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DEVELOPING A FLEXIBLE PLATFORM FOR OPTIMAL ENGINEERING DESIGN OF COMMERCIAL WIND FARMS

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ABSTRACT

In this paper, we develop a flexible design platform to account for the influences of key factors in optimal planning of commercial scale wind farms. The Unrestricted Wind Farm Layout Optimization (UWFLO) methodology, which avoids limiting assumptions regarding the farm layout and the selection of turbines, is used to develop this design platform. This paper presents critical advancements to the UWFLO methodology to allow the synergistic consideration of (i) the farm layout, (ii) the types of commercial turbines to be installed, and (iii) the expected annual distribution of wind conditions at a particular site. We use a recently developed Kernel Density Estimation (KDE) based method to characterize the multivariate distribution of wind speed and wind direction. Optimization is performed using an advanced mixed discrete Particle Swarm Optimization algorithm. We also implement a high fidelity wind farm cost model that is developed using a Radial Basis Function (RBF) based response surface. The new optimal farm planning platform is applied to design a 25-turbine wind farm at a North Dakota site. We found that the optimal layout is significantly sensitive to the annual variation in wind conditions. Allowing the turbine-types to be selected during optimization was observed to improve the annual energy production by 49% compared to layout optimization alone.

Keywords: Energy, Mixed-discrete optimization, Particle Swarm, Turbine, Wind farm

INTRODUCTION

Wind Energy

In recent years, growing climate change concerns and unstable fossil fuel prices have increased the focus on sustainable energy resources, such as wind and solar energy. The practical viability of energy production is generally governed by such factors as (i) the potential for *large scale energy production*, (ii) predictability of the power to be supplied to the grid, and (iii) the expected *return on investment*. These factors have been restraining the exploitation of the full potential of wind energy. The 2009 worldwide nameplate capacity of wind powered generators was only approximately 2% of the worldwide electricity consumption [1]. For wind to play a major role in the future

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energy market, we need steady improvement in the wind power generation technology, which can be realized in part through optimal planning of wind farms.

Wind farm planning generally includes (but is not limited to) critical decision-making, regarding

1. the layout of the turbines in the wind farm,
2. the number of wind turbines to be installed, and
3. the types of wind turbines to be installed.

Commercially available wind turbines types can be uniquely defined in terms of their (i) rated power, (ii) rated speed, (iii) rotor-diameter, and (iv) hub height. The objectives of *optimal wind farm planning* are to (i) minimize the Cost of Energy (COE), expressed in $/kW.h, and (ii) maximize the net energy production. Successful accomplishment of these objectives demands a robust and flexible wind farm optimization platform that allows appropriate consideration of the different critical factors.

In the authors' opinion, the estimated energy production from a wind farm is a guiding factor in the planning of a wind energy project. The power extracted through turbines in a wind farm is a variable quantity, which is a function of a series of parameters; the local distribution of wind speed and direction is the most important one among the parameters that cannot be regulated (through design or operation). In the literature [2, 3], the long term wind speed variation is often represented using a Weibull distribution (or variations of the same). A general overview of wind farm optimization is provided in the next section.

Wind Farm Optimization

Wind farms generally consist of multiple wind turbines located in a particular arrangement over a substantial stretch of land (onshore) or water body (offshore). It has been shown that the total power extracted by a wind farm is significantly less than the simple product of the power extracted by a standalone turbine and the number of identical turbines (N) in the farm [4]. This deficiency can be attributed to the loss in the availability of energy due to wake effects - i.e. the shading effect of a wind turbine on other wind turbines downstream from it [5]. Energy deficit due to mutual shading effects is determined using wake models that give a measure of both the growth of the wake, and the velocity deficit in the wake with distance downstream from the wind turbine. The Park wake model, originally developed by N. O. Jensen [6] and later by Katic et al. [7], is one of the most popular analytical wake models used in wind farm modeling. The modified Park wake model and the Eddy Viscosity wake model are other standard wake models. The reduction in the wind farm efficiency (loss in the effective energy available), due to this mutual shading, depends primarily on the geometric arrangement of wind turbines in a farm.

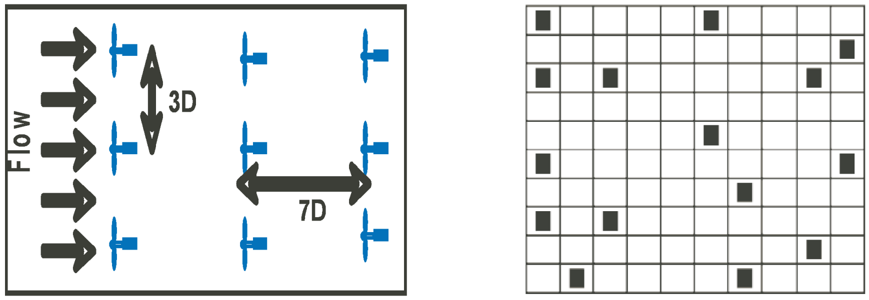
To address this energy deficiency, two popular class of approaches have been reported in wind farm layout modeling: (i) models that assume an array like (row-column) farm layout [4, 8] , and (ii) models that divide the wind farm into a discrete grid in order to search for the optimum grid locations of turbines [5, 9–11]. Figure 1 illustrates these two class of approaches. These approaches are often not readily applicable to the broad commercial scenario that requires synergistic consideration of the arrangement and the selection of turbines, as shown by Chowdhury et al. [12, 13]. A limited set of choices of wind turbines (turbine types) are available in the commercial market. The selection of an optimal combination of turbines from this set yields a mixed-discrete optimization problem. Commercial wind farms often comprise of a large number of turbines (upto a hundred or more); for a wind farm with N turbines, the optimization problem is characterized by atleast $2N$ design variables. Together with the likely multimodal nature of the power generation model [12], this characteristic leads to an appreciably challenging optimization problem. Robust optimization methodologies are necessary to address this challenge of designing large scale commercial wind farms.

