

# Probabilistic Swinging Door Algorithm as Applied to Photovoltaic Power Ramping Event Detection

Anthony Florita, Jie Zhang, Carlo Brancucci  
Martinez-Anido, Bri-Mathias Hodge  
National Renewable Energy Laboratory  
Golden, CO, USA

Mingjian Cui  
Wuhan University, China

**Abstract**—Photovoltaic (PV) power generation experiences power ramping events due to cloud interference. Depending on the extent of PV aggregation and local grid features, such power variability can be constructive or destructive to measures of uncertainty regarding renewable power generation; however, it directly influences contingency planning, production costs, and the overall reliable operation of power systems. For enhanced power system flexibility, and to help mitigate the negative impacts of power ramping, it is desirable to analyze events in a probabilistic fashion so degrees of beliefs concerning system states and forecastability are better captured and uncertainty is explicitly quantified. A probabilistic swinging door algorithm is developed and presented in this paper. It is then applied to a solar dataset of PV power generation. The probabilistic swinging door algorithm builds on results from the original swinging door algorithm, first used for data compression in trend logging, and is described by two uncertain parameters: (i)  $\epsilon$ , the threshold sensitivity to a given ramp, and (ii)  $\sigma$ , the residual of the piecewise linear ramps. These two parameters determine the distribution of ramps and capture the uncertainty in PV power generation.

*Keywords*—solar forecasting; uncertainty; Bayesian

## I. INTRODUCTION

The United States is experiencing rapid growth in renewable power generation, fueled by state renewable portfolio standards and declining renewable prices. Solar power is one of the most promising renewable energy sources due to its low operating costs and resource abundance. However, with photovoltaic (PV) power generation varying on the order of seconds to hours, power system operators have limited control over production yet must accommodate abrupt fluctuations while meeting demand. There are thus three PV characteristics of concern: its variability, forecasting uncertainty, and non-dispatchable nature, all of which impose challenges for the integration of PV power. This research addresses the first two challenges so that the third might be overcome when the technology and communication infrastructure exist.

### A. Review of Solar Characterization Literature

Studies involving solar characterization have examined the variability of irradiance or PV power from various mathematical perspectives. Typically, the raw irradiance data are transformed into clearness indices to account for

the sun's zenith angle and to obtain a weakly stationary, yet stochastic process; PV panels act as a filter to transform this process signal into power [1]. For high-level examinations of data, probability density functions (pdf) or cumulative distribution functions (cdf) have been found to be helpful. For the next finer level of granularity, it has been commonly sought to uncover macroscale relationships among the solar variables according to the application. Finally, time series analysis has allowed more detailed evaluation of observations and short-term forecasting. These various approaches are considered below, in turn.

For distributions investigations, Ibanez et al. [2] surveyed the literature involving the prediction of hourly and daily pdf and cdf of clearness indices and used ten years of data for six cities to show the bi-exponential pdf fit is superior to existing models. Jurado et al. [3] performed similar analysis with clearness indices from Seville, Spain, with irradiance data integrated over five-minute intervals, but found a mixture of normal distributions to be a superior fit to the bimodal nature of the data. Hollands and Huget [4] used classical probability theory to derive expectations for irradiation on inclined surfaces, using clearness index and diffuse fraction as random variables. Olseth and Skartveit [5] also developed an hourly irradiance (analytical) pdf model for arbitrary planes, which accounted for bimodal phenomena and was tuned to training data. Tovar et al. [6] investigated dependencies of one-minute irradiance pdfs as a function of hourly averages, harnessing earlier research [7, 8] where one-minute pdfs were conditioned on optical air mass with considerations for cloud-reflected radiation enhancement; comparable research was also conducted by Varo et al. [9]. Other authors have focused on statistics from the pdf or cdf of interest: Meek [10] estimated the maximum possible daily irradiance, and Gordon and Reddy [11] investigated hourly/daily stationary and sequential (persistence) statistics.

