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# CHARACTERIZING THE INFLUENCE OF LAND AREA AND NAMEPLATE CAPACITY ON THE OPTIMAL WIND FARM PERFORMANCE

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## ABSTRACT

*The development of utility-scale wind farms that can produce energy at a cost comparable to that of conventional energy resources presents significant challenges to today's wind energy industry. The consideration of the combined impact of key design and environmental factors on the performance of a wind farm is a crucial part of the solution to this challenge. The state of the art in optimal wind project planning includes wind farm layout design and more recently turbine selection. The scope of farm layout optimization and the predicted wind project performance however depends on several other critical site-scale factors, which are often not explicitly accounted for in the wind farm planning literature. These factors include: (i) the land area per MW installed (LAMI), and (ii) the nameplate capacity (in MW) of the farm. In this paper, we develop a framework to quantify and analyze the roles of these crucial design factors in optimal wind farm planning. A set of sample values of LAMI and installed farm capacities is first defined. For each sample farm definition, simultaneous optimization of the farm layout and turbine selection is performed to maximize the farm capacity factor (CF). To this end, we apply the recently developed Unrestricted Wind Farm Layout Optimization (UWFLO) method. The CF of the optimized farm is then represented as a function of the nameplate capacity and the LAMI, using response surface methodologies. The variation of the optimized CF with these site-scale factors is investigated for a representative wind site in North Dakota. It was found that, a desirable CF value corresponds to a cutoff "LAMI vs nameplate capacity" curve – the identification of this cutoff curve is critical to the development of an economically viable wind energy project.*

**Keywords**: Capacity factor (CF), land area per MW installed, nameplate capacity, Response surface, Wind farm layout optimization, turbine selection

## INTRODUCTION

### Engineering Design of Wind Farms

Among alternative energy resources, wind energy has received appreciable attention in research and a major impetus

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in the global energy market. However, the 2010 worldwide nameplate capacity of wind powered generators was only approximately 2.5% of the worldwide electricity consumption [1]. We believe that the development of *reliable and geographically adaptive protocols* for optimal engineering design of wind farms can play a major role in advancing the growth of wind energy. The engineering design of wind farms should generally include careful decision-making, regarding

1. the *nameplate capacity* of the wind farm
2. the *land area per kW (or per MW) installed*
3. the shape of the farm-land
4. the type(s) of turbine to be installed
5. the arrangement of turbines in the wind farm (farm layout),

The overall objectives of *optimal wind farm planning* are to (i) maximize the capacity factor (CF) and/or (ii) minimize the Cost of Energy (COE) of the farm. COE can be defined as the cost of producing energy, and is generally expressed in \$/kWh. The *nameplate capacity* of the farm is the power that the farm would produce if each turbine is operating at a wind speed greater than or equal to its rated speed (and less than its cut-off speed). However, mother nature is not so generous - wind speed varies and generally averages less than the rated speed of standard commercial turbines, even at the best sites (e.g. class 7 sites). At the same time, there are wake losses within the farm. Owing to this two major reasons, farms seldom operate at their nameplate capacity. The capacity factor (CF) of a farm represents the ratio of (i) the actual energy produced (or expected production) and (ii) the energy that could have been produced, if always operating at nameplate capacity. In the case of commercial wind farms, loss of energy is also caused by other factors such as turbine downtime for O&M, extreme weather conditions, snow accumulation, and curtailments (e.g. noise/environmental curtailment and grid curtailment). These project-specific factors are not considered in this paper.

The power extracted through turbines in a wind farm is a variable quantity, which is a function of a series of environmental factors. The variation in wind conditions, namely wind speed, wind direction, and air density, is among the most important of these factors. Generally, probability distribution models are used to represent the long term variations of a wind resource. Optimal choice of each of the engineering factors (listed above) depends on the predicted wind resource variations, and these dependencies are generally not independent of each other. Therefore, for a given wind resource, it is the *overall farm configuration* that should be carefully designed such that a most desirable performance can be extracted from the wind project. On the contrary, in conventional wind farm development, the major engineering factors are often addressed individually; overlooking the interaction among these factors can introduce substantial inaccuracies in the projected farm performance and farm economics.

For example, turbine choices are often proposed based on the estimated class of wind and wind loads for the concerned site. Such information pertains to each turbine operating as a standalone entity; their performance as part of an array of turbines, which is highly sensitive to the mutual wind-shading effects, remains largely unaccounted for. Another instance of neglecting the factor-interdependence occurs during farm layout planning; designing the arrangement of turbines in the farm is conventionally performed as an independent post-process to planning the nameplate capacity and the land area of the farm. Intuitively, it is possible to accomplish similar farm *Capacity Factors* (CF) from optimal farm layouts obtained under different combinations of nameplate capacity and farm-land area (and shape); the costs of the farms corresponding to these different combinations can however be significantly different. Investigation into such scenarios is rare in the literature. This paper seeks to lay the foundation of a more comprehensive wind farm design framework, which avoids the conventional assumptions regarding the interactions between various decision-factors involved. The key inputs (variables and conditions) and the outputs of a desirable comprehensive wind farm design framework are illustrated in Fig. 1. The input and output factors that are not explicitly addressed in this paper appear in grey font in the figure. A general overview of optimal wind farm design is provided in the next section.