The Unrestricted Wind Farm Layout Optimization (UWFLO) methodology, introduced by Chowdhury et al. [12], avoids the limiting assumptions presented by other methods, regarding the layout pattern and the selection of turbines. In the UWFLO method, the turbine location coordinates are treated as continuous variables, which provides infinite possibilities of arrangement of the turbines; in addition, turbines with differing rotor-diameters are allowed, in order to favorably modify the flow pattern within the farm (for maximum power generation). The UWFLO method was successfully applied to an experimental scale wind farm. However, the original UWFLO method considers only a unidirectional and fixed speed incoming wind. Explicit consideration of the variation of wind velocity, along with other commercial factors (described later in this paper) are necessary to extend the applicability of the UWFLO method to full scale commercial wind farms.

In this paper, we develop a methodology that can address the broad scope of wind farm optimization. This paper advances the original UWFLO methodology by

1. including the estimated distribution of the wind speed and direction at a particular location,
2. modifying the power generation model to allow turbines with different hub heights and performance characteristics,
3. evaluating the cost of the wind farm using an accurate *response surface-based wind farm cost* model (RS-WFC), and most importantly
4. implementing a newly developed mixed-discrete Particle Swarm Optimization (PSO) algorithm [13, 14].

To the best of the authors' knowledge, such a robust yet flexible *optimal wind farm design* strategy is unique in the literature. The advanced UWFLO method is summarized in Section . Sec-



(a) Array Layout Approach

(b) Grid-based Layout Approach

FIGURE 1. Existing approaches in wind farm optimization: (a) Array layout approach, and (b) Grid-based approach

tion presents the problem definition for farm optimization, and a case study to investigate the applicability of the wind farm design methodology developed in this paper.

UNRESTRICTED WIND FARM LAYOUT OPTIMIZATION (UWFLO) METHOD

In the UWFLO model, the growth of the wake behind a turbine is determined using the wake growth model proposed by Frandsen et al. [15]. The corresponding energy deficit behind a turbine is determined using the velocity deficit model presented by Katic et al. [7]; this velocity deficit model is widely adopted in wind farm modeling [10, 11, 16]. In a wind farm, the velocity of the wind approaching a turbine can be affected by the wake of multiple turbines upstream from it. Crespo et al. [17] provides a review of different methods that account for the merging of wakes (wake superposition), in calculating the wake velocity deficits. UWFLO implements the wake superposition model developed by Katic et al. [7]. In the UWFLO power generation model, we also account for the possibility of a turbine being ‘partially’ in the wake of another turbine (located upwind). The wind farm model developed in UWFLO has been successfully validated by Chowdhury et al. [12, 18], against recently published experimental data [19].

The net power generated by the wind farm, for a given wind speed and direction, is evaluated by the sum of the power generated by the individual turbines. The farm dimensions and the minimum distance required between any two turbines are treated as system constraints during optimization. In commercial wind farm planning, other factors such as (i) the topography and the terrain, (ii) the grid connection, (iii) the load bearing capacity of the soil, and (iv) the road layout in a farm [11], might further restrict the arrangement of turbines. The consideration of grid connection is particularly important in off-shore wind farms, owing to the associated costs. It is helpful to note that the generalized optimization framework proposed in this paper would still be applicable when factors (ii), (iii), and (iv) are considered. In that case, there will be additional constraints in the optimization problem, which should be formulated from appropriate characterization of these three factors. However, in order to account for the local variability of the topography, more advanced wake models are necessary. Discussion of these factors are however, not within the scope of this paper. Particle Swarm Optimization [20] is applied to optimize the farm layout with the objective of maximizing the total energy production.

The UWFLO method allows for the use of non-identical turbines in a wind farm. Chowdhury et al. [13] illustrated that a combination of wind turbines with differing rotor-diameters can significantly increase the power generation without any unfavorable effects on the cost. In this case, a mixed-discrete version of the PSO algorithm [13] was used to account for the selection of non-identical turbines. Owing to the wake effects and the variation in wind conditions, the overall wind energy available over the year might vary from turbine to turbine. At the same time, different types of turbines are suitable for differing incoming wind speeds. Hence, we explore the use of multiple turbine-types in a wind farm, which can be optimally located to enhance the overall energy production. However, using multiple turbine-types can also prove to be commercially more expensive due to the likely associated increase in purchase, installation, and maintenance costs. *This paper specifically focuses on the increase in the farm energy production, which can be accomplished by using* ***optimally placed multiple turbine-types***. As more detailed cost data becomes available from turbine manufacturers, the associated increase in cost when using multiple turbine will be explored to determine the most attractive “farm performance vs. cost” trade-offs.

In the subsequent subsections, we discuss the key components of the original UWFLO method (briefly), and the advanced features developed in this paper. These advanced features provide flexibility to the UWFLO method, and extends its applicability to the designing of full scale commercial wind farms.