Uncovering relationships among solar variables can be immensely valuable for practical applications. As such, the ubiquitous Angstrom equation, relating sunshine duration and irradiance, has been revisited by Suehrcke [12]; many researchers continually seek such valuable relationships with comparable studies at higher time resolution. Atwater and Ball [13] reviewed numerical solar radiation models from meteorological observations and found severe limitations due to the sparse and inaccurate nature of the

data available circa 1978. More recent research such as Craggs et al. [14] has focused on more localized phenomena, with quality data and focus on optimal time averaging, by monitoring irradiance levels and uncovering patterns that can be applied to design and for predictive purposes. Olseth and Skartveit [15] incorporated cloud observations to yield an hourly model for irradiance with a lag autocorrelation feature. Perez et al. have extended the research, especially at hourly time resolution, through irradiance relationships such as enhanced parameterization of hourly insolation conditions [16] and prediction of the direct component with climatic evaluation [17].

Time series investigations of solar data are inherently complex due to its nonstationary nature and the difficulty of inferring atmospheric states associated with point observations. Amato et al. [18] analyzed 20 years of Italian irradiance data to develop a daily irradiance model, with a Fourier series describing the mean and variance and a Markov process for the stochastic component. Craggs et al. [19] used a Seasonal Autoregressive Moving Average (SARIMA) model as part of an irradiance monitoring campaign to test its short-term prediction performance in Northern latitudes. Glasbey [20] developed a new form of nonlinear autoregressive time series model to summarize irradiance data from Edinburgh, Scotland and allow easy simulation.

The majority of the solar characterization literature has focused on predicting irradiance or power for longer-term performance estimates. As such, intra-hour or even inter-minute variation studies are sparse and this is crucial for quickly responding to changes from cloud cover. Gansler et al. [21] examined one-minute irradiance data for three locations and found that significant differences exist between one minute and hourly data with respect to irradiance distributions, which is attributed to inaccurate estimates of the magnitude and distributions of the diffuse component. Vijayakumar et al. [22] investigated the inaccuracies caused by using hourly rather than one to three minute irradiance data, and summarized differences in terms of regressions and utilizability. Suehrcke and McCormick [23] examined the frequency distribution of instantaneous measurements over a one year measurement period, identifying fractional time curves with distinct steps driven by clear and cloudy states. They also focused on the diffuse component [24], showing substantial differences

from integrated diffuse fractions and leading to the proposal of a new model. Woyte et al. [25] examined the fluctuation in instantaneous clearness index, using wavelets to quantify the scale and location of fluctuations.

### B. Research Objective

This research extends the swinging door algorithm, developed by Bristol [26] for data compression in trend logging, in a probabilistic fashion for uncertainty quantification and short-term PV power forecasting. As witnessed in the literature review, there are a myriad of mathematical techniques for characterizing solar power. The advantage of the swinging door algorithm is that it is computationally simple, allowing straightforward and robust extensions into Monte Carlo based approaches. Furthermore, its applicability for renewable power ramp event detection [27], and dynamic programming optimization [28] thereof, have proven fruitful.

## II. DATA AND METHODS

### A. PV Power Data

The PV power data comes from the National Renewable Energy Laboratory in Golden, CO. The system under investigation is a nameplate 1-MW PV plant, with average power [kW] and integrated energy [kWh] recorded at 15-min intervals. Only the power data are considered here, and the nighttime data has been deleted from the dataset because any measurement noise is not of interest – especially for the concerns of the swinging door algorithm which marginalizes all insignificant fluctuations in power. The data examined herein were recorded from May 21, 2013 to July 21, 2013.

### B. Swinging Door Algorithm

From the domain of data compression, the swinging door algorithm was deemed useful for simplifying trend logging and minimizing storage memory requirements [26]. It requires only one parameter in its definition and is robust to noisy data, allowing trends to be clearly extracted from measured data. It uses a linear piecewise approximation to extract the ramps from the original time series and disregards the noise inherent to the process under investigation. The approximation can be thought of as only recording significant changes in the process signal. These significant linear ramps, in terms of magnitude and duration, determine the trend according to a sequence of local derivatives.