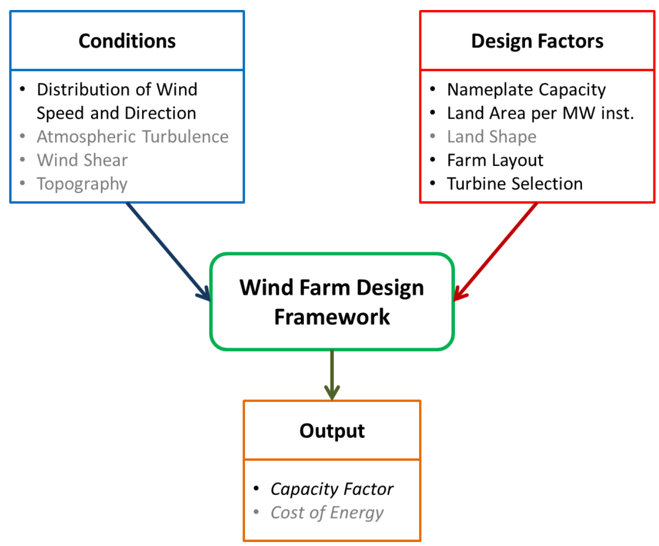



**FIGURE 1**. COMPREHENSIVE WIND FARM DESIGN FRAMEWORK

### Farm Layout Optimization

It has been shown that the total power extracted by a wind farm is significantly less than the simple product of the power extracted by a standalone turbine and the number of identical turbines (N) in the farm [2]. This deficiency can be attributed to the loss in the availability of energy due to wake effects (or wind shading effects) [3]. Energy deficit due to mutual shading effects can be determined using analytical or numerical wake models. These wake models provide a measure of the growth of the wake and the velocity deficit in the wake as functions of the distance downstream from a given wind turbine. The Park wake model, originally developed by N. O. Jensen [4] and later by Katic et al. [5], is one of the most popular analytical wake models used in wind farm modeling. The modified Park wake model and the Eddy Viscosity wake model are other standard wake models.

The reduction in the wind farm efficiency (loss in the effective energy available), due to this mutual shading, depends primarily on the geometric arrangement of wind turbines in a farm. To address this energy deficiency, two popular classes of approaches have been reported in wind farm layout modeling: (i) models that assume an array like (row-column) farm layout [2,6], and (ii) models that divide the wind farm into a discrete grid in order to search for the optimum grid locations of turbines [3,7–9]. These approaches are often not readily applicable to the broad commercial scenario, which may require synergistic consideration of the arrangement and the selection of turbines, as shown by Chowdhury et al. [10,11]. A limited set of choices of wind turbines are available in the commercial market. The selection of an optimal combination of turbines from this set yields a mixed-discrete optimization problem. At the same time, the dimensionality of layout optimization is generally proportional to the number of turbines in the farm. Together with the likely multimodal nature of the power generation model [10], these characteristics (of farm optimization) lead to a challenging optimization problem. Powerful optimization methodologies are therefore necessary to address the challenge of designing large scale commercial wind farms.

In this paper, we implement the Unrestricted Wind Farm Layout Optimization (UWFLO) method, introduced by Chowdhury et al. [12]. The UWFLO method avoids the limiting assumptions presented by other methods regarding the layout pattern and the selection of turbines. The original UWFLO method was successfully validated against data from a wind tunnel experiment on a scaled down wind farm. This method was further advanced by Chowdhury et al. [13] to include (i) the long term variation in wind conditions and (ii) the use of commercial wind turbines. In the UWFLO method, the turbine location coordinates are treated as continuous variables. The UWFLO method also allows a combination of differing turbine-types (that are selected during optimization), in order to favorably modify the flow pattern within the farm for maximum power generation. Since the focus of this paper is the influence of the site-scale factors, we use a single uniform turbine type (optimally selected) for the case study. Further description of the UWFLO method is provided in Section . The next section (Section ) discusses the influence of site-scale factors (such as land area and installed capacity) on farm performance, and develops a model to characterize that influence. Section  ends with a case study that illustrates the model using the example of a commercial-scale wind farm.

## MODELING IMPORTANT SITE-SCALE FACTORS

### Significance of Site-Scale Factors

In commercial projects, the farm layout (and/or turbine selection) is generally decided after the following have been planned:

1. What would be the shape and area of the land (together called land configuration) on which the wind farm will be constructed (boundaries of the project site)?
2. What would be the nameplate capacity of the wind farm (total installed capacity)?

Existing farm layout optimization methods hence treat decision-making regarding the “farm layout and turbine selection” as an independent post process to that regarding the site-scale factors: the *land configuration* and the *nameplate capacity* of the farm. In reality, optimal choice of what turbines to install and where to install them directly depends on the allowed farm configuration and the allowed nameplate capacity. Consequently, the decided *farm land configuration* and *installed capacity* places a ceiling on the optimal performance (e.g. maximum CF) that can be extracted from the concerned project. Moreover, both decision-making - (i) regarding “*farm layout and turbine choices*” and (ii) regarding “*land configuration and nameplate capacity of the farm*” - must carefully consider the distribution of wind speed and direction at the site. **It is therefore important to investigate the relationship between the site-scale factors (*land configuration* and *nameplate capacity*) and the maximum performance that can be obtained from a site through “*farm layout and turbine choice optimization*”.**

