 1-parameter Rayleigh distribution, the 3-parameter generalized Gamma distribution, the 2-parameter Lognormal distribution, the 3-parameter Beta distribution, the bimodal Weibull model. A handful of promising non-parametric wind distribution models have also been recently developed.⁴ The uncertainty introduced by the ill-predictability of the long term wind distribution (determined using a parametric/non-parametric method) propagates into the predicted probabilities of the sample wind conditions that are used to evaluate the annual energy production (as in Eq. 11); the uncertainties in the predicted *sample wind condition probabilities* then propagates into the the annual energy production. The primary objective of this paper is to model this (apparently two-tier) process of uncertainty propagation in wind energy.

The uncertainty in *the probability of wind approaching at a particular speed U , from a particular direction θ , and with an air density ρ* can be represented in terms of the uncertainties in the parameters of the wind distribution. In this case, the distribution parameters are themselves considered to be stochastic. The variance of a continuous stochastic parameter can be used as a measure of the uncertainty in that parameter. Considering the probability $p(U, \theta, \rho)$ of the wind to be a nonlinear function of the corresponding distribution parameters, we can express the uncertainty $\sigma_{p(U, \theta, \rho)}^2$ in the wind distribution as³⁴

$$\sigma_{p(U, \theta, \rho)}^2 = Q \Sigma^q Q^T, \quad \text{where} \quad (12)$$

$$Q_k = \frac{\partial p(U, \theta, \rho)}{\partial q_k} \quad \text{and} \quad k = 1, 2, \dots, n$$

In Eq. 12, q_k represents the k^{th} parameter of the wind distribution model; Σ^q is the covariance matrix for the wind distribution parameters, which represents the uncertainty in these parameters. The wind distribution defined by $p()$ comprises n parameters.

Equation 11 shows that, the annual energy production of a wind farm is a linear combination of *the probabilities of wind approaching with different sample combinations of wind speed, wind direction, and air density*. The subsequent uncertainty propagating into the annual energy production can then be modeled as

$$\sigma_{E_{farm}}^2 = C \Sigma^p C^T, \quad \text{where} \quad (13)$$

$$C_i = 8760 \times \Delta U \Delta \theta \times P_{farm}(U^i, \theta^i, \rho^i)$$

and where, the terms of the covariance matrix (Σ^p) for the wind probabilities $p(U^i, \theta^i, \rho^i)$ are defined by

$$\Sigma^p = J \Sigma^q J^T \quad (14)$$

The matrix J represents the Jacobian of the probabilities, where $J_{ik} = \frac{\partial p(U^i, \theta^i, \rho^i)}{\partial q_k}$.

For this uncertainty characterization, the Jacobian J depends on the type of parametric distribution that is used to represent the annual variation in wind conditions. However, in the case of non-parametric distributions such as the MMWD model,⁴ this methodology cannot be readily implemented. Hence, we also formulate a more generalized model of the propagation of the uncertainties into the annual energy production. In this generalized model, we conceive *the probability of wind approaching with a particular speed, direction, and air density* to be directly a stochastic parameter, which can be represented by an suitable distribution model. The uncertainties in the *predicted yearly sample wind probabilities*, given by the covariance matrix Σ^p , is then readily available; thereby, we avoid the derivation of Σ^p from the uncertainties in the wind distribution parameters Σ^q (as given by Eq. 14). In this paper, we model the uncertainty in *predicted sample wind probability* as a univariate Lognormal distribution. *We call this uncertainty characterization, the **Distribution of the Probability of a Sample Wind Condition (DPSWC)***.