Modeling the Net Power Generation

The power generated by a wind farm is an intricate function of the configuration and location of the individual wind turbines. The flow pattern inside a wind farm is complex, primarily due to the wake effects and the highly turbulent flow. Hence, the velocity of the wind approaching a turbine and the corresponding power generated are determined separately for each turbine. Assuming neutral conditions (negligible thermal effects), the mean velocity in the surface layer (for heights less than 100m) is commonly represented by the log profile [21]. For a known recorded wind speed U_m at a height z_m, the log profile can be expressed as

$$\frac{U}{U_m} = \frac{\ln \frac{z}{z_0}}{\ln \frac{z_m}{z_0}} \tag{1}$$

where z_0 is the average roughness length (terrain dependent) in the farm region, and U is the wind speed at a height z. In this paper, we use a uniform incoming flow equivalent to "the logarithmic velocity profile (in Eq. 1), integrated and averaged over the rotor area".

The layout modeling process in the original UWFLO method, for a wind farm comprised of N turbines, is concisely represented using an influence matrix (M). This matrix is defined as

$$\begin{aligned} M_{ij} &= \begin{cases} +1 & \text{if Turbine}-i \text{ influences Turbine}-j \\ -1 & \text{if Turbine}-j \text{ influences Turbine}-i \\ 0 & \text{if there is no mutual influence} \end{cases} \\ &\forall i,j = 1,2,\ldots,N; \quad i \neq j \end{aligned} \tag{2}$$

In this paper we allow turbines with differing hub-heights; accordingly, we have modified the turbine influence criterion which is stated as follows. Turbine-j is in the influence of the wake created by Turbine-i if and only if

$$\begin{aligned} &\Delta x_{ij} < 0 \quad \text{and} \quad \sqrt{(\Delta y_{ij})^2 + (\Delta H_{ij})^2} - \frac{D_j}{2} < \frac{D_{wake,ij}}{2}, \quad \text{where} \\ &\Delta x_{ij} = x_i - x_j, \;\; \Delta y_{ij} = y_i - y_j, \;\; \Delta H_{ij} = H_i - H_j \\ &\forall i,j = 1,2,\ldots,N; \quad i \neq j \end{aligned} \tag{3}$$

In Eq. 3, D_j and H_j are, respectively, the rotor-diameter and the hub-height of Turbine-j; $D_{wake,ij}$ represents the diameter of the wake produced by Turbine-i, and approaching Turbine-j; x_i and y_i represent the coordinates of turbine-i measured "along" and "perpendicular to" the streamwise direction, respectively.

The power generated by turbine-j (P_j), for an incoming wind speed U_j, is given by

$$P_j = k_g k_b C_p \left(\frac{1}{2} \rho \pi \frac{{D_j}^2}{4} {U_j}^3 \right) \tag{4}$$

where C_p, k_b, and k_g are the power coefficient, the mechanical efficiency, and the electrical efficiency of the turbine, respectively; and ρ represents the density of air. The power generated by each turbine is determined using approximated power curves. The power curves are represented by normalized polynomial functions that are determined (function fitting) using the *cut-in*, the *cut-out*, and the *rated* wind speeds. The normalized curves can be scaled according to the rated power of the concerned turbine-type.

The net power generated, P_{farm}, is given by

$$P_{farm} = \sum_{j=1}^{N} P_j \tag{5}$$

where P_j represents the power generated by Turbine-j. Accordingly, the farm efficiency [4] can be expressed as

$$\eta_{farm} = \frac{P_{farm}}{N P_{0j}} \tag{6}$$

where P_{0j} is the power that Turbine-j would generate if operating as a standalone entity, for the given uniform incoming wind speed.

Annual Variation in the Power Generated by the Wind Farm

The power generated by an individual turbine is a cubic function of the approaching wind speed (as seen from Eq. 4). On the other hand, the wind direction, together with the farm layout, are two major factors that regulate the overall flow pattern (wake patterns) inside the wind farm. The determination (or prediction) of the annual energy generation from a wind farm should thereby account for the variation in wind speed and direction. To this end, we can apply the following two-step procedure:

1. Estimate the annual distribution of the wind speed and direction (as a probability distribution function).
2. Integrate the power generation model (for a given wind speed/direction) over the entire annual wind distribution.

One of the most widely-used model for characterizing the wind speed is the 2-parameter Weibull distribution [2,22]. Other models used to characterize wind speed include 1-parameter Rayleigh distribution, 3-parameter generalized Gamma distribution, 2-parameter Lognormal distribution, 3-parameter Beta distribution, bimodal Weibull model, 2-parameter inverse Gaussian distribution, singly truncated normal Weibull mixture distribution, and maximum entropy probability density function [22,23]. However, the direction of wind is also a crucial factor in the design of optimum wind farm configurations. Hence, a multivariate probability distribution of the wind speed and wind direction would be particularly useful for wind farm layout modeling.

Majority of the existing wind distribution models make limiting assumptions regarding the dimensionality and the modality of the variation in wind conditions. In this paper, we use a newly developed Multivariate and Multimodal Wind Distributions (MMWD) model [24] that avoid such limiting assumptions. This model is developed using multivariate kernel density estimation (KDE) [25] that is also known as the Parzen-Rosenblatt window method [26, 27]. KDE is a non-parametric method of estimating the probability density function of random variables.

The wind data used in this paper is obtained from the North Dakota Agricultural Weather Network (NDAWN) [28]. We use the daily averaged data for wind speed and direction, measured at the Baker station between the years 2000 and 2009. Figure 2 shows the Baker station, and further details are provided in Table 1. It is helpful to note that the wind speed data is recorded at a height of 3m; the log-profile from Eq. 1 is used to determine the wind speed at the pertinent heights. The estimated annual distribution of wind speed and direction obtained by the MMWD method is illustrated by a Windrose diagram in Fig. 3. In the Windrose diagram, each of the sixteen sectors represent the respective probability of wind blowing from that direction.