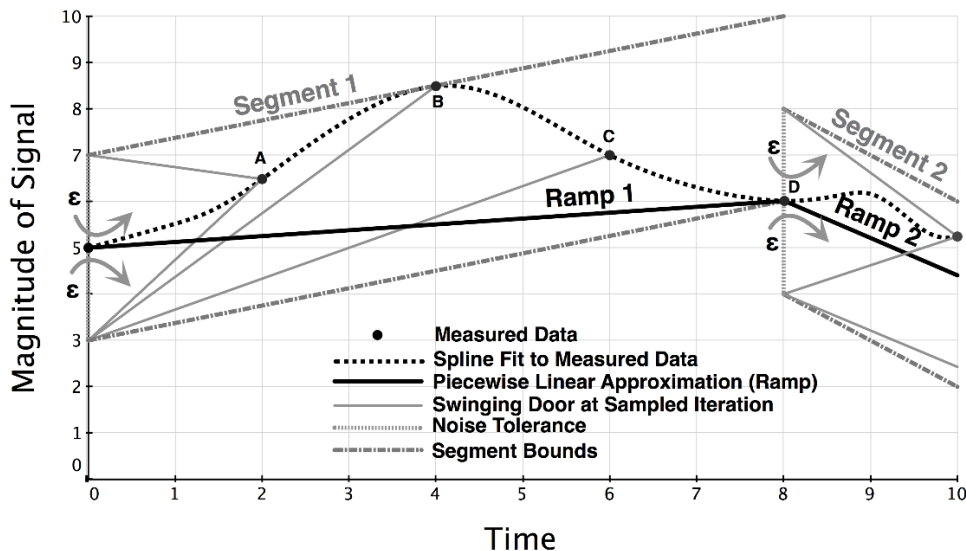


Figure 1. The swinging door algorithm for (power) ramping event detection.

The threshold parameter,  $\epsilon$ , also the width of one “door” in the algorithm as shown in Figure 1 (with arbitrary scale), is the only tunable parameter and directly influences the sensitivity to noise or other insignificant fluctuations in the signal. If the tolerance for  $\epsilon$  is very low (i.e. a small  $\epsilon$  value), the ramp extraction algorithm will identify many small ramps as it basically traces the original signal, violates the threshold, and starts over. If the tolerance is very high (i.e. a large  $\epsilon$  value), the algorithm will identify a few number of large ramps as it is under-constrained and results in a crude approximation of the original signal; a large fluctuation is required for the threshold to be violated and the algorithm to start again.

With reference to Figure 1, the swinging door algorithm is briefly described as follows:

1. The initial point (pivot), or beginning of the new ramp segment iteration of the algorithm, is on the y-axis and the doors of width  $\epsilon$  (threshold) are placed above and below it.
2. A new point ‘A’ is obtained and the doors “swing open” and extend, as indicated, to include the point; i.e., lines are drawn from the doors’ hinges to the new point.
3. A new point ‘B’ is obtained and the doors swing open further; lines are drawn to intersect at ‘B.’
4. A new point ‘C’ is obtained but there has been an inflection in the signal. The doors only swing open to accommodate new points in a ramp segment iteration of the algorithm, so the top door remains in its angle position above ‘C’ and the lower door line is drawn to ‘C.’
5. A new point ‘D’ is obtained and again the top door (line) angle position is not updated, and the lower door line is drawn to point ‘D.’ The lines are now parallel, or will not intersect at a future time, and this starts a new iteration of the algorithm; i.e., the threshold has been exceeded when the line angle from the hinges to their most open position is greater than or equal to parallel.

Changes in ramp sign indicate fluctuation; however, it is not clear what an insignificant or significant fluctuation

may be. This fact is left to the user and the specifics of the application. For instance, the  $\epsilon$  value could be set to 20% of the plant’s maximum capacity, indicating that only major fluctuations are of concern, or it may be set to 1% of the plant’s maximum capacity to indicate that there are sensitivities to only slight changes in power. Figure 2 provides an example of swinging door algorithm as applied to the PV dataset, with the  $\epsilon$  value set to 15% of the plant’s maximum capacity. This two-day time period in June shows a clear sky day followed by a cloudy day. It can be observed that the swinging door algorithm requires six or seven ramps to approximate the clear sky day and approximately triple that number for the cloudy day.