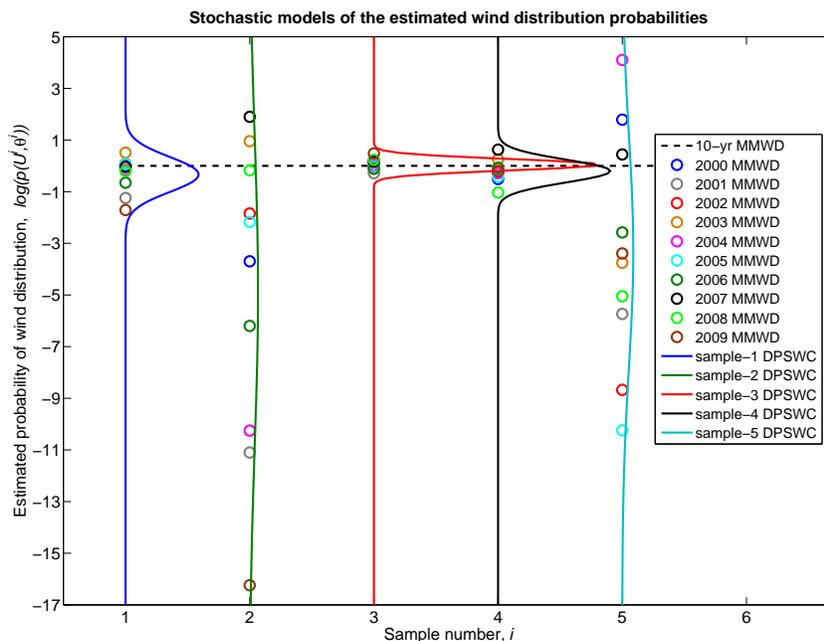


Figure 5. Sample wind probability measures (over 10 years) and their stochastic variation

Figure 5 shows the wind probabilities (both individual year estimates and the 10-year estimates) for five samples of wind speed, direction, and air density (determined using the MMWD model), and the corresponding Lognormal distributions. The wind probabilities for the individual years and for the 10-year period are represented by the circle symbols and the dashed line, respectively. The Y-axis in this figure represents the logarithm of the wind probabilities. In Fig. 5, the wind probabilities appear in their normalized

form (through division by the 10-year wind distribution value) for the ease of illustration. Interestingly, we observe from this figure (5) that, the logarithms of the *sample wind probabilities* (representing their orders of magnitude) shift up and down over the 10-year period. In this paper, we represent *the negative logarithm of the “sample wind probabilities” using a Lognormal distributions. The negative logarithm of the “sample wind probabilities* ($p(U^i, \theta^i, \rho^i)$)” are denoted by \bar{p}^i , which can be expressed as

$$\bar{p}^i = -\ln(p(U^i, \theta^i, \rho^i)) \quad (15)$$

The Lognormal distribution is a reasonably appropriate choice to represent the stochastic nature of the *predicted sample wind probabilities*, because of the following reasons.

- A continuous probability density function (pdf), as a random variable itself, is bound in the range [0,1).
- The negative logarithm of a pdf should therefore belong to the range (0,∞).
- Numerical experiments showed that the logarithms of the parameters \bar{p}^i s (for the set of sample wind conditions) generally followed the trend of a normal distribution.

The DPSWC ($p_{\bar{p}^i}$) and the corresponding uncertainty ($\hat{\sigma}_{\bar{p}^i}$) can be, respectively, expressed as

$$p_{\bar{p}^i} = \frac{1}{\bar{p}^i \sigma_{\bar{p}^i} \sqrt{2\pi}} \exp \left[-\frac{(\ln(\bar{p}^i) - \mu_{\bar{p}^i})^2}{2\sigma_{\bar{p}^i}^2} \right] \quad \text{and} \quad (16)$$

$$(\hat{\sigma}_{\bar{p}^i})^2 = \left[\exp(\sigma_{\bar{p}^i}^2) - 1 \right] \exp(2\mu_{\bar{p}^i} + \sigma_{\bar{p}^i}^2)$$

In Eq. 16, $(\hat{\sigma}_{\bar{p}^i})^2$ represents the variance of the DPSWC. In this paper, we assume that the correlation coefficients for the DPSWCs at different sample wind conditions ($i \neq j$) are zero. Future research should also account for the correlation of DPSWCs ($p_{\bar{p}^i}$). The uncertainty in the annual energy production can be simplified from Eq. 13 to

$$\sigma_{E_{farm}}^2 = \sum_i^{N_p} C_i^2 (\hat{\sigma}_{\bar{p}^i})^2 \quad (17)$$

where N_p is the number of sample wind conditions.

The cost of energy (COE) is inversely proportional to the annual energy production (as shown by Eq. 9). According to the expressions for uncertainty propagation given in the manual by Lindberg,³⁵ the uncertainty in the COE (σ_{COE}^2) can be represented by,

$$\sigma_{COE} = COE \frac{\sigma_{E_{farm}}}{E_{farm}} \quad (18)$$

It is helpful to note that the uncertainty in the wind conditions and the corresponding uncertainties in (i) the annual energy production and (ii) the COE can also be represented in terms of confidence intervals. Such confidence intervals can be generally determined using Maximum Likelihood Estimation (MLE) techniques.