Chowdhury et al. [12] investigated the variation of the farm efficiency of an experimental-scale farm with respect to the *installed capacity* (for a fixed-size land area). It was found that the farm efficiency remains same up to a threshold value of the *installed capacity* (for the given farm-land area), and then progressively decreases with increasing *installed capacity*; this observation was attributed to the increasing crowding effect of turbines, leading to increasing wake losses. The same authors also developed and reported a framework that modeled the variations of “farm efficiency and COE” with respect to the “aspect ratio and N-S-E-W orientation” of a rectangular farm-land [14]. The application of this framework to design a 25MW farm in N Dakota showed that farm efficiency and COE varied periodically with changes in farm-land aspect ratio and orientation. Other than these two research publications, *to the best of the authors’ knowledge, a wind farm design protocol that accounts for the site-scale factors within the design process is rare in the literature.* The wind farm design framework developed in this paper accounts for the influence of the site-scale factors within the context of "*farm layout and turbine choice optimization*". The specific objectives of this paper are:

1. To develop a generic modeling technique to represent the relationship between site-scale factors and optimal farm performance;
2. To illustrate the relationship and its significance in wind farm design, using an onshore farm site example.

## Comprehensive Wind Farm Design Framework

Any changes in the *land configuration* and/or the *nameplate capacity* would alter the constraints imposed during the "farm layout and turbine choice optimization" process. Hence, optimization is performed for different sample combinations of *land area per MW installed* (*LAMI*) and *nameplate capacity*. The macro- and the micro-siting of commercial wind farms are generally presented with various other site-specific constraints, such as caused by: (i) the local vegetation coverage, (ii) the construction of roadways, (iii) the local topography, and (iv) the load bearing capacity of the soil. These constraints are not explicitly considered in this paper; the direction of further mathematical development required to incorporate the effects of these constraints is however discussed at the end of this section. The overall framework, which models the relationship between the site-scale factors and the farm performance, includes the following major steps:

1. A set of sample *LAMI* and *nameplate capacities* are generated within specified practical ranges.
2. For each defined sample combination of site-scale factors, the farm layout and turbine selection are simultaneously optimized to maximize the CF. The wind farm is assumed to be circular in shape.
3. A response surface is fitted to represent the maximized CF as a function of the *LAMI* and *nameplate capacity* of the farm.
4. The wind farm that yielded the highest value of the maximized CF, among the defined samples, is identified. For this farm, we determine the smallest polygon and the smallest rectangle that enclose the optimized arrangement of turbines.

## Development of the Response Surface Models

Response surfaces comprise a class of analytical approximation functions that are popularly used to represent complex functional relationships. Such approximation functions are generally necessary for real life design problems where relationship(s) among important parameters are not analytically well defined and/or are expensive to evaluate. The dependence of the maximum CF (that can be obtained through layout optimization) on the site-scale factors is not a straightforward analytical function. Secondly, it is computationally expensive to evaluate the maximum farm for any given *LAMI* and *nameplate capacity* - an entire optimization process is required. Hence, we adopt the response surface methodology to model and investigate the relationship between the maximum CF and the site-scale factors.

The set of $N_S$ sample *LAMI* and *nameplate capacities* is generated using Sobol's quasirandom sequence generator [15]. For a sample *LAMI* and farm *nameplate capacity* given by $A_{MW}$ and $P_{NC}$, respectively, the radius of the circular farm ($R_{farm}$) can be expressed as

$$R_{farm} = \sqrt{\frac{A_{MW} \times P_{NC}}{\pi}} \tag{1}$$

The radius of the farm is used to constrain the placement of turbines during optimization (refer Section ). A circular wind farm-land is assumed to avoid any bias towards incoming wind directions. For example, a rectangular wind farm of particular aspect ratio and N-S-E-W orientation would promote lower wake losses for winds coming from a particular range of directions [14]. A circular farm-land thus helps in maintaining the focus on the generic *land area per MW installed* (*LAMI*), instead of the land shape - the latter factor is highly subjective to the site conditions as mentioned in the previous section.

The number of turbines, $N$, that can be installed in a sample farm is readily given by $N = P_{NC}/P_r$, where $P_r$ is rated capacity of the turbines to be installed. The number of design variables in wind farm optimization depends on the number of turbines to be installed. For a specified farm *nameplate capacity*, if turbines with different rated powers are allowed during optimization, it would lead to candidate farm designs with different numbers of design variables. To promote a convenient illustration of the new framework, we allow turbines of a particular rated capacity to be installed (in this paper). For each specified farm-land radius ($R_{farm}$) and number of turbines (N), the UWFLO method is applied to optimize the *farm layout and turbine selection* - a total of $N_S$ optimizations are thus performed. Using the maximum accomplished CF values from the $N_S$ wind farm optimizations, a response surface is developed to relate the *LAMI* and *nameplate capacity* with the maximum CF.

Quadratic Response Surface Methodology (QRSM), which is a popular surrogate modeling method, is used to represent the CF function in this paper. Typical interpolating surrogates (e.g. Kriging and Radial Basis Functions) are particularly useful when a locally accurate response is required, such as during optimization; whereas, approximating surrogates (e.g. QRSM) can generally provide a better understanding of the overall trend of a function. Secondly, interpolating surrogates involve a significantly larger number of parameters (generally proportional to number of training points) than QRS for low dimensional functions; analytical functions with limited parameters are more helpful in pursuing tractable trend analysis and exploration. The purpose of the CF response surface is: (i) to provide generic insights into how wind farm performance is actually influenced by site-scale factors such as *LAMI* and *nameplate capacity*, and (ii) to provide a standard guidance in decision-making regarding the site-scale factors at the commercial level. Hence, approximating surrogate models such as quadratic response surfaces (QRS) are expected to be a better choice than interpolating surrogates to model the concerned relationship in this paper. A generic quadratic response surface (QRS) for the two dimensional functional relationship (of CF) can be expressed as:

$$CF = c_0 + c_1 A_{MW} + c_2 P_{NC} + c_3 A_{MW} P_{NC} + c_4 A_{MW}^2 + c_5 P_{NC}^2 \quad (2)$$

where $c_0$, $c_1$, $c_2$, $c_3$, $c_4$, and $c_5$ are unknown coefficients that are determined by the least squares approach. The development of this response surface is illustrated using an onshore wind farm case study. The description of the case study and the results accomplished are provided in the next section.