### C. Uncertainty and Solar Power Forecasting

It is known a priori that there is uncertainty in the  $\epsilon$  value that should be used in the swinging door algorithm. If a small  $\epsilon$  value is used the residuals will be small. If a large  $\epsilon$  value is used the residuals will be large. An appropriate balance between the two is only justified by the specific application. It is envisioned that the swinging door algorithm will be used in a network fashion, allowing the communication of power states and short-term forecasts, such that local uncertainties can be considered in global uncertainty computations involving renewable power generation and demand. This allows superior contingency planning, reductions in production costs due to better planning, and a more reliable power system. Enhanced forecastability and planning also allows for more power system flexibility, mitigating the negative impacts of power ramping.

The details of all these topics are left to future research. For now, uncertainty is quantified by recognizing that there exists a balance between the  $\epsilon$  value and the residual. Thus, there are two uncertain parameters: (i)  $\epsilon$ , the threshold sensitivity to a given ramp, and (ii)  $\sigma$ , the residual of the piecewise linear ramps. These two parameters determine the distribution of ramps and capture the uncertainty of the PV power generation. The uncertainty quantification must be adaptable to changes and be able to update beliefs according to new information, and thus a Bayesian perspective was considered. Particularly, a Bayesian parameter estimation approach was applied which is easily adaptable to online applications.

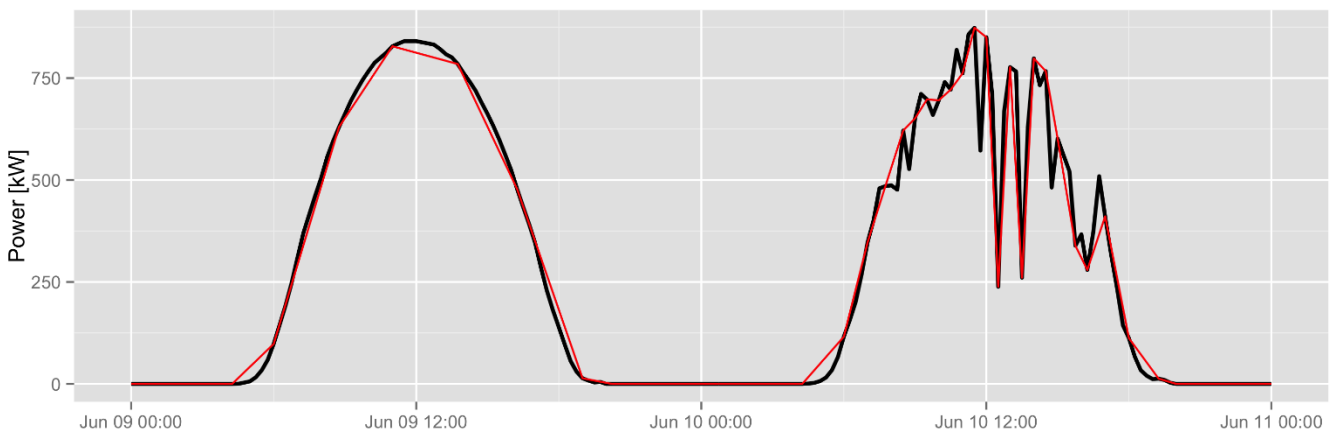


Figure 2. Example of the swinging door algorithm for power ramping event detection on clear sky and cloudy days.

#### D. Bayesian Parameter Estimation

Parameters can be estimated with point values, e.g. maximum likelihood estimation or by minimizing some distance metric. However, these approaches disregard the inherent uncertainty of the problem. Point estimation routines can also be sensitive to initial conditions, especially those involving optimization. Bayesian approaches avoid such issues through its probabilistic, degree-of-belief perspective. The benefits over traditional methods are due to its incorporation of prior knowledge, including all problem-specific information available, directly into the estimation task. Furthermore, irrelevant variables (e.g. sensor noise or other undesirable latent variables) can be marginalized over to yield a posterior probability distribution involving only relevant variables. This provides insight to the relationships between model parameters, revealing tradeoffs and compensating interactions. The probabilistic approach also lends itself to model updating, allowing the posterior distribution to act as the prior distribution once new information is available; i.e., subsequent parameter updates with new data acquisition.