TABLE 1. Details of the NDAWN station at Baker, ND [28]

Parameter	Value
Location	Baker, ND
Period of record	01/01/2000 to 12/31/2009
Latitude	48.167°
Longitude	−99.648°
Elevation	512m
Measurement height	3m

The total annual energy produced by a wind farm (in kWh), E_{farm}, at a particular location can be expressed as

$$E_{farm} = 8760 \int_{0^\circ}^{360^\circ} \int_{0}^{U^{max}} P_{farm}(U,\theta)\, p(U,\theta)\, dU d\theta \qquad (7)$$

where, U^{max} is the maximum possible wind speed at that location, and $P_{farm}(U,\theta)$ represents the power generated by the farm for a wind speed U and a wind direction θ. In Eq. 7, $p(U,\theta)$ represents probability of occurrence of wind conditions defined by speed U and direction θ. The power generated by the entire wind farm is a complex function of the incoming wind properties, the arrangement of turbines, and the turbine features. Hence, a numerical integration approach [29] is suitable for estimating the annual energy production as given by Eq. 7.

FIGURE 2. Baker station setup [28]

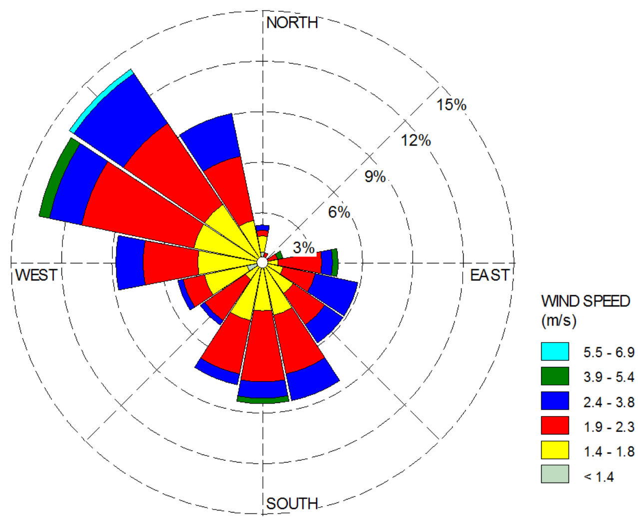


FIGURE 3. Windrose diagram for Baker station, ND (Years 2000-2009)

In this paper, we integrate using the Monte Carlo method that is implemented through the Sobol's quasirandom sequence generator. The approximated total annual energy produced by the wind farm is expressed as

$$E_{farm} = 8760 \times \sum_{i=1}^{n_p} P_{farm}\left(U^i,\theta^i\right) p\left(U^i,\theta^i\right) \Delta U \Delta\theta, \quad \text{where} \quad \Delta U\Delta\theta = U_{max} \times 360^\circ / n_p \qquad (8)$$

and where n_p is the number of sample points used; the parameters U^i and θ^i, respectively, represent the speed and direction of the incoming wind for the i^{th} sample point. Hence, the annual energy is readily determined by the summation of the estimated power generation (P_{farm}) over a set of randomly distributed n_p

wind velocities.

The Monte Carlo integration technique is expected to be more accurate and readily implementable compared to repeated one dimensional integration techniques. For a multivariate scenario, the Monte Carlo integration is appreciably sensitive to the sample size used. However, higher sample sizes, which provide more accurate quantification of the annual energy production, increases the computational expense significantly. Since the annual energy production is determined using a summation over the estimated power generation for each wind sample, the computational expense of optimization is directly proportional to the sample size. From a set of numerical experiments, we found that a sample size of 100 provides a reasonable trade-off between accuracy and computational expense; this sample size is used in the case studies presented in this paper.

Wind Farm Cost Model

Numerous techniques have been developed to evaluate the cost (installation, operation and maintenance) of both onshore and offshore wind farms in the last twenty years. Notable examples include: Short-cut model [30], cost analysis model for the Greek market [31], OWECOP-Prob cost model [32], JEDI-wind cost model [33] and the Opti-OWECS cost model [34]. In this paper, we develop and implement a *response surface-based wind farm cost* (RS-WFC) model that is founded on the principles presented by Zhang et al. [35]. Such a cost model has two major advantages - (i) the cost is represented by a continuous analytical function that can be easily used as a criterion function in optimization (irrespective of the search strategy) (ii) the estimated cost function is highly adaptive to the training data provided, and hence locally accurate. In this paper, the estimated annual cost of the farm is represented as a function of the number and the rated power of turbines in the farm. The annual farm cost is expressed in *dollars per kW installed* (*\$/kW*).

Radial Basis Functions (RBF) [36] are used to develop the RS-WFC model in this paper . These functions are evaluated using data provided by the Wind and Hydropower Technologies program (US Department of Energy) [33]. For a farm with N number of turbines, each with rated power P_r, the cost in *\$/kW installed* can be represented as

$$Cost\,(P_r, N) = \sum_{i=1}^{m_p} \sigma_i \sqrt{\left(P_r - P_r^i\right)^2 + \left(N - N^i\right)^2 + c^2} \tag{9}$$

where P_r^i and N^i represent the i^{th} training data on the turbine rated power and the number of installed turbines, respectively; m_p represents the number of training points used to develop the response surface; and c is a prescribed constant. The unknown coefficients (σ_i) are evaluated using the pseudoinverse technique. A wind farm comprising of multiple types of turbines requires modification of the cost function, represented by Eq. 9. To this end, we approximate the total annual cost in dollars (*\$*) by a more generic expression, which is given by

$$Cost_{farm} = \sum_{k=1}^{n_t} Cost\left(P_r^k, N^k\right) \times P_r^k \times N^k \tag{10}$$

where n_t denotes the number of different turbines types used in the wind farm; the parameters P_r^k and N^k represent the rated power and the number, of turbines of type-k. In this case, the total number of turbines (N) in the farm is equal to $\sum_{k=1}^{n_t} N^k$. Subsequently, the COE (in \$/kWh) can be estimated as

$$COE = \frac{Cost_{farm}}{8760 \times P_{farm}} \tag{11}$$

where P_{farm} is the net power generated by the farm (as given by Eq. 5), expressed in KW.