## Case Study: Description and Results

The case study is performed for a wind farm site at the Baker wind station in North Dakota. The corresponding wind data is obtained from the North Dakota Agricultural Weather Network (NDAWN) [16]. We use the daily averaged data for wind speed and direction, measured at the Baker station between the years 2000 and 2009. Details of the Baker station are provided in Table 1. The variation of wind conditions at this site is illustrated by a Windrose diagram in Fig. 2. In the Windrose diagram, each of the sixteen sectors represent the respective probability of wind blowing from that direction. The Multivariate and Multimodal

**TABLE 1**. DETAILS OF THE STATION AT BAKER, ND [16]

| Parameter | Value |
|---|---|
| Location | Baker, ND |
| Period of record | 01/01/2000 to 12/31/2009 |
| Latitude | 48.167° |
| Longitude | −99.648° |
| Elevation | 512m |
| Measurement height | 3m |
| Average roughness | 0.1m (grassland) |
| Average air density | 1.2 kg/m$^3$ |

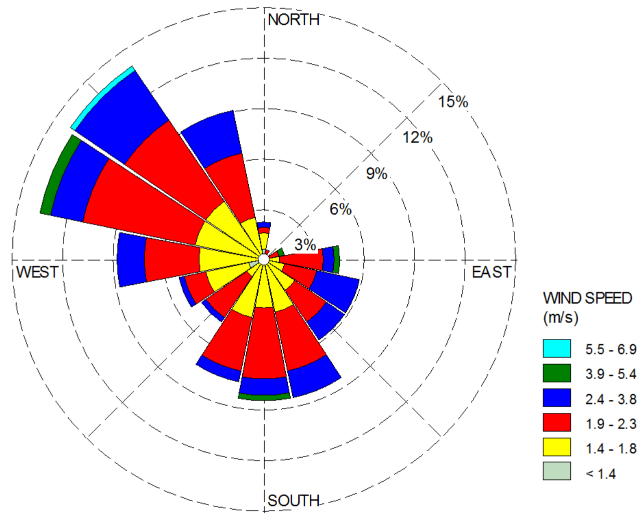



**FIGURE 2**. WINDROSE DIAGRAM FOR BAKER STATION, ND (YEARS 2000-2009)

Wind Distribution (MMWD) model [17] is used to estimate the joint distribution of wind speed and direction at this site. The estimated wind distribution serves as an input to the annual energy production (AEP) model within the UWFLO framework (refer Section ).

A set of 50 sample *LAMI* and *nameplate capacities* are generated within the following specified ranges:

$$10\frac{D^2}{P_r} < A_{MW} < 30\frac{D^2}{P_r}; \quad 10P_r < P_{NC} < 50P_r \qquad (3)$$

where $P_r$ is rated power and $D$ is the average rotor diameter of commercially available 2MW turbines: $P_r = 2$ MW, $D = 83.9$ m. The numerical ranges for *LAMI* and *nameplate capacity* are therefore specified as 35,215 m$^2$/MW - 105,646 m$^2$/MW and 20 MW - 100 MW, respectively. The specified range for *LAMI* is based on industry standards, and that for *nameplate capacity* is representative of a small-medium scale wind farm.

Wind farm optimization is performed on the 50 sample site-scale specifications, and the following response surface is obtained to represent maximum CF as a function of *LAMI* and *nameplate capacity*:

$$CF = \begin{array}{l} 7.036 \times 10^{-1} - 7.068 \times 10^{-7} A_{MW} + 4.600 \times 10^{-3} P_{NC} \\ +3.179 \times 10^{-8} A_{MW} P_{NC} - 8.055 \times 10^{-12} A_{MW}^2 \\ +3.099 \times 10^{-6} P_{NC}^2 \end{array} \qquad (4)$$

The maximum error and the root mean squared error of the QRS fit for CF (shown in Eq. 4) was estimated to be 20.6% and 9.9%, respectively. Considering that the response surface is used for investigation of the general trend in CF variation with respect to *LAMI* and *nameplate capacity*, the level of accuracy obtained for the QRS fit is acceptable. The 3D plot and the contour plot of the CF response surface are shown in Figs. 3(a) and 3(b). It is helpful to note that the range of CF obtained in this case study (40-65%) is higher than that observed in commercial wind farms (30-40%), since project-specific loss factors are not considered in this paper (as discussed in Section ).