Conditional probabilities are related through the product rule in the derivation of Bayes' Theorem, allowing consideration of "a priori" and "a posteriori." The prior probability distribution (i.e. before data) is updated with any measured data to form the posterior probability distribution (i.e. after data), which represents the state of knowledge in the inference task. From a parameter estimation perspective, the probability of parameter set,  $\beta$ , given measured data,  $x$ , and knowledge of the system,  $\theta$ , can be written as posterior probability,  $p(\beta|x\theta)$ . Bayes' Theorem computes this conditional probability as shown in Equation (1):

$$p(\beta|x\theta) = p(\beta|\theta) \frac{p(x|\beta\theta)}{p(x|\theta)}, \quad (1)$$

where  $p(\beta|\theta)$  represents prior knowledge about parameter values,  $p(x|\beta\theta)$  represents the likelihood of observing the measured dataset,  $x$ , given a particular parameter set,  $\beta$ , and background knowledge of the system,  $\theta$ , and  $p(x|\theta)$  is the probability of observing the data. In discrete form, integrated out background knowledge parameters, the numerator remains the product of likelihood and prior and the denominator is a normalization factor so that posterior probabilities sum to unity as shown in Equation (2):

$$p(\beta|x) = \frac{p(\beta)p(x|\beta)}{\sum_i p(\beta_i)p(x|\beta_i)}. \quad (2)$$

Assuming Gaussian noise about a measured datum,  $x_i$ , the likelihood of an observation can be determined from its

relative location. The normal distribution with standard deviation,  $\sigma$ , centered at  $\mu$  set equal to the measured datum,

$$p(x_i|\beta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x_i - M_i)^2}{2\sigma^2}\right), \quad (3)$$

where  $M_i$  is the model output given the parameter set  $\beta$ .

When assuming independent errors, the likelihood of the time series is simply the product of individual likelihoods. Correlated errors could be handled with a slightly different formulation, but have already been considered in one respect through the ramp's slope as extracted from the swinging door algorithm. The autocorrelation of errors has been ignored because they are assumed small due to the piecewise approximation of the swinging door. The derivation of the likelihood function, given by Equation (4), and is equivalent to the least squares equation:

$$p(x|\beta) = \frac{1}{(\sigma\sqrt{2\pi})^n} \exp\left(\frac{-1}{2\sigma^2} \sum_{i=1}^n (x_i - M_i)^2\right). \quad (4)$$

Numerical issues can arise from the evaluation of Equation (4) since a small range of  $\sigma$  values results in likelihoods with extremely small values. These numerical issues can be alleviated by computing the natural logarithm of the posterior rather than the posterior directly. To compute the natural logarithm of the posterior, the natural logarithm of Equation (2) is evaluated:

$$\ln(p(\beta|x)) = \ln\left(\frac{p(\beta)p(x|\beta)}{\sum_i p(\beta_i)p(x|\beta_i)}\right). \quad (5)$$

The right hand side of Equation (5) can be separated using logarithmic product and quotient rules:

$$\begin{aligned} \ln(p(\beta|x)) &= \ln(p(\beta)) + \ln(p(x|\beta)) \\ &\quad - \ln\left(\sum_i p(\beta_i)p(x|\beta_i)\right). \end{aligned} \quad (6)$$

The log-likelihood term of Equation (6),

$$\begin{aligned} \ln(p(x|\beta)) &= \\ &= \ln\left(\frac{1}{(\sigma\sqrt{2\pi})^n} \exp\left(\frac{-1}{2\sigma^2} \sum_{i=1}^n (x_i - M_i)^2\right)\right), \end{aligned} \quad (7)$$

is further simplified by applying product and quotient rules as shown in Equations (8) and (9), respectively,

$$\ln(p(x|\beta)) = \ln\left(\frac{1}{(\sigma\sqrt{2\pi})^n}\right) + \ln\left(\exp\left(\frac{-1}{2\sigma^2}\sum_{i=1}^n(x_i - M_i)^2\right)\right), \quad (8)$$

$$\ln(p(x|\beta)) = \ln(1) - \ln\left((\sigma\sqrt{2\pi})^n\right) + \ln\left(\exp\left(\frac{-1}{2\sigma^2}\sum_{i=1}^n(x_i - M_i)^2\right)\right). \quad (9)$$