The cost of a wind farm is however a complex function that depends on several other economic and environmental factors as well. The objective of the 2-dimensional cost function developed in this paper is to specifically explore the benefits of using multiple turbine types of a wind farm.

APPLICATION OF THE NEW UWFLO METHOD

Case Study: Designing Commercial Scale Wind Farms

In this paper, we explore and compare two different scenarios in the optimal design of a commercial wind farm, where the number of turbines and the farm size are assumed to be fixed. These two scenarios are:

Case 1 Optimize the layout of a wind farm, comprised of a defined turbine-type (suitable for the region studied).

Case 2 Simultaneously optimize the farm layout and the turbine-type of each turbine used, thereby allowing a combination of different turbine types.

Cases 1and 2 present $2N$ and $3N$ design variables, respectively. The cost constraint, g_3 is implemented only in Case 2, in order to restrict the cost of the optimized farm to that of the optimized farm in Case 1.

The “GE 1.5MW xle” turbine [37] is chosen as the specified turbine-type in Case 1, and as the reference turbine-type in Case 2. The features of this turbine is provided in Table 2. This turbine is reported to be suitable for IEC Wind Class III-b, and has an average velocity specification of 8.0m/s. For the NDAWN farm site (Baker, ND), the average velocity at the hub-height (of 80m for “GE 1.5MW xle”) is 8.89m/s and 8.92m/s from the estimated and recorded wind distributions, respectively. Although, the “GE 1.5MW xle” turbine is expected to perform well for this site, it may not be the best choice for the given (recorded/estimated) wind conditions at this site. However, the optimization framework is not sensitive to this choice, since using a particular specified turbine type is a case sub-optimal to the scenario where turbine-types are allowed to vary.

TABLE 2. Features of the "GE 1.5MW xle" Turbine [37]

Turbine Feature	Value
Rated Power (P_{r0})	1.5MW
Rated wind speed (U_{r0})	11.5m/s
Cut-in wind speed (U_{in0})	3.5m/s
Cut-out wind speed (U_{out0})	20.0m/s
Rotor-diameter (D_0)	82.5m
Hub-height (H_0)	80.0m

The case studies are performed for a wind farm site at the North Dakota location (refer Section). We consider a fixed-size (land) rectangular wind farm that is comprised of twenty-five turbines. The specified wind farm properties are given in Table 3. The farm is oriented such that the positive X-direction of the layout co-ordinate system points towards the South. The rectangular farm size/orientation corresponds to a 5x5 array configuration, with a row-wise spacing of seven times the turbine rotor-diameter and a column-wise spacing of three times the turbine rotor-diameter.

TABLE 3. Specified Wind Farm Properties

Farm Property	Value
Location	Baker, ND (refer Table 1)
Land size (length x breadth)	$(4\times 7D_0)\times(4\times 3D_0)$
Orientation	North to South lengthwise
Average roughness	0.1m (grassland)
Density of air	1.2 kg/m^3

Optimization Problem Definition

The objective of wind farm optimization (in this paper) is to maximize the annual energy production, for a specified farm size and number of turbines. Using the data (available online) from major turbine manufacturers catering to the US onshore market, we assigned an integer number code T^j to each unique turbine-type-j. A turbine-type is defined by a unique combination of rated-power, rotor-diameter, hub-height, and performance characteristics. A list of 66 turbine-types (with rated-power 0.6 to 3.6 MW) is prepared and coded from the following turbine manufacturers: GE, Vestas, Gamesa, Siemens, Mitsibishi, and Suzlon.

The selection of turbine-type(s) is expected to affect the cost of the farm. Therefore, the optimization problem is formulated such that the farm cost can be restricted to a reference cost. This reference cost should correspond to a wind farm comprised of identical turbines of a particular type (reference type). The "GE 1.5MW xle" turbine is chosen as the reference turbine, since the specifications of this turbine are compatible with the wind class (wind power density) of the location being considered.

The overall optimization problem can be defined as

$$
\begin{aligned}
&\text{Max} \;\; f(V) = \frac{\sum_{i=1}^{n_p} P_{farm}\left(U^i,\theta^i\right) p\left(U^i,\theta^i\right)\Delta U \Delta\theta}{NP_{r0}}\\
&\text{subject to}\\
&\quad g_1(V)\le 0\\
&\quad g_2(V)\le 0\\
&\quad g_3(V)\le 0\\
&\quad V=\{X_1,X_2,\ldots\ldots,X_N,Y_1,Y_2,\ldots\ldots,Y_N,T_1,T_2,\ldots\ldots,T_N\}\\
&\quad 0\le X_i\le X_{farm}\\
&\quad 0\le Y_i\le Y_{farm}\\
&\quad T_i\in\{1,2,\ldots,T^{max}\}
\end{aligned}
\tag{12}
$$

where P_{r0} is the rated-power of the reference turbine that is used for normalizing the energy production objective; P_{farm} is given by Eq. 5; the parameters T_i and T^{max}, respectively, represent the type code of the i^{th} turbine, and the total number of turbine-types considered. It is helpful to note that the constant terms in the annual energy production (as previously given in Eq. 8) have been neglected in the optimization objective definition in Eq. 12. The inequality constraint g_1 represents the minimum clearance required between any two turbines, and is given by

$$
\begin{aligned}
&g_1(V)=\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\max\left(\left(D_i+D_j+\triangle_{\min}-d_{ij}\right),0\right), \quad \text{where}\\
&d_{ij}=\sqrt{\Delta x_{ij}{}^2+\Delta y_{ij}{}^2}
\end{aligned}
\tag{13}
$$