It can be observed from the figures that the CF (accomplished through farm optimization) increases when the $A_{MW}/P_{NC}$ ratio increases. This observation can be attributed to the increasing crowding effect of turbines caused by decrease in the $A_{MW}/P_{NC}$ ratio. Even if turbines are allowed the same *land area per MW installed* (*LAMI*), a greater number of turbines (higher *nameplate capacity*) would lead to greater wake losses, leading to lower energy production (hence lower CF). In the case of commercial wind farms, greater land area generally leads to higher development costs. More importantly, the effective land area available at a site (for farm development) is limited by land ownership, environmental impact, and noise and visual impact on local population. In other words, the smaller the farm in terms of land area, the lower is its overall lower impact on its surroundings. At the same time, in order to be economically viable, a wind farm should operate above a *threshold* capacity factor (CF) during its lifetime. It is therefore important to investigate

- whether adequate land area is available at a site to construct a wind farm of a desired capacity, and/or
- what is the minimum land area required to construct an economically viable wind farm of a desired capacity.

The methodology developed in this paper offers a unique framework to perform these important investigations effectively. A contour plot of the response function, like the one shown in Fig. 3(b), provides the "*LAMI vs. nameplate capacity* cutoff curve that corresponds to the threshold CF. For example, lets assume a developer plans to construct a 60 MW wind farm at the concerned site (in Baker, ND), and a threshold CF of 0.6 is necessary to ensure a economically viable power project. In that case, it is observed from the dashed lines in Fig. 3(b) that a minimum land area of approximately 87,000m$^2$ per MW installed is required for the project. A clearer illustration of the sensitivity of the optimized farm CF to the *LAMI* and the *nameplate capacity* is provided by Figs.

A quadratic variation of the optimized farm CF with respect to the *LAMI* is observed from Fig. 4(a). Figure 4(b) shows that the optimized farm CF has more of a linear variation with respect to the *nameplate capacity* of the farm. It is observed that greater the *nameplate capacity*, higher is the rate of CF increase with increasing *LAMI*. It is also observed from Figs. 4(a) and 4(b) that below a certain *nameplate capacity*, there is negligible variation in CF with changing *LAMI*, e.g. 20 MW in this case. This observation corresponds to the scenario where, there are so few turbines that the layout optimization is successful in practically avoiding wake losses for the entire specified range of *LAMI*.

## UNRESTRICTED WIND FARM LAYOUT OPTIMIZATION (UWFLO)

### Overview of UWFLO

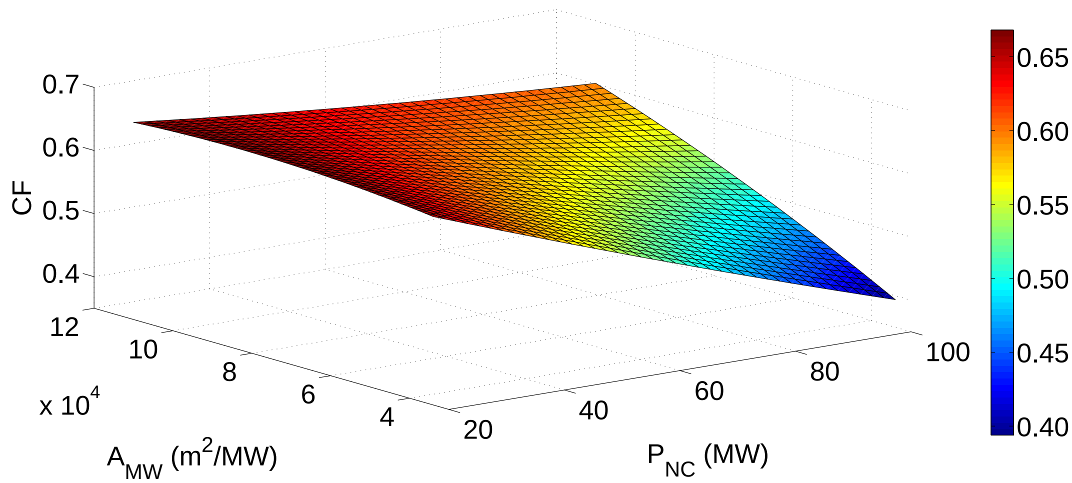
One of the most important component of the UWFLO framework is the power generation model that provides the power generated by the farm for any given incoming wind speed and direction. In the UWFLO power generation model, the growth of the wake behind a turbine is determined using the wake growth model proposed by Frandsen et al. [18]. The corresponding energy deficit behind a turbine is determined using the velocity deficit model presented by Katic et al. [5]; this velocity deficit model has been widely adopted in wind farm modeling [8,9,19]. In a wind farm, the velocity of the wind approaching a turbine can be affected by the wake of multiple turbines upstream from it. Crespo et al. [20] provides a review of methods that account for the merging of wakes (wake superposition) while estimating the wake velocity deficits. In UWFLO, the popular wake superposition model developed by Katic et al. [5] is implemented. This power generation model also accounts for the possibility of a turbine being 'partially' in the wake of another turbine located upwind.

The net power generated by the wind farm, for a given wind speed and direction, is evaluated by the sum of the power generated by the individual turbines. The cost of the farm (in $/kW installed) is estimated using a radial basis function based cost model. The farm dimensions and the minimum distance required between any two turbines are treated as system constraints during optimization. In commercial wind farm planning, other factors such as (i) the grid connection, (ii) the terrain, (iii) the load bearing capacity of the soil, and (iv) the road layout in a farm [9], might further govern the optimal arrangement of turbines. These factors are, however, not within the scope of this paper. A mixed-discrete Particle Swarm Optimization [21,22] is applied to maximize the capacity factor (CF) of the farm. In this section, we briefly discuss the key components of the UWFLO framework.