The first term drops out and the power rule can be applied to the middle term of the right hand side. The last term of the right hand side simplifies, due to the logarithmic identity, to yield Equation (10).

$$\ln(p(x|\beta)) = -n\ln(\sigma\sqrt{2\pi}) + \frac{-1}{2\sigma^2}\sum_{i=1}^n(x_i - M_i)^2. \quad (10)$$

Recombining the simplified log-likelihood of Equation (10) with the log-posterior equation of Equation (6) yields

$$\ln(p(x|\beta)) = \ln(p(\beta)) - n\ln(\sigma\sqrt{2\pi}) - \frac{1}{2\sigma^2}\sum_{i=1}^n(x_i - M_i)^2 - \ln\left(\sum_i p(\beta_i)p(x|\beta_i)\right). \quad (11)$$

The last term of the right hand side of Equation (11) is a constant that is subtracted from each individual  $\ln(p(\beta_i)p(x|\beta_i))$  value. Since the value of this constant term does not impact the shape or relative information of the posterior it can be thought of as an arbitrary constant  $C$ . Therefore,

$$\ln(p(\beta|x)) = \ln(p(\beta)) - n\ln(\sigma\sqrt{2\pi}) - \frac{1}{2\sigma^2}\sum_{i=1}^n(x_i - M_i)^2 + C. \quad (12)$$

The constant term can be moved to the left hand side of the equation to yield

$$\ln(p(\beta|x)) - C = \ln(p(\beta)) - n\ln(\sigma\sqrt{2\pi}) - \frac{1}{2\sigma^2}\sum_{i=1}^n(x_i - M_i)^2. \quad (13)$$

To avoid numerical underflow or overflow, the constant is:

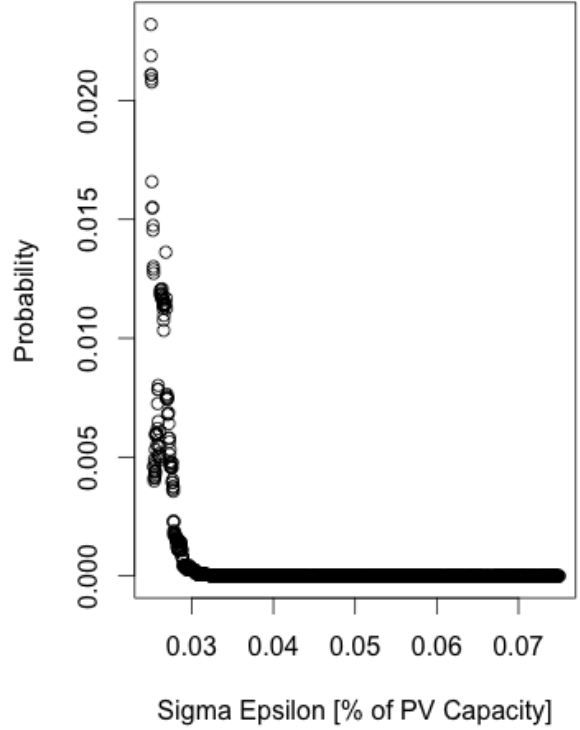


Figure 3. Typical (numerical) example of probabilities for a fixed  $\varepsilon$  value, and varying  $\sigma$ , for the probabilistic swinging door algorithm as applied to the solar PV power dataset.

$$C = \max\left(\ln(p(\beta)) - n\ln(\sigma\sqrt{2\pi}) - \frac{1}{2\sigma^2}\sum_{i=1}^n(x_i - M_i)^2\right), \quad (14)$$

which shifts all points so that the maximum is 0. The maximum value of 0 in the natural log space ensures that all values will be mapped to the interval  $[0, 1]$  when taking the exponential. After taking the exponentials, the values can be scaled (constant) so that the probabilities sum to unity.