In Eq. 13, D_i represents the rotor-diameter of Turbine-i, and $\triangle_{min}$ is the minimum clearance required between the outer edge of the rotors of the two turbines. In this paper, the value of $\triangle_{min}$ is set at zero, to allow maximum flexibility in turbine spacing. In practice, a higher value of $\triangle_{min}$ might be necessary due to factors such as dynamic loading on the turbines. The parameters X_{farm} and Y_{farm} in Eq. 12 represent the extent of the rectangular wind farm in the X and Y directions, respectively. To ensure the placement of the wind turbines within the fixed size wind farm, the X_i and Y_i bounds are reformulated into an inequality constraint, $g_2(V) \leq 0$. The constraint g_2 is expressed by

$$g_2(V) = \frac{1}{2N}\left(\begin{array}{l} \frac{1}{X_{farm}}\sum_{i=1}^{N}\max\left(-X_i, X_i - X_{farm}, 0\right) \\ +\frac{1}{Y_{farm}}\sum_{i=1}^{N}\max\left(-Y_i, Y_i - Y_{farm}, 0\right) \end{array}\right)$$

In order to restrict the cost of a feasible wind farm design to the reference cost, the constraint g_3 is applied. This constraint is defined as

$$g_3(V) = COE - COE_{Case1} \tag{14}$$

where COE is given by Eq. 11, and COE_{Case1} represents the estimated COE for the optimized farm design accomplished in Case 1.

Mixed-Discrete Particle Swarm Optimization (PSO)

PSO is one of the most well known stochastic optimization algorithms, initially coined by an Electrical Engineer (Russel Eberhart) and a Social Psychologist (James Kennedy) in 1995 [20]. Several improved variations of the algorithm have later appeared in the literature and been used in popular commercial optimization packages. In this paper, we use a mixed-discrete PSO (MDPSO) algorithm developed by Chowdhury et al [13, 38]. This algorithm has been successfully tested on a wide range of *mixed-discrete constrained optimization* test problems [38]. Prominent additional features of this mixed-discrete PSO algorithm include:

1. an ability to deal with both discrete and continuous design variables, and
2. an explicit diversity preservation capability to prevent premature stagnation of particles.

These features are uniquely helpful in addressing the wind farm optimization problem that involves a large number of discrete and continuous variables, and is likely to be multimodal in nature. Further mathematical details regarding the mixed-discrete PSO can be found in the paper by Chowdhury et al. [38].

The prescribed parameters in the mixed-discrete PSO specified for each case is summarized in Table 4. The coefficients α, β_l, β_g, and γ_0 in Table 4, respectively regulate the inertia, the personal behavior, the social behavior, and the diversity preserving behavior of the particles (swarm members). Further description of these parameters and their influence on the dynamics of swarm motion can be found in the paper by Chowdhury et al. [38].

Case Study Results

The optimization converged in both the cases. The convergence histories are shown in Fig. 4. In this figure, the objective on the Y-axis is the normalized farm power that is determined by Eq. 12. Further crucial details of the optimized wind farms in the two cases are provided in Table 5. The reference wind farm in Table 5 is comprised of a 5x5 array layout, with a 7D x 3D (row-wise x column-wise) turbine spacing. The normalized farm power, the $P_{farm}/P_{farm,r}$ ratio, and the COE for the reference wind farm was estimated to be 0.597, 0.597, and \$0.024, respectively. From Fig. 4, we observe that a significantly higher

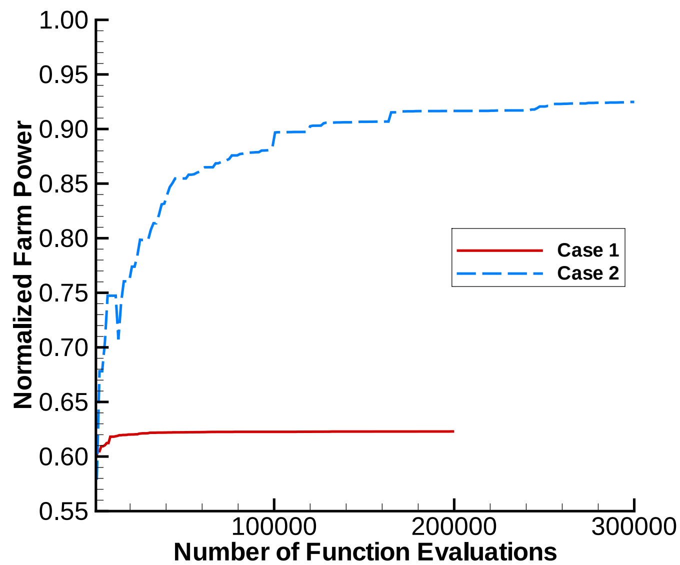


FIGURE 4. Convergence history for all the two cases

improvement in the annual energy production is accomplished in Case 2. As hypothesized by the authors, simultaneous optimization of the layout and the turbine-type selection (Case 2) provided better farm performance compared to that provided by layout optimization alone. This observation thus illustrates, *it is critically important to select turbines in coherence with optimal farm layout planning*.