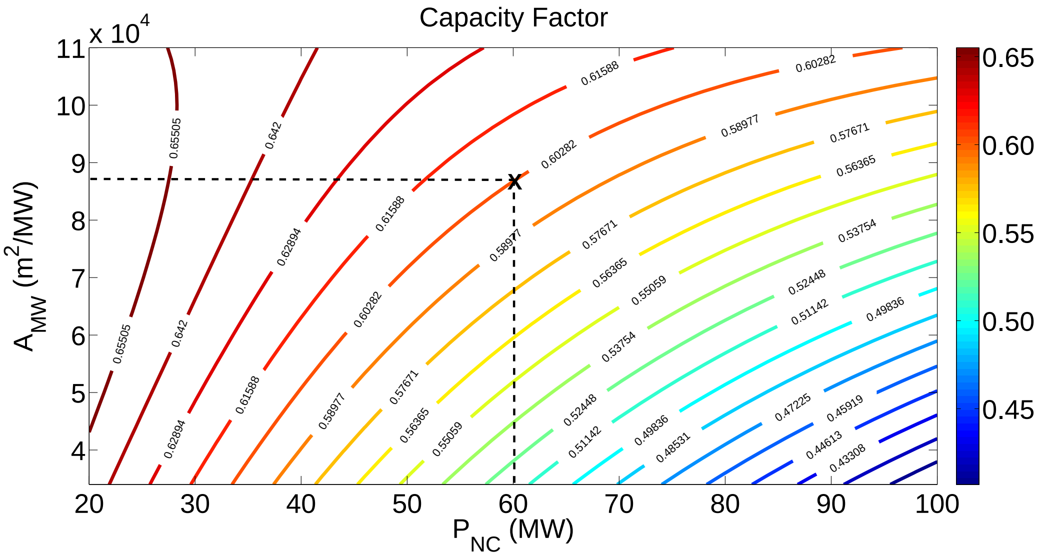
### Annual Energy Production (AEP) Model

The prediction of the annual energy production (AEP) from a wind farm should account for the correlated long term variations in wind speed, wind direction, and air density, which are together termed as "*wind condition*". To this end, the annual distribution of wind conditions is first represented using a suitable probability density function. Subsequently, the AEP is determined by integrating the power generation function over the estimated annual wind distribution.

The AEP of a wind farm in kWh, $E_{farm}$, at a particular loca-




(a) 3D response surface




(b) Contour plot of the response function

**FIGURE 3**. VARIATION OF THE OPTIMIZED FARM CAPACITY FACTOR WITH RESPECT TO *LAMI* AND *NAMEPLATE CAPACITY*

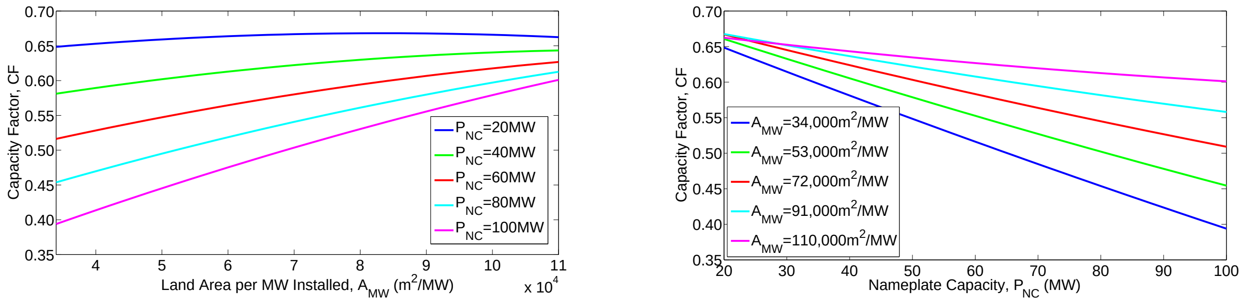
tion can be expressed as

$$E_{farm} = (365 \times 24) \int_{\rho_{min}}^{\rho_{max}} \int_{0^\circ}^{360^\circ} \int_{0}^{U_{max}} P_{farm}(U,\theta,\rho)\; p(U,\theta,\rho)\, dU d\theta d\rho \quad (5)$$

where, $U_{max}$ is the maximum recorded wind speed at that location, and $\rho_{min}$ and $\rho_{max}$ represent the minimum and the maximum air densities at the concerned location, respectively; and $p(U,\theta,\rho)$ represents the probability of occurrence of wind conditions defined by speed $U$, direction $\theta$, and air density $\rho$. In Eq. 5, $P_{farm}(U,\theta,\rho)$ represents the power generated by the farm for a wind speed $U$, a wind direction $\theta$, and an air density $\rho$. The farm power generation, $P_{farm}$, is equal to the summation of the power generated by the individual turbines, which is expressed as

$$P_{farm} = \sum_{j=1}^{N} P_j \quad (6)$$

where $P_j$ represents the power generated by Turbine-$j$, and $N$ represents the number of turbines in the farm. For any given incoming wind speed and direction, the power generated by the individual turbines is determined by the UWFLO power gener-



(a) With respect to *LAMI*, for fixed values of *nameplate capacity*

(b) With respect to *nameplate capacity*, for fixed values of *LAMI*

**FIGURE 4**. VARIATION OF THE CAPACITY FACTOR OF THE OPTIMIZED FARM

ation model. It is helpful to note that the accuracy of the analytical power generation model is significantly sensitive to that of the analytical wake models used. Discussion of the inherent assumptions in the analytical wake model, and detailed description of the power generation model can be found in the papers by Chowdhury et al. [12, 13].