This approach to model the uncertainty propagation using the sample wind probabilities directly has the following three major advantages:

1. This approach can be applied to both parametric and non-parametric wind distribution models.
2. This approach avoids the bias that might be introduced by the estimation of the non-linear uncertainty propagation (given by Eq. 14). This bias is attributed to the truncated Taylor series expansion, used to linearize the nonlinear distribution model.
3. This approach can be readily applied to univariate, bi-variate, and multivariate (including air-density) wind distribution models.

IV. Robust Wind Farm Optimization

A. Optimization Problem Definition

Several different optimization strategies can be defined to address the uncertainties within the scope of maximizing the wind farm performance. In this paper, we apply the following 2-step strategy, using the Unrestricted Wind Farm Layout Optimization (UWFLO) method:

Step-1: Apply UWFLO to minimize the Cost of energy (COE) of the farm.

Step-2: Apply UWFLO to minimize the uncertainty in the COE (σ_{COE}^2). The minimum COE obtained in the previous step is relaxed (increased) by 5%, and applied as an additional constraint.

For the case study in this paper, we consider a bivariate distribution of wind speed and wind direction. Future research should investigate the uncertainty propagation for a multivariate distribution of wind speed, wind direction, and air density. In Steps 1 and 2, for a given wind farm capacity (25MW) and a given farm-land size (rectangular farm), we simultaneously optimize the farm layout and the selection of the turbine-type (to be installed). This 2-step optimization is applied to design a wind farm at the North Dakota location described in Section III) and Table 1.

The specified wind farm properties are given in Table 2. The farm is oriented such that the positive X -direction of the layout co-ordinate system points towards the South. The objective function in Step-1 is normalized by dividing the COE of a candidate farm design by the COE estimated for a reference farm. The reference wind farm is comprised of twenty-five “GE 1.5MW xle” turbines, arranged in a 5x5 array layout. The COE of this reference farm (COE_{ref}) was reported to be \$0.023/kWh in the paper on the commercial scale application of UWFLO.¹⁷ Details regarding the features and the compatibility of the “GE 1.5MW xle” turbine for the concerned site is provided in the same paper.¹⁷ Description of the mixed-discrete Particle Swarm Optimization (PSO) algorithm and details of the specified PSO parameters can be found in the paper by Chowdhury et al.¹⁷

Table 2. Specified Wind Farm Properties

Farm Property	Value
Location	Baker, ND (refer Table 1)
Land size (length \times breadth)	2800m \times 1200m
Orientation	North to South lengthwise
Average roughness	0.1m (grassland)
Density of air	1.2 kg/m ³
Allowed turbines-types	GE 1.5 MW and 2.5MW turbines (6 variations)

The overall optimization problem for the two steps can be defined as

$$\begin{aligned}
 \text{Min } f_1(V) &= \frac{COE}{COE_{ref}} \quad \text{and} \quad f_2(V) = \frac{\sigma_{COE}}{COE} \\
 \text{subject to} \\
 g_1(V) &\leq 0 \\
 g_2(V) &\leq 0 \\
 g_3(V) &\leq 0 \\
 V &= \{X_1, X_2, \dots, X_N, Y_1, Y_2, \dots, Y_N, T_1, T_2, \dots, T_N\} \\
 0 &\leq X_i \leq X_{farm} \\
 0 &\leq Y_i \leq Y_{farm} \\
 T_i &\in \{1, 2, \dots, T^{max}\}
 \end{aligned} \tag{19}$$

where the parameters T_i and T^{max} , respectively, represent the type code of the i^{th} turbine, and the total number of turbine-types considered (6 in this paper). In the above problem definition, f_1 and f_2 represent the objective functions minimized in Steps 1 and 2, respectively. Each Step thus represents a typical single

objective optimization problem. The inequality constraint g_1 represents the minimum clearance required between any two turbines, and is given by

$$g_1(V) = \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \max((D_i + D_j + \Delta_{min} - d_{ij}), 0), \quad \text{where} \quad (20)$$