Table 5 shows that the COE for the optimized farms in Cases 1 and 2 are comparable. Progressively higher rated turbines were selected during optimization in Case 2, which is one of the factors that helped in the remarkable increase in farm power. The preference for turbines with higher rated power can be in part attributed to the inherent assumptions in the cost model - rotor diameters, hub-heights, and turbine performance characteristics are not considered explicitly in the cost model, primar-

TABLE 4. User-defined constants in PSO

Parameter	Case 1	Case 2
α	0.5	0.5
β_g	1.4	1.4
β_l	1.4	1.4
γ_0	20	10
Population Size	$20 \times N = 500$	$20 \times 3N = 750$
Allowed Number of Function Calls	200,000	200,000

TABLE 5. Attributes of the Optimized Wind Farms from the Case Studies

Parameter	Case 1	Case 2	Reference farm
Normalized Annual Energy Production (f)	0.623	0.933	0.597
Overall Power generated/Power installed ($f \times P_{farm,r0}/P_{farm,r}$)	0.623	0.635	0.597
Cost of Energy (COE)	0.023	0.023	0.024

ily owing to lack of pertinent data. The *ratio of the generated farm power to the installed power of the farm* or the overall efficiency of the farm is not sensitive to the assumptions in the cost model. The overall efficiency of the farm thus provides a reliable measure of the performance of the optimized farm. As seen from Table 5, the generated power to the installed power ratio ($f \times P_{farm,r0}/P_{farm,r}$) obtained in Cases 2 is better than that in Case 1. This ratio in the optimized farms in all the two cases is observed to be higher by 4.4 to 6.4%, when compared to the reference wind farm.

In case 2, the optimized farm is comprised of a combination of eight Vestas 1.8MW-90m, four Vestas 1.8MW-100m, two Gamesa 2.0MW-90m, one Mitsubishi 2.4MW-95m, four Mitsubishi 2.4MW-102m, one GE 2.5MW-100m, and five Vestas 3.0MW-112m wind turbines. In a commercial scenario, a wind farm might not comprise of turbines from different manufacturers. However, in this paper, we allowed this wide range of turbine options to investigate the unrestrained potential of using combination of turbine-type. The optimized farm layout for Case 1 is shown in Fig. 5, where the dashed line represents the farm boundary. The squares that represent the turbine locations are colored according to the overall power generated by the corresponding turbines. The layouts present a significantly scattered arrangement of turbines, which are also a noticeable deviation from any array pattern. Interestingly, the turbines located on the Eastern edge of the farm are observed to generate relatively less power (as seen from Fig. 5). This phenomenon agrees with the Windrose diagram (Fig. 3), which shows that over the year the expected wind from the East is minimal. We also observe that the differences in power generation between the turbines that generate the maximum and the minimum power annually is only 7.5% for Cases 1. This observation can be attributed to the appreciable reduction in wake effects achieved through layout optimization.

Figures 6(a), 6(b), 6(c), and 6(d) represent the optimized layout obtained in Case 2, where the turbine locations are colored according to the (i) overall power generation, (ii) rated power, (iii) rotor-diameter, and (iv) hub-height of the individual turbines, respectively. The overall power generation of a turbine-j (considering the annual wind variation) is given by $\sum_{i=1}^{n_p} P_j\left(U^i,\theta^i\right) p\left(U^i,\theta^i\right) \Delta U \Delta\theta$. It is most interesting to note that (from Fig. 6(b) and 6(c)): the UWFLO framework has taken advantage of the turbine selection allowance to place *higher rated-power turbines that come with larger rotor-diameters* on the Eastern edge of the wind farm. These turbines with larger rotor diameters are more suitable for lower wind speeds (lower wind classes), as experienced by the Eastern face of the farm. Their location on the Eastern edge of the farm also ensures minimal effects of the likely larger wakes created by these turbines. Nevertheless, it is challenging to provide conclusive insights into the coupled influence of the rated power, the rotor-diameter, and the hub-height of the selected turbine types on the overall farm layout. To this end, the application of advanced wake models and further analysis of the wind-flow pattern within the farm should be helpful.

Conclusion

This paper developed a platform for the *optimal design of commercial-scale wind farms*, which is an evolution from the Unrestricted Wind Farm Layout Optimization (UWFLO) methodology. This platform allows appropriate consideration of

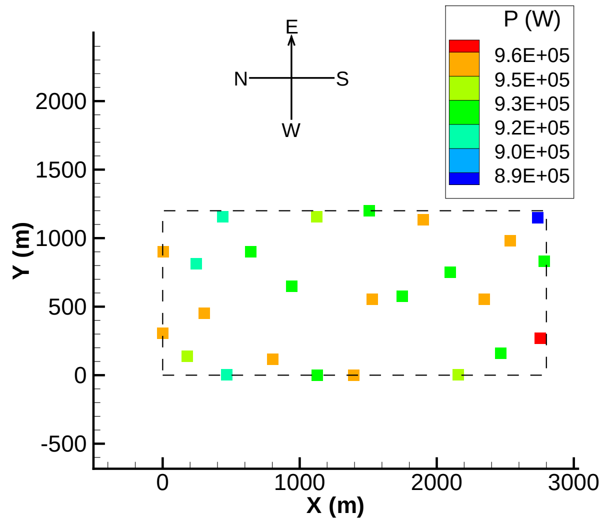


FIGURE 5. Layout of the optimized wind farm for Case 1

the local variation in wind conditions. The power generation model is modified to account for wind turbines with different hub-heights and performance characteristics. This modification provides the uniquely helpful feature - *the explicit consideration of a combination of differing turbine-types* in commercial wind farm design. In order to account for the economic viability of optimizing the selection of turbines, we developed and applied a response surface based wind farm cost model.