The power generated by the group of turbines in a wind farm is a complex function of the incoming wind attributes, the arrangement of turbines, and the turbine features; it is not a tractable analytical function that can be directly integrated. Hence, a numerical integration approach [23] is suitable for estimating the AEP (defined in Eq. 5). To this end, the Monte Carlo integration method is implemented using the Sobol's quasirandom sequence generator [15]. This class of integration methods is likely to provide greater accuracy for the same number of sample evaluations when compared to the repeated integrations using one-dimensional Quadrature Rule methods. The approximated total annual energy produced by the wind farm is expressed as

$$
\begin{aligned}
&E_{farm} = (365\times 24)\sum_{i=1}^{n_p} P_{farm}\left(U^i,\theta^i,\rho^i\right)\ p\left(U^i,\theta^i,\rho^i\right)\Delta U\Delta\theta\Delta\rho\\
&\text{where}\\
&\Delta U\Delta\theta\Delta\rho = \left(U_{max}\times 360^\circ\times(\rho_{max}-\rho_{min})\right)/n_p
\end{aligned}
\tag{7}
$$

and where $n_p$ is the number of sample points used; the parameters $U^i$, $\theta^i$, and $\rho^i$, respectively, represent the wind speed, the wind direction, and the air density of the incoming wind for the $i^{\text{th}}$ sample point. For the case study in this paper, we consider the air density to be constant. The probability of wind speed and direction, $p\left(U^i,\theta^i\right)$, is estimated by the MMWD model. Based on the AEP given by Eq. 5, the capacity factor ($CF$) of the farm can be expressed as

$$
CF = \frac{AEP}{(365\times 24)\times\sum_{j=1}^{N} P_{rj}}
\tag{8}
$$

where $P_{rj}$ is the rated power of Turbine-$j$.

### Optimization Problem Definition

The objective of wind farm optimization (in this paper) is to maximize the CF, for a specified *land area per MW installed* (*LAMI*) and *nameplate capacity* of the farm. The variables in the optimization problem are the location of each turbine $(X_j,Y_j)$ and the type of turbine to be used - a total of $2N+1$ design variables for a $N$-turbine farm. A turbine-type is defined by a unique combination of rated-power, rotor-diameter, hub-height, and performance characteristics. In this paper, we allow the use of 2MW turbines manufactured by the Vestas and Gamesa - a total of 13 commercially available 2MW turbines. The turbine types are sorted in the order of their rated power, followed by the order of their rotor-diameters and their hub-heights; accordingly, each turbine-type is assigned an integer code between 1 and 13. The overall optimization problem is defined as

$$
\begin{aligned}
&\text{Max}\ \ f(V) = CF\\
&\text{subject to}\\
&\qquad g_1(V)\le 0\\
&\qquad g_2(V)\le 0\\
&\qquad V=\{X_1,X_2,\ldots\ldots,X_N,Y_1,Y_2,\ldots\ldots,Y_N,T\}\\
&\qquad 0\le X_j\le R_{farm}\\
&\qquad 0\le Y_j\le R_{farm}\\
&\qquad T\in\{1,2,\ldots,T^{max}\}
\end{aligned}
\tag{9}
$$

where the parameter $T^{max}$ represents the total number of commercial turbine-types considered; $T^{max}=13$, in this paper.

The inequality constraint $g_1$ represents the minimum clearance required between any two turbines, and is given by

$$
\begin{aligned}
&g_1(V) = \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\max\left(\left(D_i+D_j+\triangle_{\min}-d_{ij}\right),0\right)\\
&\text{where}\\
&d_{ij} = \sqrt{\Delta X_{ij}^{\,2}+\Delta Y_{ij}^{\,2}}
\end{aligned}
\tag{10}
$$

In Eq. 10, $D_i$ and $D_j$ represent the rotor-diameters of Turbine-$i$ and Turbine-$j$, and $\triangle_{min}$ is the minimum clearance required between the outer edge of the rotors of these two turbines. The parameters, $\Delta X_{ij}$ and $\Delta Y_{ij}$, represent distances between the locations of Turbine-$i$ and Turbine-$j$ along the $X$ and $Y$ axes, respectively. In this paper, the value of the minimum spacing between turbines ($\triangle_{min}$) is set at 10% of the mean rotor diameter (of commercially available turbines) to account for the effects of dynamic loading on turbines. To ensure the placement of the wind turbines within the circular wind farm boundaries, the $X_i$ and $Y_i$ bounds are reformulated into the inequality constraint, $g_2(V) \leq 0$. The constraint $g_2$ is given by

$$g_2(V) = \frac{1}{N}\sum_{j=1}^{N}\frac{1}{R_{farm}}\max\left(\sqrt{X_j^2 + Y_j^2 - R_{farm}^2}, 0\right) \tag{11}$$

## Wind Farm Optimization Results

A total number of 100 iterations and a population of $10 \times (2N+1)$ were allowed during optimization. The values of the prescribed parameters in mixed-discrete PSO and further description of the optimization algorithm can be found in the papers by Chowdhury et al. [13, 21]. In a majority of the samples, the Gamesa G90-2.0 turbine was selected during optimization. In fifteen cases, variants of the Gamesa G80-2.0 turbine was selected. Different farm layouts were obtained for the different specified sample values of *LAMI* and *nameplate capacity*. This observation illustrates that the optimal layout pattern is likely to be strongly dependent on the specified/allowed values of the site-scale factors.