$$d_{ij} = \sqrt{\Delta x_{ij}^2 + \Delta y_{ij}^2}$$

In Eq. 20, D_i represents the rotor-diameter of Turbine- i , and Δ_{min} is the minimum clearance required between the outer edge of the rotors of the two turbines. In this paper, the value of the minimum spacing between turbines (Δ_{min}) is set at 10% of the mean rotor diameter (of commercially available turbines) to counter the effects of dynamic loading on turbines. The parameters X_{farm} and Y_{farm} in Eq. 19 represent the extent of the rectangular wind farm in the X and Y directions, respectively. To ensure the placement of the wind turbines within the fixed size wind farm, the X_i and Y_i bounds are reformulated into an inequality constraint, $g_2(V) \leq 0$. The constraint g_2 is expressed by

$$g_2(V) = \frac{1}{2N} \left(\frac{1}{X_{farm}} \sum_{i=1}^N \max(-X_i, X_i - X_{farm}, 0) + \frac{1}{Y_{farm}} \sum_{i=1}^N \max(-Y_i, Y_i - Y_{farm}, 0) \right)$$

The constraint g_3 is the COE constraint that is applied only in Step-2. This constraint is defined as

$$g_3(V) = COE - 1.05 \times COE_{min} \quad (21)$$

where COE_{min} is the minimum COE obtained through optimization in Step-1.

B. Results and Discussion

For the optimization Steps 1 and 2, we allow 100,000 evaluations (each) of the power generation model. The convergence histories of the two optimizations are shown in Figs. 6(a) and 6(b). The minimum COE obtained in Step-1 is 0.0185/ kWh . The same turbine, GE 2.5MW xl 100m/100m, was selected as the optimal type in Steps 1 and 2. The optimized farm layouts obtained in Steps 1 and 2 are shown in Figs. 7(a) and 7(b), respectively. Details of the farm performances in the two steps are summarized in Table 3.

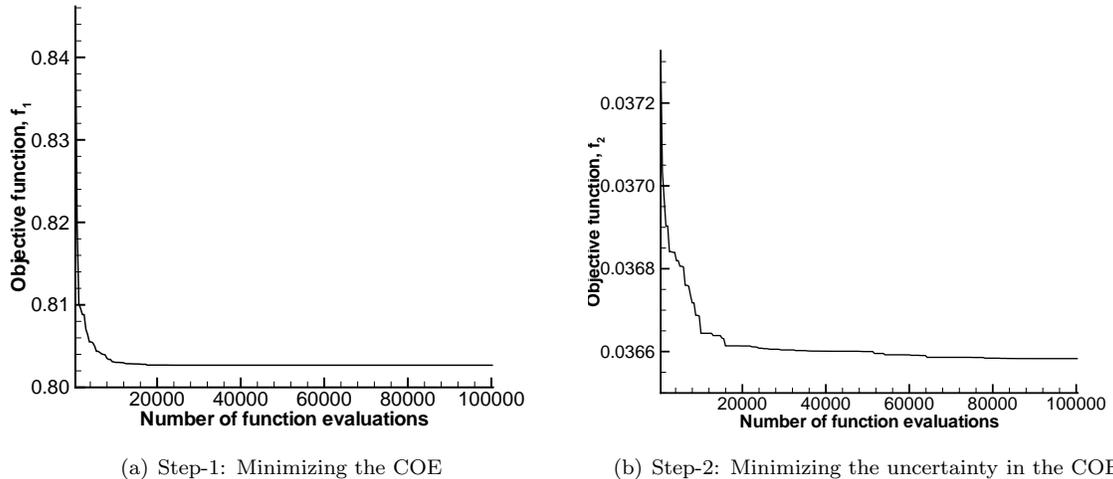


Figure 6. Convergence history of the the mixed-discrete PSO

We observe from Figs. 6(a) and 6(b) that, the *minimization of the COE* converges faster than the *minimization of the uncertainty in the COE*. In the latter case, the UWFLO method seeks to simultaneously (i) accomplish a COE less than 105% of the minimum COE obtained in Step-1, and (ii) minimize the overall