The *commercial-scale wind farm design* model presents a mixed-discrete optimization problem that is solved (in this paper) using a newly developed mixed-discrete Particle Swarm Optimization algorithm. Such a robust optimization strategy successfully addresses the high-dimensionality and the likely multi-modality of this design problem. A Case Study, for a site in North Dakota, is performed to design a wind farm with 25 turbines and a specified farm size. Two different farm design scenarios, based on the turbine selection approach, were explored. Appreciable improvement in the farm performance was observed in both the scenarios, when compared to a farm with a 5x5 array configuration. Interestingly, simultaneous optimization of (i) the farm layout and (ii) the choice of turbine-types (Case 2) provided an overall farm efficiency that was 2% higher than that accomplished by layout optimization alone (Case 1). This observation illustrates that the concept of using *a combination of optimally selected turbine-types as opposed to one specified turbine-type for the farm* can prove to be a helpful innovation in developing high-performance wind farms. An increase in the cost of the farm is however likely, when using multiple turbine types; the exploration of commercially viable trade-offs between the increase in costs and the increase in farm energy production is therefore an important future research direction.

Future work should address the selection of other global factors such as the orientation and shape of the farm and the installed capacity, within the scope of optimal wind farm planning. Together with a more comprehensive cost model, such a *optimal wind farm planning* platform should provide a strong foundation for future research in wind energy.

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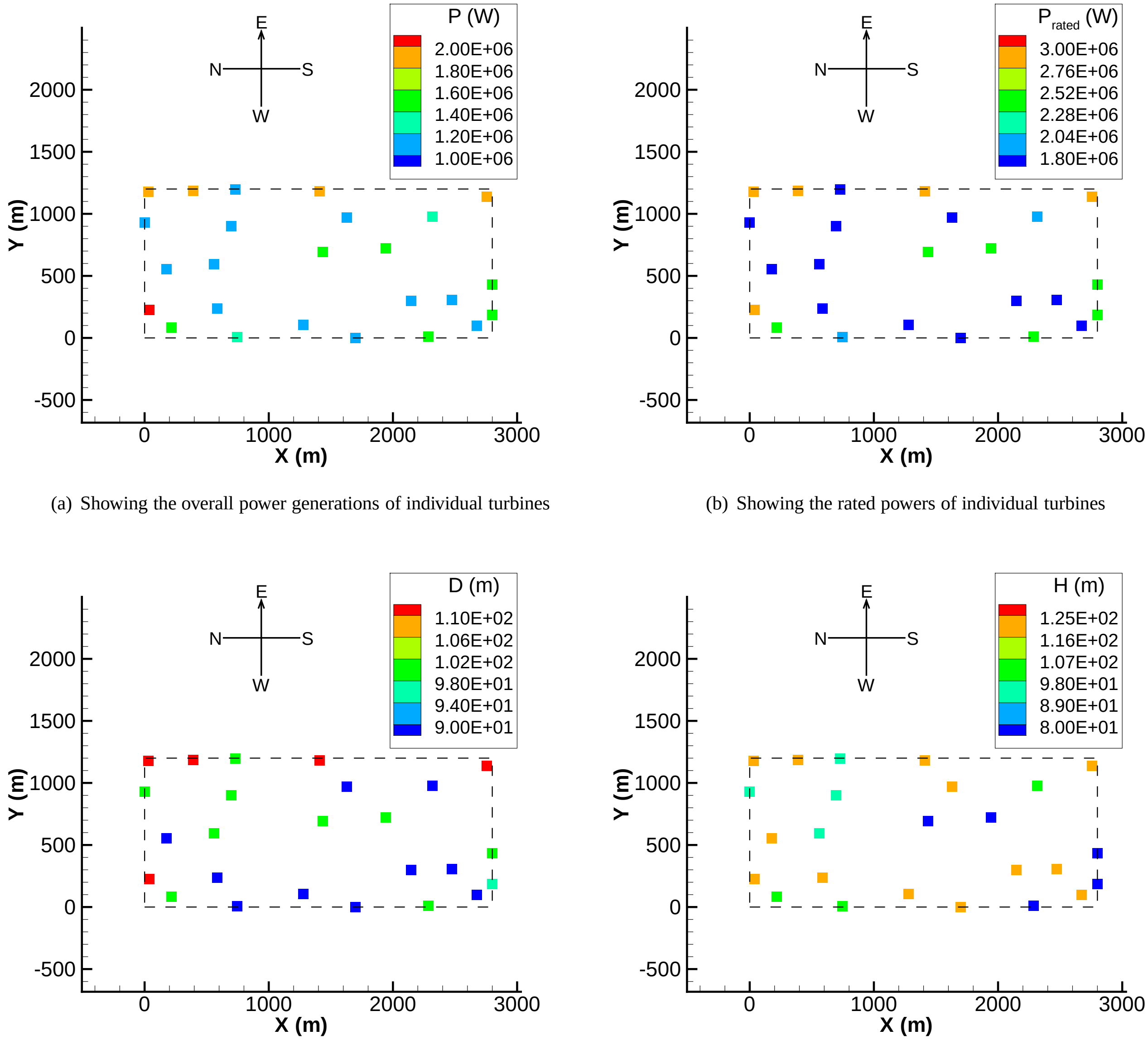

(a) Showing the overall power generations of individual turbines

(b) Showing the rated powers of individual turbines

(c) Showing the rotor-diameters of individual turbines

(d) Showing the hub-heights of individual turbines

FIGURE 6. Layout of the optimized wind farm for Case 2 (optimal combination of turbine-types)

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