The sample farm with a nameplate capacity of 26 MW (i.e 13 turbines) and a land area of 101,237 $m^2$ per MW installed (i.e. an area of approximately $29D^2$ per turbine) yielded the highest capacity factor ($CF = 0.69$), on wind farm optimization. The sample farm with a nameplate capacity of 96 MW (i.e 48 turbines) and a land area of 39,614 $m^2$ per MW installed (i.e. an area of approximately $11D^2$ per turbine) yielded the lowest capacity factor ($CF = 0.39$), on wind farm optimization. Figures 5(a) and 5(b) illustrate the optimized layouts for these two wind farm configurations. It is readily evident from the optimized layouts in Figs. 5(a) and 5(b) that, more crowding of turbines limit the maximum CF. Expectedly, in this case, the optimized wind farms with the highest and the lowest CF values are respectively associated with the highest and the lowest values of the $A_{MW}/P_{NC}$ ratio among the samples.

## CONCLUSION

This paper developed a wind farm design framework that helps to understand how the optimum farm performance is influenced by key site-scale factors, namely the *land area per MW installed* (*LAMI*) and the *nameplate capacity* of the farm. Decision making regarding the farm-scale factors (farm layout and turbine selection) are not independent of the decisions made regarding site-scale factors. The framework presented in this paper initiates important investigation into how these factors are interrelated in their collective influence on the overall energy production capacity of the farm. In the proposed method, a set of sample *LAMI* and *nameplate capacity* values are generated, and the farm layout and turbine selection are optimized for each sample configuration. The maximum capacity factor (CF) thus obtained is represented as a function of the *LAMI* and the *nameplate capacity*, using a quadratic response surface. Contour plots of the CF response function provides important insights into the suitable choice of values for the site-scale factors, required to construct an economically viable wind project.

The proposed framework was applied to designed to design a farm at a site in N Dakota. It was found that the CF of the optimized farm progressively increases with increasing value of the "*LAMI*/*nameplate capacity*" ratio. We also observed that in order to ensure a desirable threshold capacity factor, the "*LAMI-nameplate capacity*" combination should be beyond a particular cutoff curve. The determination of this cutoff curve is important to ensure a profitable wind energy project. Owing to the complexity of the farm performance with respect to the resource variations (local wind distribution), appropriate quantification of the site-scale factors is far from trivial - a comprehensive numerical/computational framework, as presented in this paper, is therefore expected to be more effective than traditional intuitive decision making in this case.

A circular wind farm shape was assumed in this paper to avoid a bias towards wind direction, and focus on the role of *land area per MW installed*. In commercial farm development, the available land at a site can be of an arbitrary shape; at the same time, based on the local wind distribution, particular land shape(s) may provide more flexibility to the optimal placement of turbines. Future work should focus on accounting for different land shapes, and modeling the joint influence of land shape and LAMI on farm performance.

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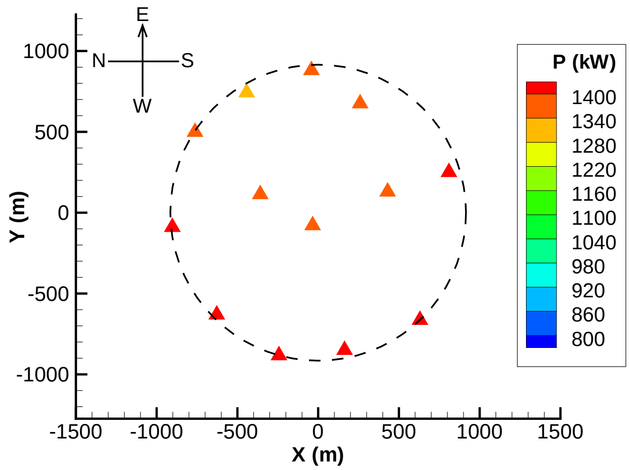
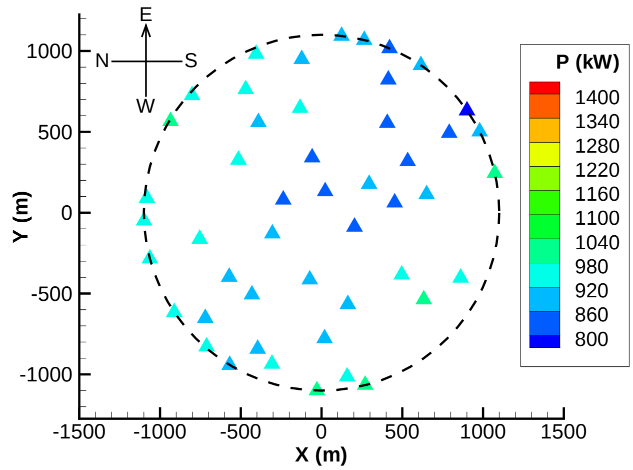



(a) Farm with the highest CF ($P_{NC} = 26$ MW and $A_{MW} = 101,237$ m$^2$/MW)




(b) Farm with the lowest CF ($P_{NC} = 96$ MW and $A_{MW} = 39,614$ m$^2$/MW))

**FIGURE 5**. OPTIMIZED FARM LAYOUTS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