### III. RESULTS AND DISCUSSION

#### A. Typical Probability Distribution

Figure 3 provides a typical (numerical) example of a probability distribution for a fixed  $\varepsilon$  value, and varying  $\sigma$ , using the probabilistic swinging door algorithm as applied to the PV power dataset. As can be seen in the figure, in this case a randomly chosen  $\varepsilon$  value leads to the majority of probability mass associated with  $\sigma$  values less than 3% of the PV plant capacity. That is, most of the residual is rather small, indicating that the  $\varepsilon$  value is rather small as well. The tradeoffs encapsulate the uncertainty in predictions, or forecasts, which can be made with the data at any given moment.

With the uncertainty in PV power generation being quantified, using a two parameter approach via the probabilistic swinging door algorithm, it is possible to understand the current state of operations. Is the current state of power conversion volatile? Does low uncertainty allow the accurate prediction of short-term solar power generation? Does high uncertainty lead to more generation reserve requirements?

#### IV. CONCLUSIONS AND FUTURE WORK

A probabilistic swinging door algorithm was considered and used to analyze PV power data. The approach allows the quantification of uncertainty associated with PV power generation using two parameters. It is envisioned the tool will be helpful for state estimation and short-term forecasting of PV power generation. Future research will sample from the uncertainty distributions to probabilistically forecast short-term PV power data. It is currently unknown how accurate the forecasts will be, without exogenous data or a network configuration of the algorithm, but it is envisioned the probabilistic swinging door algorithm will be most valuable for sub-hourly solar power forecasting.

#### ACKNOWLEDGMENT

NREL's contribution to this work was supported by the U.S. Department of Energy under Contract No. DE-AC36-08-GO28308 with the National Renewable Energy Laboratory.