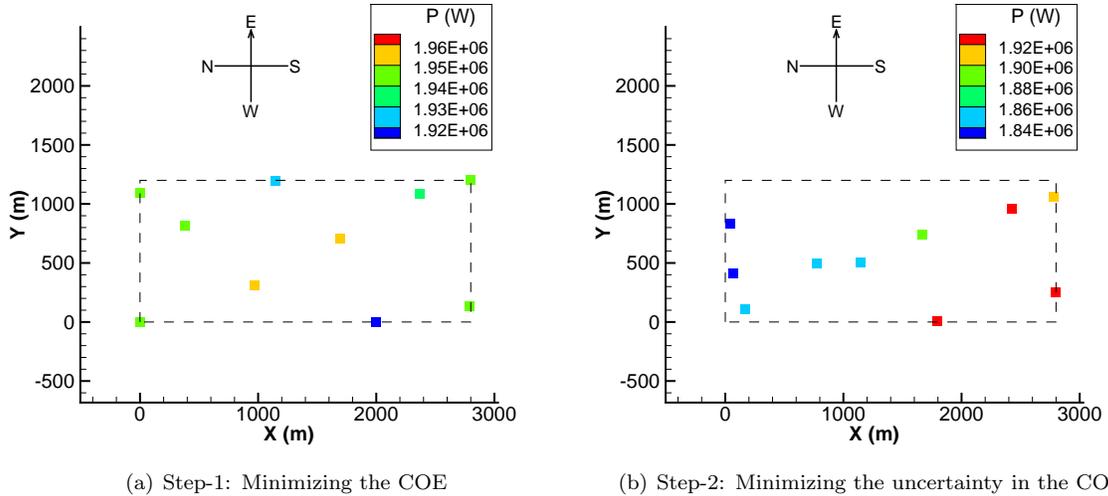


Figure 7. Optimized wind farm layouts

Table 3. Attributes of the optimized wind farms

Parameter	Step-1	Step-2
Overall Farm Efficiency	0.776	0.752
COE (\$/kWh)	0.0185	0.0191
Uncertainty in the COE ($f_2 \times 100$)	3.7%	3.6%

uncertainty in the COE. Intuitively, the *minimization of the uncertainty in the COE* should seek to reduce the *sensitivity of the “annual energy production” to the “relatively more uncertain wind conditions”* - The reduction of wake losses through layout optimization is thus expected to be biased towards wind speeds and wind directions that present relatively lower uncertainties. On the other hand, the minimization of the COE seeks to increase the overall power generation from the entire range of wind conditions. Expectedly, robust wind farm optimization presents conflicting objectives.

Table 3 shows that a better farm performance (higher energy production and lower COE) is accomplished in Step-1 (compared to Step-2). The reduction of the *uncertainties in overall farm performance* (in Step-2) is thus accompanied by a compromise in the farm performance. This observation illustrates the multi-objective nature of the robust wind farm design problem. Future research in “optimal wind farm design addressing uncertainties” should explore a typical multiobjective optimization scenario (as opposed to the 2-step method). The distinct natures of the two optimization steps (as discussed above) is also illustrated by the optimized layouts shown in Figs. 7(a) and 7(b). We observe that the minimization of the COE has produced a layout that is well-spread in all directions, thereby reducing the overall wake effects. On the other hand, the minimization of the uncertainty in the COE has produced a layout that is relatively more well-spread in the North-South direction. A comprehensive investigation of the sensitivity of the optimized layouts to the uncertainties in wind conditions should provide further insights into the crucial aspects of robust wind farm design.

V. Conclusion

The net power generation in a wind farm is highly uncertain. The efficient planning of an optimal wind farm design thereby demands an appropriate consideration of the uncertain parameters. The ill-predictability of wind conditions is one of the primary sources of uncertainty in a wind energy project. This paper presents a new methodology to (i) characterize the *uncertainties in the predicted long term wind distribution*, and (ii) model the propagation of these uncertainties into the overall farm performance. The probability (frequency

of occurrence) of a particular wind condition is itself treated as a stochastic parameter, and represented by a lognormal distribution. The *uncertainty in the annual energy production* (given by its variance) is then represented as a function of the *uncertainties in the predicted yearly probabilities of sample wind conditions* (given by the variances of the corresponding lognormal distributions). This generalized approach can be readily applied to a wide range of wind distributions (parametric and non-parametric distributions; univariate, bivariate and multivariate distributions).

The incorporation of this uncertainty propagation model into the Unrestricted Wind Farm Layout Optimization (UWFLO) framework lays the foundation of a *robust wind farm optimization* strategy. This wind farm optimization framework is used to design a 25MW wind farm (in N. Dakota) that is both high performing and reliable. The minimization of the COE and the minimization of the uncertainty in the COE produced characteristically different farms layouts. Expectedly, we also observed that maximizing the farm performance (COE minimization) and maximizing the reliability of the farm performance (minimization of the uncertainty in the COE) are conflicting objectives. Typical multiobjective optimization should provide more helpful insights into the nature of the trade-offs between these two objectives.

VI. Acknowledgements

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