#### REFERENCES

- [1] J. Marcos, L. Marroyo, E. Lorenzo, D. Alvira, and E. Izco, "From Irradiance to Output Power Fluctuations: The PV Plant as a Low Pass Filter." *Progress in Photovoltaics: Research and Applications* 19(2011); pp. 505-510.
- [2] M. Ibanez, W. Beckman, and S. Klein, "Frequency Distributions for Hourly and Daily Clearness Indices." *Journal of Solar Energy Engineering*, Vol. 124, Feb. 2002; pp. 28-33.
- [3] M. Jurado, J. Caridad, and V. Ruiz, "Statistical Distribution of the Clearness Index with Radiation Data Integrated over Five Minute Intervals." *Solar Energy* 55(6), 1995; pp. 469-473.
- [4] K. Hollands and R. Huget, "A Probability Density Function for the Clearness Index, with Applications." *Solar Energy* 30(3), 1983; pp. 195-209.
- [5] J. Olseth and A. Skartveit, "A Probability Density Model for Hourly Total and Beam Irradiance on Arbitrarily Oriented Planes." *Solar Energy* 39(4), 1987; pp. 343-351.
- [6] J. Tovar, F. Olmo, F. Batlles, and L. Alados-Arboledas, "Dependence of One-Minute Global Irradiance Probability Density Distributions on Hourly Irradiation." *Energy* 26(2001); pp. 659-668.
- [7] J. Tovar, F. Olmo, and L. Alados-Arboledas, "One-Minute Global Irradiance Probability Density Distributions Conditioned to the Optical Air Mass." *Solar Energy* 62(6), 1998; pp. 387-393.
- [8] J. Tovar, F. Olmo, F. Batlles, and L. Alados-Arboledas, "One Minute  $K_b$  and  $K_d$  Probability Density Distributions Conditioned to the Optical Air Mass." *Solar Energy* 65(5), 1999; pp. 297-304.
- [9] M. Varo, G. Pedros, P. Martinez-Jimenez, and M. Aguilera, "Global Solar Irradiance in Cordoba: Clearness Index Distributions Conditioned to the Optical Air Mass." *Renewable Energy* 31(2006); pp. 1321-1332.
- [10] D. Meek, "Estimation of Maximum Possible Daily Global Solar Radiation." *Agricultural and Forest Meteorology* 87(1997); pp. 223-241.
- [11] J. Gordan and T. Reddy, "Stationary Statistics and Sequential Properties of Normal Beam and Global Solar Radiation on Tilted Surfaces." *Solar Energy* 42(1), 1989; pp. 35-44.
- [12] H. Suehrcke, "On the Relationship Between Duration of Sunshine and Solar Radiation on the Earth's Surface: Angstrom's Equation Revisited." *Solar Energy* 68(5), 2000; pp. 417-425.
- [13] M. Atwater and J. Ball "A Numerical Solar Radiation Model Based on Standard Meteorological Observations." *Solar Energy* 21(1978); pp. 163-170.
- [14] C. Craggs, E. Conway, and N. Pearsall, "Statistical Investigation on the Optimal Averaging Time for Solar Irradiance on Horizontal and Vertical Surfaces in the UK." *Solar Energy* 68(2), 2000; pp. 179-187.
- [15] J. Olseth and A. Skartveit, "Characteristics of Hourly Global Irradiance Modelled from Cloud Data." *Solar Energy* 51(3), 1993; pp. 197-204.
- [16] R. Perez, P. Ineichen, R. Seals, and A. Zelenka, "Making Full Use of the Clearness Index for Parameterizing Hourly Insolation Conditions." *Solar Energy* 45(2), 1990; pp. 111-114.
- [17] R. Perez, R. Seals, A. Zelenka, and P. Ineichen, "Climatic Evaluation of Models that Predict Hourly Direct Irradiance from Hourly Global Irradiance: Prospects for Performance Improvements." *Solar Energy* 44(2), 1990; pp. 99-108.
- [18] U. Amato, A. Andretta, B. Bartoli, B. Coluzzi, V. Cuomo, F. Fontana, and C. Serio, "Markov Processes and Fourier Analysis as a Tool to Describe and Simulate Daily Solar Irradiance." *Solar Energy* 37(3), 1986; pp. 179-194.
- [19] C. Craggs, E. Conway, and N. Pearsall, "Stochastic Modelling of Solar Irradiance on Horizontal and Vertical Planes at a Northerly Location." *Renewable Energy* 18(1999); pp. 445-463.
- [20] C. Glasbey, "Nonlinear Autoregressive Time Series with Multivariate Gaussian Mixtures as Marginal Distributions." *Applied Statistics* 50(2001); pp. 143-154.
- [21] R. Gansler, S. Klein, and W. Beckman, "Investigation of Minute Solar Radiation Data." *Solar Energy* 55(1), 1995; pp. 21-27.
- [22] G. Vijayakumar, M. Kummert, S. Klein, and W. Beckman, "Analysis of Short-Term Solar Radiation Data." *Solar Energy* 79(2005); pp. 495-504.
- [23] H. Suehrcke and P. McCormick, "The Frequency Distribution of Instantaneous Insolation Values." *Solar Energy* 40(5), 1988; pp. 413-422.
- [24] H. Suehrcke and P. McCormick, "The Diffuse Fraction of Instantaneous Solar Radiation." *Solar Energy* 40(5), 1988; pp. 423-430.
- [25] A. Woyte, R. Belmans, and J. Nijs, "Fluctuations in Instantaneous Clearness Index: Analysis and Statistics." *Solar Energy* 81(2007); pp. 195-206.
- [26] E. Bristol, "Swinging Door Trending: Adaptive Trend Recording?" ISA National Conference Proceedings, 1990; pp. 749-756.
- [27] A. Florita, B. Hodge, and K. Orwig, "Identifying Wind and Solar Ramping Events." IEEE Green Technologies Conference, 2013; pp. 147-152.
- [28] M. Cui, J. Zhang, A. Florita, B. Hodge, D. Ke, and Y. Sun, "An Optimized Swinging Door Algorithm for Identifying Wind Ramping Events," *IEEE Transactions on Sustainable Energy*, In Press.